A Case Study of Cultivation of Teachers’ Teaching Ability in the Context of New Era

Hong FANG *

College of Basic Sciences, Tianjin Agricultural University, Tianjin 300384, China

Abstract For young teachers, the ability to transform teaching materials and the ability to control teaching is a basic teaching regulation and control ability. Through a case teaching of function limits in the mathematics course, this article studied how to explore deep into the content of textbooks, use the textbooks in a creative and proper manner, help students better understand and master the content of teaching, improve students’ autonomous learning ability, and promote comprehensive improvement of students’ knowledge, ability and caliber.

Key words Transformation ability of teaching materials, Teaching regulation and control ability, Case teaching

1 Introduction

In the Opinions of the Ministry of Education on the Implementation of First-class Undergraduate Courses, it stated that course is the core element of talent cultivation, while the quality of the course directly determines the quality of talent cultivation. Course teaching is not simply imparting knowledge, but a combination of knowledge, ability and quality. Teachers’ teaching should not just copy the textbooks. Instead, teachers should go deep exploration of the content of the textbooks, to refine the teaching content that meets the students’ cognitive level, and through the teaching of the course content, let the students understand the cutting-edge and era of the subject development, improve their comprehensive ability and advanced thinking to solve complex problems[1]. The main field of first-class undergraduate course construction is classroom teaching. The design of course teaching activities should conform to the direction of student-centered course teaching reform, focus on stimulating students’ learning interest and learning potential. In the teaching process, teachers should not instill knowledge into students, but students actively construct knowledge. Besides, teachers’ control and treatment of teaching contents, regulation of teaching process and organization and management of classroom teaching activities are particularly important.

2 Problems to be solved in the case teaching

Understanding and mastering the language of function limits is a difficult point for students. $\varepsilon - \delta$ For the students’ transition from the intuitive definition of the limit to the abstract concept, there should be a bridge. How can the teacher extend the abstract concept from the specific problem reflects the teacher’s ability to transform the teaching materials and the processing content of teaching contents. Besides, the process of demonstrating mathematical formulas, theorems, and concepts contains important methods of thinking. Teachers should not simply impart knowledge in accordance with the original textbooks. They should demonstrate their thinking processes and allow students to master new knowledge, learn to actively establish a knowledge system, and cultivate the spirit of actively seeking new knowledge.

3 Teaching mode of the case teaching

3.1 Encouraging students to learn with questions As regards the understanding of the concept language of functional limits $\varepsilon - \delta$, teachers may set the following questions for students to preview. (i) What is the relationship between $\varepsilon$ and $\delta$ in functional limit concept? How does it differ from the relationship between $\varepsilon$ and $N$ in the definition of the sequence limit? (ii) What is the difference between the geometric shape when the function limit exists and the geometric shape when the sequence limit exists? (iii) Understanding of certain sentences (or formulas) in the definition of function limits.

3.2 Teaching and learning process of the case teaching

3.2.1 Exploring new knowledge. We have learned how to describe the two infinite approximation processes $x \to \infty$ and $f(x) \to A$ with precise mathematical language and formulas. How should we define the limit processes $x \to x_0$ and $f(x) \to A$? Starting from specific problems, this lesson would guide students to summarize, understand, conclude the concept of function limits during the learning process.

Teacher: for function $f(x) = \frac{x^2 - 9}{x - 3}$, students are required to draw a graph of function $f(x)$ and think based on the graph (i); why the function $f(x)$ is undefined when $x = 3$, but there is a limit when $x$ is infinitely close to $3$? (ii) How to understand $f(x) \to 6$, when $x \to 3$? When $x \to 3$, $f(x) \to 3$, it means that with $x$ being infinitely close to $3$, the distance between $f(x)$ and $6$ is infinitely small.

Students: through observing the picture of function $f(x)$, it
found that when $x$ is closer to 3, the corresponding function value $f(x)$ is closer to 6; when $x$ is infinitely closer to 3, the corresponding function value $f(x)$ is infinitely closer to 6, but when the function $f(x)$ is undefined at $x = 3$? How to understand this question?

Teacher: the limit of function $f(x)$ limit when $x$ is close to 3 refers to seeking variation trend of $f(x)$ when $x$ is infinitely close to 3, rather than to calculating the function value of the function $f(x)$ when $x = 3$. Therefore, it is not connected with the function $f(x)$ at $x = 3$. This indicates that it is only required that the function $f(x)$ is defined in a punctured neighbourhood of $x = 3$.

Students: we can measure the degree of $x$ approaching 3 using the distance formula $|x - 3|$, and measure the degree of $f(x)$ approaching to 6 using $|f(x) - 6|$, what is the connection between the two?

Teacher: This question is a good question. How to establish a relationship between the two? We found that to realize $|f(x) - 6| = \frac{x^2 - 9}{x - 3} - 6 < 0.01$, as long as $0 < |x - 3| < 0.01$. In other words, when $x$ falls a punctured neighbourhood $x_0 = 3$ with the radius as 0.01, it is able to guarantee the corresponding function value $f(x)$ satisfies $|f(x) - 6| < 0.01$. To realize $|f(x) - 6| = \frac{x^2 - 9}{x - 3} - 6 < 0.001$ as long as $0 < |x - 3| < 0.001$. In other words, when $x$ falls a punctured neighbourhood $x_0 = 3$ with the radius as 0.001, it is able to guarantee the corresponding function value $f(x)$ satisfies $|f(x) - 6| < 0.001$.

Students: in the case (1), as long as taking the radius $\delta < 0.01$ of the punctured neighborhood of $x_0 = 3$, it is able to satisfy the requirement; in the case (2), as long as the radius $\delta < 0.001$ of the punctured neighborhood of $x_0 = 3$, it is able to satisfy the requirement.

Teacher: very good, please think that if given an arbitrarily small positive number, can we find the positive number $\delta$?

After inspiring students to think, it could be concluded that as the given positive number decreases, a positive number $\delta$ can always be found to satisfy the condition; that is, given a positive number $\varepsilon$ no matter how small, a positive number $\delta$ always exists, so when $0 < |x - 3| < \delta$, the inequality $|f(x) - 6| < \varepsilon$ holds. This is the essence of function $f(x)$ being infinitely close to 6 when $x \to 3$. Finally, the teacher guides the students to give a precise definition of the limit of the function.

### 3.2.2 Further exploration of teaching contents.

(i) After giving the definition of $\varepsilon - \delta$ of the function limit, ask students to continue their research in small groups; in $\varepsilon - \delta$ language, a positive number $\varepsilon$ is used to describe the closeness of $f(x)$ and 6, and a positive number $\delta$ is used to describe the closeness of $x$ and 3. Give a $\varepsilon$, we can find a $\delta$, then is the value of $\delta$ only value? (ii) How many cases does $x \to x_0$ in the function limit include? After careful discussions, students reached the conclusions that: The value of $\delta$ is not only, because the selection of $\delta$ depends on $\varepsilon$; $x \to x_0$ includes two cases, $x$ can approach to $x_0$ from the left side of $x_0$, and it can approach to $x_0$ from the right side of $x_0$. Therefore, it should be divided into left and right limit forms, which is different from the sequence limit.

#### 3.2.3 Application examples:

Proof: $\lim_{x \to x_0} \sin x = \sin x_0$.

Analysis: through the previous learning process, it was found that $\delta$ depends on $\varepsilon$, that is, given a positive number $\varepsilon$, then find a positive number $\delta$, so the "backward inference method" can be used in the proof. In the solution of specific problems, in order to make the calculation of the positive number $\delta$ simple, the technique of simplification or application of $|f(x) - A| < \varepsilon$ can be converted into a function $f(\frac{x - x_0}{1} < \varepsilon$ can be solved to find the positive number $\delta$ meeting the conditions.

Students: the amplification method has an influence on the selection of positive number $\delta$.

Teacher: this question examines the students' understanding of $x \to x_0$ in the concept of function limits. Please give answers after the group discussion.

### 4 Teaching introspection

In this paper, we discussed how to guide the teaching content in the teaching materials step by step through the teacher, so that students can approach the essence of the problem they are seeking step by step, and experience the process of knowledge construction. The teacher’s teaching is not simply imparting the content of the teaching materials into the students. Instead, teachers teach students efficient learning methods, ponder over the essential way of thinking through phenomena, and guide students to autonomously analyze, summarize, understand, and learn how to solve abstract problems.2 Teachers provide exercises that define the limits of the proof function to test students’ ability to use what they have learned to solve specific problems. We hope that through this form of teaching, students can interact with teachers in the classroom, dare to question, and dare to express their views, and cultivate the independent thinking ability, dare to criticize and question, and have innovative ability and innovative thinking. The overall construction of first-class undergraduates should attach importance to "making courses better, teachers stronger, students busier, management stricter, and results more effective", so as to establish a high level talent cultivation system. Therefore, it is recommended to improve teaching ability of teachers, and better guide their teaching work. Teachers should make effort to comprehensively improve students’ knowledge, ability, and caliber.

### References
