The Determination of National Retail and Wholesale Prices of Infant Formula

J. William Levedahl and Albert Reed

Economic Research Service
1800 M St. NW
Washington, DC 20036

Contact: Levedahl@ers.usda.gov

Selected Paper prepared for presentation at the American Agricultural Economics Association Annual Meeting, Providence, Rhode Island, July 24-27, 2005

Lead authorship is not assigned. Views herein are those of the authors and do not necessarily represent those of UDSA or ERS.

The Determination of National Retail and Wholesale Prices of Infant Formula

Introduction

Since 1989 State agencies have entered into cost containment contracts with manufacturers that provide infant formula used in the Women, Infant, and Children Program (*WIC).* In exchange for being selected as the sole source provider for a local market, the manufacturer receives a contract-specified *net price* for each unit of formula that is ultimately allocated to *WIC* participants at retail. The net price equals a national wholesale price minus a rebate. *WIC* agencies redeem vouchers issued to participants at the retail price and rebate funds accrue to the local *WIC* agency to supplement the cost of reimbursing retailers for accepting vouchers instead of cash from *WIC* participants.

For a given allocation of supply to *WIC*, rising *retail* infant formula prices mean higher *WIC* agency costs. However, because the net price remains constant over the life of the contract, increases in the national *wholesale* price of infant formula result in greater rebates and increased revenues for *WIC* agencies. The implication is that market and policy variables that affect infant formula wholesale and retail prices affect both the revenues and costs of the *WIC* agencies and their ability to provide infant formula to needy households. This paper represents an empirically refutable market analysis of retail and wholesale infant formula pricing.

In the framework developed here market and policy variables affect infant formula prices at both levels of the market. A key market variable is the demand for infant formula by paying customers and shifts in market demand by *WIC* participants. A key policy variable is rebates. Our analysis represents an attempt to estimate the joint impact that such variables have on *national* retail and wholesale infant formula prices. A key parameter is this analysis is the own-price elasticity of derived demand for manufacturing level infant formula. Following the formulation by Wohlgenant and Haidacher, we show that this parameter serves to transmit changes in retail costs, final demand, and *WIC* participation to wholesale infant formula prices, and it is this parameter that transmits changes in manufacturing supply to retail infant formula prices. While our market framework is national in scope, it recognizes that *WIC* interfaces with national infant formula markets at the local level by driving a wedge between the retailers' demand price for manufacturing level infant formula and the manufacturer's supply price.

Wholesale Infant Formula Prices

Consider a local market defined by the geographical area for which there is assigned a sole-source provider, a fixed net price and a rebate. In this market there are a small number of manufacturing firms that sell their brands nationally to a large number of different retail firms at nationally determined wholesale prices. One of these firms is designated the sole-source provider so that in a particular local market only its brand is allocated to *WIC* participants, while paying customers are free to choose any brand. For each *WIC* participant transaction at retail, the retailer receives the reported retail price from *WIC* and the manufacturer pays *WIC* a rebate so as to receive the contract specified net price. Levedahl and Reed construct a framework for the national infant formula market based on the structure that two manufacturing firms sell their two homogeneous but different brands at two national wholesale prices in local markets.

Limitations in obtaining firm-specific data forces us, in this study, to reduce the structure of the cited work to one with a single manufacturing firm that sells its homogeneous infant formula product to many different retailers located in $k = 1,\dots,n$ spatially dispersed local markets. For every unit of output sold the manufacturer receives the national wholesale price P_m from retailers. Because there is a single national firm, it is necessary to assume that it is the sole-source provider for *WIC* infant formula in *every* local market. This means it pays a different local-market-specific rebate r_k for each unit allocated participants. This rebate satisfies the contract's fixed net price (n_k) equation $n_k = P_m - r_k = (1-\rho_k)P_m$, where $\rho_k \equiv \frac{r_k}{P_m}$ is the local rebate-to-national price ratio.

These provisions are incorporated into the manufacturing firm's revenues as follows. If ω_k denotes the number of units of formula allocated to *WIC* participants and x_k denotes the number of units of infant formula sold to paying customers so that $s_k = \omega_k + x_k$ denotes firm's supply in local market *k*, the manufacturer's revenues in that market is

(1.1) $$R_k(s_k; \omega_k, r_k, P_m) = s_k P_m - r_k \omega_k$$

or

(1.2) $$R_k(\omega_k, x_k, \rho_k, P_m) = \omega_k(1-\rho_k)P_m + x_k P_m.$$

Because the manufacturer's output product is assumed to be homogeneous, dividing (1.2) by s_k yields the supply price

(1.3) $$p_{bk} \equiv \frac{R_k}{\omega_k + x_k} = (1-\lambda_k \rho_k)P_m$$

where $\lambda_k = \frac{\omega_k}{s_k}$ is the fraction of local supply allocated to *WIC* participants in market *k* and $(1-\lambda_k\rho_k)$ is the ratio of the firm's local supply price p_{bk} to the national wholesale price. This means that (1.2) can be re-expressed as

(1.4) $$R_k(s_k; \lambda_k, \rho_k, P_m) = (1-\lambda_k \rho_k)P_m s_k \equiv p_{bk} s_k.$$

Equation (1.4) is a convenient way to express local market revenues of the manufacturing firm because it is based on the local supply price and incorporates information on the fraction of supply allocated to *WIC* and the rebates of cost containment contracts. One of the goals of this study is to estimate the impact of *WIC* rebates on infant formula prices, and to do this we treat λ_k as a parameter and the rebate ratio ρ_k , as an exogenous variable in the local supply decisions of the national manufacturing firm.

It is important to note, however, that in a model of a single supplier that is a monopoly in every local market, we may be able to solve for an optimal vector of spatially separated rebates. In particular, consider the possibility that the manufacturer is a price-discriminating monopolist so that it faces each of the local derived demands functions for manufacturing level infant formula. Let $C\left(\sum_k s_k\right)$ denote its total national cost so that marginal costs are independent of location. Then by (1.4) its national profits are $\pi = \sum_k (1-\lambda_k \rho_k) P_m s_k - C\left(\sum_k s_k\right)$ and the optimizing conditions are

$$(1-\lambda_k \rho_k) P_m \left[1 + \frac{1}{E_{kk}}\right] = C' \qquad (k = 1,\ldots,n) \tag{1.5}$$

where $E_{kk} = \frac{\partial P_m}{\partial s_k}\frac{s_k}{P_m} < 0$ denotes the *k*th market's own-price elasticity of derived demand for manufacturing infant formula. Equation (1.3) implies these conditions reduce to

$$\frac{p_{bk}}{p_{bl}} = \frac{(1-\lambda_k \rho_k)}{(1-\lambda_l \rho_l)} = \left(\frac{1+\frac{1}{E_{ll}}}{1+\frac{1}{E_{kk}}}\right). \tag{1.6}$$

Equation (1.6) states that the supply price is smaller in the more price elastic market. In our application, this result means that $\lambda\rho$ is smaller in the more elastic market, which in turn implies that if the fraction of supply allocated to *WIC* is approximately constant across markets so that $\lambda_k \approx \lambda_l$, then the rebate-to-price ratio ρ and the rebate r would be smaller in the market with the more elastic derived demand.

Since infant formula markets are characterized by only a few manufacturing firms, the price discrimination model may be relevant for infant formula price analysis. Moreover, the solution is interesting to us for two reasons. First, it suggests one approach to solving for an optimal rebate *surface* that is defined across local markets. Second, (1.6) touches on a recurring theme of this paper: the shape of the derived demand for manufacturing level infant formula plays a central role in the determination of infant formula prices.

Finally, as a preamble to the development of the national model we derive the national analogues to some of the local variables developed above. From (1.2) national revenues are

$$R = \sum_{k=1}^{n} \omega_k (1-\rho_k) P_m + \sum_{k=1}^{n} x_k P_m = \omega (1-\rho) P_m + x P_m \tag{1.7}$$

where $x = \sum_{k=1}^{n} x_k$ is the portion of national supply allocated to paying customers, $\omega = \sum_{k=1}^{n} \omega_k$ is the portion allocated to *WIC* participants, and

$$\rho = \sum_{k=1}^{n} \left(\frac{\omega_k}{\omega} \right) \rho_k \tag{1.8}$$

is the national average rebate ratio. Given homogeneous national manufacturing output, the national supply price is

$$P_b \equiv \frac{R}{\omega + x} = (1-\lambda\rho) P_m \tag{1.9}$$

where $\lambda \equiv \dfrac{\omega}{\omega + x}$. Just as with the local level analysis, the ‘wedge’ between national supply and demand prices is $(1-\lambda\rho)$.

A general concern in infant formula markets is that by restricting supply, a small number of national firms would distort national infant formula prices. One might test this in the present framework by applying a monopoly model to the national infant formula manufacturing sector. In particular, if $C(S_m)$ denotes the monopolist's cost function where S_m is national infant formula supply, the optimization problem is

$$\pi = \max_{S} \left[(1-\lambda\rho)P_m S_m - C(S_m)\right]$$

with first-order conditions

$$P_m = \frac{C'}{\left[(1-\lambda\rho)\left(1+\frac{1}{E_{mm}}\right)\right]}$$

where $E_{mm} = \frac{\partial S_m}{\partial P_m}\frac{P_m}{S_m} < 0$ is the own price elasticity of derived demand for manufacturing level infant formula. Given $|E_{mm}| > 1$ and for a given value of λ and a given marginal cost, higher rebates unambiguously lead to higher national manufacturing prices.

The Determination of National Infant Formula Prices

The discussion in the last section is intended to serve as a building block to a more complete theory of price determination in infant formula markets given in this section. In the previous section the own-price elasticity of derived demand for infant formula at the manufacturing level is taken as an independent parameter. This section develop a more complete theory by showing that at the national level in which different retail firms compete for the manufacturer's supply, this parameter depends on the distribution of retailers' costs, and on the allocation of retail supply to *WIC* participants and to paying customers. For example, at the level of the national market our theory predicts how the shape of the derived demand function changes if the growth of relatively high cost retail outlets such as *WIC*-only stores were to continue, if the behavior of paying customers changes, and if *WIC* participation continues to grow. Furthermore, a version of our theory suggests this price elasticity plays a central role in measuring the response of retail and wholesale prices to exogenous changes that originate in either the retail or wholesale level of the market.

At the outset it is important to note the main difference in the way we conceptualize infant formula supply at retail and at the manufacturing level of the market. In particular, at the manufacturing level one firm produces a *homogenous* product and at retail different types of retail outlets (e.g., convenience stores, grocery stores, superstores, *WIC* only stores) with different cost structures sell a *heterogeneous* set of final infant formula products which we treat as a single composite commodity. However artificial this may seem, we find it to be a convenient setup for the analysis of retail and wholesale infant formula price determination.

We begin with the optimization problem of the *i*th retail firm. This firm sells q_{ri} units of a composite infant formula product by combining s_{mi} units of manufacturing level infant formula with a single non-infant formula factor X_i according to a firm-specific production function f_i. We think of X_i as a single composite comprised of a number of non-infant formula set of inputs (energy, shelf space, wages). Every retail firm pays P_m to the manufacturer for each unit of the manufacturing-level infant formula input purchased, W for every unit of X_i purchased, and it receives the (average) retail price P_r for every unit of the composite output sold. As is consistent with *WIC* provisions the retail firm receives this price either from the *WIC* agency or the paying customers. If the firm takes these prices as given, then the *i*th firm-specific profit function is defined as

$$\pi_i(P_r, P_m, W) = \max_{s_{mi}, X_i} \left[P_r f_i(s_{mi}, X_i) - P_m s_{mi} - W X_i \right].$$

By Hotelling's lemma, this firm's supply is

(2.1) $$\frac{\partial \pi_i(P_r, P_m, W)}{\partial P_r} = s_{ri} = S_{ir}\left(P_r, P_m, W\right)$$

and its demand for manufacturing-level formula is

(2.2) $$-\frac{\partial \pi_i(P_r, P_m, \mathbf{W})}{\partial P_m} = s_{mi} = D_{im}(P_r, P_m, W).$$

Furthermore, because the firm's Hessian matrix is positive semi-definite and symmetric we have $\frac{\partial S_{ir}}{\partial P_r} > 0$, $\frac{\partial D_{im}}{\partial P_m} < 0$, and the symmetry condition

(2.3) $$\frac{\partial S_{ir}}{\partial P_m} = -\frac{\partial D_{im}}{\partial P_r} \; .$$

These conditions serve as the building blocks for the two market-clearing condtions. At the level of the market, the collection of all such retail firms face the market-level consumer demand for the composite price plus an exogenous allocation of retail products θ to nonpaying *WIC* participants. Associated with paying customers is the economy-wide consumer demand function

(2.4) $$D_r(P_r, Z)$$

where Z is a shift variable associated with this paying customer demand function which implies that *total market* demand is $D_r(P_r, Z) + \theta$. Thus if there are *m* retail outlets selling infant formula, then (2.1) and (2.4) imply market clearing at the retail-level is

(2.5) $$\sum_{i=1}^{m} S_{ir}(P_r, P_m, W) - D_r(P_r, Z) - \theta = 0\,.$$

In this section we assume that national manufacturing-level supply of infant formula is predetermined at S_m so that given (2.2) and for *m* firms, market-clearing at the manufacturing level is

(2.6) $$S_m - \sum_{i=1}^{m} D_{im}(P_r, P_m, W) = 0\,.$$

We relax the assumption of a predetermined manufacturing supply in the next section.

Equations (2.5) and (2.6) summarize the structure of the infant formula market. This pair of equations account for the behavior of infant formula consumers, the *WIC* allocation, market-level manufacturing supply, and *firm-level* supply and *firm-level* derived demand functions of a number of retail firms with different cost structures. Implicit in (2.5) and (2.6) are *market-level* retail and wholesale price *functions* with well-defined partial derivatives that define the way in which infant formula prices change when the market clears.

To show this it is necessary to totally differentiate this pair of conditions and express the result in terms of *firm-level* elasticities. If we denote the proportion of retail supply allocated to *WIC* participants as γ so that $\gamma = \dfrac{\theta}{D_r + \theta}$ then the percent change in the total market demand shifter is

(2.7) $$\left(\frac{dZ}{Z}\right)^{*} = (1-\gamma)\frac{dZ}{Z} + \gamma\frac{d\theta}{\theta}\,.$$

Thus the percent change in the *total market demand* shifter is the percent change in the market demand shifter for paying customers weighted by the demand of paying customers as a proportion of retail supply, plus the percent change in *WIC* participant’s allocation weighted by participant's proportion of retail supply. Then as shown in the Appendix, total differentiation of (2.5) and (2.6) yields

(2.8)
$$\begin{bmatrix} (\xi_{rr} - (1-\gamma)e) & \xi_{rm} \\ -\xi_{mr} & -\xi_{mm} \end{bmatrix} \begin{pmatrix} \frac{dP_r}{P_r} \\ \frac{dP_m}{P_m} \end{pmatrix} = \begin{bmatrix} 1 & -\xi_{rw} & 0 \\ 0 & \xi_{mw} & -1 \end{bmatrix} \begin{pmatrix} \left(\frac{dZ}{Z}\right)^* \\ \frac{dW}{W} \\ \frac{dS_m}{S_m} \end{pmatrix}$$

where $e = \frac{\partial D_r}{\partial P_r}\frac{P_r}{D_r}$ is the own-price elasticity of demand for paying customers, and the ξ parameters are weighted sums of *firm* level supply or derived demand elasticities in which the weight associated with the firm's elasticity is its fraction of market-level retail supply of infant formula or its fraction of market-level demand for manufacturing level infant formula. For example $\xi_{rr} = \sum_{i=1}^{m}\left(\frac{\partial S_{ir}}{\partial P_r}\frac{P_r}{S_{ir}}\right)\left(\frac{S_{ir}}{S_r}\right)$ is the weighted sum of *firms'* own-price elasticities of retail supply and $\xi_{mm} = \sum_{i=1}^{m}\left(\frac{\partial D_{im}}{\partial P_m}\frac{P_m}{D_{im}}\right)\left(\frac{D_{im}}{S_m}\right)$ is the weighted sum of *firms'* own-price elasticities of derived demand for manufacturing level infant formula (see Appendix). Equation (2.8) represents a pair of market-clearing expressions associated with a competitive retail sector of the national infant formula market.

Defining ϕ_m as the fraction of retail cost or consumer dollar allocated to the manufacturing sector, the symmetry condition (2.3) can be expressed as

(2.9)
$$\xi_{rm} = -\phi_m \xi_{mr} .$$

Wohlgenant and Haidacher show that (2.9) fails to hold when firms exercise market power.

Inverting (2.8) gives

(2.10)
$$\begin{pmatrix} \frac{dP_r}{P_r} \\ \frac{dP_m}{P_m} \end{pmatrix} = \begin{bmatrix} A_{rz} & A_{rw} & A_{rm} \\ A_{mz} & A_{mw} & A_{mm} \end{bmatrix} \begin{pmatrix} \left(\frac{dZ}{Z}\right)^* \\ \frac{dW}{W} \\ \frac{dS_m}{S_m} \end{pmatrix}$$

where

(2.11a) $A_{rz} = \frac{-\xi_{mm}}{D} > 0$

(2.11b) $A_{rw} = \frac{\xi_{rw}\xi_{mm} - \xi_{mw}\xi_{rm}}{D}$

(2.11c) $A_{rm} = \frac{\xi_{rm}}{D} < 0$

(2.11d) $A_{mz} = \frac{\xi_{mr}}{D} > 0$

(2.11e) $A_{mw} = \frac{\xi_{mw}\left(\xi_{rr} - (1-\gamma)e\right) - \xi_{rw}\xi_{mr}}{D}$

(2.11f) $A_{mm} = \frac{-\left(\xi_{rr} - (1-\gamma)e\right)}{D} < 0$

(2.11g) $D = \xi_{rm}\xi_{mr} - \left(\xi_{rr} - (1-\gamma)e\right)\xi_{mm} > 0$

Because (2.10) is expressed in terms of percent changes, the $A_{r.}$ and $A_{m.}$ coefficients are *market-level* price flexibilities. Following Heiner, and Wohlgenant and Haidacher equations (2.10) represent valid market-level inverse retail supply and inverse derived demand functions, with the $A_{r.}$ and $A_{m.}$ coefficients representing market-level price flexibilities. Of particular importance is the A_{mm} coefficient because $E_{mm} = A_{mm}^{-1}$ where E_{mm} equals the market-level derived demand elasticity for manufacturing level infant formula.

Equations (2.11a) – (2.11g) suggest that the market-level flexibilities differ fundamentally from the weighted sums of *firm*-level elasticities (i.e., the ξ parameters) that appear in the market-clearing conditions of (2.8). The ξ parameters are weighted-sums of firms' elasticities, where the response holds manufacturing and retail infant formula prices fixed. It follows however, that when all firms respond to any change the market-level retail supply and derived demand functions move along the consumer demand and the manufacturing supply and infant formula prices at both levels of the market change. The price flexibilities in (2.10) incorporate the sum of firms' responses to these additional price changes.

Wohlgenant and Haidacher show there are three sets of restrictions takes apply to the price flexibilities of (2.10). First, they show that $A_{rz} > 0$, $A_{rm} < 0$, $A_{mz} > 0$, $A_{mm} < 0$.[1] Second, they show that equations (2.11c) and (2.11d) can be used to restate the symmetry condition (2.9) as

(2.12) $$A_{rm} = -\phi_m A_{mz}$$

Third, they show the constant returns restrictions are

(2.13) $$A_{rz} = -A_{rm}$$

(2.14) $$A_{mz} = -A_{mm}.$$

The constant returns restrictions follow from the argument that if retail firms face no barriers to entry, an industry-level cost function would be approximated by a constant returns industry level cost function (Diewert). Equations (2.13) and (2.14) derive from the fact that under constant returns to scale the percentage change in market-level manufacturing input demand equals the percentage change in market level retail supply.

The theory developed by Wohlgenant and Haidacher allow us to re-parameterize the market model in a very useful way. To achieve this use equations (2.12) to (2.14) to express

[1] To repeat their arguments, since A_{mm} is the inverse of the own-price elasticity of derived demand for manufacturing level infant formula $A_{mm} < 0$, so that because $(1-\gamma)e < 0$ and $\xi_{rr} > 0$, by (2.11f) $D > 0$. Also since $\xi_{mm} < 0$, $A_{rz} > 0$. Finally because we expect manufactured-level infant formula to be a normal factor of retail firms we expect, $\xi_{rm} < 0$ so that by the symmetry condition (i.e., (2.9)) $\xi_{mr} > 0$, and by (2.11d), $A_{mz} > 0$. The signs of A_{rw} and A_{mw} are ambiguous.

A_{rz}, A_{rm} and A_{mz} in terms of A_{mm}, and note that the own-price elasticity of derived demand is $E_{mm} = 1/A_{mm}$. This implies (2.10) can be represented as

$$\text{(2.15)} \qquad \begin{pmatrix} \dfrac{dP_r}{P_r} \\ \dfrac{dP_m}{P_m} \end{pmatrix} = \begin{bmatrix} -\phi_m (E_{mm})^{-1} & A_{rw} & \phi_m (E_{mm})^{-1} \\ -(E_{mm})^{-1} & A_{mw} & (E_{mm})^{-1} \end{bmatrix} \begin{pmatrix} \left(\dfrac{dZ}{Z}\right)^* \\ \dfrac{dW}{W} \\ \dfrac{dS_m}{S_m} \end{pmatrix}.$$

Next, Wohlgenant and Haidacher show that the market level elasticity of derived demand, E_{mm}, is related to the output constant derived demand elasticity

$$\text{(2.16)} \qquad E_{mm}^c = \xi_{mm} - \xi_{rm}\xi_{mr} / \xi_{rr}$$

the elasticity of price transmission $n = A_{rm} / A_{mm}$, and the own-price elasticity of consumer demand, $e^* = (1-\gamma)e$ according as

$$\text{(2.17)} \qquad E_{mm} = E_{mm}^c + (\xi_{mr} / \xi_{rr}) n e^* .$$

Then because our model is the two-factor case, multiplying and dividing E_{mm}^c by $(1-\phi_m)$ gives

$(1-\phi_m)\dfrac{E_{mm}^c}{1-\phi_m} = (1-\phi_m)\sigma$ where σ is the elasticity of substitution between the manufacturing infant formula factor and the other non-infant formula composite factor. Furthermore (2.12) to (2.14), the elasticity of price transmission n equals the manufacturing share so that $n = \phi_m$. Also given constant returns to scale $\xi_{mr} \to \infty$ and $\xi_{rr} \to \infty$ which implies $(\xi_{mr} / \xi_{rr}) \to 1$. Thus for the two factor case we have

$$\text{(2.18)} \qquad E_{mm} = (1-\phi_m)\sigma + \phi_m e^*$$

which is the familiar Marshall-Hicks rule for the two input case. Equation (2.18) allows us to express the market-level derived demand elasticity, E_{mm} in terms of the market level consumer demand elasticity and the elasticity of substitution.

Equations (2.15) to (2.18) can be used to shed light on current issues in infant formula markets. For example, it can be used to explain why the growth of *WIC* participation affects the prices faced by paying customers. Note that as the proportion of retail supply allocated to *WIC* rises, γ rises and e^* becomes smaller in magnitude so that the total market demand schedule becomes less price elastic. Then by (2.18) the derived demand function becomes less price elastic. Equation (2.15) implies that a less price elastic derived demand schedule means that any expansion in the demand for infant formula at retail (i.e., $(dZ/Z)^* > 0)$ means sharper increases in retail (and wholesale) prices of infant formula facing paying customers.

Another current concern is the growth of *WIC*-only stores. Because such stores provide extra services to *WIC* participants, they might encourage *WIC* participation. The concern is the cost of these extra services justifies vouchers being redeemed at higher retail prices. These extra costs, however, might imply such stores are less manufacturing-infant-formula intensive than other types of retail outputs in the market. In turn this might mean that the ξ^i_{mm} terms associated with the *WIC* may be smaller in magnitude than the parameters of other stores and associated with the growth of such stores are larger weights. The result is that the industry level ξ_{mm} parameter falls and by (2.16) and (2.17) the magnitudes of E^c_{mm} and E_{mm} falls. This means that associated with the growth of *WIC*-only stores, the market-level derived demand would be less manufacturing price elastic. By (2.15) this means that associated with the growth of *WIC*-only stores, an expanding retail demand for infant formula would be associated with larger increases in retail and wholesale prices of infant formula.

The Supply of Infant Formula at the Manufacturing Level

In the previous section we derived a market model of retail and wholesale infant formula prices based on a predetermined level of manufacturing supply. Based on the earlier development this necessarily entailed ignoring the role exogenous rebates may play in the determination of infant formula prices at the national level. In this section we relax the restriction of a predetermined national supply by recognizing that rebates operate at the local level. The development is based on Lewbel's general theory of aggregation, and this theory depends on a stochastic independence condition that is likely to hold because of the way in which national and local blend prices depend on the same national wholesale price of infant formula.

To begin recall from (1.3) and (1.9) above that the local-to-national blend price ratio can is $d_k = \frac{p_{bk}}{P_b} = \frac{(1-\lambda_k \rho_k)}{(1-\lambda\rho)}$, so that d_k does not depend on P_b. This suggests that d_k may be distributed independently of P_b Define $\delta_k \equiv \log(d_k) = \log(p_{bk}) - \log(P_b) \equiv \tilde{p}_{bk} - \tilde{P}_b$, $\boldsymbol{\delta}$ as the *n*-vector of (log) relative prices, and suppose $\boldsymbol{\delta}$ is *distributed* independently of $\tilde{P}_b$. Next, recall that $s_k = \omega_k + x_k$ denotes local supply of manufacturing infant formula in the *k*th local market. Let $\tilde{v}$ denote the log of one or more prices of factors used in the production of manufacturing level infant formula (i.e., casein) and define $g_k : \tilde{p}_{bk} \times \tilde{v} \to q_k$ as the *k*th local output supply function that satisfies

$$s_k = g_k(\tilde{p}_{bk}, \tilde{v}) + \varepsilon_k \qquad (3.1)$$

So that if E is the mathematical expectations operator, ε_{1k} satisifies $E(\varepsilon_k \mid \tilde{p}_{bk}, \tilde{v}) = 0$. Thus in addition to g_k representing a valid local supply function, based on information on $(\tilde{p}_{bk}, \tilde{v})$,

$g_k(\tilde{p}_{bk}, \tilde{v})$ is the best unbiased predictor of local supply. Now since a unit of manufactured infant formula is homogeneous across all markets $S_m \equiv \sum_k s_k$ denotes the national supply of manufacturing level infant formula. Then consider the regression equation

$$S_m = G(\tilde{P}_b, \tilde{v}) + u \tag{3.2}$$

where it is assumed that u satisfies $E(u \mid \tilde{P}_b, \tilde{v}) = 0$ so that based on information about national blend prices and factor prices $(i.e., \tilde{P}_b, \tilde{v})$, $G(\tilde{P}_b, \tilde{v})$ is the best unbiased predictor of national manufacturing-level infant formula supply, S_m.

These assumptions imply that $G(\tilde{P}_b, \tilde{v})$ is a valid aggregate, infant formula manufacturing-level supply function. In particular, it is shown in the Appendix that if $\boldsymbol{\delta}$ is distributed independently of $\tilde{P}_b$, G is a weighted average of the sum the *n*-local supply functions g_{1k} with weights associated with the distribution of the log of relative local blend prices. In particular, it is shown that G is zero degree homogeneous in P_{1b} and v, and that the price elasticity of aggregate supply is

$$\eta \equiv \frac{\partial \log G}{\partial \log P_b} = E\left(\sum_k \left(\frac{s_k}{S_m} \right) \eta_k \mid \tilde{P}_b, \tilde{v} \right). \tag{3.3}$$

Equation (3.3) means that the national price elasticity of supply for manufactured infant formula is a best unbiased estimate of the weighted sum of the local market supply elasticities with weights equal to local market fractions of economy-wide supply.

The establishment of an economy-wide manufacturing supply schedule allows us to incorporate rebates into the market clearing model developed in the last section. We first note from (3.2) that

$$\frac{dS_m}{S_m} = \eta \frac{dP_b}{P_b} \; + \; \eta_v \frac{dv}{v}$$

where for example $\eta = \frac{\partial G}{\partial \tilde{P}_b} \frac{1}{S_m}$. Next from (1.9) we have

$$\frac{dP_b}{P_b} = \frac{dP_m}{P_m} - \left(\frac{\lambda \rho}{1 - \lambda \rho} \right) \frac{d\rho}{\rho}$$

and from (1.8) we have $\rho = \frac{r}{P_m}$ where $r \equiv \sum_{k=1}^{n} \left(\frac{\omega_k}{\omega} \right) r_k$. This means

$$\frac{d\rho}{\rho} = \frac{dr}{r} - \frac{dP_m}{P_m} \; .$$

Together these expressions imply

(3.4)
$$\frac{dS_m}{S_m} = \eta \left(1 + \frac{\lambda \rho}{1 - \lambda \rho} \right) \frac{dP_m}{P_m} - \eta \left(\frac{\lambda \rho}{1 - \lambda \rho} \right) \frac{dr}{r} + \eta_v \frac{dv}{v}$$

Now by substituting (3.4) into (2.15) where we evaluate the parameter $\psi = \frac{\lambda \rho_o}{1 - \lambda \rho_o}$ at some point $\rho = \rho_o$ we have

(3.5)
$$\begin{pmatrix} \frac{dP_r}{P_r} \\ \frac{dP_m}{P_m} \end{pmatrix} = F^{-1} \begin{bmatrix} -\phi_m (E_{mm})^{-1} & A_{rw} & \phi_m (E_{mm})^{-1} \eta \psi & \eta_v \\ -(E_{mm})^{-1} & A_{mw} & (E_{mm})^{-1} \eta \psi & \eta_v \end{bmatrix} \begin{pmatrix} \left(\frac{dZ}{Z} \right)^* \\ \frac{dW}{W} \\ \frac{dr}{r} \\ \frac{dv}{v} \end{pmatrix}$$

where

(3.6) $$F^{-1} = \begin{pmatrix} 1 & -\phi_m \dfrac{(E_{mm})^{-1}\eta(1+\psi)}{1-(E_{mm})^{-1}\eta(1+\psi)} \\ 0 & \dfrac{1}{1-(E_{mm})^{-1}\eta(1+\psi)} \end{pmatrix}$$

.

Equations (3.5) and (3.6) implies

(3.7) $$\begin{pmatrix} \dfrac{\partial \log P_r}{\partial \log r} \\ \dfrac{\partial \log P_m}{\partial \log r} \end{pmatrix} = \begin{bmatrix} \phi_m (E_{mm})^{-1}\eta\psi \dfrac{1-2(E_{mm})^{-1}\eta(1+\psi)}{1-(E_{mm})^{-1}\eta(1+\psi)} \\ (E_{mm})^{-1}\eta\psi \quad \dfrac{1}{1-(E_{mm})^{-1}\eta(1+\psi)} \end{bmatrix}$$

Given that $\psi, \eta, \phi_m > 0$ and that $E_{mm} < 0$, (3.7) indicates that both the retail and wholesale price flexibilities with respect to rebates are negative. The idea is that rebates lead to a contraction of the national manufacturing supply function, and this leads to higher wholesale and retail infant formula prices. Hence we have the result that paying customers pay for higher rebates in the form of higher retail prices.

Empirical Implications

The goal of empirical work in this study is to evaluate as many retail and wholesale infant formula price flexibilities as possible. The information required to do so can be summarized with the following fundamental relationships from the above discussion

(4.1) $$E_{mm} = (1-\phi_m)\sigma + \phi_m e^*$$

(4.2) $$\psi = \frac{\lambda\rho_o}{1-\lambda\rho_o}$$

$$(4.3) \qquad \begin{pmatrix} 1 & -\phi_m \dfrac{(E_{mm})^{-1}\eta(1+\psi)}{1-(E_{mm})^{-1}\eta(1+\psi)} \\ 0 & \dfrac{1}{1-(E_{mm})^{-1}\eta(1+\psi)} \end{pmatrix} \begin{bmatrix} -\phi_m(E_{mm})^{-1} & A_{rw} & \phi_m(E_{mm})^{-1}\eta\psi & \eta_v \\ -(E_{mm})^{-1} & A_{mw} & (E_{mm})^{-1}\eta\psi & \eta_v \end{bmatrix}$$

The idea is to evaluate as many of the elements of the matrices of (4.3) with information summarized in (4.1) and (4.2).

Equation (4.1) is the Marshall-Hicks rule (see 2.18) that can be used to link estimates of the own-price elasticity of paying customer demand and the proportion of retail supply allocated to *WIC* (i.e., γ), and an estimates of the elasticity of substitution, σ to the own-price elasticity of derived demand, E_{mm}^{-1} for manufacturing level infant formula. Then at a given point (ψ, ϕ) and estimates of manufacturing supply elasticities, η and η_v, one can evaluate all of the price flexibilities of (4.3) except A_{rw} and A_{mw}.

References

Heiner, R.A. “Theory of the Firm in ‘Short Run’ Industry Equilibrium.” *The American Economic Review* 82(1996):555-62.

Levedahl, W.J. and A.J. Reed. “The Determination of National Retail and Wholesale Prices of Infant Formula”. Draft manuscript, February 2005.

Lewbel, A. “Aggregation without Separability: A Generalized Composite Commodity Theorem.” *The American Economic Review* 86(1996):524-43.

Wohlgenant, M.K. and R.C.Haidacher. *Retail to Farm Linkage for a Complete Demand System of Food Commodities.* U.S. Department of Agriculture, Econ. Res. Serv., TB-1774, Dec. 1989.

Appendix

In this appendix we derive the market clearing conditions at retail and the manufacturing level of the market and the national manufacturing supply function for infant formula.

1. Market Clearing Conditions. In this section we follow Wohlgenant and Haidacher in the percentage change form of the market clearing conditions at two levels of the market. We repeat the derivation here so that we can reveal the source of some of the key parameters of the model.

The first equation of (2.8) obtains by totally differentiating the market clearing condition at retail given by (2.5), and dividing the result by national supply, S_r where $S_r \equiv D_r + \theta$. Then by defining $\gamma \equiv \frac{\theta}{S_r}$ as the proportion of retail infant formula allocated to *WIC*, the total differentiation of (2.5) is

$$\left[\left(\sum_{i=1}^{m}\frac{\partial S_{ir}}{\partial P_r}\frac{P_r}{S_{ir}}\frac{S_{ir}}{S_r}\right)-\left(\frac{\partial D_r}{\partial P_r}\frac{P_r}{D_r}(1-\gamma)\right)\right]\left(\frac{dP_r}{P_r}\right)+\sum_{i=1}^{m}\frac{\partial S_{ir}}{\partial P_m}\frac{P_m}{S_{ir}}\frac{S_{ir}}{S_r}\left(\frac{dP_m}{P_m}\right)$$

$$=\left[\frac{\partial D_r}{\partial Z}\frac{Z}{D_r}(1-\gamma)\right]\left(\frac{dZ}{Z}\right)+\gamma\left(\frac{d\theta}{\theta}\right)-\sum_{i=1}^{m}\frac{\partial S_{ir}}{\partial W}\frac{W}{S_{ir}}\frac{S_{ir}}{S_r}\left(\frac{dW}{W}\right)$$

or,

$$\left(\xi_{rr}-(1-\gamma)e\right)\left(\frac{dP_r}{P_r}\right)+\xi_{rm}\left(\frac{dP_m}{P_m}\right)=(1-\gamma)e_z\left(\frac{dZ}{Z}\right)+\gamma\left(\frac{d\theta}{\theta}\right)-\xi_{rw}\left(\frac{dW}{W}\right)=\left(\frac{dZ}{Z}\right)^{*}-\xi_{rw}\left(\frac{dW}{W}\right)$$

where the second equality derives from (2.7) in the text. Totally differentiating (2.6) and dividing the result by national derived demand S_m gives

$$-\sum_{i=1}^{m}\left(\frac{\partial D_{iw}}{\partial P_r}\frac{P_r}{D_{iw}}\frac{D_{iw}}{D_w}\right)\left(\frac{dP_r}{P_r}\right)-\sum_{i=1}^{m}\left(\frac{\partial D_{iw}}{\partial P_m}\frac{P_m}{D_{iw}}\frac{D_{iw}}{D_w}\right)\left(\frac{dP_m}{P_m}\right)=$$

$$\sum_{i=1}^{m}\left(\frac{\partial D_{iw}}{\partial W}\frac{W}{D_{iw}}\frac{D_{iw}}{D_w}\right)\left(\frac{dW}{W}\right)-\frac{dS_m}{S_m}$$

or

$$-\xi_{mr}\frac{dP_r}{P_r}-\xi_{mm}\frac{dP_m}{P_m}=\xi_{mw}\left(\frac{dW}{W}\right)-\frac{dS_m}{S_m}$$

which is the result in the text.

2. National Supply of Manufacturing Level Infant Formula. The derivation of a national supply function follows many of the arguments of Lewbel's Generalized Composite Commodity Theorem. Recall from (3.1) that $s_k = g_k(\tilde{p}_{bk},\tilde{v})+\varepsilon_k$ where ε_k satisfies $E(\varepsilon_k \mid \tilde{p}_{bk},\tilde{v})=0$, '~' denotes 'log' so $\tilde{p}_{bk}=\log(p_{bk})$, and $\tilde{v}=\log(v)$. Each g_k is homogeneous of degree zero in p_{bk} and v. From (3.2) $S_m = G(\tilde{P}_b,\tilde{v})+u$ where $S_m \equiv \sum_k s_k$, and where u satisfies $E(u \mid \tilde{P}_b,\tilde{v})=0$. Denote $\delta_k=\log(p_{bk})-\log(P_b)\equiv \tilde{p}_{bk}-\tilde{P}_b$ and define the n-vector $\boldsymbol{\delta}$ as the n-vector of these log relative prices. Next we assume that $\boldsymbol{\delta}$ is distributed independently of $\tilde{P}_b$. Then we have

$$G(\tilde{P}_b,\tilde{v})=E\left(S_m \mid \tilde{P}_b,\tilde{v}\right)=E\left(\sum_k g_k(\tilde{p}_{bk},\tilde{v}) \mid \tilde{P}_b,\tilde{v}\right)=E\left(\sum_k g_k(\tilde{P}_b+\delta_k,\tilde{v}) \mid \tilde{P}_b,\tilde{v}\right)=$$

$$\int\sum_k g_k(\tilde{P}_b+\delta_k,\tilde{v})dF(\boldsymbol{\delta})$$

which shows that G is a weighted average of the sum the n-local supply functions g_k with weights formed from the distribution of the log of relative local blend prices. Then because

$$G(\tilde{P}_b - tk, \tilde{v} - k) = E\left(\sum_k g_k(\tilde{P}_b - t + \delta_k, \tilde{v} - t) \mid \tilde{P}_b, \tilde{v}\right) = E\left(\sum_k g_k(\tilde{p}_{bk} - t, \tilde{v} - t) \mid \tilde{P}_b, \tilde{v}\right)$$

$$= E\left(\sum_k g_k(\tilde{p}_{bk}, \tilde{v}) \mid \tilde{P}_b, \tilde{v}\right) = G(\tilde{P}_b, \tilde{v})$$

we have the result that G is homogeneous of degree zero in P_b and v. Finally, we have the gradient

$$E\left(\sum_k \frac{\partial g_k(\tilde{p}_{bk}, \tilde{v})}{\partial \tilde{p}_{bk}} \mid \tilde{P}_b, \tilde{v}\right) = E\left(\sum_k \frac{\partial g_k(\tilde{p}_{bk}, \tilde{v})}{\partial\left(\tilde{P}_b + \gamma_k\right)} \mid \tilde{P}_b, \tilde{v}\right) = E\left(\sum_k \frac{\partial g_k(\tilde{p}_{bk}, \tilde{v})}{\partial \tilde{P}_b} \mid \tilde{P}_b, \tilde{v}\right)$$

$$= \frac{\partial E\left(\sum_k g_k(\tilde{p}_{bk}, \tilde{v}) \mid \tilde{P}_b, \tilde{v}\right)}{\partial \tilde{P}_b} = \frac{\partial G(\tilde{P}_b, \tilde{v})}{\partial \tilde{P}_b} = S_m \eta$$

where η is the own price elasticity of national supply. Thus,

$$S_m \eta = E\left(\sum_k \frac{\partial g_k(\tilde{p}_{bk}, \tilde{v})}{\partial \tilde{p}_{bk}} \mid \tilde{P}_b, \tilde{v}\right) = E\left(\sum_k s_k \eta_k \mid \tilde{P}_b, \tilde{v}\right)$$

from which it follows that the aggregate own price elasticity of supply is

$$\eta = E\left(\sum_k \frac{s_k}{S_m} \eta_k \mid \tilde{P}_b, \tilde{v}\right).$$

This shows that the price elasticity of national manufacturing infant formula supply is the best unbiased estimate of weighted sums of local wholesale price elasticities in which the weights are local market fractions of nation-wide supply.