THE AUSTRALIAN WOOL CORPORATION'S RESERVE PRICE SCHEME:
AN ASSESSMENT OF PRICE STABILISATION AND PRODUCER BENEFITS*

R W FRASER and A J MURRELL
School of Agriculture
The University of Western Australia

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Abstract

This paper presents a new framework for evaluating the producer benefits from price stabilisation schemes. The methodology includes formulae for explicitly characterising the floor price feature typical of such schemes. This methodology is applied to an analysis of the Australian Wool Corporation's (AWC) Reserve Price Scheme (RPS) for wool. Using post intervention data to generate values of ex ante (unstabilised) and ex post (stabilised) price variability, results indicate that the coefficient of variation of wool prices has declined in response to the operation of the RPS by 41 per cent. A large decrease in price instability from historical levels to the level estimated in this paper is also observed. It is suggested that some of this reduction in price instability may be attributed to an existence effect of the RPS. As a consequence, ignoring this existence effect may lead to an understatement of the benefits from price stabilisation.

Introduction

Hinchy and Fisher (1988) use an extension of the Newbery and Stiglitz (1981) approach to assess producer benefits from price stabilisation, using the Australian wool industry as a case study. An essential feature of the framework developed in Hinchy and Fisher is that price stabilisation is modelled "as if continuous intervention occurs over the range of prices" (1988 p.12). This is in contrast to the actual operation of the Reserve Price Scheme which is based on the determination of some pre-set minimum level of price at which the Corporation will intervene to purchase wool. As a consequence, Hinchy and Fisher are careful to point out: "The results of the present study, of course, cannot be interpreted as measuring the welfare gain from the minimum reserve price scheme" (1988 p.11).

The aim of this paper is to reassess the producer benefits from the AWC's RPS using a methodology which explicitly takes account of its floor price feature. The specific formulae developed to characterise this impact are based on "winsorisation" of the assumed normal probability distribution of wool prices. An advantage of this specification is that the formulae, combined with ex post (stabilised) data, can be used to estimate ex ante (unstabilised) price variability. Ex ante price variability is defined as the level of price variability that would have prevailed in the Australian wool industry in the absence of the RPS, whereas ex post price variability refers to price variability with the scheme in operation. Estimates of ex ante and ex post price variability provide an effective means of assessing the effects of the RPS on price stability. These estimates are combined in the model of producer behaviour to assess producer benefits (see Fraser 1984 1986 1988).

The ex ante and ex post levels of price variability generated in this paper are significantly lower than historical levels of price instability.
estimated from pre-intervention price data. It is suggested here that some of this increased stability may be as a result of the mere existence of the RPS. If this is the case, then excluding this existence effect when evaluating the RPS may result in an underestimate of the actual benefits of the scheme.

The plan of the paper is as follows. First, the model of producer behaviour is outlined, and the formulae used to characterise the impact of the RPS defined. Second, the data used in the estimation procedure are specified, initially for assessing the level of price stability and subsequently for assessing producer benefits. In the third section of this paper the results are discussed. The paper concludes with a summary.

The Methodology

The Model of Producer Behaviour

The model of producer behaviour used in this paper is taken from Fraser (1988). It assumes that the only input to production is the producer’s own labour, \( l \), and that a single output is produced which is subject to multiplicative risk:

\[ x = \theta f(l) \]

where: \( x \) = uncertain actual output \([E(x) = \bar{x} = f(l)]\)
\( \theta \) = multiplicative risk term \([E(\theta) = 1]\)
\( f(l) \) = planned output \([f'(l) > 0, f''(l) < 0]\).

With price also uncertain, the producer’s random income \((y)\) is thus given by:

\[ y = px \]

where: \( p \) = uncertain price \([E(p) = \bar{p}]\)

It is further assumed that the producer’s utility is (additively) separable in income and leisure so that his objective is to maximise by choice of labour input the expected utility of income\(^1\):

\[ E[U(px)] = w l \]

where: \( w \) = (constant) marginal disutility of labour\(^2\)
\( U(px) \) = concave function representing the utility of random income \((U' > 0, U'' \leq 0)\).

It is shown in Fraser (1984) that using a second order Taylor series expansion (1) may be approximated by:

\[ U(\bar{p}x) + 0.5U''(\bar{p}x)\bar{x}^2(\sigma^2_p + \sigma^2_\theta) - \sigma_\theta \bar{x} U'(\bar{p}x)(R-1) - w l \]

where: \( \sigma^2_p \) = variance of \( p \)

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1. Newbery and Stiglitz (1981 p.81) explain that the assumption of separability "is equivalent to the assumption that income and leisure are on the borderline between being substitutes and complements".
2. Assuming \( w \) is held constant for a producer means that information about its value can be deduced from the producer’s first order condition
\[ \sigma^2_p = \text{variance of } p \]
\[ \sigma_{p\theta} = \text{covariance of } p, \theta \]
\[ R = -U''(\tilde{p}\tilde{x})/U'(\tilde{p}\tilde{x}) = \text{the producer's coefficient of relative risk aversion (evaluated at } \tilde{p}, \tilde{x}). \]

R is a standard measure of the extent of an individual's aversion to risk. For example, for a risk neutral individual \( U''(px) = 0 \) and so \( R = 0 \). Note also from (2) that whether a covariance of a given sign contributes positively or negatively to utility depends not only on the sign of the covariance, but also on whether \( R \) exceeds or is less than unity.

Fraser (1986) examined the effect on optimal labour input \((\ell)\) of the covariance between uncertain price and output. Differentiating (1) with respect to \( \ell \) gave the producer's first order condition as:

\[ \mathbb{E}[U'(px)p\ell]f'(\ell) = w \]

which, using a second order Taylor series expansion, may be approximated by:

\[ U'(\tilde{p}\tilde{x})[\tilde{p} + 0.5(\sigma^2_p\tilde{p} + \sigma^2_{p\theta})][R(R-1)\tilde{p} R'] + \sigma_{p\theta}[(R-1)^2 - \tilde{p}xR']f'(\ell) = w \]

Equation (4) shows how the parameters of the uncertain environment combine with the characteristics of the producer's risk aversion to determine optimal labour input. Since \( w \) is assumed constant, its value from (4) can be used in (2) to find the expected utility of income with and without price stabilisation.

**Price Stabilisation**

The RPS is in effect a Buffer Stock Scheme. Specifically, the AWC set a floor price at which they guarantee to enter the market and buy wool. Thus when market prices approach the floor price the AWC will buy stock to maintain wool prices at the floor level. This stock is then sold at times of unusually high prices, in effect creating a ceiling price. In this way the overall variation of price is reduced. The mean level of price may also be altered by the operation of a RPS. Assuming that wool prices are normally distributed, this would occur if the ceiling price is set further from the mean price than is the floor price. In such a case the average price would be raised by a process of stock accumulation (and vice versa). However, if the floor price and the ceiling price are symmetrical about the mean then the mean price remains unchanged in the event of an RPS.

In order to incorporate the precise impact of the RPS into the model of producer behaviour, specific formulae have been developed. These formulae assume that unstabilised wool prices have a normal probability distribution and that the effect of the buying and selling operations of the RPS is to create a double winsorised price distribution.

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3. Payne and Whan (1971 p. 97) concluded that, with respect to wool prices prior to the RPS, "there was no evidence of any significant departure from normality".
Hinchy (1987 p.2) explained the process of winsorisation as "shifting the probability mass from below the underwritten price to the underwritten price". In this paper winsorisation refers to the floor price feature of the RPS. In the case where a ceiling price is assumed, winsorisation will also shift the probability mass from above the ceiling price to the ceiling price (see Figure A in the Appendix).

The formulae for characterising the floor and ceiling price features of the RPS are shown below. A formal derivation of these formulae is contained in the Appendix.

\[ \begin{align*}
E(p_s) &= F(p_f)\varphi_f + [1 - F(p_c)]p_c + [F(p_c) - F(p_f)]\epsilon_2 \\
\text{Var}(p_s) &= [F(p_c) - F(p_f)]\sigma_f^2 + F(p_f)[p_f - E(p_s)]^2 \\
&\quad + [1 - F(p_c)][p_c - E(p_s)]^2 + [F(p_c) - F(p_f)](\epsilon_2 - E(p_s))^2
\end{align*} \]

where: \( E(p_s) \) = expected stabilised price

\( p_f \) = floor price

\( p_c \) = ceiling price

\( F(p_f) \) = cumulative probability of \( p \leq p_f \)

\( F(p_c) \) = cumulative probability of \( p \leq p_c \)

\( \epsilon_2 = \hat{p} + \sigma_p[Z(p_f) - Z(p_c)]/[F(p_c) - F(p_f)] \)

\( \hat{p} \) = mean price

\( Z(p_f) = (1/\sqrt{2\pi})\exp[-0.5((p_f - \hat{p})/\sigma_p)^2] \)

\( Z(p_c) = (1/\sqrt{2\pi})\exp[-0.5((p_c - \hat{p})/\sigma_p)^2] \)

\( \text{Var}(p_s) \) = variance of the stabilised (ex post) price distribution (= variance of the winsorised distribution)

\[ \sigma_f^2 = \sigma_p^2[1 + ((p_f - \hat{p})/\sigma_p)Z(p_f) - ((p_c - \hat{p})/\sigma_p)Z(p_c)]/[F(p_c) - F(p_f)] \]

\[ - [(Z(p_f) - Z(p_c))/(F(p_c) - F(p_f))]^2 \] = the variance of the normal distribution between \( p_f \) and \( p_c \)

\( \sigma_f^2 \) = variance of the unstabilised (ex ante) price distribution

\( \sigma_p \) = standard deviation of the unstabilised (ex ante) price distribution.

In what follows it is assumed that:

\[ \hat{p} \cdot p_f = p_c - \hat{p} \]

in which case the formulae simplify to:

\[ \begin{align*}
E(p_s) &= \hat{p} \\
\text{Var}(p_s) &= [1 - 2F(p_f)]\sigma_f^2 + 2F(p_f)(p_f - \hat{p})^2
\end{align*} \]

This assumption means that the only scope for a producer to experience any "transfer benefit" (i.e., change in expected income) from the operation of the price stabilisation scheme is through its impact on the magnitude of the covariance between price and output (\( \sigma_{px} \)). This assumption implies that...
stock adjustments are used to absorb:

(a) the effects of any differences in the elasticity of demand at times of buying and selling;

(b) the effects of any nonlinearity in the demand curve.

Note, however, that the formulas can be altered to take account of the mean price impact of these demand effects.

The level of price stability achieved as a result of the RPS is assessed by comparing ex post ($\sigma^2_{ps}$) and ex ante ($\sigma^2_p$) price variability where equation (6') allows estimation of the ex ante level of price variability ($\sigma^2_p$) from post-intervention data (i.e., $\tilde{p}$, Var($p_s$) and $p_f$).

**Data Specifications**

*Estimating the Level of Price Stability*

Weekly price data for the Market Indicator (MI) rate was analysed for the period July 1976 to December 1986. Price variability was calculated on the basis of deviations from a linear trend, and expressed in terms of the coefficient of variation. The levels of ex post and ex ante price variability generated from post intervention data are also compared to historical estimates of wool price instability derived from pre-intervention data.

The MI represents a useful benchmark for the average value of Australian wool. It is $\tilde{p}$ so the expected value upon which the AWC's aggregate measure of the price floor ($p_f$) for wool for the coming year is based. The percentage ratio of these two statistics ($p_f/MI \times 100$) provides a valuable indication of the level of support offered by the AWC in any one period of time. The percentage ratios of the floor price to the corresponding MI price for twelve-monthly intervals from July 1976 to June 1988 are shown in Table 1. In this analysis these ratios are used to derive a floor price that is indicative of the level of AWC support offered between 1976/77 and 1986/87. The ratios also provide a means of identifying periods in which the operation of the RPS is inconsistent with average AWC support levels.

An examination of the ratios in Table 1 indicates that the AWC has attempted to support wool prices (in terms of the MI rate) to within approximately 90 per cent of the expected mean price. Periods in which the level of support is significantly below the assumed benchmark level of 90 per cent (e.g., July 1979 to June 1980, and January 1987 to June 1988) are therefore considered inconsistent with typical AWC support levels. For this reason, data from these periods are excluded when estimating $\sigma^2_{ps}$, $\sigma^2_p$, $p_f$ and $p_c$. Data for years prior to July 1976, and in which the RPS was in

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4. See Hinchy and Fisher (1988) for a discussion of how (a) and (b) can affect the mean price following stabilisation.
### Table 1: Average percentage ratios between market indicator prices and corresponding floor prices -- July 1976 to June 1988

<table>
<thead>
<tr>
<th>12 Month Period</th>
<th>Average Ratio (MI/$p_F$) * 100</th>
</tr>
</thead>
<tbody>
<tr>
<td>July 76 to June 77</td>
<td>88.60</td>
</tr>
<tr>
<td>July 77 to June 78</td>
<td>93.82</td>
</tr>
<tr>
<td>July 78 to June 79</td>
<td>89.37</td>
</tr>
<tr>
<td>July 79 to June 80</td>
<td>80.57</td>
</tr>
<tr>
<td>July 80 to June 81</td>
<td>88.82</td>
</tr>
<tr>
<td>July 81 to June 82</td>
<td>94.88</td>
</tr>
<tr>
<td>July 82 to June 83</td>
<td>97.23</td>
</tr>
<tr>
<td>July 83 to June 84</td>
<td>96.83</td>
</tr>
<tr>
<td>July 84 to June 85</td>
<td>89.63</td>
</tr>
<tr>
<td>July 85 to June 86</td>
<td>93.74</td>
</tr>
<tr>
<td>July 86 to June 87</td>
<td>82.14</td>
</tr>
<tr>
<td>July 86 to Dec 86 (six month period)</td>
<td>89.09</td>
</tr>
<tr>
<td>Jan 87 to June 87 (six month period)</td>
<td>74.82</td>
</tr>
<tr>
<td>July 87 to June 88</td>
<td>65.71</td>
</tr>
</tbody>
</table>

### Table 2: Summary of information for the Market Indicator rate over the period July 1976 to December 1986

<table>
<thead>
<tr>
<th></th>
<th>Ex post (stabilised)</th>
<th>Ex ante (unstabilised)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Price ($\bar{p}$)</td>
<td>273.80</td>
<td>273.80</td>
</tr>
<tr>
<td>Price variability ($\sigma^2_{ps}$ and $\sigma^2_p$)</td>
<td>276.29</td>
<td>804.80</td>
</tr>
<tr>
<td>Standard deviation ($\sigma_{ps}$ and $\sigma_p$)</td>
<td>16.62</td>
<td>28.37</td>
</tr>
<tr>
<td>Coefficient of variation ($CV_{ps}$ and $CV_p$)</td>
<td>6.07%</td>
<td>10.36%</td>
</tr>
<tr>
<td>Ratio (MI/$p_F$ * 100)</td>
<td>92.33%</td>
<td>-</td>
</tr>
<tr>
<td>Floor price ($p_F$)</td>
<td>252.80</td>
<td>-</td>
</tr>
<tr>
<td>Ceiling price ($p_c$)</td>
<td>294.79</td>
<td>-</td>
</tr>
</tbody>
</table>
operation, are simply excluded on the basis that these data were not available in the form required. The values estimated for $p, \sigma^2_{ps}, \sigma^2_p, p_f$ and $p_c$ are summarised in Table 2.

One estimate of the historical level of price instability is derived from Harris et al (1974) and is given as 29.90 per cent. This figure was calculated from data spanning the period 1952/53 to 1972/73 in which there was no price stabilisation (in the form of the RPS). Hinchy and Fisher (1988) also estimated the coefficient of variation of wool price (as 18.00 per cent) but from data spanning the years 1953/54 to 1984/85. This estimate is therefore based on a mixture of the periods, one in which there was not and one in which there was price stabilisation.

**Producer Benefits**

Given the estimates of $\sigma^2_{ps}$ and $\sigma^2_p$ from above, additional information requirements to assess producer benefits from equation (2) are:

(a) specification of the producer's risk aversion as characterised by his utility function;

(b) specifications about the producer's initial economic circumstances including:

(i) $f(t)$ = planned output
(ii) $\sigma^2_{\theta}$ = variance of the multiplicative risk term $\theta$
(iii) $\sigma_{p\theta}$ = covariance of $p, \theta$.

It is assumed that the producer's attitude to income risk can be adequately represented by the constant relative risk aversion function:

$$U(\tilde{p}x) = \frac{\tilde{p}x}{R^1/(1-R)}$$

It should be noted that this assumption simplifies equation (4) by eliminating the terms related to whether $R$ is increasing or decreasing ($R'$). In the following analysis, a range of values of $R$ consistent with empirical estimates is considered. Newbery and Stiglitz conclude that "most individuals are risk averse, but not very risk averse" (1981, p.105), with $R$ varying typically between 0.5 and 1.2. Evidence from Bond and Wonder (1980) suggests attitudes to risk for Australian farmers consistent with $R \leq 0.45$ (see Fraser 1988 for a discussion of Bond and Wonder's results).

The specification of the producer's initial economic circumstances requires a mixture of assumptions and actual industry data. The already simplified relationship between the producer's labour input and his output [$f(t)$] requires further simplification to a precise functional form. It is assumed that this form is given by:

$$\tilde{x} = \tilde{d}^M$$

where $\tilde{x}$ is given a positioning value equal to unity ($\tilde{x} = 1$)\(^5\).

5. Since $\tilde{x}$ is given a value of unity and supply response to price stabilisation is excluded from this analysis, the value of $m$ becomes irrelevant.
For \( \hat{x} \) to represent optimal planned output over a range of values of \( R \),
the value of \( w \) must be (precisely) inversely related to the value of \( R \).
However, as the results are calculated in percentage change terms, this
additional assumption concerning \( w \) is not restrictive.

The producer’s information about the size of \( \sigma_{\theta}^2 \) and \( \sigma_{p\theta} \) is based on
actual industry data which represent details of income variability in the
Australian wool industry before the introduction of the RPS derived from
Harris et al (1974). In particular, it is assumed that both the variance of
output and the correlation coefficient (\( \rho \)) between price and output are
unchanged by the operation of the RPS.

Given the assumption of \( \hat{x} = 1 \), it can be derived that (see Harris et
al 1974 p.302):

\[
\sigma^2_x = 0.01.
\]

In addition, given an assumption of joint normality of price and output, it
can be derived that (see Harris et al 1974 p.304):

\[
\rho = -0.51
\]

Results

Price Stabilisation

The results from this paper are summarised in Table 3 where it is
shown that the coefficient of variation of ex post (stabilised) wool
prices for the period 1976/77 to December 1986 is 6.07 per cent and the
coefficient of variation of ex ante (unstabilised) wool prices for the same
data period is 10.36 per cent. These results indicate that the RPS has been
successful in stabilising wool prices, reducing the coefficient of
variation of wool prices by 41.41 per cent. This reduction in price
variability is similar to that estimated by Campbell, Gardiner and Haszler

Table 3: Levels of price stability achieved as a result of the Reserve Price Scheme

<table>
<thead>
<tr>
<th>Coefficient of Variation (per cent)</th>
<th>Percentage Difference</th>
</tr>
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<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>( CV_p )</td>
<td>( CV_ps )</td>
</tr>
<tr>
<td>Fraser and Murrell</td>
<td>10.36</td>
</tr>
<tr>
<td>Harris et al (1974)</td>
<td>29.90</td>
</tr>
<tr>
<td>Hinchy and Fisher (1988)</td>
<td>18.00</td>
</tr>
</tbody>
</table>
(1980), i.e. 44 per cent. The coefficient of variation of historical wool prices from the period 1952/53 to 1972/73, as estimated by Harris et al (1974), is 29.90 per cent. A further comparison could be made with the coefficient of variation of wool prices of 18.00 per cent estimated by Hinchy and Fisher (1988) using data from the period 1953/54 to 1984/85. However, as mentioned previously, this estimate of the coefficient of variation of historical prices is in effect an average of price variability in the pre-intervention and post-intervention periods, and is therefore not an appropriate estimate of unstaobilised wool price variation. Nevertheless, it is interesting to note that the weighted (by number of years) sum of the pre-(29.90) and post-(6.07) intervention estimates of price variability gives an average coefficient of variation of price of 20.96 per cent which is similar to the Hinchy and Fisher estimate.

These results raise some important questions. If the operation of the RPS can be shown to have reduced the coefficient of variation of wool prices from 10.36 to 6.07 per cent, resulting in a total percentage reduction in price variability of 41.41 per cent, how can the reduction in the underlying variability of wool prices from the historical level of 29.90 per cent be accounted for? It is suggested here that some of this reduced price variability may be attributed to an existence effect of the RPS. In other words, the presence of the RPS may itself be sufficient to bring about a greater level of price stability. Just knowing that the RPS will be brought into operation if price fluctuations become excessive may be enough to generate a more stable marketing of wool. This existence effect is in addition to the operating effects of the AWC that can be measured through its buying and selling activities.

Accepting such an explanation means that producer benefits assessed from the operation of the RPS alone may underestimate the actual benefits from the scheme because they do not include any measure of the benefits due to this existence effect.

Producer Benefits

To account for the possibility of this existence effect two sets of estimates of producer benefits have been generated. The first set assesses

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6. The results generated in this study assess price stability for the HI rate alone; therefore, they are not directly comparable to results produced in Connolly, MacAulay and Piggott (1987). Connolly, MacAulay and Piggott assessed the level of price stability experienced across six different grades of wool. They concluded that the price variability for these grades of wool varied above and below the 44 per cent estimated by Campbell, Gardiner and Haszler (1980). The methodology used in this paper can be applied to an analysis of other specific grades or types of wool.

7. Recent research on exchange rate target zones has identified a similar phenomenon. Specifically, "the presence of a commitment by authorities to keep the exchange rate within a band tends to stabilise the movement of the exchange rate even inside that band" (Krugman 1987 p.22).
the level of price stability (and producer benefits) using ex post and ex ante price variability. In effect this creates what is considered the minimum level of benefit from the RPS. The second set of results assesses the level of price stability achieved (and producer benefits) using historical price variability and ex post price variability. These results are considered to represent the maximum level of benefits from the RPS by assuming that the total reduction in the coefficient of variation from 29.90 to 6.07 per cent can be attributed to the RPS.

Producer benefits are expressed in terms of a producer's willingness to pay for price stabilisation (as a percentage of expected income) assessed over a range of individual attitudes to risk. The results are shown in Table 4.

Table 4: Willingness-to-pay for price stabilisation (percentage of income)

<table>
<thead>
<tr>
<th>Reduction in CV</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>10.36 → 6.07</td>
<td>0.16</td>
</tr>
<tr>
<td>29.90 → 6.07</td>
<td>0.92</td>
</tr>
</tbody>
</table>

The results show that a price stabilisation scheme that reduces the coefficient of variation of wool prices from 10.36 to 6.07 per cent increases expected income by 0.16 per cent for a risk neutral producer (R = 0). This means that a risk neutral producer is willing to pay up to 0.16 per cent of his income for the scheme (because it is reducing the negative correlation between price and output). However, the scheme also reduces the variation of prices and so a risk averse producer is willing to pay not only for the increase in expected income but also for the reduced variability of income. The results in Table 4 show that this extra willingness-to-pay increases with the risk aversion of the producer. For example, a producer with R = 0.3 is willing to pay 0.06 per cent of expected income (0.22 minus 0.16) for the reduction in income variability, whereas the producer with R = 0.6 is willing to pay 0.12 per cent of his expected income (0.28 minus 0.16).

The second set of results shows that producers are willing to pay a considerably higher percentage of their income for stabilisation of prices from the historical level of variability.
Conclusions

This paper uses a new framework for evaluating the producer benefits from price stabilisation schemes. The methodology includes formulae for explicitly characterising the floor price feature typical of such schemes and is applied to an analysis of the Australian Wool Corporation's Reserve Price Scheme for wool.

Post-intervention data for the Market Indicator are used to generate values of ex ante (unstabilised) and ex post (stabilised) price variability. Ex ante price variability is estimated from an equation used to characterise the floor price feature of the Reserve Price Scheme. The results indicate that the coefficient of variation of wool prices has declined in response to the operation of the Reserve Price Scheme by 46 per cent. However, this estimate could be considered conservative on the basis of a possible existence effect of the Reserve Price Scheme. The possibility of an existence effect is raised by observing that both ex ante and ex post price variability are significantly lower than historical levels of price instability. Moreover, calculations of producer benefits from the Reserve Price Scheme suggest that these benefits are significantly underestimated if this existence effect is ignored.
References


Appendix

Double Winsorising a Normal Distribution

Let $p_F$ be the lower and $p_C$ the upper point of Winsorising, $\hat{\mu}$ the original mean and $\sigma^2$ the original variance of a normal distribution.

Then this is equivalent to mixing three distributions in the proportions: $F(p_F)$, $[F(p_C) - F(p_F)]$ and $[1 - F(p_C)]$ where:

- for $p \leq p_F: \epsilon_1 = p_F$, $\sigma_1^2 = 0$
- for $p_F < p < p_C: \epsilon_2 = E(p|p_F < p < p_C)$
  $\sigma_2^2 = \text{Var}(p|p_F < p < p_C)$
- for $p \geq p_F: \epsilon_3 = p_C$, $\sigma_1^2 = 0$

The second of these is a double truncated normal distribution

where:

1. $\epsilon_2 = \hat{\mu} + \left[\frac{Z(p_F) - Z(p_C)}{F(p_C) - F(p_F)}\right]\sigma_p$

   Note: If $\hat{\mu} = p_F = p_C = \hat{\mu}$, then $\epsilon_2 = \hat{\mu}$.

2. $\sigma^2 = \sigma_p^2\left[1 + \left[\frac{(p_F - \hat{\mu})/\sigma_p}{Z(p_F)} - \left[\frac{(p_C - \hat{\mu})/\sigma_p}{Z(p_C)}\right]/\left[F(p_C) - F(p_F)\right]\right] - \left[\frac{Z(p_F) - Z(p_C)}{F(p_C) - F(p_F)}\right]^2\right]$
Note: If \( \tilde{p} - p_f = p_c - \tilde{p} \) then:
\[
\sigma_2^2 = \sigma_p^2 \left[ 1 - 2 \left( \frac{p_c - \tilde{p}}{\sigma_p} \right) E(p_f) \right] \frac{[1 - 2F(p_f)]]}{(see \ Johnson \ and \ Kotz \ 1970, \ pp.81-83)}.
\]

Given the following formulae for a mixture \( p_s \):
\[
E(p_s) = \sum_{i=1}^{k} p_i \epsilon_i
\]
\[
\text{Var}(p_s) = \sum_{i=1}^{k} p_i \sigma_i^2 + \sum_{i=1}^{k} p_i (\epsilon_i - \bar{\epsilon})^2
\]

where:
\[
\bar{\epsilon} = \sum_{i=1}^{k} p_i \epsilon_i = E(p_s)
\]

For \( k = 3 \) and the above information:
\[
E(p_s) = F(p_f)p_f + [1-F(p_c)]p_c + [F(p_c)-F(p_f)]E(p_s)
\]

Note: If \( \tilde{p} - p_f = p_c - \tilde{p} \), then \( E(p_s) = \tilde{p} \).

\[
\text{Var}(p_s) = [F(p_c)-F(p_f)]\sigma_2^2 + F(p_f)[p_f-E(p_s)]^2
\]
\[
+ [1-F(p_c)] [p_c-E(p_s)]^2 + [F(p_c)-F(p_f)][\epsilon_2-E(p_s)]^2
\]

Note: If \( \tilde{p} - p_f = p_c - \tilde{p} \), then:
\[
\text{Var}(p_s) = [F(p_c)-F(p_f)]\sigma_2^2 + 2F(p_f)(p_f-\tilde{p})^2
\]

(see Johnson and Leone 1964, p.129).