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ON THE USE OF DETERMINISTIC LINEAR PROGRAMMING FOR PLANNING IN A NON-CERTAIN ENVIRONMENT*

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Farm planning in the non-certain world is complex so that models which adequately represent this situation are difficult and expensive to experiment with. Thus a range of linear programming models based on simplifications has been suggested for planning under non-certainty. This discussion reviews the problem of farm system infeasibility which can arise from the use of these models and suggests an alternative approach.

1 INTRODUCTION

An individual farmer's managerial problem is one of deciding upon, and implementing, a course of action in the current period of time in order to achieve certain objectives. The optimal course, or courses as there may be alternatives, will be unique to his situation and will depend on the current state of the farm and his estimates of future conditions. Plans for future periods of time must be in the form of strategies to be implemented according to the conditions which eventuate once the future period becomes the current one. Such strategies may also change as expected future conditions change.

This problem is undoubtedly extremely complex. The economic size of the majority of farms means that the potential rewards to detailed and sophisticated planning are usually insufficient to warrant such complicated models being developed for individual farms. There are also problems for which economists do not have the techniques to adequately represent, or the computational facilities to solve. It is true that dynamic programming and stochastic linear programming [10] appear to provide model frameworks which are very similar to reality, however, dynamic programming is incapable of representing a detailed whole farm model due to the dimensionality problem. Similarly, stochastic linear programming must recognize only a very small number of states of nature and periods if the model is to be tractable. Simulation techniques can be realistic but they must rely on an experimental approach in obtaining improved systems and are therefore costly and beyond use by individual

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enterprises.[1] Taken together this means advisers will frequently need to rely upon representative farm type work using less realistic models for some time to come. Many such models will produce, hopefully, optimal solutions for a given set of conditions but will not consider transition problems of moving from the current system to the optimal (which may well be different next period). Advisers will need to extrapolate to individual farms and subjectively allow for the dynamic nature of the problem.

For this type of work there are available a range of models which transform to linear programming problems, recognizing non-certainty[2] in a variety of ways. Examples are quadratic programming [2, 5] and MOTAD programming [3] which relate expected income to variance and mean absolute deviation respectively. Other examples are the range of models which use game theory criteria as the objectives [6, 11, 2] and chance constrained [13] and focus-loss [13, 4] type models. These latter models are designed to maximize expected income subject to various risk constraints and minimum income levels.

A potential problem with all these models is that all non-certainty is assumed to be reflected in the net revenues of the activities. Such models can generally be described by the following statements:

(a) find a vector x of activity levels which maximises or minimizes some function containing, or derived from, the activity net revenues (vector c);

(b) this vector must satisfy resource and other restrictions ($Ax = b$, where A is a matrix of input-output coefficients and b is a vector of requirements);

(c) in some cases this vector must also satisfy restrictions derived from c (minimum income requirements);

(d) all the non-certainty exists within the components of the vector c.

The components of A and b are assumed to be deterministic.[3] For some problems this assumption will be realistic. The land restraint in most farm problems is an example where supply and requirements by activities are deterministic. However, in the majority of problems there will be resource relationships involving such factors as, for example, stock feed demand and supply, working cash demand and supply and labour demand and supply. In these cases it is unlikely that the demand and supply coefficients will be known with certainty so that the deterministic assumption is violated and the solution vector x will be infeasible under some states of nature.

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[1] A compromise which may be useful under some circumstances is a stochastic programming-simulation combination [12].

[2] The term "non-certainty" is used to cover both risk and uncertainty in a loose way. The latter is frequently converted to subjective risk through estimating subjective distributions.

[3] A model based on game theory ideas which is an exception to this statement is Maruyama's [9] "Truncated Maximin" model. The conceptual problem with this model will be referred to later. A practical problem relates to matrix size problems. The model has constraints representing possible combinations of the coefficients in A, b and c.

The farmer's reaction to infeasibility is, for example, to buy additional feed, borrow additional cash, to accept lower productivity from the stock and so on.[4] These adjustments could be reflected in the distributions of the individual components of the c vector. Thus, it could be argued that assuming a deterministic A matrix and b vector is acceptable where the costs and returns resulting from ensuring feasibility under all conditions are allowed for by assuming the correct distribution of the components of c. The distribution of the components of c will also, of course, depend on the distributions of the physical levels of inputs and output which can vary without affecting feasibility. Similarly, the distributions of the per unit prices and costs will affect the distributions of the net revenues even though such variations may not affect feasibility. For example, the net revenue of a wheat activity in a mixed stock-crop system can have two major components affecting its variability. First, the variability due to such factors as the yield and price variability and the requirement for weed control measures. These factors could be independent of the stubble feed provided and such variations may not require actions to ensure feasibility. If the feed supply represented in the A matrix is assumed to be constant, the possible feed purchases and other additional actions to ensure the activity provides a fixed supply under all conditions gives rise to the second source of variability. The activity could, therefore, implicitly assume a number of actions to allow for all possible conditions.

The problem with this approach is that it assumes a series of optimal actions which ensure feasibility under all conditions can be predetermined before solving the problem. The predetermination then enables the distributions of the components of c to be calculated. Some workers for example, [3], use historical series of gross margins to indicate the variability of the components of c. The assumption in this case is that the farmers, whose data are being used, have historically made the optimal infeasibility adjustments decisions. This is a doubtful assumption, though in models developed for predictive work it may be acceptable. However, shifts in the price and cost situation may mean, of course, historical data should be adjusted before use.

Whether or not predetermination of optimal infeasibility adjustments can be accepted will depend on the problem. For problems where optimal adjustments are a function of the actual activity mix in the solution vector x predetermination cannot be accepted.

For example, in a problem involving stock and feed activities there are at least four approaches to solving potential infeasibilities due to variation in the feed supply. Firstly, feed supply variations can be accepted and the feed intake per animal allowed to vary so that productivity varies. This would mean the variability is accounted for in the stock activity's c component. Similarly, and secondly, feed intake can be made constant by reducing stock numbers and this total product variability allowed for in the relevant c components. Thirdly, feed production variability can

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[4] Hanf [17] has discussed the general nature of this problem but did not show how it can be solved.

be assumed to be adjusted through feed purchasing to give a constant effective supply. In this case the feed producing activities' c components would reflect the variability. Fourthly, some combination of these approaches can be assumed. The difficulty in this example is that an optimal system to ensure feasibility cannot be predetermined. Whether it is optimal to maintain stock productivity at a constant level or allow it to vary will depend on the feed supply variability of the activities in the solution and the solution activities cannot be predetermined in a non-trivial problem. Similarly, the type of stock activities in the solution should influence the optimal infeasibility adjustment procedure.

A somewhat different example is where, given a particular solution vector, it may not be physically possible to make adjustments to give feasibility under some states of nature. Consider a problem where monthly cash reconciliation is necessary. The cost and return of monthly cash requirement and supply variability could be allowed for in the c components. However, there will be a physical limit on short-term borrowing and whether this limit might be exceeded cannot be predetermined. It will depend on the particular activity combination represented in the solution vector.

In problems where predetermination of an optimal adjustment procedure is considered to be unacceptable a model must be developed which enables the endogenous determination of such adjustments. One such model is stochastic linear programming, with its associated size and cost problems. An alternative is the relatively simple approach described below. It will be seen to have simplifying assumptions and, therefore, will not necessarily be more acceptable than the risk programming techniques discussed above. It will depend on the particular case.

2 A DETERMINISTIC LINEAR PROGRAMMING APPROACH ALLOWING FOR FEASIBILITY PROBLEMS

The basis of the system is to use a two-phase system. The first phase consists of finding the optimal deterministic solution for each state of nature assuming each state of nature can be predicted. This provides up to k solutions if there are k states of nature. Each such solution represents an alternative plan available for choice. Such plans, however, are unlikely to be feasible under all states of nature. Thus, the second phase consists of determining the optimal adjustment procedures for each plan to ensure feasibility under each of the possible condition sets, and to determine the associated objective function values. Where the objective function used is net revenue the k^2 values provide a net revenue payoff matrix which then forms the basis for choice.

2.1 The First Phase

Where we let b^s and A^s be the vector and matrix values for the s^{th} state of nature, $s = 1, 2, \ldots k$, and c^s be a vector of net revenues (usually) for the s^{th} state of nature, the problem is to find the solution vector x^s which maximizes $Z^s = c^s x^s$ . . . . (1)

subject to $b^s(\geqq = \leqq) A^s x^s$ . . . . (2)

and $x^s \geqq 0$ . . . . (3)

for each s. Let Z^s be the optimal objective function value for the s^{th} state of nature.

2.2 THE SECOND PHASE

As Z^s is unlikely to be achieved if x^s is implemented for other states of nature due to feasibility problems and net revenue changes, adjustment activities to be associated with a particular state of nature must be determined. Similarly, the associated objective function value must be derived. Let Π_s^t be the objective function value for a plan x^t which is optimal for the t^{th} ($t = 1, 2, \ldots k$) state of nature but is followed, together with adjustments, under the s^{th} state of nature. For $s = t$, $\Pi_s^s = Z^s$.

A number of approaches to determining Π_s^t and the associated adjustment activities are possible. A simple approach would be to determine the extent of any infeasibilities and to use comparative budgeting to estimate a reasonable adjustment procedure. A complex procedure would be to use a detailed simulation (policy experimentation) approach[5] and an intermediate alternative would be the use of linear programming.

Using linear programming the problem is as follows:

Let

q_s be a vector of adjustment activity levels for the s^{th} state of nature. These activities would represent processes such as additional feed purchasing, feed selling, stock buying and selling, reducing stock productivity and so on.

Q_s be a matrix of input-output coefficients associated with the components of q_s for the s^{th} state of nature.

r_s be a vector of objective function coefficients associated with the variables in q_s.

Then, the problem is to find the vector q_s for each t and s combination, except $t = s$, which maximizes (recall that x^t is the solution vector from phase one for the t^{th} state of nature):

$\Pi_s^t = c^s x^t + r_s q_s$ . . . . (4)

subject to

(i) $b^s (\geqq = \leqq) A^s x^t + Q_s q_s$ . . . . (5)

(ii) $x^t = I_m x^t$ where m is the order of x^t . . . . (6)

(iii) $q_s \geqq 0$ . . . . (7)

These $k^2 - k$ solutions together with the k solutions from the first phase provide a payoff matrix with components Π_s^t. Whether linear programming or an alternative approach is used to give Π_s^t, the payoff matrix will have the form:

[5] Similar to [12].

State of Nature

		1	2	. . . .	k
	x^1	Π_1^1	Π_2^1	. . . .	Π_k^1
	x^2	Π_1^2	Π_2^2	. . . .	Π_k^2
Plan x^t		⋮	⋮		⋮
	x^k	Π_1^k	Π_2^k	. . . .	Π_k^k

2.3 Using the Payoff Matrix

In effect each row represents an alternative strategy which consists of a basic plan together with actions to ensure feasibility. Using this payoff matrix a choice between the basic plans x^t can be made using any criteria considered appropriate, provided such criteria are formed from the objective function assumed in the linear programming problems. For example, the game theory criterion maximizing the minimum income could be used.[6]

Furthermore, as the basic plans together with the relevant adjustment activities associated with the components of a row of the payoff matrix are feasible for the particular state of nature, a convex combination of the plans will similarly be feasible. Such combination therefore forms alternative plans for consideration according to the relevant criteria.

2.4 Problems of the Approach

The suggested approach clearly has a number of imperfections. As the plans and associated adjustments for each state of nature are not simultaneously selected and not directly chosen on the basis of the particular objective which might be applied to the payoff matrix, they are unlikely to be optimal for the particular objective. However, where it is assumed only one objective is relevant this could be used in phase one, and two in some cases. Where the objective used in the linear programming models is maximum net revenue, the system provides a number of probably new optimal strategies to choose between. Furthermore, it is assumed that some basic plan is followed through all states of nature with appropriate adjustment, whereas in the dynamic real world there will be cases where major adjustments through time

[6] Tadros and Casler [11] use a payoff matrix in this way though their payoff matrix is constructed without considering feasibility problems.

may be appropriate, particularly with respect to an individual farm's starting state for example, fodder reserves. This is, of course, one of the problems associated with using the results of a case study for general use. It should also be noted that the system makes no assumptions about changing the basic plan according to the predicted state of nature. Such predictions are, of course, impossible in general. However, while the adjustment processes are activated depending on how the season progresses there could be problems of premature adjustments.

The question of adjusting plans as outcomes are observed gives rise to the problem of Maruyama's [9] model. This model assumes the components of A, b and c are random variables and has constraints for each possible state of nature so that feasibility is ensured under all conditions. However, the real case of being able to make adjustments as the period progresses is not allowed for.

Where linear programming is used in the second phase there are $k^2 - k$ solutions to be obtained giving rise to an appreciable computing cost. In some cases, however, comparative budgeting may be sufficiently accurate to provide acceptable adjustment procedures for infeasibility problems.

3 AN EXAMPLE

The method was applied to a representative farm selected from an area close to Roma in Queensland. The purpose of the exercise was to produce information on alternative short-run basic plans which could then be used by extension workers in the area. The area is bisected by the 575 mm annual rainfall isohyet and experiences considerable rainfall variability from season to season and year to year. Eighty years of annual rainfall records indicate that the distribution of annual rainfall records is relatively flat. Similar observations apply to seasonal rainfall. As rainfall is a major determinant of productivity, it was considered necessary to allow for non-certainty in formulating plans.

The soils in the area exhibit a relatively high natural fertility. This factor combined with down type topography of the area means that a large number of production activities are possible. These include both summer and winter cash and feed cropping possibilities as well as a range of sheep and cattle activities. Given this range of production processes and the extreme rainfall variability, feasibility problems were considered to be an important consideration, particularly with respect to feed supplies and planting rains. This meant a simple linear programming approach was unlikely to be useful. Instead a two phase system involving the use of linear programming to determine the plans for each state of nature in the first phase and comparative budgeting to estimate the adjustments activities for other states of nature was used.

In order to select a representative farm to obtain input-output and resource information, a survey of all farms in the area was carried out. The farm selected was modal with respect to size, soil type, cultivable area and technical competence as reflected by crop yields.

Cash feasibility was not considered to be a major problem and the results were intended for use in the year following the analysis. Accordingly the variability of prices and costs around expected values would be small. For this reason this variability was largely ignored. Allowances for non-certainty in terms of defining the possible states of nature was, therefore, restricted to physical non-certainty. Some allowance was made, however, in the comparative budgeting for prices which are correlated with the season such as the cost of purchased hay.

Nine states of nature were recognized. The planning year was divided into two periods based on the times at which major decisions have to be made, particularly with respect to summer and winter cropping decisions. For each period three climatic states of good, average (which was in fact roughly the mode and median) and bad were recognized. Thus, as the type of season in each period was not correlated, this gave nine combinations of good/good, good/average, good/bad, etc., whole year climatic states.

The phase one linear programming matrix for the average/average state of nature was of order 96 x 149. The b and c vectors and the A matrices were different for each state of nature. The c vector consisted of gross margin components. Some components of b were varied as, for example, maximum limits on planting and harvesting areas were dependent on the type of season. The components of c were varied as, for example, the yields and prices of some activities were likewise dependent on the type of season.

The phase two procedure consisted of taking each phase one plan and for this estimating the total net revenue for each alternative state of nature. The constraint violations or surpluses were also estimated for each state of nature. Comparative budgeting was then used to determine what appeared to be the optimal way of making the plan feasible or to utilize surplus resources depending on the case. The cost (or return) of these adjustments was then added to the recalculated net revenue to give Π_s^t .

While it is not the purpose of this discussion to consider the detail of the plans determined,[7] it is interesting to consider the payoff matrix obtained. This is presented in Table 1.

The payoff matrix does not exhibit a completely regular pattern. This follows from the fact that the plans place a different emphasis on the general types of activities included so that some tend to dominate in better seasonal situations while being poor in others. An example of an apparent anomaly is given by the payoffs for the good/good state of nature. Plan x_4, for example, has a greater payoff than plan x_1, which is specifically designed for the good/good state of nature. The reason for this is that the high rainfall in this state of nature gives rise to harvesting and underground grain storage problems. The diagonal elements give the returns for the state of nature for which the plans have been specifically designed but clearly these are exceeded under some conditions. Using

[7] Appendix 1 contains some sample plans to indicate the type of plan variation.

TABLE 1

The Payoff Matrix ($100 *units*) *of Total Gross Margins*

Plan	State of nature (probabilities are in brackets)								
	Good Good (·07)	Good Average (·05)	Average Good (·12)	Average Average (·12)	Good Bad (·12)	Bad Good (·12)	Bad Average (·14)	Average Bad (·12)	Bad Bad (·14)
x_1	361	287	251	133	140	51	1	37	−54*
x_2	350	296	193	151	138	56	−3*	41	43
x_3	373	306	296	151	108	96	41	35	−51*
x_4	386	300	281	185	101	194	120	34	−54*
x_5	337	292	231	183	182	48	22	96	− 2*
x_6	284	195	231	93	48	167	51	−28	−83*
x_7	304	226	278	186	26	228	142	− 6	−50*
x_8	336	290	235	187	187	108	28	125	11*
x_9	190	164	144	113	118	102	94	84	41*

* Indicates minimum, for plan.

the conservative criterion of maximizing the minimum income, plan x_9 is preferred. But the higher minimum income is achieved at the expense of relatively low incomes in better seasons. Where fixed costs are high other plans worth considering would be x_2, x_5 and x_8 as their minimum incomes tend to be relatively high and they also have relatively high payoffs in good states. For many farmers a consideration of the chances of each season occurring would be important. For example, if the probability on the seasons average/bad and bad/bad were low the x_4 plan could be attractive as the two low incomes occur in these seasons.

On the basis of rainfall records probabilities were placed on the states of nature. These are given in Table 1. Where the low income possibilities are important useful information is the chance of a plan achieving a particular level of income or greater (similar to chance constrained programming). An example of this information is given in table 2, which also contains the expected income and standard deviation of the plans.

TABLE 2

Parameters of the Income Distributions of the Plans

Plan	Chance of achieving at least a given level of total gross margin		Expected income $	Standard deviation of income $
	$4,000	$12,000		
x_1	·60	·48	10,421	12,434
x_2	·86	·48	10,132	11,474
x_3	·74	·35	11,637	12,464
x_4	·74	·62	14,536	12,601
x_5	·72	·48	12,894	10,694
x_6	·74	·35	8,501	11,311
x_7	·62	·62	12,945	12,179
x_8	·72	·60	14,375	9,811
x_9	1·00	·23	10,731	3,561

The method of using this information will depend on the objectives. Two examples are:

(a) If achieving $4,000 was the primary aim plan x_9 would be chosen.

(b) Taking $12,000 as the desired level of income x_4 would dominate x_7.

Such figures, however, ignore other important parameters of the alternatives. Two of these are the expected income and standard deviation. Plan x_4 has the greatest expected income but also the highest standard deviation though this must be weighed against the reasonable chances of achieving the $4,000 and $12,000 levels. Considering all these parameters x_8 is also attractive, with its near optimal expected income but lower standard deviation.

The data presented indicate the useful range of information that can be estimated. But this is achieved, of course, at the expense of a considerable time input. In this example case nine linear programming solutions were obtained for constructing the payoff matrix though many more were obtained for other reasons. In most linear programming problems experimentation would normally require at least this number of solutions. If linear programming had been used in the second phase an additional seventy-two solutions would have been required. Thus it is important that careful consideration be given to the importance of the infeasibility problem and, therefore, whether the imperfections of the risk models discussed earlier could be accepted in view of the cost of overcoming them. Another possibility that should also be considered is the use of nine simplified two-stage stochastic programming models.[8] This is a model where decisions are determined in the first stage before the outcome of the random variables are known. In the second stage a set of decisions to ensure feasibility is determined for each possible state of nature, the two stages being solved simultaneously. This model suffers from size problems in a similar way to stochastic linear programming [10]. The approach presented in this paper achieves similar ends but reduces the size of each problem at the expense of more problems. Furthermore, the two-stage model does not produce a payoff matrix, the advantage of which is to allow a range of objectives to be used.

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[8] Wagner [13, p. 658] discusses such a model. Madansky [7] also discusses the two-stage model compared with the use of coefficient expected values.

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APPENDIX 1

A Summary of Optimal Plans for a Range of Example States of Nature

Plan details	State of nature		
	GOOD	AVERAGE	BAD
Native pasture (ha)	413	413	413
Continuous grain oats (ha)	40	..	..
Continuous grain sorghum (ha)	19	84	..
Continuous sunflower seed (ha)	113	40	..
Continuous grain wheat (ha)	138	45	78
Wheat (grain)—lucerne rotation (ha)	..	..	40
Crop area grazed (ha)	64	157	113
Wheaten Hay (ha)	16	..	44
Lucerne Hay (ha)	..	58	54
Silage—oats (ha)	..	..	76
Silage—sorghum (ha)	13	20	..
Merino ewes (fat lamb sire)	750	750	750
Merino replacements purchased	210	210	337
Xbred lambs—fat at 9 months	305	175	62
Xbred lambs—sold at stores	265	320	118
Cows—calves sold at 8 months	26	..	17
Replacement cows	6	..	21
Weaners crop fattened	48	30	38
Steers crop fattened	..	30	30
Steers grain fattened	30	30	..
Hay sold (tonnes)	80	80	80