Talent Utilization and Search for the Appropriate Technology

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Abstract

This paper analyzes a model of economic development which is propagated by matching between technologies and the talents they require. Differences in productivity across countries are amplified by three dimensions of talent utilization. First, the range of different talents utilized. Second, the density of a specific talent utilized. Third, the average match quality in the economy. In our setup, the equilibrium variety of technologies increases with productivity. A larger number of technologies enables higher match quality between individuals’ talents and requirements of technologies and thus increases the returns to search. More intensive search further contributes to talent utilization.

Keywords: income level, total factor productivity, technological density, appropriate technology, talent utilization, search.

JEL Classifications: J21, L16, O11, O33, O47.

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1 Introduction

Total factor productivity (TFP) is an important determinant of development. However, measured by the Solow residual, it is no more than “The Measure of Our Ignorance” (Abramovitz 1956). This research provides a theoretical explanation of differences in TFP and, thus, income differences across countries. It harnesses Adam Smith’s idea of the division of labor to explain how small differences in productivity across countries are amplified through search and matching. The premise of the paper is that each technology requires a different set of talents, which are distributed across individuals. When more individuals are properly matched to the appropriate technology, talent utilization increases, and with it output. The paper describes an economy in which exogenous productivity affects overall talent utilization through technological variety and individuals’ search for the appropriate technology.

The paper has the following results. Higher productivity yields a larger variety of technologies, which enables a larger range of talents to be utilized and better matches on average. This induces individuals to increase their search effort, contributing to the extent of talent utilization. Our main result shows that small differences in economies’ productivity are amplified through higher talent utilization and higher average match quality.

The idea of the paper is presented by a model of economic development where the final output is produced by many intermediate goods. Each country produces a different variety of intermediate goods. Each intermediate good corresponds to a specific technology and is produced by a continuum of entrepreneurs with heterogeneous talents. To implement a particular technology a specific entrepreneurship talent is required. The extent to which entrepreneur’s talent matches the technology requirements determines the efficiency units of labor that an entrepreneur supplies. Entrepreneurs’ efficiency units of labor are combined with raw labor to produce Intermediate goods.

With decreasing returns to accumulated factors, a fixed set up cost determines the number of intermediate varieties. Higher productivity increases entrepreneurial profits. As a result, a smaller continuum of entrepreneurs is needed to cover the fixed cost, yielding higher match quality on average and a larger number of varieties, both
of which induce higher incentives to search. Thus, investment in search increases with development, acting as another source of amplification by increasing the extent to which talents are being utilized.

This paper belongs to a strand of literature which tries to explain why some countries are so much richer than others. The answer the empirical literature provides lies between factor accumulation and the efficiency with which these factors are used. Nonetheless, this literature identifies an important role for TFP in explaining cross-country differences in income.

Given the importance of TFP in explaining large cross-country differences in income leaves us with the need to understand the underlying technological differences across countries. Zeira (1998), Basu and Weil (1998) and Acemoglu and Zilibotti (2001) are theoretical contributions that emphasize the role of appropriate technologies for explaining TFP differences. Zeira (1998) focuses on the range of technologies adopted due to differences in capital labor ratios. Basu and Weil (1998) addresses the role of learning-by doing that influences technological progress at the capital labor ratio. Acemoglu and Zilibotti (2001) emphasizes skill supply for utilizing advanced technologies. In these papers, differences in factor distribution across countries drive the adoption or invention of the appropriate technology. In our model, appropriateness is at the micro level. Each individual can be appropriately matched to a technology, or not. Thus, countries may have the same factor distribution, yet differ in the appropriateness of technology.

Our work is related to the burgeoning literature on search and matching between heterogeneous workers and firms. Sharing our concern regarding the mismatch between workers and jobs, this literature investigates the search and assignment process itself, focusing at times on the wage offer dynamics (Crawford and Knoer 1981), or the search process and its direction (Shimer 2005, Gautier and Teulings 2004).

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1 For an updated survey of such development accounting literature see Caselli (2005).
3 Our research is motivated by evidence on match at the individual level (Baumgardner 1988a, Baumgardner 1988b, Garicano and Hubbard 2009).
Decreuse 2008). We take an alternative black box approach to the assignment problem. Instead of specifying the search process, we allow the mismatch level in the economy to be determined by the endogenous structure of the economy. Given the number of sectors in equilibrium and the optimal search effort, the constrained efficient assignment results in wage heterogeneity and mismatch.\footnote{See also Lentz and Mortensen (2010) for an unemployment search and matching model with product variety.}

The rest of the paper is organized as follows. Section 2 formalizes the arguments, section 3 solves for the equilibrium, section 4 provides a cross country analysis, section 5 presents some concluding remarks and proofs appear in the Appendix.

## 2 The Model

Consider a small open economy in a world with one final good, which is used for consumption only. This final good is produced using a continuum of intermediate goods. For simplicity the model assumes no physical capital and, therefore, intermediate goods are produced using labor only. All markets are assumed to be perfectly competitive. The final good as well as each intermediate good is assumed to be perfectly tradable, but labor is not tradable, and its market is domestic. For simplicity there is no population growth and population size is normalized to one.

### 2.1 Production

#### 2.1.1 Production of the final good

The final good is produced by the following continuous log-linear production function

\[
\log Y = \int_0^1 \log x(j) \, dj, \tag{1}
\]

where \(Y\) is the total output produced in an economy, \(x(j)\) is the input of intermediate good \(j\).
2.1.2 Production of intermediate goods

Each country produces a discrete variety of intermediate goods out of a potential continuum, which is the interval \([0, 1]\). Each point on this unit segment represents a different type of intermediate good which requires a specific talent to operate the technology by which it is produced. This specific talent will be henceforth called the “job requirements”.

Individuals are indexed on the unit segment with uniform density. The index of each individual represents her talent. As job requirements represent the location of an entrepreneur whose talent accurately matches these requirements, individuals and intermediate goods are both indexed on the same unit segment without any ambiguity.

Each intermediate good is produced by a continuum of entrepreneurs, each endowed with a specific talent which matches to some extent the job requirements of the technology used. The extent to which an entrepreneur’s talent matches the job requirements determines the number of efficiency units of labor this entrepreneur supplies, according to the following function.

\[
h(j, i) = h_0 - b d(j, i),
\]

where \(h(j, i)\) is the ex-post efficiency units of labor that entrepreneur \(i\) supplies for producing intermediate good \(j\), \(h_0\) is the maximum efficiency units of labor that an entrepreneur can have and \(d(j, i)\) is the distance between the location of intermediate good \(j\), which reflects its job requirements, and that of entrepreneur \(i\), which reflects her entrepreneurship talent. This distance expresses the level of mismatch between the two. The larger the distance is, the greater the mismatch.

Each individual is a potential entrepreneur, who can produce an intermediate good.

\(^5\)A different location on the unit segment reflects a different type of talent, and not a better one. More specifically, the location of a specific individual at the extreme end of the talent distribution, \((i = 1)\), does not indicate a maximum level of talent; rather, it represents a different talent from any \((i \neq 1)\).
\[ x(j, i) = A \left[ l(j, i) \right]^\alpha [h(j, i)]^{(1-\alpha)} , \]

where \( \alpha \in (0, 1) \), \( x(j, i) \) is the output of intermediate good \( j \) produced by entrepreneur \( i \), \( l(j, i) \) is the number of workers employed by her and \( A \) is a productivity parameter, which is country specific. This coefficient may reflect geography: land quality, climate and access to sea, resource endowments: land abundance and natural resources or even infrastructure, and therefore differs across countries.

Each intermediate good is produced by a continuum of entrepreneurs taking prices as given. Namely, each entrepreneur \( i \) takes the equilibrium wage, \( w \), the cost \( r(j) \) of technology \( j \)'s blueprint, and the price \( P(j) \) of a unit of intermediate good \( j \) and maximizes:

\[ \pi(j, i) = P(j)A \left[ l(j, i) \right]^\alpha [h(j, i)]^{(1-\alpha)} - w[l(j, i)] - r(j). \]

2.1.3 The monopolistic market for technologies

Each intermediate good requires knowledge of a specific technology. This knowledge is owned by a monopoly. Since intermediate goods are substitutes in the production of the final good, competition arises among monopolies.

The market for technologies operates as follows. A monopolistic owner of a technology incurs a setup cost, \( C \). This cost, which is measured in terms of the final good, can be interpreted as the cost of importing on the shelf technology for producing intermediate good \( j \). Her revenues are \( R(j) \), which consist of total payments collected from all entrepreneurs using technology \( j \). Assuming an owner does not observe entrepreneurs’ talents, she cannot discriminate and thus charges a uniform price, \( r(j) \). Therefore, profit generated by monopolistic owner \( j \) is

\[ \pi(j) = R(j) - C, \]

where \( R(j) = \int_{i \in E(j)} r(j) \, di \) and \( E(j) \) is the set of entrepreneurs using technology \( j \).
2.2 Individuals

Each individual derives utility from consuming the final good and, thus, individuals’ maximization problem collapses to income maximization problem. An Individual can either work as an entrepreneur, utilizing her talent and earning some profits or be employed as a simple worker, earning the equilibrium wage, $w$.

For a non trivial number of technologies to arise in equilibrium, an entrepreneur must earn at least as much as a simple worker. However, to be an entrepreneur, an individual must search and find an appropriate technology. The information friction is such, that each individual does not know how well her talent matches with the existing technologies. This could either be because she does not know her own talent or she does not know the technological requirements of $j$.

**Assumption 1** The probability that entrepreneur $i$ finds the closest technology $j$ is independent of her distance from technology $j$.

This assumption captures the symmetry in individuals’ ignorance regarding technological requirements. Individuals are as likely to find the most appropriate technology for their talents whether they are very close to it or further away.\(^6\) This assumption implies that investment in search is equal across individuals.

An individual invests $s$ in search, incurs a cost $g(s)$ and finds the closest technology with probability $q(s)$, where $g(s)$ is increasing and convex and $q(s)$ is increasing and concave. Individuals choose search effort to maximize their expected income,

$$I = [1 - q(s)]w + q(s)I_{Informed} - g(s),$$

where $I_{Informed}$ is the average income of the set of individuals who find the location of the closest technology $j$,

$$I_{Informed} = [E(\pi(j,i) | \pi(j,i) > w)]\rho + w(1 - \rho).$$

\(^6\) As will be seen later, the average distance is shorter in more developed countries. Thus, relaxing this assumption and allowing for higher success probability for shorter distances will add another dimension of amplification.
Denote by $\rho$ the probability that labor market clearing conditions enable an informed individual to operate as an entrepreneur, that is, $\pi(j, i) > w$.

2.3 Labor market

Labor market consists of entrepreneurs producing intermediate goods using a specific technology and employing workers. Let $J$ denotes the equilibrium number of technologies and $\phi(j, i)$ is the density of entrepreneurs of talent $i$ using technology $j$. Each entrepreneur $i$ producing with technology $j$ demands $l(j, i)$ workers. Let $\{j_1, \ldots, j_J\}$ be the set of technologies arising in equilibrium. Aggregate demand for labor is

$$\sum_{j \in \{j_1, \ldots, j_J\}} \int_{i \in E(j)} \phi(j, i)l(j, i) \, di,$$

and aggregate supply of labor is

$$1 - \sum_{j \in \{j_1, \ldots, j_J\}} \int_{i \in E(j)} \phi(j, i) \, di.$$

3 Equilibrium

An equilibrium is a vector $\{s, r(j), E(j), P(j), l(j, i), w, J\}$ of search effort, price of technology, set of entrepreneurs utilizing each technology, prices of the intermediates goods, employment of workers by entrepreneurs, wage of workers, and number of technologies, which is a solution to (i) individuals’ maximization of income; (ii) the monopolistic owners of technologies profit maximization and (iii) zero profits condition; (iv) the final good maximization problem; (v) the intermediate goods maximization problem; (vi) a threshold condition on individual’s choice of employment; and (vii) labor market clearing condition. In the rest of the section we solve for the equilibrium.
3.1 Final good market

Let the final good serves as a numeraire. Profit maximization by final good producers leads to the following first-order condition

\[ P(j) = \frac{\partial Y}{\partial x(j)} = \frac{Y}{x(j)}. \]  

(8)

Substituting equation (8) into (1) we get that the condition \( \int_0^1 \log P(j) \, dj = 0 \) must hold at the optimum. Due to symmetry and to the world competition in markets for intermediate goods all prices must be equal. Hence \( P(j) = P = 1. \)

3.2 Intermediate goods market

Profit maximization by entrepreneur \( i \) who produces intermediate good \( j \) leads to the following demand for labor

\[ l(j, i) = \left( \frac{\alpha A}{w} \right)^{1-\alpha} [h_0 - bd(j, i)]. \]  

(9)

The profits of entrepreneur \( i \) decreases with the distance to technology \( j \). An entrepreneur would like to set up a firm producing intermediate good \( j \) as long as it is profitable. The threshold condition for the marginal entrepreneur in sector \( j \), who is indifferent between being an entrepreneur or working as an employee in any firm is:

\[ \pi(j \pm \bar{d}(j)) = (1 - \alpha)A \left[ l(j) \right]^\alpha [\bar{h}(j)]^{(1-\alpha)} - r(j) = w; \]  

(10)

where \( \bar{h}(j) \) is the number of efficiency units of labor that the marginal entrepreneur has and \( \bar{l}(j) \) is the number of workers employed by her. Recall from equation (4) that \( \bar{h}(j) = h_0 - \bar{d}(j) \), where \( \bar{d}(j) \) is the maximal distance between the requirements of sector \( j \) and the talent of the marginal entrepreneur.
3.3 The monopolistic market for technologies

Since entrepreneurs join sector \( j \) from both sides, the size of sector \( j \) is represented by the width of that sector which is the interval \([j - \bar{d}(j), j + \bar{d}(j)]\). The size of each sector, which is \(2\bar{d}(j)\), represents the continuum of firms that produces the same intermediate good \( j \). An additional intensive margin is given by the density of entrepreneurs of talent \( i \) working in sector \( j \), \( \phi(j, i) \).

**Lemma 1** At the macro level \( \phi(j, i) = q(s_{ij}) \).

**Proof.** Follows directly from the law of large numbers. ■

**Corollary 1** The density of entrepreneurs of talent \( i \) is independent of her distance from her closest technology \( j \), i.e. \( \forall i, j \text{ s.t. } i \in E(j), \phi(j, i) = q(s) \).

**Proof.** Follows from assumption (1) and lemma (1) ■

The density of entrepreneurs for a given skill-technology match is the probability that an entrepreneur matches with her closest technology. This probability depends on search effort \( s \) which is the same for all individuals. Hence, density does not depend on \( i \), and it is now convenient to integrate using the distance \( t \) defined by: \( t = |j - i| \). Then (5) becomes:

\[
\pi(j) = r(j) \cdot 2 \left[ \int_0^{\bar{d}(j)} q(s) \, dt \right] - C. \tag{11}
\]

Where \( q(s) \) is the density of entrepreneurs at any given location. This density is a function of search effort, \( s \), and it is independent of distance, \( t \). From equation (10), it follows that the price that owner \( j \) charges for selling her technology to other entrepreneurs, \( r(j) \), affects entrepreneurs’ profits and therefore affects the size of sector \( j \). The first order condition with respect to the monopolistic price yields:

\[
\frac{\partial \bar{d}(j)}{\partial r(j)} r(j) + \bar{d}(j) = 0. \tag{12}
\]
Substituting equation (9) into (10) and applying the implicit function theorem implies that:

$$\frac{\partial \bar{d}(j)}{\partial r(j)} = \frac{-w^{\frac{\alpha}{1-\alpha}}}{\alpha^{\frac{1}{1-\alpha}}(1-\alpha)bA^{\frac{1}{1-\alpha}}}.$$  \hspace{1cm} (13)

Substituting equation (13) into equation (12), isolating \(w\),

$$w = \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}b^{\frac{\alpha}{1-\alpha}}A^{\frac{1-\alpha}{\alpha}} \left(\frac{\bar{d}(j)}{r(j)}\right)^{\frac{1-\alpha}{\alpha}}.$$  \hspace{1cm} (14)

Substituting equation (14) and (9) into (10) and isolating \(r(j)\) yields:

$$r_j = \gamma \frac{b\bar{d}(j)}{(h_0 - 2bd(j))^{\alpha}}A,$$  \hspace{1cm} (15)

where \(\gamma = \alpha^{\alpha}(1 - \alpha)^{(1-\alpha)}\).

Another potential entrepreneur \(j^t\), located far from \(j\), finds it profitable to initiate a new sector that produces a different intermediate good. She incurs the setup cost, \(C\), and through the above described market for technologies she sells the blueprint to other entrepreneurs close to her. Ultimately, many sectors are being established, where each sector produces a unique intermediate good by a continuum of firms. The larger the variety of intermediate goods, the smaller the profits for each monopolistic owner. This conclusion is driven by the assumption of substitution of the intermediates in producing the final good. As a result, in equilibrium, the variety of intermediate goods in an economy is determined by applying the zero profit condition for all owners, which yields:

$$2q(s)\gamma \frac{b[\bar{d}(j)]^2}{[h_0 - 2bd(j)]^{\alpha}}A = C.$$  \hspace{1cm} (16)

**Corollary 2** All sectors are symmetric, i.e., each sector has the same size and, therefore, charges the same price for selling technology.

(i) \(\forall j, r(j) = r\).

(ii) \(\forall j, \bar{d}(j) = \bar{d}\).
Proof. Follows directly from equation (15) and (16).

Thus (16) becomes

$$2q(s)\gamma \frac{b [\hat{d}]^2}{[h_0 - 2bd]^\alpha} A = C. \quad (17)$$

Using corollary (2) and substituting (15) into (14) yields

$$w = \gamma (h_0 - 2bd)^{1 - \alpha} A. \quad (18)$$

### 3.4 Labor market clearing

Recall that \( t \) is the distance between an entrepreneur and her technology. Given that \( J \) is the equilibrium number of sectors, and by corollary (2) labor market clearing implies that

$$2J \left[ \int_0^{\hat{d}} q(s)l(t) \, dt \right] = 1 - 2J \left[ \int_0^{\hat{d}} q(s) \, dt \right]. \quad (19)$$

The term \( 2J \int_0^{\hat{d}} q(s)dt \) represents the size (measure) of entrepreneurs out of a normalized population. Therefore, the left hand side of (19) represents the demand for labor and the right hand side of (19) represents the supply for labor.

Using corollary (2) and substituting equation (18) into (19), equation (19) can be rewritten as a function of the distance, \( t \).

$$l(t) = \frac{\alpha}{1 - \alpha} \frac{h_0 - bt}{h_0 - 2bd}. \quad (20)$$

**Proposition 1** Firm size is positively affected by the match quality.

Proof. Follows from equation (20).
Substituting equations (20) into (19), yields:

\[ J = \frac{1}{q(s)d \left( \frac{\alpha}{1-\alpha} \cdot \frac{2h_0-\beta d}{h_0-2\beta d} + 2 \right)}. \]  

(21)

### 3.5 Individuals’ Optimization:

Recall that \( \rho \) is the probability that an informed entrepreneur finds it profitable to set up a firm. Corollary (2) yields that \( \rho = 2dJ. \) Along with (7), (6) could be rewritten as

\[ I = [1 - q(s)]w + q(s)\{2dJE(\pi) + (1 - 2dJ)w\} - g(s), \]  

(22)

where \( E(\pi) \) is the expected profits from being an entrepreneur, which can be described by

\[ E(\pi) = \int_0^d \{ (1 - \alpha)A[l(t)]^\alpha[h(t)]^{1-\alpha} - r \} f(t) \, dt, \]  

(23)

where \( f(t) \) is the density function of talent with distance \( t. \) Given that \( t \) is uniformly distributed on \([0, \bar{d}], \) \( f(t) = (1/\bar{d}). \)

Substituting equation (20) and (15) into (23) yields

\[ E(\pi) = \gamma \frac{h_0 - \frac{3}{2}bd}{(h_0 - 2bd)^\alpha} A. \]  

(24)

Maximizing equation (22) yields the following first order condition

\[ q'(s)2dJ[E(\pi) - w] = g'(s). \]  

(25)

The intuition behind equation (25) is straightforward. The left hand side of (25) is the gain from a marginal increase in \( s \) and the right hand side is its cost.
Substituting (17), (18), (21) and (24) into (25) yields

\[
\frac{q'(s)}{q(s)g'(s)} \frac{C}{2d \left( \frac{a}{1-a} \frac{2h_0-bd}{h_0-2bd} + 2 \right)} = 1. \tag{26}
\]

Ultimately, (17) and (26) solve for the equilibrium values of \( s \) and \( \bar{d} \), and (21), in turn, solves for \( J \).

## 4 Talent Utilization Across Countries

This section examines cross country differences in talent utilization. More specifically, we will show next how small differences in productivity are amplified through different channels. (i) Higher productivity yields more diversification reflected by a larger variety of technologies. Such an environment potentially allows for better matching between individuals’ talents and technologies. (ii) Within this better environment, the range of different talents being utilized is larger. That is, more developed economies utilize some talents that are wasted in less developed economies. (iii) This better environment also increases the marginal cost of mismatch, yielding better matches by employing a smaller range of talents in each technology. (iv) Finally, this better environment, under reasonable assumptions, induces individuals to increase their search effort, resulting in a higher intensity of talent utilization, and therefore better match quality.

The two core mechanisms underlying the above channels are diversification and search. Although these two mechanisms are interrelated, it will prove useful to isolate the role of diversification by holding search effort constant. Thus, initially, we analyze an economy in which the density of entrepreneurs, \( q(s) \), and hence the intensity of talent utilization is constant.

### 4.1 The Quality of Matches

The match quality in the economy could be measured by the average mismatch of entrepreneurs, \( \bar{d} \).
Proposition 2 In more developed economies the average match quality is higher, which is reflected by a smaller continuum of talents employed in each sector. Formally, $\frac{\partial \bar{d}}{\partial A} < 0$.

Proof. Follows directly from applying the implicit function theorem on (17), which shows that

$$\frac{\partial \bar{d}}{\partial A} = -\frac{\bar{d}}{2A \left(1 + \frac{\alpha bd}{h_0 - 2bd}\right)} < 0.$$  \hspace{1cm} (27)

Corollary 3 Monopolistic price for selling technologies increases with development. Formally, $\frac{\partial r}{\partial A} > 0$.

Proof. Differentiating (15) with respect to $A$ and substituting (27) yield,

$$\frac{\partial r}{\partial A} = \frac{\gamma bd}{(h_0 - 2bd)^{\alpha}} \left(1 - \frac{1 + \frac{2\alpha bd}{h_0 - 2bd}}{2 + \frac{2\alpha bd}{h_0 - 2bd}}\right) > 0.$$  \hspace{1cm} (28)

The intuition of the result described in proposition (2) is as follows. Entrepreneurs in more developed economies are more productive and thus are not only willing to pay higher prices, but their willingness to pay declines more steeply with their distance $d_i$. The monopoly which faces a steeper demand, sets a higher price. In addition, as we shall see later, wages in this economy are higher. Thus, the marginal entrepreneur at distance $\bar{d}$ faces both higher alternative wages and higher prices, and thus must be more productive, i.e. better matched.

Next we show how development increases both the variety of technologies and the variety of talents utilized.

4.2 The Variety of Technologies

Proposition 3 Higher productivity induces more diversification, that is, a larger number of intermediate goods. Formally, $\frac{\partial d}{\partial A} > 0$. 

Proof. Follows directly from (27) and (21). ■

Intuitively, in more developed countries, both entrepreneurs and workers are more productive factors. Since in equilibrium zero profit condition holds, it requires less factors in order to cover the same fixed costs, \( C \). As each technology captures a smaller share of the factors of production, more technologies arise.

### 4.3 The Range of Talents

**Proposition 4** Higher level of development is associated with a larger range of talents utilized. Formally, \( \frac{\partial B}{\partial A} > 0 \), where \( A = 2 \frac{h_0 - b \bar{d}}{\bar{d} - 2b \bar{d}} \).

Proof. Rewriting (21) as
\[
B = 2J\bar{d} = \frac{2}{q(s) \left( \frac{a - 2b - b\bar{d}}{1 - a} - 2b\bar{d} + 2 \right)},
\]
shows that \( B \) decreases with \( \bar{d} \). ■

Proposition (4) states that although the size of each sector is smaller in more developed economies, the increase in the variety of sectors dominates. Thus, higher productivity increases the share of entrepreneurs in the population. Since in this model talents play a role only through entrepreneurial activities, it turns out that in more developed countries a larger variety of talents are utilized, albeit the same ex-ante distribution of talents in all countries.

Proposition (2) and Proposition (4) together, imply that more individuals enjoy higher match quality in more developed countries. Another way to relate these two results is that individuals are more likely to receive returns to their skill, which implies less randomness in income.

### 4.4 Amplification through Search

In this section we would like to learn how individuals’ choice of search effort for the appropriate technology varies across economies. As described above, different
economies foster different environments which shapes the incentives individuals face when searching for the appropriate technology.

**Corollary 4** The size of each sector, \(2d\), and the investment in search, \(s\), are negatively correlated. That is, in an economy where the average match at the sectoral level is higher, individuals invest more in search for the appropriate technology. Formally, \(\frac{\partial s}{\partial d} < 0\).

**Proof.** See the Appendix. ■

To elaborate the results of our model with endogenous search, we assume that the weakly concave probability function takes the general form \(q(s) = s^\beta\), where \(\beta \in (0, 1]\), and the weakly convex cost function takes the general form \(g(s) = s^\delta\), where \(\delta \in [1, \infty)\).

**Proposition 5** For any weakly concave probability function and any weakly convex cost functions given above, higher level of development is associated with higher investment in search if \(\alpha \frac{\beta + \delta}{\beta} > 1\).

**Proof.** See the Appendix. ■

Three remarks should be made concerning the condition in proposition (5). First, this is a sufficient condition for search effort to increase with development for any weakly concave probability function and weakly convex cost function from the form described above. Second, the left hand side of this condition increases with \(\delta\) and decreases with \(\beta\). Thus, the range of \(\alpha\) for which this condition holds increases with the concavity and the convexity of the probability and cost functions, respectively. The smallest range of \(\alpha\) for which this condition still holds is when we take \(\beta\) and \(\delta\) to the corners and assume linear functions, \(\beta = \delta = 1\). In this case, the condition becomes \(\alpha > 1/2\). Third, recall from the production function of the intermediate goods that \(\alpha\) describes the share of labor in output. Empirical work which estimated the share of labor found that it is far above 1/2 for all countries (Gollin 2002, Bernanke and Gurkaynak 2002).
Equation (21) implies that holding the average mismatch $\bar{d}/2$ constant, an increase in search decreases $J$. However, as $\bar{d}$ declines labor market clearing condition implies that $J$ increases. To see the overall impact on $J$ we substitute the probability and cost functions in equation (26), isolating $s$ and substituting it in equation (21), which yields

$$J = \frac{\left(\frac{2h_0}{N_0}\right)^{\frac{1}{2+s}}}{\bar{d}\left(\frac{1-h_0}{2h_0} + 2\right)^{\frac{1}{2+s}}}.$$ 

As the average mismatch $\bar{d}/2$ declines, the number of technologies $J$ increases. Finally, with endogenous search, the measure of entrepreneurs is $2J\tilde{d}q(s)$ which increase as $\tilde{d}$ declines. Thus, we have shown that for our general cost and probability functions, with a reasonable labor share ($\alpha > 0.5$), higher level of development is associated with a larger variety of technologies $J$, larger search effort $s$, larger measure of talents being utilized $2\tilde{d}Jq(s)$ and a lower average mismatch in the economy $\bar{d}/2$.

5 Concluding Remarks

This paper argues that small differences in productivity are amplified by talent utilization. Talent utilization is a result of matching between technologies’ requirements and individuals’ talent. The amplification process works through three different channels. First, the variety of different talents utilized. Second, the density of a specific talent utilized. Third, the average match quality in the economy.

The analysis provides a tool to understand differences in economic structure across countries. It describes the forces that determine three different dimensions related to the structure of the economy: first, the number of sectors, each identified with a different technology; second, the size of each sector, which is reflected by the continuum of entrepreneurs utilizing their talents; third, the distribution of firms’ size mirrored by the distribution of workers employed by entrepreneurs.
The model could also be used as a tool to analyze income inequality within economies. This inequality is measured by the difference between the wage of simple workers and the differentiated income of entrepreneurs, as well as the relative sizes of these two groups, as determined by the structure of the economy. Moreover, the model could be extended to deal with unemployment, an interesting dimension that we leave for future research.
References


APPENDIX

Proofs

Proof of corollary 4.

First, rewrite equations (17) and (26) as the following nonlinear system

\[
\begin{aligned}
F(s, \bar{d}, A) &= 2q(s)\gamma \frac{b\bar{d}^2}{(h_0-2bd)} A - C = 0 \\
G(s, \bar{d}) &= \frac{q(s)}{q(s)\bar{d}} \frac{d}{2d\left(\frac{C}{1-\alpha} \frac{2b\bar{d}}{h_0-2bd} + 2\right)} - 1 = 0
\end{aligned}
\]

(29)

The derivatives of \(s\) and \(\bar{d}\) with respect to \(A\) are calculated as

\[
\begin{aligned}
\frac{\partial s}{\partial A} &= -\operatorname{det} \begin{pmatrix}
2q\gamma \frac{b\bar{d}^2}{(h_0-2bd)^\alpha} & 2q\gamma \frac{2bd + 2q\bar{d}^2}{(h_0-2bd)^\alpha} \bar{d} \\
0 & \frac{q\bar{d}}{q\bar{d}^2} D(\bar{d})
\end{pmatrix} \\
\frac{\partial \bar{d}}{\partial A} &= -\operatorname{det} \begin{pmatrix}
2q\gamma \frac{b\bar{d}^2}{(h_0-2bd)^\alpha} A & 2q\gamma \frac{b\bar{d}^2}{(h_0-2bd)^\alpha} \\
Q(s) \frac{C}{2d\left(\frac{C}{1-\alpha} \frac{2b\bar{d}}{h_0-2bd} + 2\right)} & \frac{q\bar{d}}{q\bar{d}^2} D(\bar{d})
\end{pmatrix}
\end{aligned}
\]

(30) (31)

and

where

\[
Q(s) = \frac{q^4 q^2 g t - 2qq_t g t - q^2 q g t}{q^4 g t^2}
\]

(32)
and

\[ D(\bar{d}) = -\left( \frac{\alpha}{1-\alpha} \frac{2h_0 - b\bar{d}}{h_0 - 2b\bar{d}} + 2 \right) + \left( \frac{\alpha}{1-\alpha} \frac{3b\bar{d}h_0}{(h_0 - 2b\bar{d})^2} \right) C \]

Notice that \( D(\bar{d}) < 0 \) always holds and that \( Q(s) < 0 \) holds for any weakly concave probability function, \( q(s) \), and weakly convex cost function, \( g(s) \). Thus the sign of the numerator of equation (30) is negative while the sign of the numerator of equation (31) is positive, which yields the corollary.

**Proof of proposition (5).**  Substituting \( q(s) = s^\beta \) in equation (17), isolating \( s \) and substituting it along with \( g(s) = s^\delta \) in equation (26) and rearranging yields

\[
\frac{\delta}{\beta} \left( \frac{C}{2} \right)^{\frac{\delta}{\beta}} \left( \frac{1}{\gamma bA} \right)^{\frac{\delta}{\beta}} = \frac{d^{\beta+2\delta}}{\bar{d}^{\beta+2\delta}}
\]

\[
= \frac{\alpha}{1-\alpha} \left[ (2h_0 - b\bar{d}) + 2\frac{1-\alpha}{\alpha} (h_0 - 2b\bar{d}) \right] \frac{(h_0 - 2b\bar{d}) \alpha^{2+\delta/\beta} - 1}{H(\bar{d})}
\]

While the left hand side decreases with \( A \), the right hand side increases with \( \bar{d} \) if \( \alpha^{\frac{\beta+\delta}{\beta}} \). Thus, this condition is a sufficient assumption so as to get \( \frac{\partial\bar{d}}{\partial A} < 0 \) and, thus, \( \frac{\partial\bar{d}}{\partial A} > 0 \). 

\[ \blacksquare \]