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An Alternative Method for Deriving Optimal Fertilizer Rates

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A model of fertilizer response is outlined which makes a distinction between conventional fertilizer response curves, and the relationship between product yield and maintenance application of fertilizer. The derivation of optimal fertilizer rates for two enterprises on three soil types is used to illustrate the model. A simple rule-of-thumb, which can be used to avoid some computations, is also discussed. In the Australian context, the model has implications for the derivation of optimal superphosphate rates, and also has important implications for the type of applied superphosphate research which should be conducted in the future.

1 Introduction

Response surface analysis, integrating estimated response surfaces with the marginal principle of profit-maximization, is a standard theoretical tool for economists to analyse, *inter alia*, farm production decisions (*e.g.*, Dillon, 1977). The acceptance of this tool by non-economists is still relatively rare; the most commonly used farm management tool for guiding fertilizer decision-making, for example, still appears to be partial budgeting using a small number of point estimates of fertilizer response. An operational model of fertilizer response surface analysis for use at the farm level has been developed in Western Australia (Bowden and Bennett, 1974); this model ("Decide") has been generalized in the present paper.

The "modified Decide" model reported below has a number of advantages over its predecessors. Firstly, it better integrates the biology of fertilizer response with conventional response surface analysis. The modified model integrates a wider range of experimental data relating to fertilizer use and, by highlighting problems in integrating technical knowledge about fertilizers, provides important guidelines for future fertilizer research. Secondly, the modified model is operational at various levels of sophistication, ranging from full-scale application to a simple rule-of-thumb. Thirdly, the modified model, being a more satisfactory integration of biology and economics than its predecessors, is a useful teaching tool.

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This paper:

(a) examines two contributions to the analysis of fertilizer response against the background of traditional response surface analysis, and discusses the shortcomings of both the traditional analysis and these two contributions;

(b) proposes an alternative view of the integration of the biology of fertilizer response and response surface analysis, indicates how an analysis of optimal fertilizer use proceeds using this integrated approach, and illustrates the modified approach for different production systems and soil types; and

(c) shows how the model may be simplified in practice by using a rule-of-thumb.

2 Two Contributions to Fertilizer Response Analysis

In conventional response analysis, yield response Y to fertilizer input F is represented by a functional relationship

(1) $$Y = g(F)$$

Using marginal analysis, the profit-maximizing fertilizer input level is derived as that point where the differential of (1) is equated to the ratio of fertilizer price to output price; and, since fertilizer input and yield response are separated in time, a discount term reflecting time preference is generally included in the price ratio. However, as most fertilizers have residual effects, yield response is commonly expressed as a function of *available nutrient*, and the optimal level of nutrient is deflated by the estimated contribution of past fertilizer applications to current nutrient availability (*e.g.*, Anderson, 1967).

2.1 Bowden and Bennett's "Decide" model

The "Decide" model (Bowden and Bennett, 1974) was developed as an operational model of conventional response analysis. It defined a response curve in Mitscherlich form[1] in period t as

(2) $$Y_t = A(1 - B \exp(-C.F_t))$$

where yield Y_t and fertilizer application F_t have been made time dependent; A is maximum yield; $(1 - B)$ is the proportion of maximum response occurring at zero fertilizer input; and C influences the curvature of the response curve. The optimal fertilizer input is then determined by optimizing the net return (NR) equation[2]

(3) $$NR = Y_t.PY_t - (1 + R).F_t.PF_t + PF_t.(V.F_t + \Delta I_t.(1 + R))$$

where Y_t and F_t are as previously defined; PF_t is the cost of fertilizer as spread; PY_t is the price per unit of output net of all variable costs other than PF_t; V is the proportion of one year's applied fertilizer F_t available in the following year; ΔI_t is the depletion of initial fertility during the current response period; and R is the appropriate discount rate.

[1] See Bowden and Bennett (1974) for a discussion of the appropriateness of the Mitscherlich functional form; and also Heady and Dillon (1961), Dillon (1977) and footnote 5.

[2] The form of equation (3) has been altered from that of Bowden and Bennett (1974) by reducing all terms to common dollar terms at time of harvest (assuming fertilizer application occurs one year prior to harvest).

The residual effects of fertilizer are accounted for in two ways by this model. Firstly, the contribution of past fertilizer applications to current nutrient availability is estimated by the relation

$$(4) \qquad I_o = \sum_{t=-m}^{-1} F_t/(-t + 1)$$

where I_o is the amount of residual fertilizer becoming available in the current year, and m is the maximum period over which residual fertilizer effects occur.[3] The residual value I_o is used to deflate the current optimal fertilizer rate derived from the response curve. Secondly, if the summation in equation (4) is extended to $t = 0$, the residual effect next year (I_1) of a fertilizer history including this year's fertilizer application (F_o) may be calculated; this enables the future value of currently applied fertilizer to be determined as the reduction in future applications of fertilizer resulting from the carry over of current applications.

As can be inferred from equation (4), this year's fertilizer has residual effects for a sequence of future years $t = 1, \ldots, m$. The future value of total fertilizer residuals is the accumulated effect of these residuals on the value of production net of the cost of the residuals. The value of the residuals resulting from fertilizer applied this year may be derived from the effect of this year's fertilizer on the value of the existing residuals. The use of reductions in next period's fertilizer costs as a measure of the value of fertilizer residuals derives from the assumption that future fertilizer decisions are unaffected by residuals except that less fertilizer is applied in the future.

The "Decide" approach has two weaknesses. Firstly, the value of fertilizer residuals is considered only as the contribution to nutrient availability *in the following year*. However, even if no further fertilizer applications were made after the current year $t = 0$, residual effects of previous fertilizer applications on yield would be forthcoming not only in the following year $t = 1$, but also in subsequent years $t = 2, \ldots, m$. Further, there is a cost of having these fertilizer residuals "locked up" in the ecosystem; this cost is the opportunity cost of foregone investment of this "fertilizer capital" in alternative enterprises. Note also that the cost of fertilizer capital does not relate solely to the amount of fertilizer released in a subsequent year, but the total potential release of fertilizer in all subsequent years.

Secondly, ecosystems have an inherent tendency towards a balance of competing processes. In an earlier paper, this balance was referred to as an "equilibrium" (Helyar and Godden, 1977); a preferable terminology used in this paper is a "steady state" balance between yield and applied fertilizer, where there are continual flows of nutrient among pools in the ecosystem. The Bowden and Bennett formulation implicitly assumes a steady state balance between yield and applied fertilizer by only considering past and current fertilizer decisions; however the question as to whether or not the current steady state balance is appropriate, and how the optimum steady state balance should be calculated and achieved, cannot be easily derived from Bowden and Bennett (1974).

[3] We have preferred the slightly clumsy notation $t = -m, \ldots, -1$ to represent past fertilizer applications since we will later extend the notation to current and future years—*viz.*, $t = 0, 1, \ldots, m$.

2.2 Dynamic programming approach

An alternative approach to analysing fertilizer use concerning sorghum ratoons was reported on the Ord River (Kennedy *et al.*, 1973). The authors proposed a simple residual value function:

(5) $I_{t-1} = V_t.Q_t$

where Q_t is the amount of fertilizer (currently applied plus residual) available when there are t periods before the end of the production process; and V_t is the proportion of available fertilizer that is carried over into the following period, *i.e.*, when there are $(t - 1)$ periods until the end of the production process.

The authors derived a recurrence relation from which the optimal allocation of fertilizer can be found. This relation (in our notation) is

(6) $f_t\{I_t\} = \overset{max}{Q_t} (R.PY_t.Y_t\{Q_t\} - PF_t.(Q_t - I_t) + R.f_{t-1}\{V_t.Q_t\})$

where $f_t\{I_t\}$ is the return from following an optimal fertilizer policy in each of t future periods given that current residual fertilizer is I_t; yield $Y_t\{Q_t\}$ in time t is a function of available fertilizer Q_t; R is the time preference discount factor; and PF_t and PY_t are fertilizer and output prices, respectively. By induction, the optimal fertilizer rate in a period where there will be t future fertilizer decisions is derived from

(7) $dY_t/dQ_t = (PF_t - R.V_t.PF_{t-1})/R.PY_t$

This formulation partially overcomes the two problems of the Bowden and Bennett (1974) approach discussed above. Firstly, by analysing fertilizer application through a decision horizon extending t periods into the future, the dynamic programming approach accounts for the value of fertilizer residuals beyond the year succeeding the current application. However, the dynamic programming formulation does not account for the cost of maintaining eco-system fertilizer stocks from which the fertilizer residuals are derived. Note further, that the simple solution to the problem of deriving optimal fertilizer rates represented by equation (7) is dependent on the particular form of the residuals equation (5); if the latter is not an appropriate residual value function, a simple solution to equation (6) as represented by equation (7) may not be forthcoming.

Secondly, the dynamic programming formulation avoids the issue of the ecosystem's movement towards a steady state by specifying the existence of a final period in the fertilizer decision horizon beyond which fertilizer residuals can be ignored. However, specification of the appropriate period beyond which these residuals are negligible requires a model by which that time period may be determined.

2.3 Summary

Three related issues not satisfactorily integrated by previous analyses of fertilizer response are:

(i) the value of fertilizer residuals as a function of the total contribution of these residuals to future response;

(ii) the (opportunity) cost of maintaining a fertilizer stock in the eco-system giving rise to residuals; and

(iii) the economic effects of the approach of the nutrient system towards a steady state balance in the ecosystem.

The general framework of Bowden and Bennett's model has been used to develop the model outlined below. The discussion begins with an outline of a more comprehensive synthesis of biological and economic theories of fertilizer response. Detailed discussion of the biological basis for the model has appeared elsewhere (Helyar and Godden, 1976, 1977; *c.f.* Battese, 1978; and Helyar and Godden, 1978), and hence will be treated only briefly here.

3 Modified Model of Fertilizer Response

Application of fertilizer to a soil-plant system results in applied nutrient being used in three basic biological processes:

(i) available nutrient is lost from the system as current production is increased through the absorption and utilization of increased quantities of available nutrient, and as losses such as leaching, runoff and erosion occur;

(ii) there are inorganic chemical conversions of available nutrient to temporarily unavailable inorganic ("fixed") forms; and

(iii) there are organic processes which convert currently available nutrient to temporarily unavailable organic forms.

Because processes (ii) and (iii) are reversible, slow conversion of temporarily unavailable nutrient to available forms also occurs.

The quantitative relationships between applied nutrient, and production losses and soil pools, are also of three basic types:

(a) in an agricultural "development" phase, the rate of fertilizer application results in increased yields, and build-up of organic and inorganic nutrients in the various forms involved in the biological cycle. The build-up of currently available and unavailable nutrient forms (or soil reserves) is a capital investment process;

(b) fertilizer application rates may exactly equal the loss of nutrients in product removal, leaching, erosion, and losses to non-cycling pools. Such fertilizer applications are conventionally called "maintenance" applications and occur where a steady state balance between inputs and losses occurs;[4] and

(c) the rate of fertilizer application may be less than the rate at which soil nutrients are lost from the ecosystem. This constitutes a capital depreciation process since the total quantity of nutrients in the biological cycle is being depleted with time.

3.1 Fertilizer response curves

A fertilizer response curve associates product yields with a range of fertilizer rates applied to areas of identical base fertility. Such fertilizer response curves include aspects of each of the three types of quantitative relationships outlined in (a)–(c) above. That is, if a non-virgin soil is assumed, "low" rates of fertilizer application will not only result in low associated yields as calculated from the response curve, but also result in depreciation of fertilizer stocks in the ecosystem. Similarly, the "maintenance" fertilizer rate will give steady state

[4] In Helyar and Godden (1977) this concept was referred to as "equilibrium balance". As outlined in the text (Section 2.1), we now prefer the terminology "steady state", since it better communicates our concept of balance between yield and applied fertilizer where there are continual flows of nutrient among pools in the ecosystem.

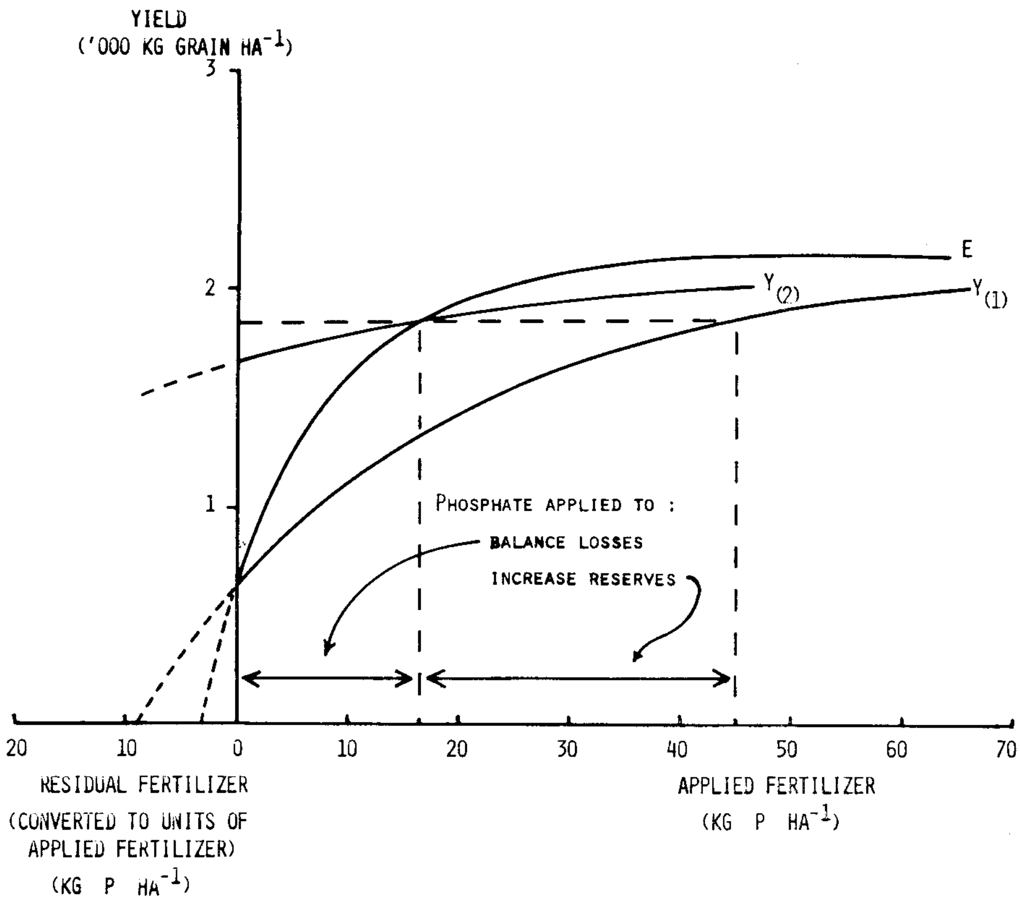
product yields, exactly balance nutrient losses and not change the level of the fertilizer stocks; and "high" fertilizer rates will have high associated yields as determined from the response curve, and also result in investment in fertilizer stocks in the ecosystem.

To depict fertilizer response curves, Bowden and Bennett's use of the Mitscherlich curve has been retained[5], but with their constant B (*c.f.* equation (2) above) shown as an explicit fertilizer input. This modified representation of fertilizer response is

(8) $Y_t = A(1 - \exp(-C(F_t + I_t + b)))$

where b represents the non-depletable soil nutrient status (or the steady state nutrient status without fertilizer application). All other terms are as previously defined. Conventional response curves $Y_{(1)}$ and $Y_{(2)}$ are shown in Figure 1.

Figure 1: *Response and Maintenance Requirement Curves*



[5] Our choice of the Mitscherlich functional form should not be inferred as implying that it is the only possible functional form for fertilizer response curves. We have chosen the Mitscherlich only for convenience; other functional forms for representing fertilizer response may be used equally well with our concepts. Additionally, our analysis is strictly deterministic because we feel that the need for stochastic analysis only arises from (a) our inability to adequately describe the system; and/or (b) our inability to predict future events. Our belief is that stochastic analysis should only be imposed upon our best deterministic understanding of the soil-plant animal-fertilizer system; whereas our paper attempts to improve our deterministic understanding of fertilizer response.

3.2 Residual value function

Equation (4) above usefully approximates residual fertilizer effects in a range of agricultural production processes (Bowden and Bennett, 1974). A more generalized residual value function which may be fitted to a wider range of production processes is

$$(9) \qquad I_t = \sum_{i=t-m}^{t-1} (a.F_i/(t - i + a))$$

where m is the period over which residual fertilizer effects occur and a is a characteristic value related to fertilizer, soil type and the type of production.[6]

This function describes the way fertilizer applications lose effectiveness with time. For a particular fertilizer, different values of a denote different soil types and production systems. The functional form is a compromise between the simplicity of equation (4) and better but more complex representations of nutrient behaviour in the ecosystem. It may be desirable to model in more detail the different ways in which applied nutrients are used or lost—for example, Russell (1977) separately modelled losses, and nutrient redistribution toward less available, more slowly cycling pools—but such detail is not essential to the objective of the present paper.

3.3 Maintenance requirement curve

Economic analysis of fertilizer response curves allows the determination of optimal current fertilizer applications while taking into account nutrient inputs from soil reserves. However, it is also desirable to determine the optimal level and timing of investment in fertilizer stocks in order to plan fertilizer development, or disinvestment, programmes.

The calculation of investment in fertilizer stocks requires determination of the difference between a current fertilizer application, and losses from biologically cycling reserves by product removal, leaching, erosion, *etc*. Since strictly speaking, all nutrient in the system is cycling at some rate, an arbitrary separation has been made between cycling and non-cycling nutrient. Fertilizer residuals remaining physically in the system are regarded as having been lost to non-cycling pools once their residual effect on plant yields becomes negligible.

As defined above, losses from the system are those that occur when there is neither investment nor disinvestment in stocks of fertilizer in the ecosystem. That is, if true *maintenance* fertilizer rates (the long term fertilizer rates required to sustain steady state yield levels) are related to corresponding yields, the relationship between fertilizer losses and yield is obtained. This yield relationship is denoted as the "maintenance requirement curve".

While the maintenance requirement curve exists in the same yield—fertilizer space as the response curve, the maintenance requirement curve is a logically separate concept from that of a response curve. The latter relates yield and applied fertilizer given a common initial fertility level. A maintenance requirement curve, however, connects one unique point on every response curve; it is the locus of points on each response curve where there are no net changes in fertilizer stocks.

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[6] A potential criticism of the functional form of equation (9) is that it is not "level dependent" (Battese, 1978: *c.f.* Helyar and Godden, 1978). Clearly, however, equation (9) is dependent on a series of past fertilizer rates and thus past yield levels through the factor F_i: $i = t - m, \ldots, t - 1$. Note that equation (9), like the comparable residual value functions—equation (4) in text (from Bowden and Bennett, 1974), and equation (5) in text (from Kennedy *et al.*, 1973)—do NOT purport to measure the amount of fertilizer *held* in the ecosystem but, rather, the amount of fertilizer in the ecosystem that is *available for use* by the plant in some specified period.

As previously indicated (Helyar and Godden, 1977, p. 26), empirical estimates of any two of the three fertilizer relationships—*viz.*, response curve, maintenance requirement curve and residual value function—enable estimation of the third. Battese (1978) illustrated this for the derivation of a maintenance requirement curve from the other two relationships. The possibility of deriving one of these three fertilizer relationships from knowledge of the other two does not mean the third relationship is redundant. There are three arguments supporting the usefulness, in particular, of the concept of a maintenance requirement curve.

Firstly, the dynamics of the system can be represented on a single diagram—that is, the various levels of the response curve through time can be shown together with their unique steady state points on the maintenance requirement curve. Secondly, the maintenance requirement curve is more readily identifiable with a text book response curve since there are no net changes to stock levels of fertilizer inputs held within the production system. Thirdly, in association with a further assumption as to how to estimate actual nutrient losses (see below), it is possible to calculate net additions to the stock of fertilizer held in the ecosystem but not used for current production.

The relationship between steady state yield and maintenance fertilizer rate (the maintenance requirement curve) has also been modelled by a Mitscherlich curve[7] of the form

(10) $$E = A\,(1 - \exp(-\gamma\,(FV + \varepsilon)))$$

where E is steady state yield; A is as previously defined; FV is the maintenance fertilizer requirement for the corresponding steady state yield; γ influences curvature[8]; and ε represents the non-depletable soil nutrient status[9]. A maintenance requirement curve, E, is shown in Figure 1.

A number of experimental techniques for estimating maintenance requirement curves were discussed in Helyar and Godden (1977). It should also be noted that, where maintenance requirement curves are estimated from the response curve and residual value function, the calculated values of the coefficients of the maintenance requirement curve may vary according to the simplifying assumptions used.

3.4 Determining investment in fertilizer stocks

In order to distinguish additions to stocks of fertilizer in the ecosystem from current use by plants and other losses of applied fertilizer, a major simplifying assumption is invoked—*viz.*, that losses of fertilizer from the production system are that function of yield described by the maintenance requirement curve *whether or not the system has reached a steady state balance.*

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[7] See footnote 5.

[8] Problems arise in the empirical determination of γ. The time taken for a steady state relationship to be achieved between yield and an actual maintenance fertilizer rate may be very long. In our previous paper (Helyar and Godden, 1977) we drew a distinction between the true value of γ for a system, and empirical estimates of γ which assumed that a steady state was reached after n years—*viz.*, γ_n. In the present paper we have assumed that γ and γ_n are identical.

[9] Since there is a unique steady state point on each response curve, there is a unique simultaneous solution to equation (8) and (10) of the form $F_t = (C.(I_t + b) - \gamma.\varepsilon)/(\gamma - C)$, since $F_t = FV_t$ at the steady state point, and each of the elements on the right-hand side are either constants (C, b, γ, ε), or a unique response curve parameter (I_t).

At a yield level Y_t in time t, with corresponding response curve and maintenance requirement curve fertilizer rates F_t and FV, respectively, additional investment in fertilizer stocks at time t (ΔFC_t) is assumed to be

(11) $$\Delta FC_t = F_t - FV_t$$

Investment in fertilizer, as opposed to losses from the system, is also illustrated in Figure 1. By solving equations (8)–(11) simultaneously, the stock of fertilizer (FC_t) held in the ecosystem *at steady state* can be derived (Helyar and Godden 1977, Appendix 1)[10] as

(12) $$FC_t = FV_t.(t - t.C/\gamma - (C/\gamma)(\sum_{i=1}^{t} \sum_{m=1}^{i-1} a/(m + a)))$$

The term in outermost brackets is a constant for any production system and, since it relates maintenance fertilizer rates to fertilizer investment, it has been denoted the Maintenance Requirement Multiplier (MRM).

4 Economic Analysis Using Modified Model of Fertilizer

4.1 Analytical framework

Assuming profit maximization in a non-stochastic world,[11] optimal fertilizer decisions can be determined in a multiperiod model by considering yield response, actual fertilizer losses from the system and changes in fertilizer stocks. These effects are summarized in the following equation for Net Returns (NR)

(13) $$NR = \sum_{t=1}^{n} ((Y_t.PY_t - (1 + R).PF_t.FV_t - R.PF_t.FC_t)/(1 + R)^t)$$

The first term in the summation is gross return in period t; the second term is the variable cost of fertilizer in period t; the third term is an interest charge on stocks of fertilizer in the ecosystem in period t; and the divisor is a conventional discount factor. Monetary values within a time period are discounted to values at the time of first fertilizer application.

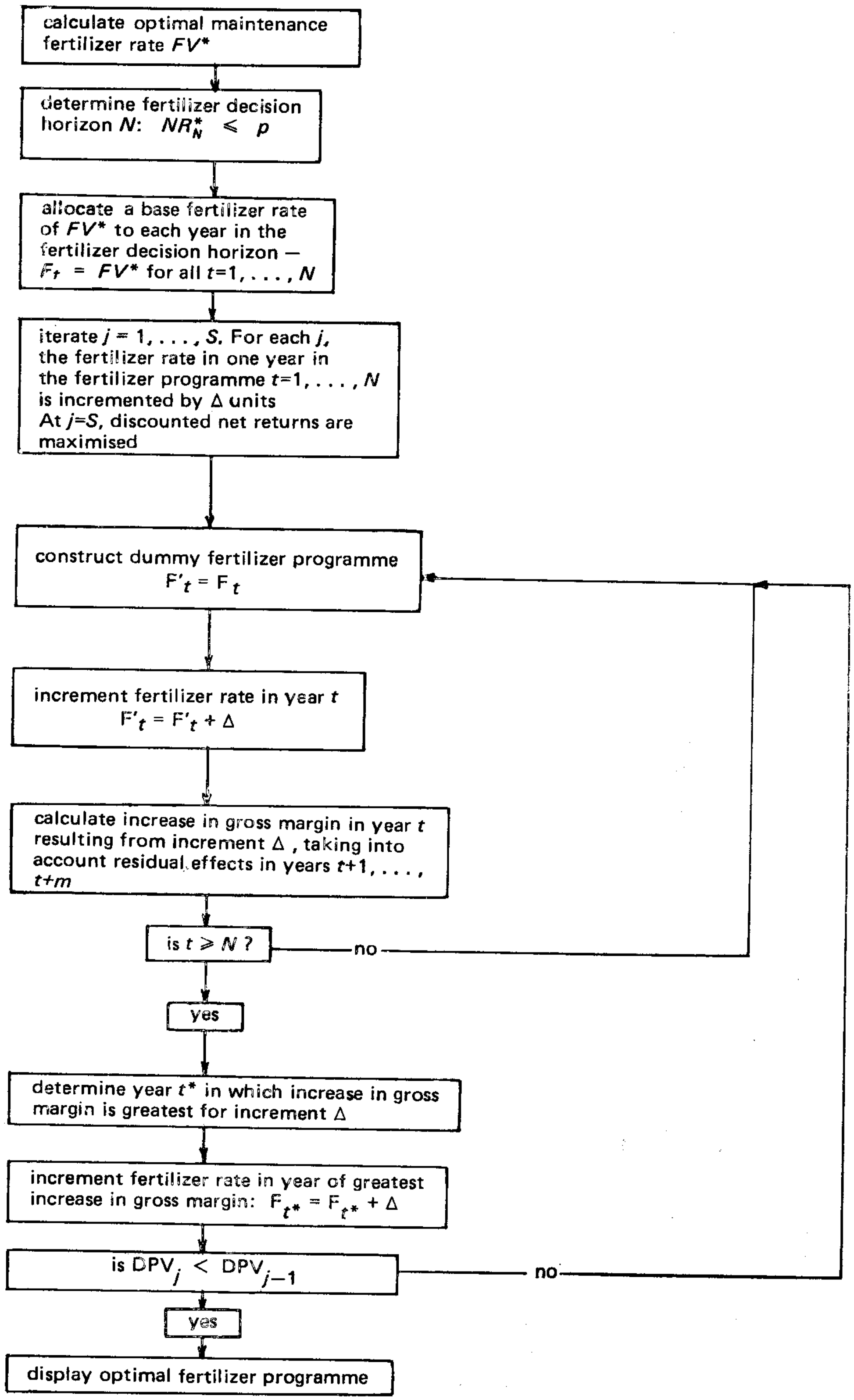
Since Y_t is a function of current and previous fertilizer rates, analytic solutions to equation (13) are complex, and it is easier to iterate solutions for varying rates of fertilizer investment. An outline of the method used for iteratively deriving an optimal solution to the fertilizer decision problem of equation (13) is shown in the flowchart of Figure 2 and was developed for a mainframe computer; similar programmes have been run on mini-computers. The optimal *maintenance* fertilizer rate was estimated using conventional maximising calculus techniques on the term *inside* the summation sign of equation (13), where the yield relation Y_t was replaced by the maintenance requirement curve E. It was assumed that the level of fertilizer stocks once steady state is achieved can be approximated by equation (12). Thus net return at steady state ($NRSS$) was optimized by the following relation

(14) $$NRSS = E.PY - (1 + R).PF.FV - R.PF.FV.MRM$$

[10] The method of analytically deriving fertilizer stocks outlined in our previous paper uses a mathematical trick of assuming approach to steady state at a constant fertilizer rate. For reasons we do not entirely understand, the level of fertilizer investment FC_t is dependent on the method used to approach the steady state—*i.e.*, the method used to estimate the maintenance requirement curve if empirical estimates are not available. In our empirical analysis, we initially estimated a steady state optimum by assuming an investment in fertilizer stocks derived from equation (12). In computing the actual optimum by marginal increments in fertilizer application, we used equation (11).

[11] *c.f.* last two sentences of footnote 5 above.

Figure 2: Flowchart for determining optimal fertilizer programme



for some value $FV = FV^*$, where E is the maintenance requirement curve defined in equation (10). The variables in equation (14) do not have a time dimension since the estimation was of an optimum at steady state.

In order to determine the length of time over which the fertilizer decision was to be optimized, a value $t = N$ was chosen for which Net Return in that period (NR_t^*), defined by

$$(15) \qquad NR_t^* = (E^*.PY - (1 + R).PF.FV^* - R.PF.FV^*.MRM)/(1 + R)^t$$

was less than some arbitrary value, $p: p \rightarrow 0$. The yield E^* was the steady state yield corresponding to the previously estimated optimal fertilizer rate FV^*.

An initial fertilizer programme was established for which each year had a fertilizer rate set at the estimated optimal fertilizer rate estimated from equation (14). That is, initial values $F_t = FV^*$ were chosen for all $t = 1, \ldots, N$.

An iterative routine was followed which selected the year in which an incremental fertilizer unit of size Δ maximized the increase in net returns defined by equation (13). This iterative routine was continued for $j = 1, \ldots, S$. At $j = S$, equation (13) was maximized and $S.\Delta$ units of fertilizer were included in the fertilizer programme *in addition* to the initial allocation of $F_t = FV^*$ for $t = 1, \ldots, N$.[12]

4.2 Empirical application

Data required to derive optimal fertilizer strategies for three different production systems are shown in Table 1. Response data for a Northern Territory sorghum enterprise are taken from Arndt and McIntyre (1963); response data for a New England fat lamb enterprise have been collated from a number of experiments (Helyar, unpublished). A history of no previous fertilizer application was assumed; calculation of the residual value function in year t used the previous 30 years' fertilizer applications. Price data were estimates for each production system in 1977 and a discount rate of 10 per cent was used.

Results of applying the method of deriving optimal fertilizer rates described in the previous sections to these sets of data are shown in Table 2. These results describe the optimal fertilizer programmes for the three production systems.

[12] Three assumptions implied by the iterative approach outlined in the text are—

(a) the optimum maintenance level is also the long run optimal allocation no matter what the initial position;

(b) that the objective function has a global optimum, so that the proposed search algorithm will not simply find a local optimum; and

(c) that all application levels should be at least as large as the maintenance level.

In empirical applications, assumptions (a) and (c) were tested by assuming different initial fertilizer histories, and determining at each iteration step whether the objective function could be increased by reducing any of the annual fertilizer rates constituting the optimal fertilizer programme at that iteration. These two assumptions were not shown to be invalid. A crude test of the existence of a global optimum was made by searching for an optimal solution beginning with an initial programme of fertilizer rates significantly less than the estimated optimal long run maintenance fertilizer rates. For each production system described in Table 1, the optimal fertilizer programme was the same regardless of the initial levels of the fertilizer programme.

The computing required to prepare this footnote very nearly validated Dillon's Third Law of Simulation: "Once started, simulation will continue until available funds are exhausted."

Table 1: Production systems evaluated using the model

	New England fat lambs [p]		Northern Territory sorghum[q]
	Basalt	Granite	Laterite
Fertilizer data—			
Response curve—			
A (production/ha)[r]	12.0	12.0	2,129.0
b (kg of P/ha)	3.2	2.24	8.8
C (ha/kg of P)	0.03	0.05	0.04
Maintenance requirement curve—			
γ (year/ha/kg of P)	0.16	0.16	0.13
ε (kg of P/ha/year)	0.7	0.7	2.7
Residual values—			
a (unitless)	1.6	0.8	0.8
MRM (unitless)[s]	6.3	3.8	4.2
Price data—			
PY (\$/unit of output)[u]	12.0	12.0	0.08
PF (\$/kg of P)	0.83	0.83	0.83
Interest rate—			
R (%/100)	0.1	0.1	0.1

[p] Helyar, K. R. (unpublished).
[q] derived from Arndt and McIntyre (1963).
[r] fat lambs: ewes/ha; sorghum: kg of grain/ha.
[s] calculated over 30 years (*i.e.*, $t = 30$ in equation (12)).
[u] fat lambs: \$/ewe; sorghum: \$/kg of grain.

The most interesting feature of Table 2 is that there is little deviation of optimal yield levels throughout the fertilizer programme from the initial estimate of the optimal steady state yield. This result was examined in more detail for the fat lambs on basalt enterprise, for which results are reported in Table 3. As suggested in that Table, wide discrepancies between optimal yields at steady state calculated via the maintenance requirement curve (equation 14) and using the full simulation approach (outlined in Figure 2) only occur at high input:output price ratios; that is in the more steeply sloping section of the maintenance requirement curve.

4.3 Suggested rule-of-thumb

The results presented in Tables 2 and 3, together with the recognized insensitivity of yields and net returns to "sub-optimal" fertilizer rates (Anderson, 1975; Jardine, 1975), suggest that the following method may be used to approximate optimal fertilizer programmes:

(i) estimate the optimal maintenance fertilizer rate (as outlined in the discussion surrounding equation (14) above);

(ii) calculate the corresponding steady state yield level; and

(iii) calculate the fertilizer rates from the appropriate response curves which result in this steady state yield level. These fertilizer rates will *approximate* the optimal fertilizer programme.

This approach may be summarized as the following "rule-of-thumb": "fertilize in each year of the programme to achieve the optimal yield derived from the maintenance requirement curve".

Table 2: Optimal fertilizer programmes for three production systems

	New England fat lambs				Northern Territory sorghum	
	Basalt		Granite			
Optimal maintenance fertilizer rate (kg P/ha/year)	16.2		17.6		19.3	
Optimal steady state yield	11.2[a]		11.4[a]		2 008[b]	
Length of fertilizer decision programme (N, years)	34		34		36	
Optimal discounted net return ($)	1,161		1,210		1,451	
Years	Fertilizer rate	Yield	Fertilizer rate	Yield	Fertilizer rate	Yield
1	86.2	11.2	57.6	11.4	66.3	2 015
2	36.2	11.3	27.6	11.3	34.3	2 008
3	26.2	11.2	27.6	11.4	29.3	2 008
4	26.2	11.2	22.6	11.3	24.3	1 997
5	21.2	11.2	22.6	11.3	24.3	2 001
6	21.2	11.2	22.6	11.4	24.3	2 008
7	21.2	11.2	17.6	11.3	24.3	2 015
8	21.2	11.2	22.6	11.4	24.3	2 022
9	16.2	11.1	17.6[e]	11.3[f]	19.3[g]	2 006[h]
10	21.2	11.2				
11	16.2	11.2				
12	21.2	11.3				
13	16.2[c]	11.2[d]				

[a] Yield in ewes per ha.

[b] Yield in kg of sorghum per ha.

[c] Optimal fertilizer rate for years 13–34.

[d] Mean Absolute Deviation from optimal steady state yield (years 13–34) = 0.06 ewes per ha (0.5 per cent).

[e] Optimal fertilizer rate for years 9–34.

[f] Mean Absolute Deviation from optimal steady state yield (years 9–34) = 0.09 ewes per ha (0.8 per cent).

[g] Optimal fertilizer rate for years 9–36.

[h] Mean Absolute Deviation from optimal steady state yield (years 9–36) = 16.8 kg per ha (0.8 per cent).

In the sorghum enterprise for which the optimal fertilizer programme is presented in Table 2, fertilizer rates for the first seven years calculated by the rule-of-thumb are 62.9, 34.9, 29.4, 26.6, 24.9, 23.5 and 22.7 kg of phosphorus per hectare per year. While not identical to the rates shown in Table 2, the rule-of-thumb fertilizer rates are similar and have a similar pattern over time.

Table 3: Mean absolute deviation of yields in optimal development programme as a percentage of optimal steady state yield[a]

R	*PY, PF*		
	6, 1.66	12, 1.66	12, 0.83
0.05	6.6	4.2	2.6
0.10	4.8	4.8	2.9
0.20	8.1	4.4	2.4

[a] To reduce computing costs, a fertilizer horizon of 15 years was assumed, compared to previous analyses where a 30 year fertilizer horizon was assumed.

5 Summary

A model drawing upon two approaches for determining optimal fertilizer use has been developed by a re-examination of the concept of fertilizer response. The distinction between short-term response (one-period response curve) and steady state response (maintenance requirement curve) has been shown to be useful. Short-term and steady state response have been combined in a biologically meaningful model and used to estimate optimal fertilizer rates. The model can be used at two levels—using relatively sophisticated computational facilities, or using a simple rule-of-thumb.

The model developed above is also capable of being used in standard extensions of the profit maximizing approach. For example, the case of limited finance or the integration of fertilizer decisions on a number of different sites can be handled. Additionally, stochastic decision models could also be used in conjunction with the basic concepts of the model.[13]

The usefulness of the model suggests that agricultural economists should be concerned with maintenance requirement curves as well as with conventional one-period fertilizer response curves, and should encourage agricultural scientists both to develop experimental techniques for estimating maintenance requirement curves, and to estimate such curves for major agricultural systems. Estimation of maintenance requirement curves necessitates long-term experiments so that agricultural systems can be observed at steady state.

[13] Fertilizer decision-making with limited finance or where there are a number of sites with different characteristics has been incorporated into the operational programme of this model available at the Agricultural Research Centre, Wollongbar. Climatic or other variability could be incorporated into the response and maintenance requirement curves and the residual value function.

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