

# Supply Chain Optimized Strategies in the Mode of External Financing

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**Abstract** In the circumstance that market demand is uncertain, it studies the decision-making problem of supply chain financial system consisting of the single supplier, a capital constraint retailer and a bank. Considering the mode of external financing, we obtain the optimal order decision of the capital constraint retailer, the optimal financing rate and the optimal wholesale price of the supplier and analyze the effects of owned capitals of retailer on the optimized decision-making of supply chain financial system. At last, it demonstrates the effectiveness of conclusion by numerical examples.

**Key words** Capital constraints, External financing, Supply chain finance, Demand uncertainty

## 1 Introduction

In the 21st century, the development of globalization has made the economy enter the rapid development period, the purchasing of each material, production and transportation of products and other economic activities have operated in an increasing speed. For the shortage of funds cannot keep up with the speed of economic development, partial enterprises have been posed in front of cleaning and acquisition. Enterprise fund flow is on the move rapidly, which inevitably causes numerous contradictions in the upstream and downstream enterprises. For example, logistics enterprises cannot operate normally due to insufficient funds, leading to the termination of logistics. Enterprises cannot complete product purchases for the shortage of funds, etc. In the morning of December 3, 2014, manufacturers of Dutch sports car Spyker applied for bankruptcy protection. The reason was its failure of the transaction of purchasing Sabo in 2010, leading to the problem of short fund flow while in the subsequent production. Until the factory defaulting approximate 125 trillion Euros (about 155 thousand dollars) used the rent, it announced the bankruptcy protection. Dutch Spyker, established in 2000, focused on production of sports cars. With the business development, it adopts a series of production decisions to confront funding limitation and finally leads to the company bankruptcy. The problem of fund shortage has become one of greatest resistance limiting the company, enterprise and even the economic development, and the function of fund flow cannot be ignored. Therefore, it equips with stronger realistic meanings to involve external bank into the system of supply chain for the study of the problem of fund shortage. The structure of the paper is as below. The first part reviews the related literature. The second part establishes the model framework and interprets related signs. Specific explanation and analysis of supply chain financial system optimized decision model of the external financing will be

given in the third part, which is also the core part of this paper. The forth part makes the value demonstration on solving results. In the last part, it is the conclusion and the further research direction in the future.

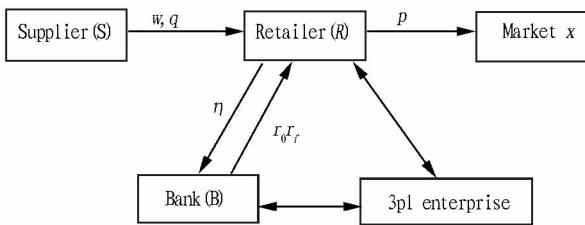
## 2 Literature review

The current capital of newly founded companies or enterprises is far less abundant than that of old companies, they tend to be more prone to meet fund shortage. Archibald *et al.* studied the product procurement of the newly founded companies, the analysis found that procurement in newly founded companies are more careful compared with mature companies, that is because the newly founded companies haven't have so much abundant capital. Xu *et al.* established procurement models of production decision-making, the analysis found that funding shortfalls have great impact on production decision of enterprises. In the early 1970s, people have taken the delayed payment as a way of short-term credit, because financial intermediation has more advantages than financial institutions. In 1985, Goyal *et al.* studied the delayed payment in the EOQ model for the first time. Yigang Zhang studied the models of wholesale price contract on condition of delayed payment and assumed the nature of short life cycle products. Dada *et al.* analyzed the newsboy problem of capital constraint, they founded that companies will purchase products from borrowing money if the borrowing costs of the bank is not high, or they'd rather give up this economic activity. Xu *et al.* founded that the bank can provide retailer of capital constraint loan contract under certain conditions. Buzacott *et al.* suggested the connection between enterprise operation of capital constraint and financing when market demand is uncertain. Chen *et al.* considered the product procurement decision-making in medical supply chain. Raghavan *et al.* studied the co-financing decision-making of a two-level supply chain which is constituted by a supplier and a retailer, both parties involved are facing capital restraint, the analysis found that the co-financing decision-making can improve the performance of supply chain.

Xiangfeng Chen *et al.* developed the operational decision-making of a supply chain which is constituted by a single supplier and a single retailer, among which the capital restraint still exists in retailer, the analysis found that the capital restraint can have an impact on enterprises' operational decision-making. The above literature considered the capital constraint problems existing in enterprises, they only analyzed these conditions from the perspective of supply chain system but not involved the outer bank into supply chain financial system. How does the bank rate affect enterprises' optimal decision? How does self-owned capital affect upstream enterprises' operational decision-making? These will be the key points of this article. Our innovation is mainly concentrated in the following two points. Firstly, we try to build system framework of the supply chain financing and secondly, analyze how self-owned capital affects upstream enterprises' operational decision-making within the system framework of the supply chain financing.

### 3 Model framework and symbol explanation

When the market demand is uncertain, supply chain financial system which is composed by single supplier, capital restraint retailer as well as the bank is shown in Fig. 1.



**Fig. 1 Framework of supply chain finance system based on external financing**

In this framework, assuming that retailer has capital constraint in produce procurement, namely his self-owned capital do not absolutely guarantee product procurement, then both the retailer and the commercial bank will find a common third party logistics company to provide credit guaranty. The retailer should credit the remaining payment to commercial bank and promise to return the original sum and part of interest to the bank after selling seasons. If the sales revenue of the retailer not quite cover commercial bank' credit, they may declare themselves to be bankrupt, what's more, commercial bank will obtain all the sales revenue and the residual value of the products. Assuming that the supplier, retailer and commercial bank are risk neutral and perfectly rational, they will consider to maximize their own interests first, all the information in the supply chain financial system is symmetrical, so there is not hidden and opaque information. Supplier' s unit production cost is  $c$ , the unit product wholesale price (decision variables) is  $w$ , retailer's unit sales price is  $p$ , retailer' order quantities (decision variables) is  $q$ , retailer's residual value of unsold products is  $v$ , retailer's self-owned capital is  $B$ , commercial bank' interest rates (decision variables) is  $r$ , risk-free interest rates of capital market is  $r_f$ , market demand  $x$  is random variables,  $f(x)$  is the probability density functions of demand,  $F(x)$  is the probability distribution function of demand,  $F^{-1}(x)$  is the inverse function of

probability distribution function of market demand, definition  $\bar{F}(x) = 1 - F(x)$ , distribution function of demand  $F(x)$  is continuous, derivative and strictly increasing and is in line with the distribution of increasing failure rate (IFR).

## 4 Optimal decision models of external financing's supply-chain finance system

**4.1 Retailer's optimal decisions** When the retailer in supply-chain finance system is short of funds, his finance from external bank according to the free funds, then the expected profits of the retailer can be expressed as:

$$E\pi_R(q; r, w) = E[p\min(q, x) + v(q-x)^+ - \eta(1+r)] \quad (1)$$

Lemma 1: As for the retailer with capital constraints, the conditions that they finance from the bank and don't bankrupt.

$$\text{Critical quantity demand: } \tilde{x}(q) = \frac{w(1+r) - v}{p-v}q - \frac{B(1+r)}{p-v}$$

$$\text{Critical order quantity: } \tilde{q} = \frac{B(1+r)}{w(1+r) - v}$$

Proof: When capital constraint retailer's expected income can pay back bank financing principal and interest, the retailer won't bankrupt, then:

$$p\min(q, x) + v(q-x)^+ - \eta(1+r) \geq 0 \quad (2)$$

(i) When  $\frac{B}{w} < x < q$ , (2) is equivalent to  $px + v(q-x) - \eta(1+r) \geq 0$ , that's to say, when  $x \geq \tilde{x} = \frac{w(1+r) - v}{p-v}q - \frac{B(1+r)}{p-v}$ , (2) exists.

(ii) When  $q \geq \frac{B}{w}$  and  $x = 0$ , (2) is equivalent to  $vq - \eta(1+r) \geq 0$ , that's to say, when  $q \leq \tilde{q} = \frac{B(1+r)}{w(1+r) - v}$ , (2) exists.

From Lemma 1, we can see that,  $\tilde{x}$  expresses the critical quantity demand when capital constraint retailer don't bankrupt, capital constraint retailer can get more sales revenue, only when market demand  $x \geq \tilde{x}$ . With principal and interest of bank loans paid back, then the bank can provide capital constraint retailer with financing services to get extra benefits. On the contrary, if market demand  $x < \tilde{x}$ , capital constraint retailer cannot pay back principal and interest of bank loans due to the low sales salaries, retailer pronounce to be bankrupt. Therefore, if final market's demand is low, capital constraint retailer will face the bankrupt risks due to high financial costs. In the another aspect, capital constraint retailer cannot order too much order quantity, just because they can finance from external bank. Too much order quantity means larger overstock inventory. Once the market demands change, retailer may bankrupt, for dull sales bring about more and more overstock goods.

Theorem 1: As for the wholesale price of given bank financing interest rates and retailer, capital constraint retailer's optimal order quantity under external financial terms is  $q^*(r, w) = F^{-1}$

$$\left[ \frac{p - vF\left[\frac{B}{w}\right] - w(1+r)}{p-v} \right].$$

Proof: Formula (1) can be simplified to:

$$E\pi_R(q; r, w) = pq - vpF\left[\frac{B}{w}\right] - (p-v)\frac{B}{w}F\left[\frac{B}{w}\right] - (wq - B)(1+r) - (p-v)\int_{\frac{B}{w}}^x F(x) dx \quad (3)$$

Calculate order quantity's first and second derivative according to (3), then.

$$\frac{dE\pi_R}{dq} = p - vF\left[\frac{B}{w}\right] - (p-v)F(q) - w(1+r) \quad (4)$$

$$\frac{d^2E\pi_R}{dq^2} = -(p-v)f(q) < 0 \quad (5)$$

From (5), the expected profit of capital constraint retailer is a concave function on order quantity, so optimal order quantity  $q^*$  satisfies  $\frac{dE\pi_R}{dq} = 0$ , then:  $p - vF\left[\frac{B}{w}\right] - (p-v)F(q) - w(1+r) = 0$ , with simplifications and clear-up.

$$q^*(r, w) = F^{-1}\left[\frac{p - vF\left[\frac{B}{w}\right] - w(1+r)}{p-v}\right] \quad (6)$$

Theorem 1 is proved. Theorem 1 expresses that, under the circumstances that the supplier gives wholesale price and the bank loans interest rate, the order quantity of the retailer is certain. Order quantity is a decreasing function on bank rate, as well as on the wholesale price, while the influence wholesale price has on retailer's order quantity is more sensitive.

**4.2 Optimal decision problems of commercial bank** Commercial bank takes part in the economic activity, which requires that their expected profit should be greater than the reservation utility. If benefits can be received, risk-free return of this activity will not be taken part in, thus, expected profits of commercial bank can be expressed as:

$$\max E\pi_b(r; q, w) = E\{\min[\eta(1+r), p \min(q, x) + v(q-x)^+ ] - \eta(1+r_f)\} \quad (7)$$

Lemma 2: Optimal order quantity of capital constraint retailer is a decreasing function on commercial bank's financing rates.

Then it is  $\frac{d(q^*)}{dr} < 0$ .

Proof: According to above (6), we can see that

$$(p-v)F(q) = p - vF\left[\frac{B}{w}\right] - w(1+r) \quad (8)$$

Calculate financing rate's first derivative according to both sides of (8), then it is  $\frac{d(q^*)}{dr} = \frac{w}{(p-v)f(q)} < 0$ , thus, Lemma 2 is proved. Then optimal decision of commercial bank can be simplified as:

$$\max E\pi_b(r; q, w) = (p-v)\bar{x}F(\bar{x}) - (p-v)\frac{B}{w}F\left[\frac{B}{w}\right] + vq \left[ F(\bar{x}) - F\left[\frac{B}{w}\right] \right] - (p-v)\int_{\frac{B}{w}}^{\bar{x}} F(x) dx + (wq - B)(r - r_f) \quad (9)$$

Theorem 2: As for the retailer with initial capitals  $B$ , in consideration of bankrupt risks, bank provides retailer with optimal financing rates.

$$r^* = r_f + \frac{(p-v)(wq - B)f(q) + vF\left[\frac{B}{w}\right]}{w^2}$$

Proof: The proving methods of Theorem 1 can be imitated.

Theorem 2 expresses that, bank's optimal financing rates can be expressed the additions of bank's risk-free rates and a certain constant, while this constant is exactly the obtained rate of return that the bank takes part in this economic activity, and influenced by decisions of supplier and retailer in supply-chain system. On the contrary, if the bank doesn't take part in the economic activity, they can obtain their own risk-free reservation utility.

**4.3 Optimal decision problems of supplier** This paper assumes that the supplier doesn't have capital constraints, then his decisions are to maximize his own profits, then the decision problems can be expressed as:

$$E\pi_s(w; q) = (w-c)q \quad (10)$$

Lemma 3: When the supplier's repurchase price is between  $\left[0, \frac{w^2(1+r)}{Bf\left[\frac{B}{w}\right]}\right]$ , optimal order quantity of capital constraint retailer is a decreasing function on supplier's wholesale price, then  $\frac{d(q^*)}{dw} < 0$ .

Proof: Calculate supplier's wholesale price's first derivative according to both sides of (8), then:

$$\frac{d(q^*)}{dw} = \frac{Bf\left[\frac{B}{w}\right] - w^2(1+r)}{w^2(p-v)f(q)} \quad (11)$$

When  $v < \frac{w^2(1+r)}{Bf\left[\frac{B}{w}\right]}$ ,  $\frac{d(q^*)}{dw} < 0$  permanently exists. Then

Lemma 3 is proved.

Lemma 3 expresses that, newsvendor's order quantity under standard newsvendor models is a decreasing function on wholesale price, when the retailer have capital constraints, once repurchase price scope of upstream supplier is stipulated, order quantity of capital constraint retailer is still a decreasing function on wholesale price.

Theorem 3: As for optimal order quantity that commercial bank gives to financing rates and capital constraint retailer, supplier's optimal wholesale price is:

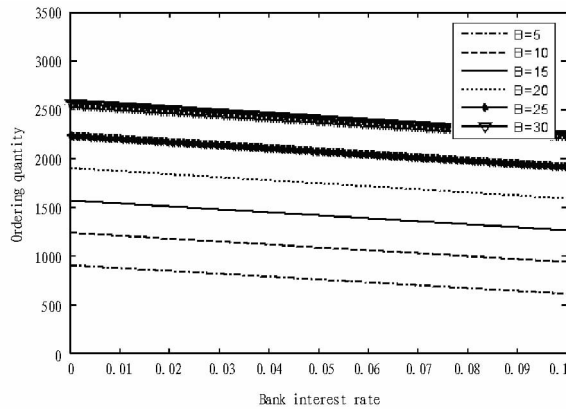
$$w^* = \frac{c + \sqrt{c^2 + 4(p-v)q^*f(q^*)}}{2} > c$$

Proof: The proving methods of Theorem 1 can be imitated.

## 5 Numerical analysis

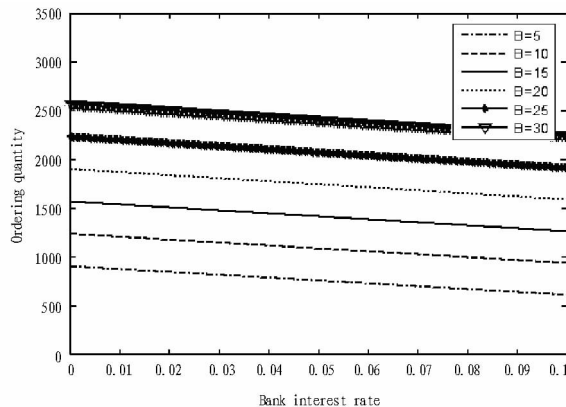
In real life, the supply chain financial system composed by a supplier, a retailer and commercial bank should be considered, assuming that retailer of automobile parts faced with capital constraint in parts procurement, he should obtain external financing model of security credibility from a third party logistics enterprise to loan from commercial bank.  $c = 100$ ,  $B = 100000$ ,  $p = 500$ ,  $v = 200$ ,  $x \in [500, 2000]$ ,  $r_f = 3.6\%$ .

Fig. 2 shows that in the supply chain system involved in outer bank, the bigger the retailer's self-owned capital is, the faster the operated capital current is and the bigger the order quantities are.



**Fig. 2** The relationship between the retailer's ordering quantity and bank interest rate under the different funds

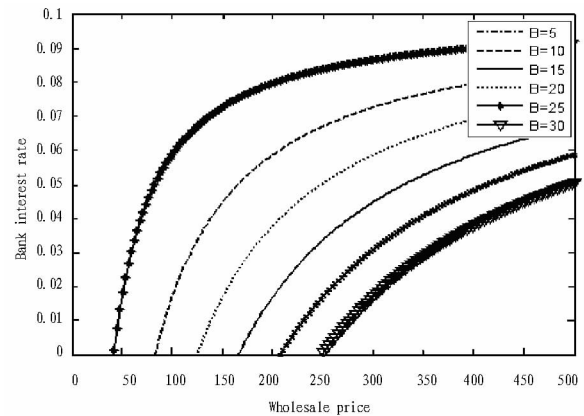
But in the same level self-owned capital, retailer's order quantity is the decreasing function of bank rates. While self-owned capital increasing on equal terms, retailer's order quantities have comparably increased, under the circumstance that the refinancing rate of outer bank is determined, if self-owned capital increase in the form of arithmetic progression, the order quantities will also increase in the form of arithmetic progression. It further illustrates that the size of self-owned capital affects the space of retailer's decision space.



**Fig. 3** The relationship between the retailer's ordering quantity and the supplier's wholesale price under the different funds

Fig. 3 demonstrates that in the supply chain system with the participation of external bank, the more the owned capitals of retailer, the more quantity its booking will be. However, in the same owned capital level, the booking quantity of retailer is the decreasing function of the wholesale price of supplier. It is consistent with the conclusion in the label newsboy model, or with the increase of the wholesale price, the booking quantity decreases. The lower the wholesale price is, the more booking quantity will be. With the increase of owned capitals, the range of booking quantity increase of retailer has become smaller and smaller in the same wholesale price level, the more owned capital influenced the decision-making rationality instead, and he will consider the size of risk losses more and more. This is quite different from the mood

of retailer with smaller owned capitals, they will be more careful and cautious while making decisions because they are afraid of losses. However, retailers with less funds, are insensitive to the quantity of funds, even the loss will not bother them and make them feel regretful.



**Fig. 4** The relationship between bank interest rate and the supplier's wholesale price under the different funds

Fig. 4 demonstrates that in the supply chain system with the participation of external bank, the wholesale price from supplier can influence the loan decisions of bank. In other words, wholesale price can affect the loan interest rate. The greater the owned capital of retailer, the lower the decided loan interest of bank will be, for at this time retailer lack less funds. The benefits acquired by the part of loan purchases of retailer cannot be used to pay the interest of bank for the higher loan interest, so it rather not choose loaning. In contrary, if owned capital of retailer is certain, supplier can select to reduce the wholesale price, and retailer can purchase more products. In this way, the fund restraint level of retailer will decrease, and the bank will search for potential loaners by choosing to lower the loan interest, which also fits the general economic phenomenon.

## 6 Conclusions

In the environment that market demands are uncertain, it establishes the supply chain financial system comprised of the single supplier, fund-restrained retailer and bank. By building the financial model of external financing supply chain, it analyzes and obtains the optimal booking decision of fund-restraint retailer, the optimal financing rate of commercial bank and the optimal wholesale price of supplier, which can provide stronger guiding meanings for the decision-making of supply chain financial system enterprises. In addition, in our research, we assume that supplier, retailer and commercial bank are risk neutral and entirely rational. In the future, risk attitude can be introduced. Whether risk attitude can influence the logistics of supply chains and its effects on financial decisions of supply chains will be studied later.