SPECIFICATION OF SUPPLIES AND DEMANDS FOR SUBSECTOR AND SUBMARKET STUDIES*

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Regional supply-adjustment studies are either underway or completed for milk, feed grain-livestock, wheat, rice, cotton, and beef. Other related studies take subsectors as units of analysis for studying markets for such inputs as labor and fertilizer, or for studying regional comparative advantage.

This paper reviews the specification of supply and demand for both kinds of studies and develops the concept of effective subsectoral demands and supplies from the standard Walrasian adjustment behavioral assumptions. These concepts are applied to both the input markets and the product markets. Two commonly used specifications of subsectoral functions: (1) the method of equal elasticity as the aggregate, and (2) the method of equal slope as the aggregate, are found to be incorrect specifications of subsectoral functions in perfectly competitive adjustment studies, except in very restrictive circumstances. The theoretically consistent specifications are developed and shown to be operationally useable for future studies.

SUBSECTORAL AND SUBMARKET STUDIES

Subsectoral adjustment and submarket equilibrium studies are frequently conducted: (1) under the conditions of perfect competition, (2) as *adjustment* studies wherein a market equilibrium is implicitly sought, and (3) with either a *subsectoral emphasis* or a *submarket emphasis.* The analysis that follows applies to both types of studies.

Let us denote a *subsector* as a group of economic actors who enter their product markets as sellers and enter their input markets as buyers and account for less than the total supply of products and demand for inputs in the respective markets. Let us further denote a *submarket* as a portion of a market wherein buyers are drawn from one set of subsectors and sellers from another set of subsectors not necessarily coincident with the first. Submarkets and subsectors are frequently delineated along both geographic and commodity lines. In subsectoral studies the emphasis is upon the actions in the subsector; in submarket studies the emphasis is upon the actions of opposite sides of the submarket, both of which are subsectors.

For example, consider a submarket for (say) steak. Steak is supplied as a part of an aggregate "all beef" and demanded as a part of an aggregate "all table cuts of meat." Steak need not comprise the same proportion of the demand aggregate as the supply aggregate. Any market exhibiting very high substitutability in supply with one group of products and very high substitutability in demand with another group of products may be viewed as a submarket. Furthermore, the submarket may comprise different proportions of the total market on the demand than on the supply side.[1] An analogous situation exists for a submarket defined along geographic lines, such as (say) the market for beef in the South.

As an example of a subsector we can take the producers of beef in the South (a subsector on both commodity and geographic lines). In the product markets they compete directly against producers of beef in other regions; in the input markets they compete directly against purchasers of agricultural inputs in the same region. Obviously the producers of beef in the South make up different shares of the markets for their inputs and products.

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[1] This occurs in any region which is either an excess supplier or a deficit supplier of a commodity.

The assumptions in this paper of perfectly competitive conditions apply to an undifferentiated product with a single price (exclusive of transportation costs) prevailing over all subsectors or submarkets. Such conditions might prevail in: (1) a geographic definition of subsectors, (2) in the input markets of subsectors defined on commodity lines, or (3) in commodities such as feed grains which face an on-farm feeding demand as well as a market demand.

THE GENERALIZED WALRASIAN ADJUSTMENT EQUATION

The Walrasian behavioral assumptions describe the process of adjustment of price and quantity toward equilibrium in terms of excess demand.[2] These assumptions are: (1) that buyers raise their bids when they perceive positive excess demand, and (2) that sellers lower their asking prices when they perceive negative excess demand. This adjustment mechanism is at work in both the product and factor markets. In each market we will restrict our discussion to cases for which the Walrasian adjustment mechanism results in a stable equilibrium.[3]

Excess demand is the difference between quantity demanded and quantity supplied at a given price. It is zero at the equilibrium price, positive at all lower prices and negative at all higher prices. For an aggregate market this mechanism yields the familiar market clearing condition:

$$(1) \qquad Q_d - Q_s = 0 \; .$$

To apply this aggregate market relationship to subsectoral or submarket studies both the aggregate market demand and the aggregate market supply can be decomposed into K subsectors.[4]

$$(2) \qquad Q_d = \sum_{k=1}^{K} \tilde{d}_k\,(p_k)$$

$$(3) \qquad Q_s = \sum_{k=1}^{K} \tilde{s}_k\,(p_k)$$

where $\tilde{d}_k(p_k)$ and $\tilde{s}_k(p_k)$ refer to the K subsectoral demand functions and supply functions respectively.

Under conditions of perfect competition with an undifferentiated product, price will vary at most by the transportation or processing costs. Thus some arbitrary price p, can be chosen as a base price and all functions $\tilde{d}_k(p_k)$ and $\tilde{s}_k(p_k)$ adjusted to the chosen base price to yield $d_k(p)$ and $s_k(p)$.[5]

Substituting (2) and (3) into (1) gives the aggregate equilibrium condition (4):

$$(4) \qquad \sum_{k=1}^{K} d_k\,(p) - \sum_{k=1}^{K} s_k\,(p) = 0 \; .$$

Excluded Subsectors.

One may wish to concentrate his analysis on a single subsector (say 1), and may have very little information about the remaining subsectors, or may wish to treat them in a highly aggregated manner. The K-1 sectors which are not the center of concern can be referred to as excluded subsectors. If one can reasonably assume that:

1. the average slope or elasticity of response of the excluded subsectors is known or estimable,
2. the historic quantities in the included subsector and the sum of the excluded subsectors are known, and
3. that the conditions of perfect competition and a single price prevail over all subsectors.

then we can state (2) and (3) as:

$$(5) \qquad Q_d \equiv d_1(p) + \bar{D}_k + (p - \bar{p})\frac{\partial}{\partial p}\,[D_k(p - \bar{p})]$$

and

$$(6) \qquad Q_s \equiv s_1(p) + \bar{S}_k + (p - \bar{p})\frac{\partial}{\partial p}\,[S_k(p - \bar{p})]$$

[2] These assumptions are consistent with the Marshallian assumptions (which are stated in terms of excess demand price) except in cases where: (1) both supply and demand functions are positively sloped or (2) both supply and demand functions are negatively sloped [3, pp. 109-113].

[3] An equilibrium is stable in the Walrasian sense if: (1) a positively sloped demand curve is more steeply inclined than its corresponding positively sloped supply curve, (2) a negatively sloped supply curve is more steeply inclined than its corresponding negatively sloped demand curve, and (3) the demand curve is negatively sloped and the supply curve is positively sloped.

[4] The aggregate market demand and supply functions need not be decomposed into the same number of subsectors; however, by using dummy subsectors, it is always possible for both sides to be decomposed into the same number. Thus, for convenience in exposition, we will assume the same number of subsectors on each side of the aggregate market. No conclusions will be changed by this assumption.

[5] If a region (or subsector) changes role from an excess supplier to an excess demander, or *vice versa* the price gradient will be reversed, hence the price deviations from the arbitrary base price will change necessitating readjustment of its demand and/or supply function.

where: $\bar{D}_k$ and $\bar{S}_k$ are historic quantities observed in the excluded sectors (estimates of $\sum_{k=2}^{K} \overline{qd}_k$ and $\sum_{k=2}^{K} \overline{qs}_k$ from assumption 2), $\bar{p}$ is the historic price corresponding to $\bar{D}_k$ and $\bar{S}_k$ (from assumption 3), and the derivatives of $D_k(p - \bar{p})$ and $S_k(p - \bar{p})$ are estimates of the slope of response of the excluded sectors stated as deviations from the historic price, $\bar{p}$ (from assumption 1).

Substituting (5) and (6) into the Walrasian equilibrium condition (1) and rearranging terms yields:

$$(7) \quad \underbrace{d_1(p) - s_1(p)}_{A} + \underbrace{\bar{D}_k + (p - \bar{p})\frac{\partial}{\partial p}[D_k(p - \bar{p})] - \bar{S}_k - (p - \bar{p})[S_k(p - \bar{p})]}_{B} = 0 \ .$$

The first two terms (A) represent the included subsector functions, and the last 4 terms (B) on the left of the equality represent the excluded subsectoral functions in highly aggregated form. Simple rearrangement of the terms of (7) gives the effective subsectoral demand and supply functions for the subsector under analysis.[6] If $s_1(p)$ is moved to the right of the equilibrium condition, then the remaining terms on the left define the effective subsectoral or submarket demand, $d_1'(p)$, (equation 8):

$$(8) \quad d_1(p) + \bar{D}_k + (p - \bar{p})\frac{\partial}{\partial p}[D_k(p - \bar{p})] - \bar{S}_k - (p - \bar{p})\frac{\partial}{\partial p}[S_k(p - \bar{p})] = s_1(p) \ .$$

Similarly if $d_1(p)$ is moved to the right of the equilibrium condition and the signs of the equation changed, the terms remaining on the left define the effective subsectoral or submarket supply, $s_1'(p)$, (equation 9):

$$(9) \quad s_1(p) + \bar{S}_k + (p - \bar{p})\frac{\partial}{\partial p}[S_k(p - \bar{p})] - \bar{D}_k - (p - \bar{p})\frac{\partial}{\partial p}[D_k(p - \bar{p})] = d_1(p).$$

In equations (8) and (9) equilibrium will occur at the price and quantity for which effective subsectoral demand (or supply) equals $s_1(p)$ (or $d_1(p)$) as they are developed in the study. For example, in a programmed supply study (say, for beef in the South), the programmed supply functions constitute $s_1(p)$, and the effective demand for beef is specified by $d_1'(p)$, the left side of equation (8). The effective subsectoral demand requires only the additional information specified in assumptions (1) and (2) above.

Shifts in Subsectoral Functions.

One may also be interested in the effects of differential shifts in the subsectoral demand and supply functions upon equilibrium prices and quantities. If the subsectoral functions in equation (4) are subject to multiplicative shifters $\alpha_1, \alpha_2, \ldots, \alpha_K$ on the demand side, and $\beta_1, \beta_2, \ldots, \beta_K$ on the supply side one obtains equation (10):

$$(10) \quad \sum_{k=1}^{K} (1 + \alpha_k)\, d_k(p) - \sum_{k=1}^{K} (1 + \beta_k)\, s_k(p) = 0 \ .$$

For additive shifters $a_1, a_2, \ldots, a_K$ and $b_1, b_2, \ldots, b_K$ one obtains the equilibrium condition of equation

$$(11) \quad \sum_{k=1}^{K} (d_k(p) + a_k) - \sum_{k=1}^{K} (s_k(p) + b_k) = 0 \ .$$

Both equations (10) and (11) may be rearranged to obtain expressions for effective subsectoral functions for a single subsector as was done with equation (7). By moving $s_k(p)$ and its appropriate shift term, $(1 + \beta_k)$, or b_k, to the right of the equality one obtains the effective subsectoral demand function, $d_k'(p)$. Similarly, by moving $d_k(p)$ and its appropriate shift term, $(1 + \alpha_k)$, or a_k, to the right of the equality and changing all signs, one obtains the effective subsectoral supply function, $s_k'(p)$.

IMPLICATIONS OF THE GENNERALIZED WALRASIAN ADJUSTMENT EQUATION

From expressions (8) and (9) we can draw implications concerning adjustments along the subsectoral functions. From equations (10) and (11) we can draw implications about the effects of shifts in the subsectoral functions.

Adjustments Along Functions.

Expressions (8) and (9) demonstrate the inappropriateness and limited applicability of two commonly used methods of deriving subsectoral supply and demand functions from prior estimates

[6] The effective subsectoral demand and supply functions describe the relationship between quantity supplied or demanded by *one* subsector and the single price prevailing over *all* subsectors.

for aggregate market relationships. These methods are:

1. The method of equal slope as the aggregate, wherein the aggregate function is relocated to pass through an historic price and quantity observed in a subsector under analysis, i.e., the subtraction of a constant quantity representing the excluded subsectors.
2. The method of equal elasticity as the aggregate wherein a function having an elasticity equal to the aggregate (at some historic price) is established through an historic price and quantity point in a subsector under analysis, i.e., the subtraction of a constant market share representing the excluded subsectors.[7]

The first method ignores the excluded subsectoral response terms $(p - \bar{p})\frac{\partial}{\partial p}[D_k(p - \bar{p})]$ and $(p - \bar{p})\frac{\partial}{\partial p}[D_k(p - \bar{p})]$ and hence is appropriate only if these terms vanish from the true Walrasian equilibrium condition. These terms vanish only when $D_k(p - \bar{p})$ and $S_k(p - \bar{p})$ are nonresponsive to price, and their partial derivatives vanish. Therefore, the first method is appropriate only when the excluded subsectors are nonresponsive to price.

The second method results in market sharing behavior on the side of the market to which it is applied. The assumption is made that the effective subsectoral function for sector i is some constant, m, times the aggregate market relationship. The constant, m, takes the value $q_i / \sum_{k=1}^{K} q_k$, which is the historic market share of subsector i, thus implying a constant market share at the new equilibrium. Market sharing behavior can result from the perfectly competitive adjustment mechanism only when there is equal elasticity of response over all subsectors as occurs in the case of:

1. power function relationships of equal elasticity in each subsector, and
2. linear relationships with equal price intercepts in each subsector.

Another situation in which the equal elasticity method is applicable occurs with analytic models for which the mobility mechanism is specified elsewhere in the model. An example is the set of transportation activities in a spatial equilibrium model. The partitioning of the problem inherent in the methodological approach allows use of the equal elasticity method.[8] It is not a redefinition of the effective subsectoral demand and supply relationships.

In all of the situations wherein market sharing behavior occurs, the true effective subsectoral relationships remain as defined by expressions (8) and (9). The subsectoral relationships derived by the equal elasticity method, by contrast, are the loci of subsectoral prices and quantities implied by market sharing behavior. Figure 1 illustrates this important distinction for a situation in which both methods give the same price and quantity results. In the upper figure lines s_1, s_2, d_1, and d_2 have elasticities equal to aggregate functions S_A and D_A, respectively. In the two lower figures, lines d_1', d_2', s_1', and s_2' are derived through application of equations (8) and (9) assuming s_1, s_2, d_1 and d_2 are the true subsectoral functions. Although prices and quantities are the same in all three methods, the effective subsectoral functions in the lower figures describe subsectoral adjustment possibilities, whereas those in the upper figure apply only to the market sharing situation. Observe that the effective subsectoral functions are much more responsive to price than the conventionally defined functions of either the equal slope or equal elasticity methods.

Shifts of Subsectoral Functions.

Let us turn now to the implications of shifts in the subsectoral functions. The effective subsectoral demand and supply functions derived from equations (10) and (11) must be used to specify the new equilibrium of a submarket experiencing shifts in its functions except in cases where market sharing behavior occurs in response to the shift. Market sharing behavior can occur on one side of a market if all multiplicative shifters α_k or β_k are equal. This applies regardless of the functional forms describing the subsectoral relationships. Equal values of multiplicative shifters might occur due to a pervasive and equal change in technology on the supply side or a pervasive change in tastes and preferences on the product demand side. Market sharing behavior can also occur if all additive shifters a_k or b_k are proportional to the historic quantities in their respective subsectors. Proportional additive shifters influence the shape of the effective subsectoral demand and supply functions. Additive shifters negate the conditions of equal price intercepts of linear functions or equal elasticity of power

[7] Expanding a regional demand function as the product of per capita consumption and population in the area under consideration is also an application of the equal elasticity method. Thus the following discussion applies equally to this method.

[8] However, the problem still appears between *included* and *excluded* subsectors.

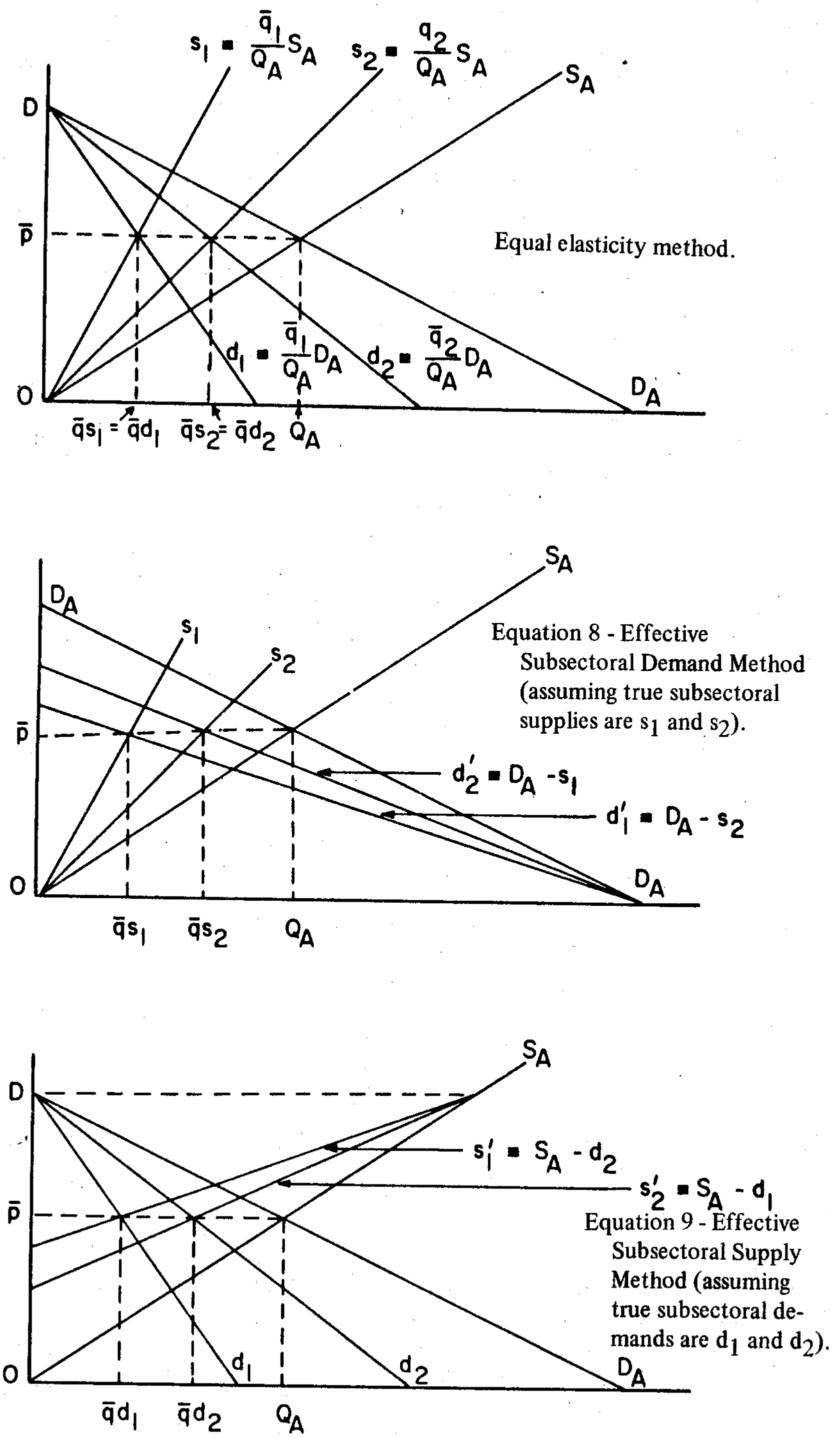

FIGURE 1. GRAPHIC COMPARISON OF EQUAL ELASTICITY SPECIFICATION AND EFFECTIVE SUBSECTORAL FUNCTION SPECIFICATIONS OF SUBSECTORAL DEMAND AND SUPPLY FUNCTIONS.

functions,[9] thus negating the conditions for market sharing adjustments along subsectoral functions.

Thus, using the method of equal slope or equal elasticity creates the same problems when studying the effects of shifts upon a single subsector as were observed with adjustments along subsectoral functions. The equal slope method is appropriate only if the excluded subsectors are nonresponsive, and the equal elasticity method implies market sharing behavior. The latter in turn implies pervasive proportional shifts by all subsectors.

In conclusion, a subsectoral study employing the commonly used specifications either gives you no more information than could have been obtained by an aggregate market study, hence negating the need for a subsectoral basis; or contains an inconsistency between the application and the implicit assumptions of the methods. In the latter case the specification of subsectoral relationships is incorrect and exhibits a strong bias toward inelasticity. The effective subsectoral function method, proposed here, gives theoretically consistent results in all of the situations cited.

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[9] Indeed the subsectoral functions are no longer simple power functions, but obtain an additive constant term.