PMP BASED PRODUCTION MODELS-DEVELOPMENT AND INTEGRATION

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Paper prepared for presentation at the PMP, Extensions and Alternative Methods

Organised Session of the XIth EAAE Congress

(European Association of Agricultural Economists)

The Future of Rural Europe in the Global Agri-Food System

Copenhagen, Denmark, August 23-27, 2005

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PMP Ten Years Later

It is now ten years since the first conceptual article on PMP was published (Howitt 1995), and the basic idea of using dual calibration values to impute the implicit costs of agricultural production seems to be quite robust. While there have been many improvements on the original idea, there are few published criticisms of the basic approach. One criticism termed the Heckelei-Wolff critique is discussed later. The development of PMP approaches has been covered by the paper by Henry de Frahan, but can be characterized as a gradual evolution of methods that are able to match the improvement in the estimation methods and data resources available to modelers. The use of maximum entropy methods has enabled optimization modelers to establish a continuum, between traditional optimization models and econometric models. The advances have taken the form of a wider range of functional forms, improved input shadow value estimation, and more formal and explicit use of prior information.

A Note on the Heckelei-Wolff Critique

In their paper in the European Review of Agricultural Economics, Heckelei & Wolff (2003) introduce their estimation paper with a short review and critique of positive mathematical programming (PMP). They conclude that because the original PMP specification uses a constrained optimization model, usually linear, to generate values for the binding resource constraints they will be inconsistent with the quadratic form of the model that is correctly defined as the process generating model. Heckelei and Wolff conclude that:

"..the set of equations (2) cannot be seen as unbiased estimating equations and will generally yield inconsistent parameter estimates if the true data generating process is correctly described by the quadratic model"

The point of this note is to show that under the original PMP specifications, the dual values derived from the linear constrained model for a given observation set are numerically identical to the dual values that the quadratic model would generate for the same observations. In short, we show that the original PMP specification (Howitt 1995) will generate a set of resource shadow values that are consistent and unbiased for calibration to a single observation. However, this does not detract from the central point of the Heckelei & Wolff paper, namely the desirability and efficiency of the simultaneous estimation of shadow values and parameters.

Using the notation in Heckelei & Wolff and using their partition into preferable activities (l^p) and marginal activities (l^m), the Lagrangian for the constrained linear problem can be written as:

(1) $L = (p^p - c^p)'l^p + (p^m - c^m)'l^m + \lambda'(b - A^p l^p - A^m l^m) + \rho'((l^0 + \varepsilon) - l)$

(2) $(\frac{\partial L}{\partial l^p}) l^p = (p^p - c^p)'l^p - \lambda' A^p l^p - \rho^{p\prime} l^p = 0$

therefore $\rho^p = p^p - c^p - A^{p\prime}\lambda$

(3) $(\frac{\partial L}{\partial l^m}) l^m = (p^m - c^m)'l^m - \lambda' A^m l^m = 0$

Since $A^m = B$

(4) $\lambda = B'^{-1}(p^m - c^m)$

This is exactly equation (5) in Heckelei and Wolff.

Now, formulating the quadratic problem and deriving the resource duals in the same manner using the Kuhn-Tucker complementary slackness condition. The Lagrangian in an unpartitioned form is:

(5) $L = p'l - d'l - 0.5 l'Ql + \lambda'(b - Al)$

(6) $(\frac{\partial L}{\partial l}) l = p'l - d'l' - l'Ql - \lambda' Al = 0$

therefore $A'\lambda = p - d - Q'l$

If this expression is premultiplied by AQ^{-1} and has the binding resource constraint equation substituted in, then one obtains equation (4) in Heckelei and Wolff.

Since, at the calibrated optimum, the preferred land allocations are within ε of the base acres so that $l^p \approx l^0$, and using the marginal cost calibration equation (2) from Heckelei and Wolff:

(7) $d + Ql^0 = c + \rho$

Substituting this into the first order conditions for the PMP problem we obtain:

(8) $A'\lambda = p - c - \rho$

For comparison with the LP model, this expression can be partitioned into the sets of preferred and marginal activities. Since $\rho^m = 0$ this yields the same expression for $A'\lambda$ as the calibrated LP model, namely:

(8) $A^{p\prime}\lambda = p^p - c^p - \rho^p$

and

(9) $A^{m\prime}\lambda = p^m - c^m$

Equations (8) and (9) show that for the general elasticity based calibration model and basic linear quadratic PMP model in Howitt (1995) and Henry de Frahn et al (2005), the shadow values derived from the constrained linear model are identical to those from the resulting quadratic PMP model.

However, the central point that Heckelei and Wolff are concerned about is not the calibration problem, but the problem of fitting a cost function and estimating the shadow values for a set of observed land allocations. Application of the two stage approach calibration approach to a data set depends on what one assumes about the data available. Heckelei and Wolff correctly define the true underlying model to be the quadratic model. They point out that if standard accounting costs are used in the first stage, the set of dual values assuming linear accounting costs will differ from the dual values generated by the “true’ quadratic model. It’s clear that the changes in the crop allocations between sample observations will generate different costs, and consequently the dual values for the quadratic model will differ from the first stage LP model with constant accounting costs.

In the above situation the QP model has additional information over the LP model, in that it assumes knowledge of the true quadratic cost function, and thus how costs change with changed land allocations. If the LP data set included the crop marginal production cost for each observation, then each LP run based on the observed data, including the marginal cost will have the same shadow value as the QP model. Since the data set for both models will contain observations on the reported marginal costs, and not the "true" cost function, the above approach seems to put the two stage model on the same empirical basis as the combined estimation approach. GME estimation can then be directly applied to the marginal cost conditions using the sets of shadow values generated by the constrained model as data as in equations (16) and (17) in Heckelei (2005). This two stage approach may have operational advantages over the more elegant Heckelei-Wolff approach, shown in Heckelei's (2005) equations (16) and (17) and (18), since the resulting bi-level optimization problem, with the embedded complementarity conditions, may be hard to solve. Heckelei briefly discusses potential problems.

A very simple empirical example of the two stage estimation is demonstrated in a Gams program available from the author. In the empirical test, a base quadratic cost function is generated from a single data point and prior elasticity estimates. This cost function is used with a set of Monte-Carlo generated marginal revenues to optimize the QP problem for a set of optimum levels and shadow values. A calibrated LP model is then run for the same set of marginal revenues and resource constraints. If a constant accounting cost is used for the marginal crops the shadow values differ from the maintained model as correctly stated by Heckelei and Wolff. However, if the cost information in the LP model is updated by using the cost function from the maintained model, the resulting LP shadow values are identical to the maintained model results. The resulting set of LP shadow values are now used to estimate the cost function. Using a separate stage to generate a set of shadow values seems redundant, but may be more efficient operationally than the one stage solution.

To conclude this point, the Heckelei and Wolff approach is an efficient estimation approach, where sufficient data sets are available. In those cases where data is minimal, and calibration has to be used, users of the two-step calibrated LP approach can be assured that the resulting dual values for single realizations are consistent between the LP and PMP models.

PMP-GME Production Function Models

Over the past several years I have been weaned away from my past work with dual symmetric formulations of production problems, and have concentrated on primal production functions for two main reasons, namely data reliability and process model interaction. Clearly there is the same information in a cost or production function approach to agricultural models, however, in my experience, farmers in all countries have better information on their yields and input quantities than they have on their marginal costs. It is very likely that this farmer preoccupation with yields rather than costs will lead to more precise responses to data questions.

The production function approach explicitly models policy response at both extensive and intensive margins of production. In addition, the GME estimates of production functions parameters have to satisfy the marginal production first order conditions for all inputs and the average product yield condition for land. This latter condition is not usually used in traditional estimation, but has the advantage of assuring that when the marginal conditions are integrated back to yields, they are consistent with the data that the decision makers and farmers are most familiar with.

The general production function specification has advantages where the environmental outcomes from the use of different inputs have different social impacts. In addition, the production function approach uses crop and factor specific prior estimates of factor demand elasticities to define the maximum entropy support values, thus has a wider range of prior knowledge than a single supply elasticity.

Heckelei and Britz (1999) propose an alternative to the maximum entropy (ME) formulation originally proposed by Paris and Howitt. In their approach, Heckelei and Britz impose the curvature on the cost or production function by using two constraints to define the Cholesky decomposition. This enables them to define the support values for the parameters of the Hessian directly, rather than through the Cholesky decomposition. In turn, this direct definition of the supports enables them to be defined using prior knowledge of the elasticities.

While there are priors to help define the support values for the parameters, the definition of the support values for the error terms are more ad hoc. The recommendations that have been put forward by various authors for handling this issue have varied widely, and as yet, no clear consensus has emerged from the literature about how to best to address this problem. Many authors invoke the 3 σ rule advanced by Pukelsheim (1994). Others recommend that error bounds are set as widely as possible, an approach that is potentially misleading and could introduce additional bias in the resulting GME estimates. Msangi and Howitt (2005) propose an alternative to this approach by introducing a moment constraint into the GME problem, to minimize the inherent bias in GME estimates, and to remove the influence of the errors supports on the resulting estimates. A moment-constrained GME estimation procedure, while adhering closely to the properties of classical estimators under 'good' data conditions and preserving un-biasedness, is still able to exploit the desirable properties of an entropy estimator, namely, its clear advantage in cases where there exists a high degree of collinearity in the data or even negative degrees of freedom. Constraining the GME-PMP estimates to be unbiased requires a simple summing constraint over each set of error components, and does not seem to alter the solution of the GME problem. Given the common occurrence of small sample bias and collinearity in production data sets the addition of a moment constraint seems a sensible precaution.

The PMP-GME production function models are readily understandable to research scientists in associated disciplines. Somehow, showing a colleague a three dimensional plot of a production surface and explaining the alternative trade offs greatly aids communication between disciplines. Currently this type of production function model is being applied to several interdisciplinary research projects. Specifically the research projects include: the intertemporal analysis of irrigation water use, the effect of global climate change on cropping patterns, and the optimal use of precision farming and cover crops on carbon sequestration.

Measuring the Information Gain from the Disaggregation of Production Models

The developments of entropy modeling methods over the past ten years has provided modelers with the ability to reconstruct models at any level of aggregation from calibrated individual farm models to large sample aggregate national models. This modeling ability raises the question of what the optimal level of aggregation is for a given data set and model purpose.

Optimal aggregation is the trade off between the heterogeneity and the noise in the data. If the sample is drawn from a truly homogenous population, then all variation is due to noise in the sample and there is no information gain by disaggregating the model. However, this situation is rare for agricultural and environmental models, where the inherent variability in the resource and climatic environment introduces some heterogeneity in the sample.

A way of measuring the information gains from disaggregation of multicrop systems is proposed by Howitt and Reynaud (2003). The measure is termed the Disaggregation Informational Gain (DIG) measure. The DIG measure is based on the Shannon (1948) measure of information, and has the following properties. (1) It increases monotonically with the heterogeneity of the disaggregated sample. (2) The gain from disaggregating a uniform set of samples is zero. (3) The DIG measure is invariant to changes in the number of disaggregated samples and the variability of the aggregated sample.

If there are “k” crop types and “i” disaggregated sets, the total number of observation being k*i , the cross-entropy between the aggregate observed land shares, y_k , and the true disaggregate land shares, y_k^i , is:

$$(10) \qquad CE = \sum_i \sum_k y_k \cdot \ln\left(\frac{y_k}{y_k^i}\right)$$

If an estimate of the crop k in region i is $\hat{y}_k^i$, then the cross-entropy between the estimated land shares, $\hat{y}_k^i$, and the true disaggregate land shares, y_k^i , is:

$$(11) \qquad C\hat{E} = \sum_i \sum_k \hat{y}_k^i \cdot \ln\left(\frac{\hat{y}_k^i}{y_k^i}\right)$$

As a polar case, assume that there is no information at the district level, and the estimation merely allocates the aggregate land share distribution y_k to each district. The information content for this crude aggregated estimate is the *CE* measure defined above.

Now assume that we obtain an estimator of regional crop land allocations $\hat{y}_k^i$. The $C\hat{E}$ measure is an aggregate measure, in term of entropy, of how far the distributions of estimates $\hat{y}_k^i$ are from the true distributions y_k^i . Note that the estimate $\hat{y}_k^i$ incorporates both the information gain from disaggregation, and the information loss from bias as the sample size is reduced. The Disagregation Informational Gain (DIG) from the disaggregated estimation is defined as:

$$(12) \qquad DIG = 1 - \frac{\sum_i \sum_k \hat{y}_k^i \cdot \ln\left(\frac{\hat{y}_k^i}{y_k^i}\right)}{\sum_i \sum_k y_k \cdot \ln\left(\frac{y_k}{y_k^i}\right)} = 1 - \frac{C\hat{E}}{CE}$$

The *DIG* is a measure of the proportion of district-level heterogeneity that is recovered by the estimator $\hat{y}_k^i$. In the case of a perfect disaggregated estimate where $\hat{y}_k^i = y_k^i \quad \forall k,i$, the *DIG* is equal to 1. In this case, we recover all the heterogeneity. In the case of full aggregation, $\hat{y}_k^i = y_k \quad \forall k,i$ and the *DIG* is equal to 0, and we recover no information at the district level. In all other cases, the *DIG* is between 0 and 1. The DIG measure increases as the disaggregated estimates get closer to the true district land use distributions y_k^i .

Currently, the optimal level of disaggregation for production function estimates is being empirically investigated for a small survey data set for northern Mexico crop production. Now that a continuum between robust econometric estimates and disaggregated calibration models is avialable to policy modelers, it seems reasonable that the level of aggregation is justified on a formal basis. Since policy modeling is not concerned with testing fundamental hypotheses, it seems logical to ignore the tyrrany of degrees of freedom and justify the level of model aggregation on the basis of information added to the policy question.

Initial Thoughts on PMP- CGE model Interaction

Subscribers to the Gams list-server will have noticed the growing popularity of CGE models for policy analysis. CGE models have many strengths that are derived from their generality. However, the general specification invariably means that the specification of the agricultural and environmental sectors have to be aggregated both by commodity and region. This aggregation limits the ability of CGE models to interact with physical process models that are often highly disaggregated. An ongoing area of work is to use a disaggregated PMP-GME model of agricultural production as a link between physical process models and regional CGE models. In one study, the policy question is: How do changes in environmental regulations impact the well being of farmers of different sizes and regions? The location in northern Mexico has farms that range from large commercial operations to subsistence farms where off-farm employment and remittances provide most of the household income.

While the regional models can link environmental policies to aggregate regional impacts on labor, input costs and secondary effects, linked CGE models provide the endogenous reaction of input and output prices to changes in regional allocations.

There are advantages and disadvantages to performing policy analysis with linked component models. The main disadvantage is in the lack of simultaneity in the price response between the models. Currently, we rely on an iterative interchange between the CGE and production model that is transmitted through vectors of input and output prices. While price convergence occurs much of the time, a general theory that ensures convergence does not seem to be available. However, such linkages enable interesting policy questions such as the following to be analysed. What would be the net effect on the income of different farm types in Northern Mexico of a simultaneous reduction in the energy subsidy for pumping irrigation water, and an opening of the corn market to international pressures from the NAFTA trade agreement?

The advantage of having decoupled models is in model calibration, estimation and debugging. Modelers who have been faced with reconciling the results of a massively connected model often react by breaking the combined model into its component parts, so why not leave them that way? In addition, different disciplines often work on different spatial and temporal scales. Aggregation and disaggregation methods enable model results to be consistently transferred between models electronically, and thus with fewer errors.

Conclusions

This short paper started with the view that agricultural production and environmental modeling has moved to an interesting stage where there is a formal continuum between econometric and calibrated optimization models. The development of maximum entropy estimation methods and better data sets has opened the potential for many developments. In this paper I have reviewed four topics of personal interest. I will leave it to my colleagues to cover the many other important topics that the development of agricultural policy models has introduced.

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