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Sensitivity analysis for average treatment effects

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Abstract. Based on the conditional independence or unconfoundedness assumption, matching has become a popular approach to estimate average treatment effects. Checking the sensitivity of the estimated results with respect to deviations from this identifying assumption has become an increasingly important topic in the applied evaluation literature. If there are unobserved variables that affect assignment into treatment and the outcome variable simultaneously, a *hidden bias* might arise to which matching estimators are not robust. We address this problem with the bounding approach proposed by Rosenbaum (*Observational Studies*, 2nd ed., New York: Springer), where `mhbounds` lets the researcher determine how strongly an unmeasured variable must influence the selection process to undermine the implications of the matching analysis.

Keywords: st0121, mhbounds, matching, treatment effects, sensitivity analysis, unobserved heterogeneity, Rosenbaum bounds

1 Introduction

Matching has become a popular method to estimate average treatment effects. The method is based on the conditional independence or unconfoundedness assumption, which states that the researcher should observe all variables simultaneously influencing the participation decision and outcome variables. This is a strong identifying assumption and must be justified case by case.¹ Hence, checking the sensitivity of the estimated results with respect to deviations from this identifying assumption becomes an increasingly important topic in the applied evaluation literature.

If there are unobserved variables that simultaneously affect assignment into treatment and the outcome variable, a *hidden bias* might arise to which matching estimators are not robust (Rosenbaum 2002). Since estimating the magnitude of selection bias with nonexperimental data is not possible, we address this problem with the bounding approach proposed by Rosenbaum (2002).² The basic question is whether unobserved factors can alter inference about treatment effects. One wants to determine how strongly an unmeasured variable must influence the selection process to undermine the implications

1. Caliendo and Kopeinig (2005) provide a survey of the necessary steps when implementing (propensity score) matching methods.

2. See Ichino, Mealli, and Nannicini (2006) for a related approach and the user-written command `sensatt` by Nannicini (2006) for an implementation in Stata.

of the matching analysis. The bounding approach does not test the unconfoundedness assumption itself, because this would amount to testing that there are no (unobserved) variables that influence the selection into treatment. Instead, Rosenbaum bounds provide evidence on the degree to which any significance results hinge on this untestable assumption. If the results turn out to be sensitive, the researcher might have to think about the validity of his identifying assumption and consider other estimation strategies. DiPrete and Gangl (2004) provide an ado-file (`rbounds`) that lets the researcher test sensitivity for continuous-outcome variables, whereas our command `mhbounds` focuses on binary-outcome variables (e.g., employment versus unemployment), which are often used in the evaluation literature.³ You can find recent applications of this approach in Aakvik (2001) or Caliendo, Hujer, and Thomsen (2005). We outline this approach briefly in section 2; you can find an extensive discussion in Rosenbaum (2002) and Aakvik (2001). Section 3.1 presents the syntax and section 3.3, the options of `mhbounds`. In section 4, we illustrate the command with some examples. This article does not aim to present or discuss the estimation of treatment effects with matching estimators. Instead, we assume that you are familiar with this literature. You can find good overviews in Heckman et al. (1998), Imbens (2004), or Smith and Todd (2005). Stata programs to estimate treatments effects are `att*` (Becker and Ichino 2002), `psmatch2` (Leuven and Sianesi 2003), and `nnmatch` (Abadie et al. 2004).

2 Sensitivity analysis with Rosenbaum bounds

Checking the sensitivity of estimated treatment effects has become an increasingly important topic in the applied evaluation literature; see Caliendo and Kopeinig (2005) for a recent survey of different methods to do so. Here we are interested in what happens when there are deviations from the underlying identifying conditional independence assumption.

2.1 Model

Let us assume that the participation probability is given by $P_i = P(x_i, u_i) = P(D_i = 1 \mid x_i, u_i) = F(\beta x_i + \gamma u_i)$, where x_i are the observed characteristics for individual i , u_i is the unobserved variable, and γ is the effect of u_i on the participation decision. If the study is free of hidden bias, γ will be zero and the participation probability will be determined solely by x_i . However, if there is hidden bias, two individuals with the same observed covariates x have different chances of receiving treatment. Let us assume that we have a matched pair of individuals i and j and further assume that F is the logistic distribution. The odds that individuals receive treatment are then given by $P_i/(1 - P_i)$ and $P_j/(1 - P_j)$, and the odds ratio is given by

3. `mhbounds` is also applicable to binary transformations of the outcome variable for continuous outcomes.

$$\frac{\frac{P_i}{1-P_i}}{\frac{P_j}{1-P_j}} = \frac{P_i(1-P_j)}{P_j(1-P_i)} = \frac{\exp(\beta x_i + \gamma u_i)}{\exp(\beta x_j + \gamma u_j)} \quad (1)$$

If both units have identical observed covariates—as implied by the matching procedure—the x vector cancels out, implying that

$$\frac{\exp(\beta x_i + \gamma u_i)}{\exp(\beta x_j + \gamma u_j)} = \exp\{\gamma(u_i - u_j)\}$$

But still, both individuals differ in their odds of receiving treatment by a factor that involves the parameter γ and the difference in their unobserved covariates u . So, if there are either no differences in unobserved variables ($u_i = u_j$) or if unobserved variables have no influence on the probability of participating ($\gamma = 0$), the odds ratio is one, implying the absence of hidden or unobserved selection bias. Sensitivity analysis now evaluates how changing the values of γ and $(u_i - u_j)$ alters inference about the program effect. We follow Aakvik (2001) and assume for simplicity that the unobserved covariate is a dummy variable with $u_i \in \{0, 1\}$. Rosenbaum (2002) shows that (1) implies the following bounds on the odds ratio that either of the two matched individuals will receive treatment:

$$\frac{1}{e^\gamma} \leq \frac{P_i(1-P_j)}{P_j(1-P_i)} \leq e^\gamma$$

Both matched individuals have the same probability of participating only if $e^\gamma = 1$. Otherwise, if for example $e^\gamma = 2$, individuals who appear to be similar (in terms of x) could differ in their odds of receiving the treatment by as much as a factor of 2. In this sense, e^γ is a measure of the degree of departure from a study that is free of hidden bias (Rosenbaum 2002).⁴

2.2 MH test statistic

For binary outcomes, Aakvik (2001) suggests using the Mantel and Haenszel (MH, 1959) test statistic. To do so, some extra notation is needed. We observe the outcome y for both participants and nonparticipants. If y is unaffected by different treatment assignments, treatment d is said to have no effect. If y is different for different assignments, then the treatment has some positive (or negative) effect. To be significant, the treatment effect has to cross some test statistic $t(d, y)$. The MH nonparametric test compares the successful number of individuals in the treatment group with the same expected number, given that the treatment effect is zero. Aakvik (2001) notes that the MH test can be used to test for no treatment effect both within different strata of the sample and as a weighted average between strata. Under the null hypothesis of no treatment effect, the distribution of y is hypergeometric. We describe N_{1s} and N_{0s} as the numbers of treated and nontreated individuals in stratum s , where $N_s = N_{0s} + N_{1s}$. Y_{1s} is the number of successful participants, Y_{0s} is the number of successful nonparticipants,

4. You can find a related approach in Manski (1990, 1995), who proposes *worst-case bounds*, which are somewhat analogous to letting $e^\gamma \rightarrow \infty$ in a sensitivity analysis.

and Y_s is the number of total successes in stratum s . The test statistic Q_{MH} follows asymptotically the standard normal distribution and is given by

$$Q_{\text{MH}} = \frac{|Y_1 - \sum_{s=1}^S E(Y_{1s})| - 0.5}{\sqrt{\sum_{s=1}^S \text{Var}(Y_{1s})}} = \frac{|Y_1 - \sum_{s=1}^S (\frac{N_{1s}Y_s}{N_s})| - 0.5}{\sqrt{\sum_{s=1}^S \frac{N_{1s}N_{0s}Y_s(N_s - Y_s)}{N_s^2(N_s - 1)}}} \quad (2)$$

To use such a test statistic, we must first make the individuals in the treatment and control groups as similar as possible, because this test is based on random sampling. Since our matching procedure accomplishes this task, we can discuss the possible influences of $e^\gamma > 1$. For fixed $e^\gamma > 1$ and $u \in \{0, 1\}$, Rosenbaum (2002) shows that the test statistic Q_{MH} can be bounded by two known distributions. If $e^\gamma = 1$ the bounds are equal to the base scenario of no hidden bias. With increasing e^γ , the bounds move apart, reflecting uncertainty about the test statistics in the presence of unobserved selection bias. Two scenarios are especially useful. Let Q_{MH}^+ be the test statistic, given that we have overestimated the treatment effect, and Q_{MH}^- , the case where we have underestimated the treatment effect. The two bounds are then given by

$$Q_{\text{MH}}^+ = \frac{|Y_1 - \sum_{s=1}^S \tilde{E}_s^+| - 0.5}{\sqrt{\sum_{s=1}^S \text{Var}(\tilde{E}_s^+)}} \quad (3)$$

and

$$Q_{\text{MH}}^- = \frac{|Y_1 - \sum_{s=1}^S \tilde{E}_s^-| - 0.5}{\sqrt{\sum_{s=1}^S \text{Var}(\tilde{E}_s^-)}} \quad (4)$$

where \tilde{E}_s and $\text{Var}(\tilde{E}_s)$ are the large-sample approximations to the expectation and variance of the number of successful participants when u is binary and for given γ .⁵

3 The mhbounds command

3.1 Syntax

```
mhbounds outcome [if], gamma(numlist) [treated(newvar) weight(newvar)
support(newvar) stratum(newvar) stratamat]
```

5. The large-sample approximation of \tilde{E}_s^+ is the unique root of the following quadratic equation: $\tilde{E}_s^2(e^\gamma - 1) - \tilde{E}_s\{(e^\gamma - 1)(N_{1s} + Y_s) + N_s\} + e^\gamma Y_s N_{1s}$, with the addition of $\max(0, Y_s + N_{1s} - N_s) \leq \tilde{E}_s \leq \min(Y_s, N_{1s})$ to decide which root to use. \tilde{E}_s^- is determined by replacing e^γ with $1/e^\gamma$. The large-sample approximation of the variance is given by $\text{Var}(\tilde{E}_s) = \{1/\tilde{E}_s + 1/(Y_s - \tilde{E}_s) + 1/(N_{1s} - \tilde{E}_s) + 1/(N_s - Y_s - N_{1s} + \tilde{E}_s)\}^{-1}$.

3.2 Description

`mhbounds` computes MH bounds to check sensitivity of estimated average treatment effects on the treated.

3.3 Options

`gamma(numlist)` is required and specifies the values of $\Gamma = e^\gamma \geq 1$ for which to carry out the sensitivity analysis. Estimates at $\Gamma = 1$ (no hidden bias) are included in the calculations by default.

`treated(newvar)` specifies the name of the user-provided treatment variable. If no name is provided, `mhbounds` uses `_treated` from `psmatch` or `psmatch2`.

`weight(newvar)` specifies the name of the user-provided variable containing the frequency with which the observation is used as a match. If no name is provided, `mhbounds` uses `_weight` from `psmatch` or `psmatch2`.

`support(newvar)` specifies the name of the user-provided common support variable. If no name is provided, `mhbounds` uses `_support` from `psmatch` or `psmatch2`.

`stratum(newvar)` specifies the name of the user-provided variable indicating strata. Aakvik (2001) notes that the MH test can be used to test for no treatment effect both within different strata of the sample and as a weighted average between strata. This option is particularly useful after stratification matching, using, for example, `atts`.

`stratamat`, combined with `stratum(newvar)`, keeps in memory not only the matrix `outmat` containing the overall/combined test statistics but also the matrices `outmat_j` containing the strata-specific test statistics, $j = 1, \dots, \#\text{strata}$.

3.4 Examples

1. Running `mhbounds` after `psmatch2`:

```
. psmatch2 college, outcome(wage) pscore(pscore) caliper(.25) common
> noreplacement
. mhbounds wage, gamma(1 (0.05) 2)
```

Here `mhbounds` performs sensitivity analysis at $\gamma = 1, 1.05, 1.10, \dots, 2$.

2. Running `mhbounds` with user-defined treatment, weight, and support indicators:

```
. mhbounds outcome, gamma(1 (0.05) 2) treated(mytreat) weight(myweight)
> support(mysupport)
```

3. Running `mhbounds` with user-defined treatment, weight, and support indicators with different strata in the population:

```
. mhbounds outcome, gamma(1 (0.05) 2) treated(mytreat) weight(myweight)
> support(mysupport) stratum(mystratum) stratamat
```

`mhbounds` is suited for k th nearest neighbor matching without replacement and for stratification matching.

4 Applying `mhbounds`

To illustrate `mhbounds`, we give two examples. The first is taken from Rosenbaum (2002) and the second one relates to the well-known and much-discussed studies by Lalonde (1986), Dehejia and Wahba (1999), and Smith and Todd (2005).

4.1 Rosenbaum's data

The first example is given in Rosenbaum (2002, 130, table 4.11) and comes from a medical study of the possible effects of the drug allopurinol as a cause of rash (Boston Collaborative Drug Surveillance Program 1972). The treatment here is the use of the drug ($D \in \{0, 1\}$) and the binary-outcome variable is having a rash or not ($Y \in \{0, 1\}$). Table 1 summarizes the available data from a case-referent study, where treated and control group are already comparable and we distinguish two strata of the population ($S = 1$ for males and $S = 2$ for females).

Table 1: Case-referent study data

Stratum	D_i	$Y_i = 0$	$Y_i = 1$
$S = 1$ (Males)	1	33	5
	0	645	36
$S = 2$ (Females)	1	19	10
	0	518	58

Source: Adapted from Rosenbaum (2002, 130).

A first look at the distribution of outcomes between treated and control units would suggest that the treatment in fact has a positive effect on the outcome variable, since, e.g., $5/33 \approx 15\%$ of the treated males have an outcome of 1, whereas this is true for only $36/645 \approx 6\%$ of the control individuals. To replicate the example, we generate a sample of individuals according to the distribution of D and Y in table 1.

```
. clear
. set obs 719
obs was 0, now 719
. gen s = 1
. gen d = _n<=38
. gen out = _n<=5
```

```

. replace out = 1 if _n>38 & _n<75
(36 real changes made)
. save s1.dta, replace
file s1.dta saved
. clear
. set obs 605
obs was 0, now 605
. gen s = 2
. gen d = _n<=29
. gen out = _n<=10
. replace out = 1 if _n>29&_n<88
(58 real changes made)
. save s2.dta, replace
file s2.dta saved
. append using s1.dta
. gen myweight = 1
. gen mysupport = 1
. by s, sort: tabulate out d

```

```
-> s = 1
```

out	d		Total
	0	1	
0	645	33	678
1	36	5	41
Total	681	38	719

```
-> s = 2
```

out	d		Total
	0	1	
0	518	19	537
1	58	10	68
Total	576	29	605

Since we have two strata (males and females) in the population, we are going to use the `stratum()` option of `mhbounds`. Furthermore, we specify that we are interested in the sensitivity of the results up to a situation where $\Gamma = e^\gamma = 8$. Since the data are already matched, we do not have to run any of the available matching routines in Stata. However, for `mhbounds` to work, we must define a treatment indicator (`treated()`), the weight assigned to each individual of both groups (`weight()`), and furthermore identify the individuals who are within the region of common support (`support()`). To keep the example simple, we assume equal weights and that all the individuals lie within the common support region.

```
. mhbounds out, gamma(1 (1) 8) treated(d) weight(myweight) support(mysupport)
> stratum(s)
Mantel-Haenszel (1959) bounds for variable out
Gamma      Q_mh+      Q_mh-      p_mh+      p_mh-
-----
  1         4.18665    4.18665    .000014    .000014
  2         1.80445    7.05822    .035581    8.4e-13
  3         .515322     9.09935    .303164     0
  4         .074087    10.7675    .470471     0
  5         .787917    12.2124    .215372     0
  6         1.37611    13.5046    .084394     0
  7         1.87943    14.6841    .030093     0
  8         2.32133    15.7759    .010134     0
Gamma : odds of differential assignment due to unobserved factors
Q_mh+ : Mantel-Haenszel statistic (assumption: overestimation of treatment effect)
Q_mh- : Mantel-Haenszel statistic (assumption: underestimation of treatment effect)
p_mh+ : significance level (assumption: overestimation of treatment effect)
p_mh- : significance level (assumption: underestimation of treatment effect)
```

In a study free of hidden bias, i.e., where $\Gamma = 1$, the Q_{MH} test statistic is 4.19 and would constitute strong evidence that using allopurinol causes rash. If we have a positive (unobserved) selection, in that those most likely to use the drug also have a higher probability of getting a rash, then the estimated treatment effects overestimate the true treatment effect. The reported test statistic Q_{MH} is then too high and should be adjusted downward. Hence, we will look at Q_{mh}^+ and p_{mh}^+ in the Stata output. The upper bounds on the significance levels for $\Gamma = 1, 2$, and 3 are 0.0001, 0.036, and 0.30 (see also Rosenbaum 2002, 131). The study is insensitive to a bias that would double the odds of exposure to allopurinol but sensitive to a bias that would triple the odds. Our example also highlights that in some applications the significance level on the bounds might fall first and then rise again. If we look, e.g., at the situation for $\Gamma = 8$, we get a significance level p_{mh}^+ of .0101, indicating a significant effect once again. This second significant value of p_{mh}^+ indicates a significant negative treatment effect because we assume a large positive unobserved heterogeneity, which turns our previously significant positive treatment effect into a negative one.

4.2 NSW data revisited

To illustrate `mhbounds` in a more common evaluation environment, we use the data also used by Dehejia and Wahba (DW99, 1999) and Smith and Todd (2005). The first study was influential in promoting matching as an evaluation method, whereas the second one raised some doubts on the reliability of the results in nonexperimental evaluation settings.

The data come from Lalonde's (1986) evaluation of nonexperimental evaluation methods and combine treated units from a randomized study of the National Supported Work (NSW) training program with nonexperimental comparison groups from surveys as the Panel Study of Income Dynamics (PSID) or the Current Population Survey (CPS).⁶ We restrict the sample to the experimental treatment group ($n = 185$) and the PSID

6. The data are available at Dehejia's web site: <http://www.nber.org/~rdehejia/nswdata.html>.

control group ($n = 2,490$). The outcome of interest in DW99 is the postintervention real earnings in 1978 (RE78). Since we are interested in binary outcomes, we define a new outcome variable `employment` taking the value of 1 if the individual had positive real earnings in 1978 and 0 otherwise. The distribution of the outcome variable is the following:

```
. use lalonde, clear
. gen employment = .
(2675 missing values generated)
. replace employment = 1 if re78>0 & re78!=.
(2344 real changes made)
. replace employment = 0 if re78==0
(331 real changes made)
. tabulate employment d
```

employment	d		Total
	0	1	
0	286	45	331
1	2,204	140	2,344
Total	2,490	185	2,675

To make the samples comparable, we use propensity score matching by running `psmatch2` on the same specification as DW99.

```
. psmatch2 d age age2 education educ2 married black hispanic re74 re75 re742
> re752 blacku74, logit out(employment) noreplacement
Logistic regression                               Number of obs   =       2675
                                                    LR chi2(12)    =       935.35
                                                    Prob > chi2    =       0.0000
Log likelihood = -204.97537                       Pseudo R2      =       0.6953
```

d	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
age	.3316904	.1203295	2.76	0.006	.0958489	.5675318
age2	-.0063668	.0018554	-3.43	0.001	-.0100033	-.0027303
education	.8492683	.3477041	2.44	0.015	.1677807	1.530756
educ2	-.0506202	.0172492	-2.93	0.003	-.084428	-.0168124
married	-1.885542	.2993282	-6.30	0.000	-2.472214	-1.298869
black	1.135973	.3517793	3.23	0.001	.446498	1.825447
hispanic	1.96902	.5668567	3.47	0.001	.8580017	3.080039
re74	-.0001059	.0000353	-3.00	0.003	-.000175	-.0000368
re75	-.0002169	.0000414	-5.24	0.000	-.000298	-.0001357
re742	2.39e-09	6.43e-10	3.72	0.000	1.13e-09	3.65e-09
re752	1.36e-10	6.55e-10	0.21	0.836	-1.15e-09	1.42e-09
blacku74	2.144129	.4268089	5.02	0.000	1.307599	2.980659
_cons	-7.474742	2.443502	-3.06	0.002	-12.26392	-2.685566

Note: 22 failures and 0 successes completely determined.
 There are observations with identical propensity score values.
 The sort order of the data could affect your results.
 Make sure that the sort order is random before calling `psmatch2`.

Variable	Sample	Treated	Controls	Difference	S.E.	T-stat
employment	Unmatched	.756756757	.885140562	-.128383805	.024978843	-5.14
	ATT	.756756757	.664864865	.091891892	.047025406	1.95

Note: S.E. for ATT does not take into account that the propensity score is > estimated.

psmatch2: Treatment assignment	psmatch2: Common support On suppor	Total
Untreated	2,490	2,490
Treated	185	185
Total	2,675	2,675

The output shows that we get a significant positive treatment effect on the treated of 0.0919. That is, the employment rate of participants is 9.2% higher than that of matched control group members. Since `psmatch2` automatically produces the variables `_treated`, `_weight`, and `_support`, we do not have to specify those when using `mhbounds`.

```
. mhbounds employment, gamma(1 (0.05) 1.5)
Mantel-Haenszel (1959) bounds for variable employment
Gamma      Q_mh+      Q_mh-      p_mh+      p_mh-
-----
  1          1.83216    1.83216    .033464    .033464
 1.05        1.62209    2.04761    .052392    .020299
  1.1        1.41978    2.2511    .077836    .01219
 1.15        1.22673    2.44599    .109961    .007223
  1.2        1.04213    2.63301    .148677    .004232
 1.25        .865226    2.81282    .193457    .002455
  1.3        .695397    2.98601    .243403    .001413
 1.35        .532076    3.15309    .297337    .000808
  1.4        .374766    3.31449    .353917    .000459
 1.45        .223022    3.47064    .411759    .00026
  1.5        .076449    3.62189    .469531    .000146

Gamma : odds of differential assignment due to unobserved factors
Q_mh+ : Mantel-Haenszel statistic (assumption: overestimation of treatment effect)
Q_mh- : Mantel-Haenszel statistic (assumption: underestimation of treatment effect)
p_mh+ : significance level (assumption: overestimation of treatment effect)
p_mh- : significance level (assumption: underestimation of treatment effect)
```

Under the assumption of no hidden bias ($\Gamma = 1$), the Q_{MH} test statistic gives a similar result, indicating a significant treatment effect. The two bounds in the output table can be interpreted in the following way: The Q_{MH}^+ statistic adjusts the MH statistic downward for positive (unobserved) selection. For the given example, positive selection bias occurs when those most likely to participate tend to have higher employment rates even without participation and given that they have the same x vector as the individuals in the comparison group. This effect leads to an upward bias in the estimated treatment effects. The Q_{MH}^- statistic adjusts the MH statistic downward for negative (unobserved) selection. In other examples, the treatment effects at $\Gamma = 1$ might be insignificant and

the bounds tell us at which degree of unobserved positive or negative selection the effect would become significant.

Given the positive estimated treatment effect, the bounds under the assumption that we have underestimated the true treatment effect (Q_{MH}^-) are somewhat less interesting. The effect is significant under $\Gamma = 1$ and becomes even more significant for increasing values of Γ if we have underestimated the true treatment effect. However, looking at the bounds under the assumption that we have overestimated the treatment effect, i.e., Q_{MH}^+ , reveals that already at relatively small levels of Γ , the result becomes insignificant. To be more specific: with a value of $\Gamma = 1.1$ the result would no longer be significant at the 5% significance level; with $\Gamma = 1.15$ it is not even significant at the 10% significance level, since the p -value is 0.109961. From these findings, one must interpret the results carefully.

However, these are worst-case scenarios. Hence, a critical value of $\Gamma = 1.15$ does not mean that unobserved heterogeneity exists and that there is no effect of treatment on the outcome variable. This result states only that the confidence interval for the effect would include zero if an unobserved variable caused the odds ratio of treatment assignment to differ between the treatment and comparison groups by 1.15. This test cannot directly justify the unconfoundedness assumption. Hence, we cannot state whether the conditional independence assumption does (not) hold for the given setting (including among others the used data, the chosen covariates, and the specification of the propensity score). However, the results are sensitive to possible deviations from the identifying unconfoundedness assumption, and hence we advise some caution when interpreting the results.

5 Saved results

`mhbounds` produces the matrix `outmat` containing the MH test statistics for all values of Γ specified by the user. When the option `stratamat` is specified in conjunction with `stratum(newvar)`, `mhbounds` keeps in memory not only the matrix `outmat` containing the overall/combined test statistics but also the matrices `outmat_j` containing the strata-specific test statistics, $j = 1, \dots, \#strata$.

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7 References

- Aakvik, A. 2001. Bounding a matching estimator: The case of a Norwegian training program. *Oxford Bulletin of Economics and Statistics* 63: 115–143.
- Abadie, A., D. Drukker, J. Leber Herr, and G. W. Imbens. 2004. Implementing matching estimators for average treatment effects in Stata. *Stata Journal* 4: 290–311.

- Becker, S. O., and A. Ichino. 2002. Estimation of average treatment effects based on propensity scores. *Stata Journal* 2: 358–377.
- Boston Collaborative Drug Surveillance Program. 1972. Excess of ampicillin rashes associated with allopurinol or hyperuricemia. *New England Journal of Medicine* 286: 505–507.
- Caliendo, M., R. Hujer, and S. Thomsen. 2005. The employment effects of job creation schemes in Germany. IZA Discussion Paper No. 1512. Bonn, Germany.
- Caliendo, M., and S. Kopeinig. 2005. Some practical guidance for the implementation of propensity score matching. IZA Discussion Paper No. 1588. Bonn, Germany. <http://ftp.iza.org/dp1588.pdf>.
- Dehejia, R. H., and S. Wahba. 1999. Causal effects in nonexperimental studies: Re-evaluation of the evaluation of training programs. *Journal of the American Statistical Association* 94: 1053–1062.
- DiPrete, T., and M. Gangl. 2004. Assessing bias in the estimation of causal effects: Rosenbaum bounds on matching estimators and instrumental variables estimation with imperfect instruments. *Sociological Methodology* 34: 271–310.
- Heckman, J., H. Ichimura, J. Smith, and P. Todd. 1998. Characterizing selection bias using experimental data. *Econometrica* 66: 1017–1098.
- Ichino, A., F. Mealli, and T. Nannicini. 2006. From temporary help jobs to permanent employment: What can we learn from matching estimators and their sensitivity. IZA Discussion Paper No. 2149. Bonn, Germany.
- Imbens, G. 2004. Nonparametric estimation of average treatment effects under exogeneity: A review. *Review of Economics and Statistics* 86: 4–29.
- Lalonde, R. J. 1986. Evaluating the econometric evaluations of training programs. *American Economic Review* 76: 604–620.
- Leuven, E., and B. Sianesi. 2003. PSMATCH2: Stata module to perform full Mahalanobis and propensity score matching, common support graphing, and covariate imbalance testing. Boston College Department of Economics, Statistical Software Components. Downloadable from <http://ideas.repec.org/c/boc/bocode/s432001.html>.
- Manski, C. 1990. Nonparametric bounds on treatment effects. *American Economic Review* 80: 319–323.
- . 1995. *Identification Problems in the Social Sciences*. Cambridge, MA: Harvard University Press.
- Mantel, N., and W. Haenszel. 1959. Statistical aspects of the analysis of data from retrospective studies. *Journal of the National Cancer Institute* 22: 719–748.

Nannicini, T. 2006. sensatt: A simulation-based sensitivity analysis for matching estimators. Boston College Department of Economics, Statistical Software Components. Downloadable from <http://ideas.repec.org/c/boc/bocode/s456747.html>.

Rosenbaum, P. R. 2002. *Observational Studies*. 2nd ed. New York: Springer.

Smith, J., and P. Todd. 2005. Does matching overcome Lalonde's critique of nonexperimental estimators? *Journal of Econometrics* 125: 305–353.

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Revised and improved versions of the programs may become available in the future on our web pages (<http://www.sobecker.de> and <http://www.caliendo.de>).