Pushing and Pulling Environmental Innovation: R&D Subsidies and Carbon Taxes

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Abstract

We use a novel modeling framework that incorporates free entry into the R&D sector and uncertainty about technological opportunity to evaluate three policy regimes (relative to *laissez faire*) designed to address a market with negative environmental externalities: a carbon tax, an R&D subsidy, and a mix of the two instruments. We show carbon taxes on their own are sufficient to obtain most of the welfare gains achieved by an optimal policy mix, and that the optimal carbon tax level is relatively robust to changing modeling assumptions, in contrast to optimal R&D subsidies. We also show R&D subsidies tend to produce more disperse outcomes than a carbon tax: either more R&D entrants (when technological opportunity is favorable) or none at all (when technological opportunity is unfavorable).

Key Words: Carbon tax, Incentive, Innovation, Renewable energy, R&D subsidy, Welfare.

JEL codes: H23, O31, Q42, Q55, Q58

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1. Introduction

Promoting the development and use of alternative forms of energy is a standard component of policies aimed at adapting to or mitigating climate change. At present, most alternative energy cannot be produced at competitive-enough costs to capture a large share of the energy market, which remains dominated by conventional fossil fuels. The need for policies to promote environmental research and development (R&D) activities stems from the existence of two major sets of market failures. As for any innovation undertaken by the private sector, indivisibilities, risk, and imperfect appropriability can severely reduce the incentive to innovate to below what is socially desirable (Arrow 1962). But for environmental innovations, this problem is compounded by the fact that the presence of uncompensated externalities reduce the private value of new technology (Jaffe, Newell and Stavins 2005, Newell 2010). What are the most appropriate policies to promote innovation in this setting? Broadly speaking, innovation policies may be grouped into two categories: push and pull policies (Nemet 2009). Push policies operate at the level of the R&D decision, for example, by directly subsidizing R&D. Pull policies operate at the level of the market for the R&D output, for example, by subsidizing use of new technologies. For environmental externalities, policies that lead agents to internalize the external cost of pollution—such as a carbon tax—also work as a pull policy vis-à-vis the promotion of alternative (clean) technologies.

The relative effectiveness of carbon taxes and R&D subsidies to promote environmental innovation has been investigated in a few studies. A key finding relates to their complementarity: because of the need to address multiple market failures, and their different mode of action, both instruments are desirable components in an optimal policy portfolio (Arrow et al. 2009; Popp, Newell and Jaffe 2010). In a two-sector growth model with endogenous technical change, Acemoglu et al. (2012) show that the joint use of an R&D subsidy and a carbon tax is crucial for directing the economy away from using inputs from the dirty sector. There is also some consensus that a carbon tax is the most important of the two policies. By simulating a climate policy model, Popp (2006) finds that the carbon tax alone can achieve as much as 95% of the welfare gains from the combined policies, whereas the R&D subsidy alone achieves only 11% of such benefits. Fischer and Newell (2008) consider a broader set of six policies and, in a numerical calibration of the US electricity sector, find the carbon tax to be the most efficient instrument, and the R&D subsidy to be the least effective one. Such conclusions appear supportive of the results of Parry, Pizer, and Fischer (2003), whose numerical analysis finds that the gains from correcting the R&D market failure are smaller than those that arise from correcting the externality. By contrast, Acemoglu et al. (2012) warn against the danger of relying solely on the carbon tax and emphasize the critical role of the R&D subsidy for directing technological change.

In this paper we revisit the question of the relative effectiveness of carbon taxes and R&D subsidies to promote environmental innovation in the context of a new model that maintains some critical features of innovation in renewable energy. The analysis relies on the modeling framework of Clancy and Moschini (2015), who build an original stochastic innovation model to study the effectiveness of quantity mandates as innovation incentives. The model's structure is meant to capture some essential long-run features of the innovation process and envisions three distinct stages: the choice of policy instrument and its level; the forward-looking decision of innovators to invest in R&D, given the policy context and their information about technological opportunity; and, *ex post* licensing of successful innovations to adopters, followed by production and consumption decisions. Two sources of uncertainty are explicitly represented: R&D uncertainty (innovation is stochastic), and policy uncertainty about the outlook for technological innovation. More specifically, the model considers two sources of energy, renewable energy and fossil fuels. The latter impose a negative externality on society. Renewable energy has no such externality, and the cost of producing it can be lowered through R&D.

As in Parry (1995), Laffont and Tirole (1996) and Denicolo (1999), the R&D sector is distinct from the production sector adopting the new technology. The profit opportunity that motivates innovators is directly influenced by policies that penalize dirty energy use (e.g., the carbon tax) or directly promote innovation (e.g., the R&D subsidy). The presumption is that innovations are patented, and successful innovators profit from licensing their technology to the production sector. The licensing framework implemented in the model permits a novel free-entry representation of the R&D enterprise where the number of innovators is endogenously determined, following the approach of Spulber (2013) and Clancy and Moschini (2015). Multiple innovators can raise welfare through two channels: an increase in the number of innovating firms increases the expected quality of the best innovation that will be discovered, and, the *ex post* royalty rate for the best innovation is reduced by the presence of competitors. This licensing formulation also effectively captures the welfare spillover effect of innovations and the associated appropriability problem that is one of the roots of R&D under-provision. The model also maintains a plausible presumption about the innovation process: by the time they choose R&D investments, firms have better information than policymakers did when they set the policy. We note at this juncture that the underlying assumption is that the regulator can commit to their policy choice. Of course, this is without loss of generality for the R&D subsidy, an inherently *ex ante* tool. But the carbon tax, in principle, could be changed *ex post*, that is after the realization of the innovation. To keep the comparison meaningful, in this paper we assume that the government can commit to the carbon tax.[1]

In the context of this model, we consider three policy regimes (in addition to *laissez faire*) to address the sub-optimal provision of R&D discussed earlier: a carbon tax, an R&D subsidy, and a mixed policy that uses both subsidies and taxes. The carbon tax is a "pull" policy that induces innovation by raising the price of conventional energy and therefore the price of clean energy (which is priced to compete with it). The R&D subsidy is a "push" policy that induces innovation by reducing the cost of R&D. We use two methods to compare and contrast the impact of these policies. Under some simplifying assumptions, analytical results are possible. We complement the analytical approach with a numerical simulation that relaxes a number of assumptions and that allows us to characterize optimal (welfare-maximizing) policies.

Our analytical results show the optimal R&D subsidy does not depend on the shape of demand or the outlook for technological opportunity when there is a single innovator. The optimal carbon tax does depend on these parameters, and we show that the optimal tax will be higher when innovation is taken into account. We also discuss how the choice of policy impacts the distribution of outcomes; in general, an R&D subsidy will induce a greater variance in R&D. Indeed, even though an R&D subsidy operates directly on the R&D decision, we find a carbon tax does a better job of ensuring innovation occurs when technological opportunity is low.

We supplement these results with a numerical simulation that allows us to compare the welfare implications of alternative policies. Our numerical simulations agree with much of the earlier literature. R&D subsidies on their own achieve only a fraction of the welfare gains attained by a carbon tax on its own, and adding R&D subsidies to a carbon tax leads to minor additional welfare gains. We also use our numerical simulation to assess the sensitivity of optimal policies to different parameter assumptions. We find the optimal carbon tax is relatively robust to changing assumptions, in contrast to the optimal R&D subsidy.

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[1] The well-known time consistency problem of environmental policies is analyzed, among others, by Laffont and Tirole 1996, Denicolo 1999, Kennedy and Laplante 1999, and Montero, 2011.

2. The Model

The stochastic innovation model developed in Clancy and Moschini (2015) presumes that R&D is carried out by specialized firms which, if successful, can license their innovations to a competitive sector. The purpose of R&D is to devise more efficient technologies for the production of renewable energy, which represents a cleaner alternative for conventional energy. Without much loss of generality, renewable energy is assumed to have zero emissions. The amount of conventional energy is denoted Q_1, and that of renewable energy is denoted Q_2. These two sources of energy are perfect substitutes from the consumer's perspective: consumers' (inverse) demand function is $P(Q)$, where $Q = Q_1 + Q_2$ represent total energy used. Total damage from emissions is $X = xQ_1$, where x is the (constant) marginal environmental damage rate.[2] On the supply side, both forms of energy are produced by competitive industries, with industry cost functions $C_1(Q_1)$ and $C_2(Q_2, \theta)$, where $\theta \geq 0$ denotes the quality of the innovation. We maintain the presumption that renewable energy, even after innovation, is unlikely to completely supplant conventional energy. To capture this asymmetry, conventional energy is assumed to be produced with constant marginal cost, whereas the new clean technology displays an upward-sloping (industry) marginal cost function, i.e.,

$$\frac{\partial C_1(Q_1)}{\partial Q_1} = c_1 \tag{1}$$

$$\frac{\partial C_2(Q_2, \theta)}{\partial Q_2} = c_2 - \theta + Q_2 \tag{2}$$

where c_1 and c_2 are fixed parameters, with $c_2 > c_1$. These marginal costs are illustrated in Figure 1.

The quality of innovation, denoted by θ, represents the realization of a random variable. Specifically, a research firms that conducts an R&D projects, upon incurring a (fixed) cost $k > 0$, gets a draw $\theta \geq 0$ from the conditional distribution function $F(\theta|\omega)$. Here, the parameter $\omega \in (0, \bar{\omega}]$ captures the outlook for innovation, and permits the model to represent an asymmetry between what researchers know when they make the R&D investment, and what the policy maker

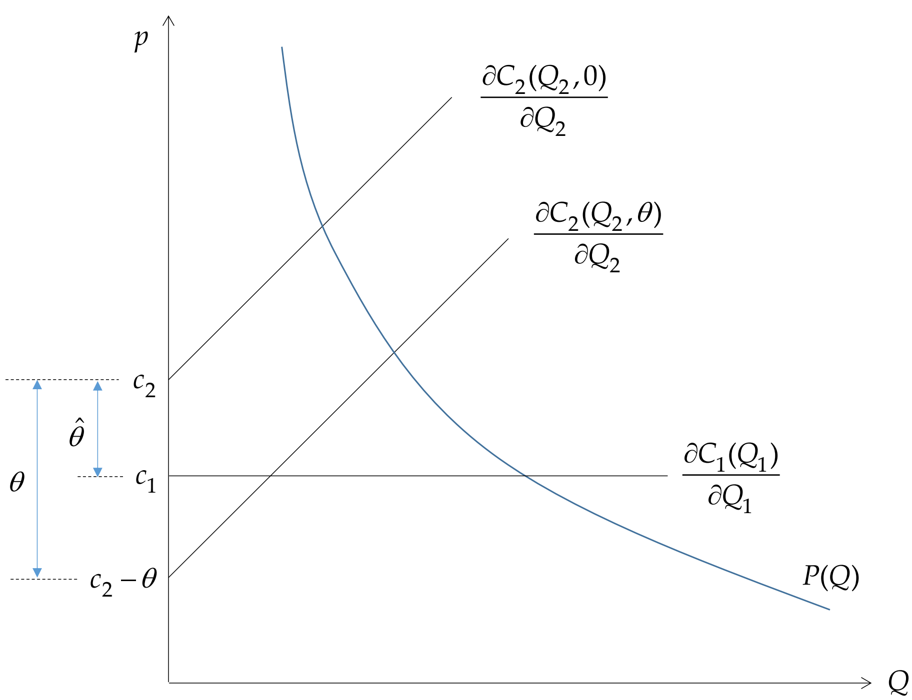
[2] This commonly invoked condition that the marginal environmental damage of the externality is constant, together with the assumption that the conditional distribution of firms' innovation outcomes is uniform (see below), simplifies the analysis and permits the derivation of explicit results.

knows when policies are chosen. In particular, the timeline we consider is as follows. First, the policymaker chooses the policy: either an R&D subsidy rate $s \in [0,1]$ that reduces the cost of R&D from k to $(1-s)k$, a per-unit carbon tax $t \geq 0$, or both. When making this choice the policymaker is uncertain about the outlook for innovation ω and only knows its distribution function $G(\omega)$. Conversely, research firms observe the actual realization of the parameter ω, and of course know the chosen policy parameter prior to making their R&D investment. Whereas the distribution function $G(\omega)$ is unrestricted, apart from the standard monotonicity and continuity properties, the analytical results that we present rely on postulating that $F(\theta|\omega)$ is a uniform distribution. The density function of this distribution is:

$$f(\theta|\omega) = \begin{cases} 1/\omega & \text{if } \theta \in [0,\omega] \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

This representation provides a straightforward interpretation of the technological opportunity parameter: both the expected value and the upper bound of the innovation draw θ are increasing in ω. But because even the most promising innovation can fail, the lower bound on innovation quality is always zero, regardless of the prospect for innovations.

Figure 1. Conventional and renewable energy: Innovation, supply and demand



In this setting, we want to evaluate the effectiveness of an R&D subsidy and/or a carbon tax as policy tools to promote innovation. For a meaningful benchmark, we compare these policy scenarios to the *laissez faire* (no policies) situation.

2.1 Innovation with a single innovator

For all cases considered—*laissez faire* situation (absence of government policy), R&D subsidy or carbon tax—the residual inverse demand curve facing producers of renewable energy is:

$$P_2(Q_2) = \begin{cases} c_1 + t & \text{if } Q_2 \le P^{-1}(c_1+t) \\ P(Q) & \text{otherwise} \end{cases} \tag{4}$$

where t denotes the carbon tax (per unit of dirty energy). For the *laissez faire* and R&D subsidy cases, $t = 0$. If clean energy is priced below the cost of dirty energy ($c_1 + t$), then it captures the entire market; if it is priced above the cost of dirty energy, demand for clean energy falls to zero; and, any quantity $Q_2 \in \left[0, P^{-1}(c_1+t)\right]$ can be sold when clean energy is priced at the cost of dirty energy.

As noted earlier, the realistic scenario is that the new renewable energy source does not completely replace the pre-existing conventional source. That is, the innovation is "nondrastic" in Arrow's (1962) terminology. The following condition, which we maintain throughout (but which is relaxed in the numerical analysis), will guarantee this outcome.

Condition 1. The upper bound on technological innovation satisfies $\bar{\omega} \le c_2 - c_1 + P^{-1}(c_1)$.

In this section we consider the case when there is only one firm capable of innovating (this assumption is relaxed in later sections). This setting is common in the literature, and relevant for applied policy analysis when there are substantial barriers to entry into the innovation market or when the human capital required to conduct R&D is scarce and concentrated. To characterize the innovator's decision problem, consider first the licensing stage for an arbitrary innovation of quality θ. The innovator essentially acts as a monopolist with a competitive fringe, and sets the per-unit royalty r to maximizes profits conditional on the adoption constraint by the competitive producers of renewable energy (which, given the foregoing considerations, face a perfectly elastic demand at price equal to $c_1 + t$). Thus, the innovator's optimal royalty maximizes rQ_2, where the demand from the competitive adopting clean energy sector, for $Q_2 > 0$, satisfies $c_2 - \theta + Q_2 + r = c_1 + t$. When $c_2 - \theta \geq c_1 + t$ there is no strictly positive license fee that can result in any adoption: the innovation is insufficient to be cost-competitive with the dirty technology. Thus, profitable licensing only occurs if the innovative step is sufficiently large. More specifically, let $\hat{\theta} \equiv c_2 - c_1 - t$ define the minimum innovative step beyond which the innovation becomes profitable (see Figure 1 for the *laissez-faire* case for which $t = 0$). For $\theta \geq \hat{\theta}$, the optimal royalty is $r^* = (\theta - \hat{\theta})/2$, and at this price the quantity licensed is $Q_2 = (\theta - \hat{\theta})/2$. The maximum profit an innovator with technology θ can obtain, when $\theta \geq \hat{\theta}$, is $\pi = (\theta - \hat{\theta})^2/4$ (and, of course, $\pi = 0$ when $\theta < \hat{\theta}$). We will restrict attention to situations when innovation is required to make renewable energy cost competitive with conventional energy:

Condition 2. The minimum inventive step is non-negative, i.e., $\hat{\theta} \geq 0$.

Clearly, a researcher with technological opportunity $\omega \leq \hat{\theta}$ expects zero profit. For $\omega > \hat{\theta}$ the innovation can still yield zero profit whenever $\theta < \hat{\theta}$, which happens with probability $\hat{\theta}/\omega$, and thus the researcher expects to make positive profit with probability $1 - \hat{\theta}/\omega$. Expected licensing profit conditional on ω, denoted $\pi(\omega)$, can therefore be written as:

$$\pi(\omega) = \left(1 - \frac{\hat{\theta}}{\omega}\right)\left[\frac{1}{4(\omega - \hat{\theta})}\int_{\hat{\theta}}^{\omega} (\theta - \hat{\theta})^2 d\theta\right] = \frac{(\omega - \hat{\theta})^3}{12\omega} \tag{5}$$

A risk neutral innovator will choose to conduct research if this expected licensing profit exceeds the subsidized costs of R&D, i.e., when $\pi(\omega) \geq (1-s)k$ where $s \in [0,1]$ denotes the R&D subsidy rate. Note that $s = 0$ for the *laissez-faire* case and for the carbon tax only regime. This implies the existence of a threshold $\hat{\omega}$, which satisfies $\pi(\hat{\omega}) = (1-s)k$, such that innovation is undertaken if and only if $\omega > \hat{\omega} \geq \hat{\theta}$.

Expected welfare can be expressed in terms of the pre-innovation static allocation and changes to this allocation that are brought about by innovation. Note that, given the presumption that innovation is non-drastic, energy is always priced at $c_1 + t$. Accordingly, the total quantity of energy Q, and consumer surplus, are not affected by innovation. Instead, innovation affects the share of energy produced by renewable sources, and reduces the damage from externalities relative to the *status quo ante* by xQ_2. Accounting for the minimum innovation step, and proceeding analogously to (5), expected clean energy is $E[Q_2] = (\omega - \hat{\theta})^2 / 4\omega$. License profits are given in equation (5). Clean producer profits can be shown to be $(\omega - \hat{\theta})^3 / 24\omega$ in expectation. We assume subsidies and taxes are possible as frictionless lump-sum transfers. All told, therefore, expected welfare is

$$E[W] = \int_0^Q (P(q) - (c_1 + x))dq + \int_{\hat{\omega}}^{\bar{\omega}} \left\{ \left[\frac{(\omega - \hat{\theta})^3}{12\omega} + \frac{(\omega - \hat{\theta})^3}{24\omega} + x\frac{(\omega - \hat{\theta})^2}{4\omega} - k \right] \right\} dG(\omega) \qquad (6)$$

where the first integral denotes pre-innovation welfare and the second integral in (6) is the expected contribution of innovation to welfare.

Equation (6) illustrates three potential market failures: the environmental externality, the innovation market failure, and the interaction of the two. First, absent government intervention, consumers consume too much dirty energy, so that $P(Q) = c_1 < c_1 + x$. This misallocation is captured by the first integral. Second, firms conduct R&D if private licensing profits (the first term under the second integral) exceed k, but do not take into account the spillovers of their discovery to the private sector. This private spillover is reflected in the gains to clean producers, given by the second term in the second integral. Third, firms do not take into account the reduction of environmental damages that their discoveries enable. This is captured by the third term under the second integral. These terms illustrate the reasons why innovation is doubly under-provisioned, as discussed by Jaffe, Newell and Stavins (2005). Innovation here is socially desirable for some $\omega < \hat{\omega}$, but under *laissez faire* is not conducted.

We now consider two policies to address these market failures.

2.2 R&D Subsidies

A social planner would like R&D to occur whenever:

$$\frac{(\omega - \hat{\theta})^3}{12\omega} + \frac{(\omega - \hat{\theta})^3}{24\omega} + x\frac{(\omega - \hat{\theta})^2}{4\omega} \geq k \qquad (7)$$

But as discussed above, under *laissez faire* the lower bound $\hat{\omega}$ is computed to take into account only the first term in equation (7). We model the R&D subsidy as the fraction $s \in (0,1)$ of the R&D fixed cost that is paid by the government, i.e., the innovator receives a lump-sum subsidy sk. Note that, because technological opportunity is not known to policymakers when the policy choice is made, the subsidy cannot be tailored to ω. If $\hat{\omega}_s$ denotes the threshold such that equation (7) holds as an equality, then $\hat{\omega}_s < \hat{\omega}$ and the optimal subsidy to R&D is such that the innovator's expected licensing profit is equal to the unsubsidized portion of R&D cost, that is s solves:

$$(1-s)k = \frac{(\hat{\omega}_s - \hat{\theta})^3}{12\hat{\omega}_s} \tag{8}$$

Note that the optimal R&D subsidy does not depend on knowledge of $P(Q)$ or $G(\omega)$. Because $G(\omega)$ describes the shape of beliefs about technological opportunity, its form may be highly uncertain and disputed. A virtue of the optimal R&D subsidy is that it does not require a consensus for $G(\omega)$.

> **Remark 1:** The optimal R&D subsidy does not depend on demand parameters $P(Q)$ or the policy-maker's beliefs about technological opportunity $G(\omega)$.

An R&D subsidy improves welfare by inducing innovators to take bigger risks and choose to conduct R&D even when $\omega \in [\hat{\omega}_s, \hat{\omega}]$ (i.e., technological opportunity is relatively low). However, it is important to note that, similar to the *laissez faire*, only innovations with $\theta > \hat{\theta}$ will be profitably licensed. These innovations are less likely to occur when $\omega \in [\hat{\omega}_s, \hat{\omega}]$. Hence, using an R&D subsidy increases the frequency of failures (*ex post*, the public might perceive such subsidies to be bad investments).

> **Remark 2:** With a single innovator, R&D subsidies induce more innovation that is unlikely to be useful (i.e., with $\theta < \hat{\theta}$).

Unfortunately, an R&D subsidy does not address the environmental externality except indirectly through the third term in equation (7). A carbon tax, in contrast, directly addresses this issue, but only indirectly impacts the incentives to innovate.

2.3 The naïve carbon tax

For both the *laissez faire* and the R&D subsidy cases, welfare is suboptimal because, *inter alia*, the uncompensated negative externality means there is excess production of dirty fuel. The canonical solution to an externality of this type is a Pigouvian tax on the dirty fuel, e.g., a carbon tax. Because use of fossil fuels incurs a social cost x per unit, if one ignores the prospect of innovation the tax should be set at $t = x$. This tax induces the optimal mix of dirty and renewable energy in the absence of innovation, but induces insufficient innovation. This is because, as noted earlier, innovating firms only take into account the impact of R&D on their own licensing profits, not the positive spillovers enjoyed by producers and consumers (in the form of reduced dependence on dirty fuel). Indeed, it can be shown:

> **Remark 3:** The optimal carbon tax with a single innovator is greater than the naïve tax $t = x$.

To see why, differentiate the expected welfare in (6) with respect to the tax (recall that $\hat{\theta} \equiv c_2 - c_1 - t$). Via Leibnitz rule, recalling that $\pi(\hat{\omega}) = k$, and simplifying, we obtain:

$$\frac{\partial E[W]}{\partial t} = \frac{\partial S_0}{\partial t} + \int_{\hat{\omega}}^{\bar{\omega}} \left\{ \left[\frac{9(\omega - \hat{\theta})^2}{24\omega} + x \frac{(\omega - \hat{\theta})}{2\omega} \right] \right\} dG(\omega) - \left[\frac{(\hat{\omega} - \hat{\theta})^3}{24\omega} + x \frac{(\hat{\omega} - \hat{\theta})^2}{4\omega} \right] \frac{\partial \hat{\omega}}{\partial t} \tag{9}$$

where $S_0 \equiv \int_0^Q (P(q) - (c_1 + x)) dq$. The optimal tax satisfies $\partial E[W] / \partial t = 0$. At $t = x$, however, $\partial S_0 / \partial t = 0$. Moreover, we note that profit is increasing in t and so $\partial \hat{\omega} / \partial t < 0$. Thus;

$$\left. \frac{\partial E[W]}{\partial t} \right|_{t=x} = \int_{\hat{\omega}}^{\bar{\omega}} \left\{ \underbrace{\left[\frac{9(\omega - \hat{\theta})^2}{24\omega} + x \frac{(\omega - \hat{\theta})}{2\omega} \right]}_{(+)} \right\} dG(\omega) - \underbrace{\left[\frac{(\hat{\omega} - \hat{\theta})^3}{24\omega} + x \frac{(\hat{\omega} - \hat{\theta})^2}{4\omega} \right]}_{(+)} \underbrace{\frac{\partial \hat{\omega}}{\partial t}}_{(-)} > 0 \tag{10}$$

where the signs of each term are displayed below them. Thus, expected welfare can be increased by raising the carbon tax above its naïve level. Intuitively, it is worth reducing the total consumption of energy by setting $P(Q) > c_1 + x$ because doing so induces more innovation, and also induces more production of clean energy (which is otherwise underprovisioned because the innovator has market power).

3. Multiple innovators

Whereas the preceding discussion pertains to the case of a single research firm, a more realistic description of the research industry should consider many (competing) innovators engaged in R&D projects. Clancy and Moschini (2015) model this case by postulating the existence of a large number of potential innovators and free entry into the renewable energy innovation sector. Innovators are *ex ante* identical and observe a common technological opportunity signal ω. If they choose to conduct R&D, they obtain independent θ draws from $f(\theta|\omega)$. The innovator who draws the highest θ, denoted θ_1, has the best technology and becomes the exclusive licensor to the renewable energy production sector. However, as in Spulber (2013), the choice of royalty by the innovator who draws θ_1 is now constrained by the presence of competing innovators. Under Bertrand competition, the second-highest θ draw, denoted θ_2, is the binding constraint. Essentially, as compared with the foregoing analysis, θ_2 plays the same role as the pre-innovation production technique $\theta = 0$ for the single innovator case. But, of course, in the multiple innovator setting θ_2 is endogenous.

The pricing of innovation in this multiple-innovators setting is characterized by Clancy and Moschini (2015). Consider first the *laissez faire* setting. For low realizations of θ_2, the constraint imposed by the second-best technology does not bind, the single-innovator results continue to hold, and the solution is $r^* = Q_2 = (\theta_1 - \hat{\theta})/2$. But whenever $\theta_2 > (\theta_1 + \hat{\theta})/2$, the optimal royalty is $r^* = \theta_1 - \theta_2$, and $Q_2 = \theta_2 - \hat{\theta}$. The best innovator's maximum profit, denoted π_1, is therefore given by:

$$\pi_1 = \frac{(\theta_1 - \hat{\theta})^2}{4} \qquad \text{if } \theta_2 \le (\theta_1 + \hat{\theta})/2 \tag{11}$$

$$\pi_1 = (\theta_1 - \theta_2)(\theta_2 - \hat{\theta}) \qquad \text{if } \theta_2 > (\theta_1 + \hat{\theta})/2 \tag{12}$$

The expected profit of a potential entrant now depends on the distribution of θ_1 and θ_2, which are best described by the concepts of "order statistics" widely used in auction theory (Krishna 2010). Specifically, given n innovators, the probability that an innovator's draw of θ is the maximum draw is equal to the probability that the $n-1$ other draws are smaller than θ. Because we have assumed a uniform distribution for the innovation projects, this probability equals $(\theta/\omega)^{n-1}$.

Moreover, conditional on a given θ being the maximum draw, the second highest realization θ_2 is the maximum of $n-1$ independent draws from the uniform distribution on the support of $[0,\theta]$. Hence, the second highest realization θ_2 has cumulative distribution function $(\theta_2/\theta)^{n-1}$ and density function $\left((n-1)/\theta_2\right)\left(\theta_2/\theta\right)^{n-1}$. Using these results on the distribution of the first and second best innovations, we can determine the expected profitability of participating in the R&D contest. Specifically, with n entrants, the expected licensing profit of each innovator, given technological opportunity ω, can be written as:

$$\pi(\omega,n) = \int_{\hat{\theta}}^{\omega} \left\{ \left(\frac{\theta_1 + \hat{\theta}}{2\theta_1} \right)^{n-1} \frac{(\theta_1 - \hat{\theta})^2}{4} + \int_{(\theta_1+\hat{\theta})/2}^{\theta_1} (\theta_1 - \theta_2)(\theta_2 - \hat{\theta}) \frac{n-1}{\theta_2} \left(\frac{\theta_2}{\theta_1} \right)^{n-1} d\theta_2 \right\} \left(\frac{\theta_1}{\omega} \right)^{n-1} \frac{1}{\omega} d\theta_1 \qquad (13)$$

This term integrates over the range of values for θ that are both feasible and earn positive profit. Within the integral, profits are divided into two terms. When $\theta_2 \le (\theta_1 + \hat{\theta})/2$, which occurs with probability $\left[(\theta_1 + \hat{\theta})/2\theta_1 \right]^{n-1}$, profit is given by equation (11). This is the first term under the integral. Conversely, whenever $\theta_2 > (\theta_1 + \hat{\theta})/2$, profit is given by equation (12). This is captured by the second term, itself an integral over possible values of θ_2.

The equilibrium number of innovators is determined by the free entry condition. In equilibrium, noting that n is an integer, the number of innovators n^* satisfies:

$$\pi\left(\omega,n^*\right) \ge \left(1-s\right)k \ge \pi\left(\omega,n^*+1\right) \qquad (14)$$

Under free entry, the choice of policy may have a significant impact on the distribution of outcomes.

> **Remark 4:** A carbon tax on its own induces (weakly) more entry of innovators for low values of technological opportunity ($\omega < c_2 - c_1$) than an R&D subsidy on its own.

To see why, recall that the minimum innovative step is defined to be $\hat{\theta} = c_2 - c_1 - t$. Under a pure R&D subsidy policy, $t = 0$ and therefore licensing profit is zero for $\omega < c_2 - c_1$. A carbon tax lowers this threshold, allowing for positive licensing profit for lower levels of technological opportunity. Provided k is not too large relative to these licensing profits, there will be additional entrants under a carbon tax relative to an R&D subsidy for $\omega < c_2 - c_1$.

The opposite effect takes place for high values of technological opportunity.

> **Remark 5:** For a given tax t and subsidy s, for sufficiently high technological opportunity ω, an R&D subsidy induces more innovation entrants than a carbon tax.

A proof is presented in the appendix, but the intuition here is that, for large innovations, the expected profits of a potential entrant under an R&D subsidy are close to those under a carbon tax (the additional price premium provided by the tax is less important). Although the expected licensing profits are of similar magnitude under either policy tools, however, entrants in an R&D subsidy regime compare profits to $(1-s)k$ while entrants in a carbon tax regime compare them to the full cost k. *Ceteris paribus,* the former supports more innovators.[3]

4. Numerical Analysis

The foregoing analysis is unable to assess the relative magnitude of the welfare effects, or the gains that come from an optimal mix of both policies. This is because welfare conclusions are bound to depend on the particular shape of the demand function $P(Q)$ and on the distribution of technological opportunities $G(\omega)$. In addition, our analytical results have been contingent on the assumptions that clean energy cannot capture the entire market (Condition 1). In this section we relax this condition and specify explicit functional forms for $P(Q)$ and $G(\omega)$ so that we may consider the impacts of the policy instruments of interests in a more general context by means of a numerical analysis.

4.1 Parameterization

We employ the same parameterization as Clancy and Moschini (2015). The marginal cost of conventional energy is normalized to $c_1 = 100$, so that a tax on dirty energy can be interpreted as a percent of the *laissez-faire* price level. In the baseline parameterization the externality is calibrated to

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[3] Note that Remark 5 only applies when ω is greater than some ω_0, as discussed in the appendix. If $\omega_0 > \bar{\omega}$ then there may be no $\omega \in [0, \bar{\omega}]$ that satisfies Remark 5.

$x = 20$, so that it amounts to 20% of the private cost of dirty energy,[4] and we put $c_2 = 120$ so that renewable energy is on the cusp of being socially desirable, but still requires innovation. Next, we postulate the inverse demand function $p(Q) = (a - \ln Q)/b$ or, equivalently, that the direct demand function for energy takes the semi-log form:

$$\ln Q = a - bp \tag{15}$$

This is a convenient parameterization which, among other desirable features, can accommodate various hypotheses concerning demand elasticity $\eta \equiv -\partial \ln Q/\partial \ln p$. For this function $\eta = bp$, hence the parameter b can be varied to implement alternative elasticity values. The parameter a is calibrated so that total demand for energy at price $p = c_1$ (and at the baseline elasticity value) is equal to $Q = 100$, that is we put $a = bc_1 + \ln 100$. As for $G(\omega)$, we assume that ω is distributed on $\left[0, \bar{\omega}\right]$ by an appropriately scaled beta distribution. The probability density function $g(\omega)$ is therefore given by:

$$g\left(\omega;\alpha,\beta\right) \propto \left(\omega / \bar{\omega}\right)^{\alpha-1}\left(1 - \omega / \bar{\omega}\right)^{\beta-1} \tag{16}$$

where the parameters α and β determine the moments of this distribution and govern its shape. This distribution is very flexible, and alternative choices of α and β can yield both symmetric and skewed density functions. We normalize $\bar{\omega} = 120$ so that, under all possible innovation, the marginal cost of clean energy remains non-negative everywhere.

Given the foregoing functional form assumptions and parametric normalizations, there are four free parameters that can be varied to gain some insights in the nature of the results. The first of these is the elasticity of demand η. Because this value depends on the evaluation price, for clarity we will

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[4] This value for the externality cost is meant to be somewhat representative of estimates for the social cost of carbon relative to the cost of transportation fuel. The US government's estimate for the 2015 social cost of carbon, in 2007 dollars, is \$37/ton of CO2 if a 3% discount rate is used, and \$57/ton of CO2 if a 2.5% discount rate is used (US Government 2013, p. 3). These discount rates have been criticized for being too high (Johnson and Hope 2012), and so we use the figure associated with the lower 2.5% discount rate as our baseline. Converting this estimate to 2015 dollars yields a social cost of \$65/ton of CO2. The carbon emission coefficient is 8.9 kg CO2/gallon of gasoline (EPA 2014), which implies a social cost of carbon is \$0.58 per gallon. Taking the benchmark price of gasoline to be \$3.00/gallon, then the damage imposed by the carbon externality is approximately 20% of the cost of fuel, which is reflected in our baseline value of $x = 20$.

always measure elasticity with reference to the *laissez-faire* price of energy, where $p = c_1$. For our baseline, we set b so that $\eta = 0.5$. We also consider the cases where $\eta = 0.25$ and $\eta = 1$ (these values reflect the widely-held belief that energy demand is inelastic; see Toman, Griffin and Lempert 2008, p. 18). Second, we vary the cost of the externality x. As noted, for the baseline we set $x = 20$, but we also consider the cases of $x = 10$ and $x = 40$. Third, we vary the R&D cost k. To calibrate this parameter we relate it to the magnitude of profits that innovation can produce in the *laissez-faire* baseline. Under the highest level of technological opportunity, the expected profit for a single innovator, in view of (5) and the chosen normalizations, is equal to $\pi_t(\bar{\omega}) = 6{,}250/9$. We consider values of k equal to 3%, 6%, and 12% of this profit level, with 6% corresponding to the baseline.

Fourth, we vary the shape of the distribution of technological opportunity $G(\omega)$. The first moment of the assumed beta distribution is $E[\omega] = \bar{\omega}\alpha/(\alpha + \beta)$. We set $\alpha + \beta = 2$ and, by varying the parameters α and β, we obtain both different values for $E[\omega]$ and different shapes. The baseline parameters are $\alpha = 0.5$ and $\beta = 1.5$, which yield $E[\omega] = 30$. This is a positively skewed distribution (low draws of ω are more likely than high ones), which reflects the belief that technological opportunity is more likely to be consistent with incremental innovation than major breakthroughs. The other two cases we consider are $\alpha = 0.25$ and $\beta = 1.75$, which yield $E[\omega] = 15$ (and correspond to an even more positively skewed distribution), and $\alpha = 1$ and $\beta = 1$, which yield $E[\omega] = 60$ (and correspond to a uniform distribution where high draws of ω are equally likely as low ones). As for the policies t and s, for each set of parameters that we consider, we numerically solve for the value of the policy instrument that maximizes welfare (expected Marshallian surplus) for an R&D subsidy on its own, a carbon tax on its own, and a combination of R&D subsidy and carbon tax. All calculation are coded in Matlab.

4.2. Results

Some basic descriptive results for the baseline parameters are reported in Table 1. For the single innovator case the expected number of innovators $E[n]$ can be interpreted as the probability that R&D will be conducted. In the baseline setting, under a *laissez-faire* policy, R&D is conducted with probability 0.25 for the single innovator case. The expected quality of innovation $E[\theta_1]$ is 9.6, which improves to 15.9 with multiple innovators. Hence, in either case the "average" technology under *laissez faire* is insufficient to compete with fossil fuels (the minimum inventive step here is $\hat{\theta} = 20$). Still, some innovation does take place under *laissez faire*, because some better-than-average draws will be profitable. The expected quantity of clean energy consumed is small but not negligible, at 2.6 and 8.7 under the single innovator and free entry conditions respectively (recall that the *laissez-faire* quantity of total energy consumed was normalized to 100).

Table 1. Numerical Results for Baseline

	Laissez Faire		R&D Subsidy		Carbon Tax		Mixed Policy	
Entrants	Single	Multiple	Single	Multiple	Single	Multiple	Single	Multiple
Subsidy rate	-	-	0.83	0.65	-	-	0.15	0.21
Carbon Tax	-	-	-	-	23.4	23.4	23.5	22.4
$E[n]$	0.25	1.52	0.36	3.52	0.56	3.07	0.57	3.58
$\sqrt{Var(n)}$	0.44	3.10	0.48	6.07	0.50	3.95	0.49	4.52
$E[\theta_1]$	9.6	15.9	11.8	20.2	14.3	23.9	14.4	24.5
$\sqrt{Var(\theta_1)}$	20.4	29.9	20.8	31.8	20.3	29.7	20.2	29.9
$E[Q_2]$	2.6	8.7	2.9	11.9	9.6	22.9	9.7	22.9
$\sqrt{Var(Q_2)}$	6.9	19.0	7.0	21.7	9.6	26.6	9.6	27.2
$E[W]$	18,123	18,402	18,129	18,455	18,310	18,676	18,310	18,679
$\sqrt{Var(W)}$	414.28	977	415.14	988.76	536.61	1098.1	537.0	1094.2

Note: the baseline parameters are $\eta = 0.5$, $x = 0.2\,c_1$, $k = 0.06\,\pi(\bar{\omega})$, and $\alpha = 0.5$ and $\beta = 1.5$ (i.e., $E[\omega] = 30$).

An optimal policy (tax or R&D subsidy) raises all these quantities, and also improves welfare. The expected quality of innovation $E[\theta_1]$, as well as the expected quantity of clean energy produced $E[Q_2]$, is significantly increased by the carbon tax, but less so by the R&D subsidy, despite the fact that the optimal level of the subsidy is quite high (83% for the single innovator case and 65% under free entry of innovators). R&D subsidies are particularly poor at increasing the use of clean energy. These features reflect the fact that R&D subsidies induce lower-quality R&D projects that would not otherwise be pursued, and which are less likely to exceed the minimum inventive step, as noted in Remark 2. Note also that, in the single innovator setting, even though the subsidy is quite large, R&D is more likely to occur under a carbon tax ($E[n]$ is 0.56 instead of 0.36). This reflects the superior ability of a carbon tax to induce innovation for $\omega < \hat{\theta}$. In both single and multiple innovators cases, welfare is also higher under an optimal carbon tax, but of course highest under the optimal mixed policy.

To gain further insights into the performance of each policy, Table 2 illustrates the sensitivity of optimal policies to changes in the calibrated parameters.

Table 2. Optimal Policy Instruments Under Alternative Assumptions

	R&D Subsidy				Carbon Tax			
Entrants	Single		Multiple		Single		Multiple	
Policy Mix	S	S+T	S	S+T	T	S+T	T	S+T
Baseline	0.83	0.15	0.65	0.21	23.4	23.5	23.4	22.4
$\eta = 0.25$	0.83	0.10	0.65	0.20	24.3	24.4	23.4	22.8
$\eta = 1$	0.83	0.20	0.63	0.23	22.5	22.4	22.7	21.7
$x = 10$	0.69	0.16	0.53	0.24	13.9	13.7	14.4	12.9
$x = 40$	0.92	0.00	0.77	0.15	47.4	47.4	42.9	42.7
$k = 0.03\overline{\pi}$	0.88	0.09	0.67	0.20	23.9	23.9	22.3	22.0
$k = 0.12\overline{\pi}$	0.79	0.20	0.63	0.25	23.0	23.0	24.0	22.6
$E[\omega] = 15$	0.83	0.30	0.65	0.28	21.7	21.7	21.8	21.4
$E[\omega] = 60$	0.83	0.26	0.59	0.17	29.2	29.2	24.8	24.0

Note: Each row changes one parameter, all other parameters as in the baseline

The first row of Table 2 reiterates the optimal policies for the baseline parameterization reported in Table 1. Each subsequent row presumes the same parameters as the baseline, except along one dimension. For example, in the second row the elasticity of demand, evaluated at the *laissez-faire* price, is changed to $\eta = 0.25$. Each column gives the optimal policy value for an R&D subsidy or carbon tax in the presence of a single or multiple entrants, and when the policy is considered alone ("S" for subsidy or "T" for carbon tax) or as part of a pair (denoted "S+T").

On its own, the optimal R&D subsidy is substantial and everywhere greater than 50%. The subsidy is largest when there is a single innovator. Note the optimal R&D subsidy for a single innovator is unaffected by variations of parameters pertaining to $P(Q)$ and $G(\omega)$, which are given by the first three and last two rows, as noted in Remark 1. The optimal policy is substantially impacted by the presence or absence of a complementary carbon tax. When a carbon tax is also in place, the optimal R&D subsidy rate drops to 30% or less in all cases considered.

In contrast, the optimal carbon tax is only significantly impacted only by changes in the level of the externality, but it is very robust otherwise. In particular, it is insensitive to factors that perturb the optimal subsidy, including the number of innovators and the presence or absence of a complementary R&D subsidy. The optimal carbon tax is everywhere greater than the naïve carbon tax of $t = x$, as noted in Remark 3.

In general, the optimal carbon tax is reduced by a small amount when paired with a complementary R&D subsidy, while the subsidy is substantially reduced. This suggests the carbon tax is relatively more important for the welfare improvement due to the optimal policy. To examine this conjecture explicitly, Table 3 computes welfare gains relative to *laissez faire* under each of the policies.

Table 3. Welfare Gains Under Alternative Assumptions

	R&D Subsidy		Carbon Tax		Both Policies	
Entrants	Single	Multiple	Single	Multiple	Single	Multiple
Baseline	5	53	187	274	187	277
$\eta = 0.25$	5	53	141	228	141	231
$\eta = 1$	5	36	152	177	152	179
$x = 10$	2	25	55	88	55	91
$x = 40$	12	124	851	1072	851	1073
$k = 0.03\bar{\pi}$	3	40	190	265	190	267
$k = 0.12\bar{\pi}$	8	65	183	275	183	281
$E[\omega] = 15$	3	30	145	196	145	199
$E[\omega] = 60$	6	97	306	428	306	431

Table 3 strongly supports the idea that carbon taxes do the majority of the "work" in improving welfare. In all cases considered, a carbon tax on its own outperforms an R&D subsidy on its own by a substantial margin. Moreover, adding a carbon tax to a pure subsidy program leads to substantial welfare gains – typically multiple times as large as the gains from an R&D subsidy program on its own. In contrast, adding an R&D subsidy to a pure carbon tax program leads to only minor improvements. The largest gains (proportionally) are when the externality is smallest.

Finally, as noted in section 3, the choice of policy may have significant effects on the distribution of outcomes. To more closely examine these distributional concerns, table 4 displays the standard deviation of the number of R&D entrants, assuming free entry and under various parameter combinations. The last column also displays the relative size of the R&D subsidy standard deviation compared to the carbon tax standard deviation.

Table 4: Standard Deviation of Entrants Under Alternative Assumptions and Free Entry

	R&D Subsidy	Carbon Tax	Both Policies	R&D Subsidy / Carbon Tax
Baseline	6.07	3.95	4.52	1.54
$\eta = 0.25$	6.07	3.96	4.49	1.53
$\eta = 1$	5.22	3.07	3.58	1.70
$x = 10$	5.07	3.73	4.36	1.36
$x = 40$	7.57	4.07	4.44	1.86
$k = 0.03\bar{\pi}$	9.12	5.78	6.51	1.58
$k = 0.12\bar{\pi}$	3.81	2.60	3.08	1.46
$E[\omega] = 15$	4.36	3.00	3.63	1.45
$E[\omega] = 60$	7.65	4.80	5.30	1.59

As noted in section 3, carbon taxes can induce more R&D entrants when technological opportunity is low (Remark 4), but fewer entrants when technological opportunity is high (Remark 5). This suggests the variance of entrants will be larger under an R&D subsidy than under a carbon tax, which is borne out in Table 4. Moreover, we would anticipate the difference between the two will be largest when technological opportunity itself has a wider dispersion, a finding consistent with the last two rows of Table 4.

5. Conclusion

Given the perceived need to promote environmental innovations, both "pull" and "push" policy tools can in principle help. In this paper we examine the efficacy of two such policies in the context of a model incorporating free entry and uncertainty about technological opportunity at the time of the policy choice. Our numerical results show a carbon tax on its own suffices to obtain most of the welfare gains that an optimal mix of carbon taxes and R&D subsidies achieves.

Our model also allows us to make some claims about the robustness of different policies to changing parameters. While the optimal R&D subsidy for a single innovator does not depend on the shape of demand or forecast technological opportunity, this turns out to be a special case. In general, the optimal R&D subsidy is highly contingent on whether free entry is modeled and whether the subsidy is paired with a carbon tax. In contrast, while the optimal carbon tax *does* depend on the shape of demand and the outlook for technological opportunity, the magnitude of these effects is small. Moreover, the optimal carbon tax level is fairly robust to changing assumptions about free entry or whether or not an R&D subsidy is also being implemented. In general, the optimal carbon tax is slightly larger than a naïve tax that exactly offsets the environmental externality.

Finally, our model also allows us to study the impact of different policy choices on the distribution of outcomes, in addition to their expected values. Compared to a carbon tax, subsidies are more likely to yield R&D that produces unused innovations. We further show numerically and analytically that R&D subsidies are associated with more disperse outcomes when innovation involves a minimum inventive step. This is because R&D subsidies are ineffective for low levels of technological opportunity, but tend to induce more innovators than carbon taxes when technological opportunity is so high that taxes become of second-order importance.

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Appendix: Proof of Remark 5

The proof draws on the following two lemmas.

Lemma 1: *For any* $m > 1$ *and polynomial* $\sum_{i=0}^{n} b_i x^i$ *where* $b_n > 0$*, there is some* x_0 *such that for all* $x > x_0$*:*

$$mb_n x^n > \sum_{i=0}^{n} b_i x^i \tag{17}$$

Proof:

Rewrite equation (17) as:

$$b_n x^n + \sum_{i=0}^{n-1} \left(\frac{m-1}{n} \right) b_n x^n > \sum_{i=0}^{n} b_i x^i \tag{18}$$

Re-order equation (18) as follows:

$$\sum_{i=0}^{n-1} x^i \left\{ \left(\frac{m-1}{n} \right) b_n x^{n-i} - b_i \right\} > 0 \tag{19}$$

The above is satisfied for:

$$x_0 = \max \left\{ \left(\frac{b_i}{b_n} \frac{n}{m-1} \right)^{1/(n-i)} \right\} \tag{20}$$

Lemma 2: *For any* $m > 1$ *and polynomial* $\sum_{i=0}^{n} b_i x^i$ *where* $b_n > 0$*, there is some* x_0 *such that for all* $x > x_0$*:*

$$\sum_{i=0}^{n} b_i x^i > b_n x^n / m \tag{21}$$

Proof:

Let $\tilde{b}_i \equiv -mb_i$. By Lemma 1, there exists some x_0 such that for all $x > x_0$ the following is true.

$$mb_n x^n > b_n x^n + \sum_{i=0}^{n-1} \tilde{b}_i x^i \tag{22}$$

Rewrite equation (22) as:

$$\begin{aligned} b_n x^n &> b_n x^n / m + \left(-\sum_{i=0}^{n-1} b_i x^i \right) \\ b_n x^n + \sum_{i=0}^{n-1} b_i x^i &> b_n x^n / m \end{aligned} \tag{23}$$

This completes the proof.

Remark 5: *For a given tax* t *and subsidy* s*, for sufficiently high technological opportunity* ω*, an R&D subsidy induces more innovation entrants than a carbon tax.*

Proof:

For expositional clarity, we include the term $\hat{\theta}$ in the expected licensing equation (13) as follows:

$$\pi(\omega,n,\hat{\theta}) = \int_{\hat{\theta}}^{\omega} \left\{ \left(\frac{\theta_1 + \hat{\theta}}{2\theta_1} \right)^{n-1} \frac{(\theta_1 - \hat{\theta})^2}{4} + \int_{(\theta_1+\hat{\theta})/2}^{\theta_1} (\theta_1 - \theta_2)(\theta_2 - \hat{\theta}) \frac{n-1}{\theta_2} \left(\frac{\theta_2}{\theta_1} \right)^{n-1} d\theta_2 \right\} \left(\frac{\theta_1}{\omega} \right)^{n-1} \frac{1}{\omega} d\theta_1$$

(24)

Where $\hat{\theta} = c_2 - c_1 - t$.

To reiterate section 3, the n^{th} innovator will enter if:

$$\pi\left(\omega, n, \hat{\theta} \right) \geq \left(1 - s \right) k \tag{25}$$

For the case of pure carbon tax, $\hat{\theta} = c_2 - c_1 - t$ and $s = 0$, while for a pure R&D subsidy, $\hat{\theta} = c_2 - c_1$ and $s > 0$. Equation (25) can be re-expressed for a carbon tax as:

$$\pi\left(\omega, n, c_2 - c_1 - t \right) \geq k \tag{26}$$

And for an R&D subsidy as:

$$\pi\left(\omega,n,c_2-c_1\right)/\left(1-s\right)\geq k \tag{27}$$

We will establish that for any s,t,n, there is a ω_0 such that:

$$\pi\left(\omega,n,c_2-c_1\right)/\left(1-s\right)>\pi\left(\omega,n,c_2-c_1-t\right)\text{ for all }\omega>\omega_0 \tag{28}$$

When this condition is satisfied, there are situations where equation (27) holds but equation (26) does not (but never the reverse). In these situations, R&D subsidies induce the entry of more innovators than a carbon tax.

We expand equation (24) in order to complete the proof. To simplify notation and keep things compact, define:

$$x\equiv\frac{\theta_1+\hat{\theta}}{2\theta_1} \tag{29}$$

The inner integral of equation (24) can be written as:

$$\int_{(\theta_1+\hat{\theta})/2}^{\theta_1}(\theta_1-\theta_2)(\theta_2-\hat{\theta})\frac{n-1}{\theta_2}\left(\frac{\theta_2}{\theta_1}\right)^{n-1}d\theta_2=\left(n-1\right)\left[\theta_1^2\left(\frac{1-x^n}{n}-\frac{1-x^{n+1}}{n+1}\right)+\theta_1\hat{\theta}\frac{1-x^n}{n}-\hat{\theta}\frac{1-x^{n-1}}{n-1}\right] \tag{30}$$

With some substitution and simplification, we can rewrite equation (24) as:

$$\pi(\omega,n,\hat{\theta})=\frac{1}{\omega^n}\int_{\hat{\theta}}^{\omega}\left\{\theta_1^{n+1}\alpha_2+\theta_1^n\alpha_1+\theta_1^{n-1}\alpha_0\right\}d\theta_1 \tag{31}$$

Where we define the following for compactness:

$$\alpha_2=\frac{1}{4}x^{n-1}-\frac{n-1}{n}x^n+\frac{n-1}{n+1}x^{n+1}+\frac{n-1}{n\left(n+1\right)} \tag{32}$$

$$\alpha_1=-\frac{\hat{\theta}}{2}x^{n-1}-\frac{n-1}{n}\hat{\theta}x^n+\frac{n-1}{n}\hat{\theta} \tag{33}$$

$$\alpha_0 = \left(\frac{\hat{\theta}^2}{4} + \frac{\hat{\theta}}{n-1} \right) x^{n-1} - \frac{\hat{\theta}}{n-1} \tag{34}$$

To expand equation (31), we need to solve many integrals of the form $\int_{\hat{\theta}}^{\omega} \theta_1^n x^m d\theta_1$. Substituting in equation (29) and using the binomial theorem, the expanded solution to these terms takes the following form:

$$\int_{\hat{\theta}}^{\omega} \theta_1^n x^m d\theta_1 = \left(\frac{1}{2} \right)^m \sum_{i=0}^{m} \binom{i}{m} \hat{\theta}^i \left(\frac{\omega^{n-i+1} - \hat{\theta}^{n-i+1}}{n-i+1} \right) \tag{35}$$

Equation (35) expresses $\int_{\hat{\theta}}^{\omega} \theta_1^n x^m d\theta_1$ as a polynomial. As shown in Lemmas 1 and 2, as ω is increased, the ω raised to the highest exponent comes to dominate all other terms. Let $L\left(f\left(x \right) \right)$ denote the largest exponent term of polynomial $f\left(x \right)$. That is:

$$L\left(\sum_{i=0}^{n} b_i x^i \right) = b_n x^n \tag{36}$$

Equation (35) implies:

$$L\left(\int_{\hat{\theta}}^{\omega} \theta_1^n x^m d\theta_1 \right) = \left(\frac{1}{2} \right)^m \frac{1}{n+1} \omega^{n+1} \tag{37}$$

Since equation (31) is the sum of polynomials, it too is a polynomial. By equation (37), the largest exponent of (31) is:

$$L\left(\pi(\omega, n, \hat{\theta}) \right) = \frac{1}{\omega^n} \int_{\hat{\theta}}^{\omega} \left\{ \theta_1^{n+1} \left(\frac{1}{4} x^{n-1} - \frac{n-1}{n} x^n + \frac{n-1}{n+1} x^{n+1} + \frac{n-1}{n\left(n+1 \right)} \right) \right\} d\theta_1 \tag{38}$$

This can be simplified to be:

$$L\left(\pi(\omega, n, \hat{\theta}) \right) = \frac{n - \left(1 - \left(1/2 \right)^n \right)}{n\left(n+1 \right)\left(n+2 \right)} \omega^2 \tag{39}$$

By Lemmas 1 and 2, equation (39) means for any $m>1$ there exists ω_0 such that for all $\omega > \omega_0$:

$$\frac{1}{m}\frac{n-\left(1-(1/2)^n\right)}{n(n+1)(n+2)}\omega^2 < \pi\left(\omega,n,\hat{\theta}\right) < m\frac{n-\left(1-(1/2)^n\right)}{n(n+1)(n+2)}\omega^2 \tag{40}$$

For any m, there is some ω_0 such that for $\omega > \omega_0$:

$$\pi\left(\omega,n,c_2-c_1-t\right) < m\frac{n-\left(1-(1/2)^n\right)}{n(n+1)(n+2)}\omega^2 \tag{41}$$

And

$$\frac{1}{m(1-s)}\frac{n-\left(1-(1/2)^n\right)}{n(n+1)(n+2)}\omega^2 < \pi\left(\omega,n,c_2-c_1\right)/\left(1-s\right) \tag{42}$$

Set $m<\sqrt{\frac{1}{(1-s)}}$, which implies:

$$m\frac{n-\left(1-(1/2)^n\right)}{n(n+1)(n+2)}\omega^2 < \frac{1}{m(1-s)}\frac{n-\left(1-(1/2)^n\right)}{n(n+1)(n+2)}\omega^2 \tag{43}$$

The conjunction of equations (41), (42), and (43) implies:

$$\pi\left(\omega,n,c_2-c_1\right)/\left(1-s\right) > \pi\left(\omega,n,c_2-c_1-t\right) \text{ for all } \omega > \omega_0 \tag{44}$$

Completing the proof.