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ESTIMATION OF TECHNOLOGICAL PROGRESS ON THE BASE OF FLEXIBLE COST FUNCTION

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One of the most popular approaches which allows for the measurement of technological progress is based on the estimation of production, cost or profit function.
There are some important methodological requirements which one should take into account when the modelling of technological progress using the concept of production function (or its' dual form i.e. cost function) is considered. The necessary requirement is that technological progress should bring about a diminution of the average costs of production, given constant output and input price level.

Moreover technological progress should be allowed to:
1. affect the elasticity of substitution between two inputs, given constant output and input price ratio (technological progress embodied in i-factor),

2. affect the magnitude and the rate of economies of scale (e.q. effect of learning by doing),

3. cause the bias in output-mix (technological progress output-biased),

4. appear as an exogenous variable independent on output and price level (autonomous technological progress),

5. change over time at a non-constant rate.

However, only few empirical studies are known with application to agriculture in which the effects of technological progress on the change of production technology were modelled and tested statistically using above criteria. In a number of studies which dealt with the measurement of technological progress usually only some of these criteria are preselected (rather ambiguously), the rest is simply ignored. New developments in the establishing flexible forms as well as the advantages stemming from duality theory made possible to weaken strong a priori assumptions recognized earlier as a straitjacket of economic analysis of technology. However, application of flexible forms for the analysis of producer behaviour and for the measurement of technological progress can be the subject to massive critique unless the appropriate theoretical requirements necessary to represent an optimization approach are ensured.

In order to represent producer's optimization behaviour (i.e. cost minimization approach) the cost function must possess the following properies:

1. positivity i.e. the cost function is positive for positive input prices and positive level of output,

2. homogeneity i.e. the cost function is homogenous of degree one in input price,

3. monotonicity i.e. the cost function is increasing in input prices and level of output,

4. concavity i.e. the cost function is concave in input prices.

Employment of flexible forms (i.e. translog. cost function) for the modelling of producer behaviour and technological progress does not ensure, however, that all of these properties will be a priori fulfilled. Some of these requirements can be ensured by the appropriate restrictions imposed on the parameters (i.e. homogeneity, symmetry restrictions), unfortunately monotonicity and concavity conditions are checked usually in applied work given parameters estimated i.e. ex post. If the estimated flexible form of the cost function, given input price ratio, fulfils all desirable properties it can be said to be a reasonable representation of a producer behaviour around this point. However, when concavity and (or) monotonicity conditions are not fulfilled the employment of a given expenditure function as a representative of the cost minimizing producer behaviour is problematic.

First, the whole interpretation of the obtained results is questionable since these would only be valid under the assumption of producer's optimization behaviour (cost minimization or profit maximization approach). Second, if monotonicity and (or) concavity conditions in a dual approach are not fulfilled also a primal optimization problem is not retrievable (both problems are in one-to- one correspondence only under appropriate regularity conditions).

Estimation of technological progress on the base of flexible form (i.e. translog cost function) implies that within this framework technological progress can be considered as a non-systematic factor which affects the rate of elasticities of substitution and economies of scale, which can be input and output biased etc. and therefore fulfils all desirable requirements of its own characteristics mentioned above.

In general one can distinguish three major assumptions which decisively impact upon econometricaly estimated rate of technological progress:

1. first set assumptions regards a general specification of technology and particulary allowing for non-homotheticity, variable elasticity of substitution and non-constant rate of technological progress;

2. second set of assumptions concerns the functional relationship between technological progress and other characteristics of technology i.e. allowing for technological progress which can be scale augmenting, input and output biased, autonomous etc.;

3. third set of assumption concerns an assumed functional form of estimated production, cost, profit or price function (different functional specifications of flexible forms which include time as a proxy for technological progress are presented in Appendix 1).

However, allowing for non-neutral technological progress within more general description of technology (as presented by flexible forms) can cause additional concavity problems which do not appear under different technological progress specification. It is well known that if the production process is a subject to Hick's neutral technological progress the isoquants move towards the origin having their shapes unaffected. However, if technological progress is not Hick's neutral and if technological progress is simultaneously allowed to affect the rate of economics of scale a completely different set of equilibria can occur.

An affirmative answer on the question whether concavity properties of the used cost function could depend on the form and characteristic of technological progress was already given in some studies (comp. MORONEY/TRAPANI, 1981; BERNDT/WOOD, 1985). It has been found that when technological progress is not neutral, the estimated translog cost functions is not locally concave. However, when the statistically rejected Hick's neutral technological progress restrictions are imposed, the estimated translog cost function is found to be locally well-behaved.

Of course local concavity properties can be imposed on the function in different ways. However, using these approaches one has to be aware of the trade-offs which usually exist between concavity conditions on one side and the flexibility of the used form on the other side. An example which shows that these methods often fail to yield satisfactory results in practice is the study of JORGENSON & FRAUMENI (1981). After employing JORGENSON & FRAUMENI concavity conditions which are based in the Cholesky decomposition of positive definite matrix BERNDT/WOOD 1985 found that with data used these restrictions imply C-D representation of technology. Not only the estimated bias of technological progress was dramatically altered when J-F global concavity conditions were imposed. Additionally to the alteration of the specification of technological progress by imposing negative semidefiniteness on the Hessian matrix severe restrictions on the own and cross price elasticities had to be imposed.

Choosing therefore in practice between forms of production function and simultaneously having in mind the necessity of estimation technological progress in agriculture in its most unrestricted form the relevant question which appears is: what considerations are relevant in the selection of one algebraic form over another using only a priori information, which is not specific to the particular data set?

There are several criteria that can be used as the base for the choice of adequate functionall form in general and specific which allow to estimate technological progress in its more unrestricted form. These are (LAU, 1986): a) theoretical consistency, b) domain of applicability, c) flexibility, d) computational facility, e) factual conformity. In practice it means that production/cost/profit function which is used for an estimation of technological progress should posses all these desirable characteristics. Unfortunately, comparison between well-known functional forms such as C-D, CES, translog, Generalized Leontief unit cost function according to above mentioned criteria has led LAU (1986) to the conclusion that there exist no algebraic functional form which simultaneously satisfies at least two of the most important criteria i.e. global extrapolative domain of applicability and flexibility, thus in general one should not expect to find a algebraic form which satisfies all five criteria.

Since the most traditional functional specifications of technology are too restrictive for modelling different types of technological progress, an alternative would be to develop functional forms which are globally 'well behaved' and still flexible enough to analyze the basic characteristics of technology i.e. technological progress, economics of scale, elasticities of substitution in their most unrestricted form. A possible solution might be an estimation of technological progress using generalized Barnet cost function which is globally concave in input prices and simultaneously accounts to flexible forms. However, the main disadvantages associated with this form are (comp. DIEWERT, WALES, 1987):

1. it requires the imposition of a large number of inequality constraints on the coefficients,

2. a priori it requires many more parameters than other flexible forms,

3. even with the large number of parameters it cannot be proved that such a functional form is completely flexible (it is quasi flexible relative to the numerary good k).

Another possible solution to overcome this dilemma is to impose on the functional form certain regularity conditions without destroying its' flexibility.

One of the recent studies of DIEWERT, WALES 1987, suggests that this is possible using a technique proposed by WILEY & SCHMIDT (1973) which is based on the decomposition of a symmetric negative semidefinite matrix as a product of minus lower triangular matrix and its transpose. Employment of this method should ensure that the estimated flexible cost function will always be concave in input prices and therefore it will possess a property of the extrapolative domain of applicability even when technological progress is assumed to be non- neutral and estimated function exhibits non-constant returns to scale.

However, an example will show that also this method can be regarded as too restrictive and therefore limited for the empirical purpose.
In order to explain this we assume that the real cost function can be approximated by an non-homothetic version of translog cost function with not Hick's-neutral technological progress.

$$\begin{aligned}\ell nc = {} & \alpha_o + \sum_i \alpha_i \cdot \ell n\, p_i + \alpha_y \cdot \ell n_y + \alpha_t \cdot t + \tfrac{1}{2} \sum_i \sum_j \gamma_{ij} \cdot \ell n\, p_i \cdot \ell n\, p_j + \\ & + \sum_i \alpha_{yi} \cdot \ell ny \cdot \ell n\, p_i + \sum_i \alpha_{ti} \cdot \ell n\, p_i \cdot t + \tfrac{1}{2} \alpha_{yy} \cdot (\ell ny)^2 + \alpha_{yt} \cdot \ell_{ny} \cdot \tfrac{1}{2}\alpha_{tt} \cdot t^2\end{aligned} \tag{1.1}$$

where: t = time as a proxy for t.p., y = output, p = i-factor price.

If cost function is (quasi)concave in input prices its Hessian matrix must be negative (semi)definite. It means that matrix $S = [S_{ij}]$ with off-diagonal elements:

$$S_{ij} = \frac{\partial^2 c}{\partial p_i \cdot \partial p_j} = \frac{\partial(w_i \cdot c/p_i)}{\partial p_j} = \frac{c \cdot (\gamma_{ij} + w_i \cdot w_j)}{p_i \cdot p_j} \tag{1.2}$$

and diagonal elements:

$$S_{ii} = \frac{\partial^2 c}{\partial p_i \cdot \partial p_i} = \frac{\partial(w_i \cdot c/p_i)}{\partial p_i} = \frac{c \cdot (\gamma_{ii} + w_i^2 - w_i)}{p_i^2} \tag{1.3}$$

where w_i = factor i share

is negative semidefinite, or
the matrix of substitution elasticity (Allen-Uzawa) $[\delta_{ij}]$ defined as:

$$\sigma_{ij} = \frac{c \cdot (\frac{\partial^2 c}{\partial p_i \cdot \partial p_j})}{\partial c/_{\partial p_i} \cdot \partial c/_{\partial p_j}} = \frac{c \cdot \Sigma_{ij}}{\partial c/_{\partial p_i} \cdot \partial c/_{\partial p_j}} \quad \text{is negative semidefinite} \tag{1.4}$$

where Σ_{ij} are the elements of Slutsky matrix.

Both matrices represent the equivalent illustration of concavity of the cost function.
Since 1.4 holds and

$$\sigma_{ij} = \frac{\gamma_{ij}}{w_i \cdot w_j} + 1 \tag{1.4.1}$$

$$\sigma_{ij} = \frac{\gamma_{ii} + w_i^2 - W_i}{w_i^2} \tag{1.4.2}$$

the conditions for the Slutsky matrix to be negative semidefinite can be easily expressed in terms of appropriate conditions imposed on the Allen-Uzawa substitution elasticity $[\delta_{ij}]$ matrix.
The necessary conditions for matrix $[\delta_{ij}]$ to be negative semidefinite (which also ensure that all own-price substitutions elasticities are negative) are that K_{ii} is smaller than zero where:

$$K_{ii} = \gamma_{ii} + w_i^2 - w_i$$

$$K_{ii} < 0 \tag{1.5}$$

Sufficient condition is that all principal minors of $[\delta_{ij}]$ matrix will alternate their signs such that odd-numbered principal minors are negative and even-numbered principal minors are positive. For example for a 3 x 3 symmetric matrix of substitution elasticities $[\delta_{ij}]$, given homogeneity restrictions, the sufficient condition is that

$$\sigma_{11} \cdot \sigma_{22} - \sigma_{12}^2 > 0 \tag{1.6}$$

(negativity of the first principal minor is already ensured via 1.5).

From above conditions it follows that:

1. even given negative estimates of the diagonal elements of $[\delta_{ij}]$ matrix it does not yet ensure that the cost function is concave (until 1.6 holds).
2. from 1.5 we can see that necessary conditions for the cost function to be globally concave do not require that the diagonal elements of Γ matrix ($\Gamma = [\gamma_{ij}]$ are to be negative as it is in the case of the method proposed by WILEY, SCHMIDT (1973) in: DIEWERT, WALES (1987).

In order to ensure global concavity of the cost function DIEWERT & WALES (1987) propose to redefine matrix Γ to be negative definite and decompose it (based on Cholesky decomposition) such that Γ can be expressed as a lower triangular matrix times its transpose:

$$\Gamma = - [\hat{\gamma}_{ij,L}] \cdot [\hat{\gamma}_{ij,L}]^T \tag{1.7}$$

where: is the transformed lower triangular matrix with γ_{ij} lements from the cost function and to estimate Γ matrix in the form (example for 3 x 3 matrix)

$$\Gamma = \begin{bmatrix} -\hat{\gamma}_{11}^2 & -\hat{\gamma}_{11} \cdot \hat{\gamma}_{12} \\ -\hat{\gamma}_{11} \cdot \hat{\gamma}_{12} & -\hat{\gamma}_{11}^2 - \hat{\gamma}_{22}^2 \end{bmatrix} \tag{1.8}$$

where: (1.9)

$$\gamma_{11} = -\hat{\gamma}_{11}^2$$
$$\gamma_{12} = -\hat{\gamma}_{11} \cdot \hat{\gamma}_{12}$$
$$\gamma_{22} = -\hat{\gamma}_{11}^2 - \hat{\gamma}_{22}^2$$

Since (1.9) holds this method implies that all estimated elements of the Γ matrix must be negative, what is a much stronger condition compared to (1.5).

An imposition of the negative values on the diagonal elements τ of the Γ matrix in the case when in the free estimation τ_{ij} show statistically significant positive signs can lead to problems of estimation, especially when the gradient methods are used. In this case most what we can obtain by employing the method proposed in DIEWERT & WALES (1987) are the parameter estimates which will tend to zero (the change of the statistically significant sign of the parameter is imposed). This, however, reduces appropriate elasticities of substitution to 1 and our function to C-D form with significant loss of fit.

For an illustration of this problem we used the data of bookkeeping farms from Schleswig-Holstein (West-Germany) over the period 1961/62 - 1983/84 in order to estimate the non-homothetic version of translog cost function with Hick's non-neutral technological progress (given as in 1.1).

Since estimated translog function was not well-behaved for a certain range of input prices, in order to ensure global concavity of the cost function the method proposed by WILEY/SCHMIDT (1973) in DIEWERT & WALES (1987) was employed.

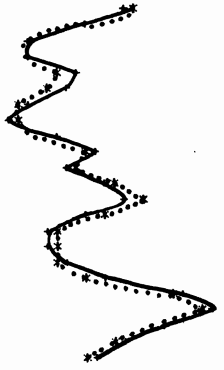
The unrestricted estimates of the cost function show a positive statistically significant value of the diagonal element of Γ matrix (τ_{11})
where: τ_{11} = τ_{11} and τ_{11} = o.1591, t = 17.2 and very high R^2 coefficient of the estimated budget share equation w_1 (graph 1). Important is that even for such a high positive value of τ_{11} estimated within a function which allows for Hick's non-neutral technological progress

Graph 1: Unrestricted estimation of the budget share w1

PLOT OF ACTUAL(*) AND FITTED(+) VALUES

R^2=0.9521

ID	ACTUAL	FITTED	RESIDUAL
2	0.2400	0.2334	0.0066
3	0.2100	0.2080	0.0020
4	0.1700	0.1698	0.0002
5	0.1700	0.1706	-0.0006
6	0.1900	0.2027	-0.0127
7	0.1800	0.1954	-0.0154
8	0.1600	0.1667	-0.0067
9	0.1600	0.1548	0.0052
10	0.1900	0.1903	-0.0003
11	0.2100	0.2139	-0.0039
12	0.2000	0.1967	0.0033
13	0.2300	0.2186	0.0114
14	0.2500	0.2337	0.0163
15	0.2200	0.2089	0.0111
16	0.1900	0.1840	0.0060
17	0.1900	0.1844	0.0056
18	0.1900	0.1966	-0.0066
19	0.2100	0.2177	-0.0077
20	0.2700	0.2725	-0.0025
21	0.2900	0.2937	-0.0037
22	0.2600	0.2632	-0.0032
23	0.2300	0.2356	-0.0056
24	0.2100	0.2114	-0.0014



the appropriate cost function is concave for the range of the budget shares given by eq. 1.5-1.6.

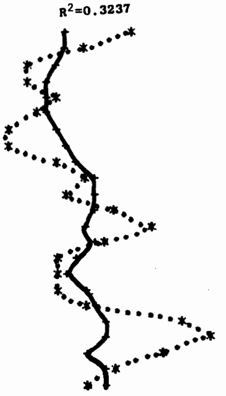
In our next step we imposed the statistically rejected hypothesis of Hick's neutrality of technological progress on the estimated cost function in order to compare the number of historical price ratios for which both estimated cost functions were locally concave. After imposing Hick's-neutral specification of technological progress the number of observations for which estimated cost function was locally concave increased by 44 %.

Further, in order to ensure that estimated cost function will be concave for all range of input prices we decomposed matrix $\Gamma = [\tau_{ji}]$ according to the method proposed by SCHMIDT & WILEY in: DIEWERT & WALES (1987). As expected, because the free estimate of the diagonal element of the matrix was originally positive and statistically significant after imposing restrictions 1.8-1.9 new estimated value of τ_{11} where $\tau_{11} = \tau_{11}^2$ appeared to be almost zero (τ_{11} = 2.89 E-08). Additional expense of the restrictions imposed on the cost function in [1.1] was a significant loss of fit of the estimated budget share. From $R^2 = 0.9521$, goodness of fit decreased to $R^2 = 0.3237$ (graph 2).

Graph 2: Concavity restricted estimation of the budget share w1

PLOT OF ACTUAL(*) AND FITTED(+) VALUES

ID	ACTUAL	FITTED	RESIDUAL
2	0.2400	0.1962	0.0438
3	0.2100	0.1965	0.0135
4	0.1700	0.1931	-0.0231
5	0.1700	0.1867	-0.0167
6	0.1900	0.1830	0.0070
7	0.1800	0.1834	-0.0034
8	0.1600	0.1936	-0.0336
9	0.1600	0.1977	-0.0377
10	0.1900	0.2041	-0.0141
11	0.2100	0.2153	-0.0053
12	0.2000	0.2187	-0.0187
13	0.2300	0.2136	0.0164
14	0.2500	0.2090	0.0410
15	0.2200	0.2139	0.0061
16	0.1900	0.2030	-0.0130
17	0.1900	0.1981	-0.0081
18	0.1900	0.2085	-0.0185
19	0.2100	0.2184	-0.0084
20	0.2700	0.2200	0.0500
21	0.2900	0.2232	0.0668
22	0.2600	0.2107	0.0493
23	0.2300	0.2246	0.0054
24	0.2100	0.2225	-0.0125



We conclude that the method proposed by SCHMIDT & WILEY in DIEWERT & WALES (1987) similar to the other methods which are based on the Cholesky decomposition of the negative definite matrix (comp. JORGENSON, FRAUMENI, 1981), can substantially affect flexibility of the estimated function. However, since the measurement of technological progress in its most unrestricted form simultaneously requires the maintenance of the principal properties of the estimated cost function, the basic dilemma which exists between the modelling of unrestricted form of technological progress and maintenance of concavity of the cost function has to be solved via further development of the functional forms which are at the same time globally theoretically consistent and flexible.

Appendix

Time (t) as a proxy for technological progress in different specifications of flexible cost functions:

The Translog Cost Function

$$\ln C(p,y,t) = a_o + \Sigma a_i \cdot \ln p_i + a_y \cdot \ln y + a_t \cdot t + (1/2)\Sigma\Sigma a_{ij} \cdot \ln p_i \cdot \ln p_j + \Sigma a_{iy} \cdot \ln p_i \cdot \ln y + \Sigma a_{it} \cdot t \cdot \ln p_i + (1/2) a_{yy} \cdot \ln y \cdot \ln y + a_{yt} \cdot \ln y \cdot t + (1/2) a_{tt} \cdot t^2, \quad (1)$$

where: $a_{ij} = a_{ji}$ for all i, j.

$$\ln C(p,y,t) = a_o' + \Sigma a_i' \cdot \ln p_{it} + 1/2\, \Sigma\Sigma \tau'_{ij} \cdot \ln p_i \cdot \ln p_j + \alpha_y' \cdot \ln y_t + 1/2\, \tau'_{yy'}(\ln y)^2 + \Sigma \tau'_{yi} \cdot \ln y_t \ln p_{it}$$

where:

$$\alpha'_i = \alpha_i + \alpha_i \cdot t$$
$$\alpha'_o = \alpha_o + \alpha_o \cdot t$$
$$\tau'_{ij} = \tau_{ij} + \tau_{ij} \cdot t \quad (2)$$
$$\alpha'_y = \alpha_y + \alpha_y \cdot t$$
$$\tau'_{yy} = \tau_{yy} + \tau_{yy} \cdot t$$
$$\tau'_{yi} = \tau_{yi} + \tau_{yi} \cdot t$$

The Generalized Leontief Cost Function

$$C(p, y, t) = \Sigma\Sigma b_{ij} \cdot p_i^{1/2} \cdot p_j^{1/2} \cdot y + \Sigma b_i \cdot p_i + \Sigma b_{it} \cdot p_i \cdot t \cdot y + b_t \cdot (\Sigma \alpha_i \cdot p_i) \cdot t + b_{yy} \cdot (\Sigma \beta_i \cdot p_i) \cdot y^2 + b_{tt} \cdot (\Sigma \tau_i \cdot p_i) \cdot t^2 \cdot y \quad (3)$$

A Generalized McFadden Cost Function (4)

$$C(p, y, t) = g1(p) \cdot y + \Sigma b_{ii} \cdot p_i \cdot y + \Sigma b_i \cdot p_i + \Sigma b_{it} \cdot p_i \cdot t \cdot y + b_t \cdot (\Sigma \alpha_i \cdot p_i) \cdot t + b_{yy} \cdot (\Sigma \beta_i \cdot p_i)2 + b_{tt} \cdot (\Sigma \tau_i \cdot p_i) \cdot t^2 \cdot y$$

where: $g1(p) \equiv (1/2)p_1^{-1} \cdot \Sigma\Sigma c_{ij} \cdot p_i \cdot p_j$

A Generalized Barnett Cost Function (5)

$$C(p, y, t) = g^1(p) \cdot y + \Sigma\, b_{ii} \cdot p_i \cdot y + \Sigma\, b_i \cdot p_i + \Sigma\, b_{it} \cdot p_i \cdot t \cdot y + b_t \cdot (\Sigma\, \alpha_i \cdot p_i) \cdot t + b_{yy} \cdot (\Sigma\, \beta_i \cdot p_i) + b_{tt} \cdot (\Sigma\, \tau_i \cdot p_i) \cdot t^2 \cdot y$$

where:

$$g1(p) = \Sigma\Sigma\, b_{ij} \cdot p_i^{1/2} \cdot p_j^{1/2} - \Sigma\Sigma\, d_{ij} \cdot p_1^{2} \cdot p_i^{-1/2} \cdot p_j^{-1/2} - \Sigma\, d_i \cdot p_1^{2} \cdot p_i^{-1},\ b_{ji} = b_{ij} \geq 0,\ d_{ij} = d_{ji} \geq 0,$$

$$b_{ij}\, d_{ij} = 0 \text{ for all } i, j.$$

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