AJAE appendix for ‘Credit Market Imperfections and the Distribution of Policy Rents’

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Note: The material contained herein is supplementary to the article named in the title and published in the American Journal of Agricultural Economics (AJAE).
A1. Proof of Proposition 1

To show: \( \frac{dr}{ds} > 1 \) with \( \alpha_s > 0 \).

We show the case when farms remain credit constrained with the subsidy.\(^1\)

With area payments the farm credit constraint is given by \( kK \leq S = \alpha_w W + \alpha_s sA \). In equilibrium the following conditions must be satisfied:\(^2\)

\[(A1.1) \quad pf_A + pf_K \frac{\alpha_s s}{k} - r + (1 - \alpha_s) s = 0 \]

\[(A1.2) \quad A = A^r. \]

Totally differentiating (A1.1) and (A1.2) yields:

\[(A1.3) \quad MdA + Rds - dr = 0 \]

\[(A1.4) \quad dA = 0 \]

where

\[(A1.5) \quad R = \frac{\alpha_s A^T}{k} \left( pf_{AK} + pf_{KK} \frac{\alpha_s s}{k} \right) + pf_K \frac{\alpha_s}{k} + (1 - \alpha_s) \geq 1 \]

\[(A1.6) \quad M = \left( pf_{AA} + pf_{AK} \frac{\alpha_s s}{k} + pf_{KA} \frac{\alpha_s s}{k} + pf_{KK} \frac{\alpha_s^2 s^2}{k^2} \right) < 0. \]

Solving for \( \frac{dr}{ds} \) yields:

\[(A1.7) \quad \frac{dr}{ds} = \frac{\alpha_s A^T}{k} \left( pf_{AK} + pf_{KK} \frac{\alpha_s s}{k} \right) + pf_K \frac{\alpha_s}{k} + (1 - \alpha_s) \geq 1. \]
With constant returns to scale in the production function it follows that \( \frac{A}{K} f_{AK} = -f_{kk} \), which implies that \( \frac{\alpha_s A^T}{k} \left( pf_{AK} - pf_{AK} \frac{\alpha_s A^T}{k} \right) \geq 0.3 \) From the first order condition with credit constraint binding \( (pf_k - k(1 + \lambda) = 0) \) it follows that \( pf_k \frac{1}{k} \geq 1 \). Hence:

1. if \( \alpha_s = 0 \) then \( \frac{dr}{ds} = 1 \)

2. if \( \alpha_s > 0 \) then \( \frac{dr}{ds} > 1 \).

Q.E.D.

**A2. Proof of Proposition 2**

To show: \( \frac{d\Pi}{ds} < 0 \) with \( \alpha_s > 0 \).

We show the case when farm remains credit constrained with the subsidy.\(^4\)

Farm profits are: \( \Pi = pf(A,K) - (r - s)A - kK \). It follows that:

\[(A2.1) \quad \frac{d\Pi}{ds} = -A^r \frac{\alpha_s A^T}{k} \left( pf_{AK} + pf_{kk} \frac{\alpha_s s A^T}{k} \right) \leq 0. \]

With \( pf_{AK} + pf_{kk} \frac{\alpha_s s A^T}{k} > 0 \) (see proposition 1) it follows that \( \frac{d\Pi}{ds} < 0 \) if \( \alpha_s > 0 \). If \( \alpha_s = 0 \) then \( \frac{d\Pi}{ds} = 0 \).

Q.E.D.
A3. Proof of Proposition 3

To show: \( \frac{dU}{ds} > 0 \) with \( \alpha_s > 0 \).

We show the case when farm remains credit constrained with the subsidy.\(^5\)

Total welfare \((U)\) is the sum of farm profits \((\Pi)\), landowners total rents \((\Pi^L = rA^T)\), and minus taxpayers costs \(sA^T\), i.e. \( U = \Pi + \Pi^L - sA^T \). The effect of subsidies on welfare is then:

\[
(A3.1) \quad \frac{dU}{ds} = \frac{d\Pi}{ds} + \frac{d\Pi^L}{ds} - A^T.
\]

Using \((A2.1), (A1.7)\) and the effect of subsidies on landowners’ rent: \( \frac{d\Pi^L}{ds} = A^T \frac{dr}{ds} \), it follows that:

\[
(A3.2) \quad \frac{dU}{ds} = A^T pf_k \frac{\alpha_s}{k} - A^T \alpha_s = \frac{\alpha_s A^T}{k} \left( pf_k - k \right) > 0.
\]

Welfare increases with \( \alpha_s > 0 \), otherwise if \( \alpha_s = 0 \), \( \frac{dU}{ds} = 0 \).

A4. Proof of Proposition 4

We analyze the general case when both farms are and remain credit constrained (and \( \alpha_s^1 = \alpha_s^2 > 0 \).\(^6\))

To show: \( \frac{d\Pi^1}{ds} < 0 \) and \( \frac{d\Pi^2}{ds} \leq 0 \) or \( > 0 \) if farm 2 is more credit constrained than farm 1, (and vice versa).

Profit of farm \( i \) is \( \Pi^i = pf^i(A^i, K^i) - (r - s)A^i - kK^i \). Then it follows that:
\( \frac{d \Pi^i}{ds} = \frac{\alpha^i_i A^i}{k} \left( pf^i_k - k \right) - A^i \frac{dr}{ds} + A^i. \)

With area payments, farm \( i \)'s credit constraint is as follows:

\( kk' \leq S^i (W^i) + \alpha^i_i s A^i. \)

In equilibrium the following condition must be satisfied:

\[ (A4.3) \quad pf^i_A + pf^i_k \frac{\alpha^i_i s}{k} - r + \left( 1 - \alpha^i_s \right) s = 0 \quad \text{and} \quad \sum_{i=1}^{2} A^i = A^T. \]

Totally differentiating (A4.3) yields:

\[ (A4.4) \quad M' dA^i + R' ds - dr = 0 \quad \text{and} \quad \sum_{i=1}^{2} dA^i = 0 \]

where

\[ (A4.5) \quad R^i = \frac{\alpha^i_i A^i}{k} \left( pf^i_{Ak} + pf^i_{kk} \frac{\alpha^i_i s}{k} \right) + pf^i_k \frac{\alpha^i_i}{k} + \left( 1 - \alpha^i_s \right) \geq 1 \]

\[ (A4.6) \quad M^i = \left( pf^i_{AA} + pf^i_{Ak} \frac{\alpha^i_i s}{k} + pf^i_{kA} \frac{\alpha^i_i}{k} + pf^i_{kk} \frac{\alpha^i_i s^2}{k^2} \right) < 0. \]

Using (A4.4) it follows that:

\[ (A4.7) \quad \frac{dr}{ds} = \frac{R^1 M^2 + R^2 M^1}{M^1 + M^2} \geq 1. \]

A necessary condition for maximum profit is that \( \Pi^i_{AA} < 0 \), implying that \( M^i < 0 \). With credit constraints it holds that \( pf^i_k - k > 0 \) and that \( \Pi^i_{AK} = pf^i_{Ak} + pf^i_k \frac{\alpha^i_i s}{k} > 0 \)

implying that \( R^i \geq 1 \), hence \( \frac{dr}{ds} \geq 1. \)
The more farm $i$ is credit constrained the less fertilizers it can use, implying (a) that the higher is the increase in land marginal productivity, $pf_{Ak}^i + pf_{kk}^i \frac{\alpha^i_s}{\kappa}$ when adding additional fertilizers, and (b) the higher is the difference between fertilizer marginal value product and fertilizer price, $pf_k^i - k$. Hence, for a given $\alpha^i_s > 0$, $R^i$ is higher the more farm $i$ is credit constrained.

Then it follows that for $\alpha^1_s = \alpha^2_s$: if $R^2 > R^1$ (if farm 2 is more credit constrained than farm 1) then $\frac{d \Pi^1}{ds} < 0$, $\frac{d \Pi^2}{ds} \leq 0$ or $> 0$.

Q.E.D.

**A5. Proof of Proposition 5**

To show:

a. $\frac{dr}{ds} > 1$ with $(\alpha_s > 0, \alpha_w > 0)$

b. $\frac{d \Pi}{ds} < 0 \geq 0$ with $(\alpha_s > 0, \alpha_w > 0)$.

We show the case when farm remains credit constrained with the subsidy.\(^8\)

**Case a:**

If farm credit is based on gross profitability and subsidies, in equilibrium conditions (A1.2) must be satisfied, as well as:

(A5.1) $pf_A + pf_k \frac{\alpha_s s + \alpha_w \pi^G}{\kappa} - r + (1 - \alpha_s)s - \alpha_w \pi^G = 0$. 
Totally differentiating (A1.2) and (A5.1) and solving for $\frac{dr}{ds}$ yields:

$$\frac{dr}{ds} = \left( pf_{kk} \left( \frac{\alpha_s + \alpha_w \pi^G}{k} \right) A + pf_{ak} \frac{A}{k} + pf_k \frac{1}{k} - 1 \right) \left( 1 - \alpha_w \right) \alpha_s$$

$$+ 1 + \alpha_w \left[ pf_{ak} \frac{A}{k} + pf_{kk} \frac{\left( \alpha_s + \alpha_w \pi^G \right) A}{k^2} - 1 \right]$$

where $\left( pf_{ak} + pf_{kk} \frac{\left( \alpha_s + \alpha_w \pi^G \right)}{k} \right) \geq 0$, the intuition is the same as shown in the proof of proposition 1 in Appendix A1.

In order to have a stable equilibrium situation, it must be the case that:

$$0 < 1 + \alpha_w \left[ pf_{ak} \frac{A^T}{k} + pf_{kk} \frac{\left( \alpha_s + \alpha_w \pi^G \right) A^T}{k^2} - 1 \right] < 1$$

This implies that with $\alpha_w < 1$,

$$\frac{pf_{kk} \left( \alpha_s + \alpha_w \pi^G \right) A^T}{k^2} + pf_{ak} \frac{A^T}{k} + pf_k \frac{1}{k} \quad 1 - 1$$

$$\frac{dr}{ds} > 1$$

The impact of subsidies on gross profit and on total credit:

$$\frac{d\pi^G}{ds} = - \frac{pf_{kk} \left( \alpha_s + \alpha_w \pi^G \right) A^T}{k} + pf_{ak} \frac{A^T}{k} \frac{\alpha_s A^T}{k}$$

$$< 0$$

$$\frac{dS}{ds} = \frac{\alpha_s A^T \left( 1 - \alpha_w \right)}{1 + \alpha_w \left[ pf_{ak} \frac{A^T}{k} + pf_{kk} \frac{\left( \alpha_s + \alpha_w \pi^G \right) A^T}{k^2} - 1 \right]} > 0$$
Gross profits decline, and with $\alpha_w < 1$ overall credit increases with subsidies.

If farm credit is based on own land assets and subsidies, in equilibrium condition (A1.2) must be satisfied, as well as:

\[(A5.6)\quad pf_A + pf_k \frac{\alpha_s s}{k} - r + (1 - \alpha_s)s = 0.\]

Totally differentiating (A1.2) and (A5.6) and solving for $\frac{dr}{ds}$ yields:

\[(A5.7)\quad \frac{dr}{ds} = \left( pf_{AK} + pf_{kk} \frac{\alpha_s s}{k} \right) \frac{\alpha_s A^T}{k} + pf_k \frac{\alpha_s}{k} + (1 - \alpha_s) \left( 1 - pf_{kk} \frac{\alpha_s A_o \partial R}{k} - pf_{AK} \frac{\alpha_w A_o \partial R}{k} \right) > 1 \]

In order to have stable equilibrium, $0 < pf_{AK} \frac{\alpha_w A_o \partial R}{k} + pf_{kk} \frac{\alpha_s A_o \partial R}{k} < 1$.

\[\left( pf_{AK} + pf_{kk} \frac{\alpha_s s}{k} \right) > 0,\] the intuition is the same as shown in the proof of proposition 1 in Appendix A1. This implies that land rent increases by more than the size of the subsidy.

**Case b:**

If farms credit is based on gross profitability and subsidies, total differentiating profits

\[(\Pi = pf(A,K) - (r - s)A - kK)\] yields:

\[(A5.8)\quad \frac{d\Pi}{ds} = \left[ pf_k \frac{\alpha_w A^T}{k} - \alpha_w A^T \right] \frac{d\pi^G}{ds} + \left[ pf_k \frac{\alpha_s A^T}{k} + (1 - \alpha_s)A^T \right] - A^T \frac{dr}{ds} \]

where
\( \frac{d \pi^G}{ds} = \frac{p f_k \alpha_s}{k} \left( 1 - \frac{\alpha_s}{\alpha_w} \right) \frac{dr}{ds} \).

From equations (A5.2), (A5.8) and (A5.9) it follows that:

\[
(A5.10) \quad \frac{d \Pi}{ds} = -\frac{\alpha_s A^T}{k} \left[ \begin{array}{c}
\frac{p f_{Ak} + p f_{kk}}{k} \left( \alpha_s + \alpha_w \pi^G \right) A^T \frac{\alpha_w}{k} \\
1 + \alpha_w \left( p f_{Ak} A^T - p f_{kk} \left( \alpha_s + \alpha_w \pi^G \right) A^T \frac{\alpha_w}{k} \right) - 1
\end{array} \right] < 0
\]

With

\[
\left( p f_{Ak} + p f_{kk} \left( \frac{\alpha_s + \alpha_w \pi^G}{k} \right) \right) \frac{\alpha_w}{k} A^T < 1
\]

\[
\left( p f_{Ak} - p f_{Ak} \left( \frac{\alpha_s + \alpha_w \pi^G}{k} \right) \right) \geq 0, \quad \frac{d \Pi}{ds} < 0.
\]

If farm credit is based on agricultural land assets and subsidies total farm income is

\[ \Pi = pf(A, K) - (r - s)A - kK + rA_o. \]

Then from equations (A5.7) it follows that:

\[
(A5.11) \quad \frac{d \Pi}{ds} = \frac{\alpha_s A}{k} \frac{\partial R}{\partial r} \left( p f_k \frac{\alpha_s}{k} \left( 1 - \frac{\alpha_s}{\alpha_w} \right) \right) - \left( A \frac{\alpha_s A}{k} + A \frac{\alpha_s A}{k} \frac{\partial R}{\partial r} \left( p f_{kk} \frac{\alpha_s}{k} + p f_{Ak} \right) \right) + A_o \frac{dr}{ds} < 0
\]

or \( \geq 0. \)

Q.E.D.
Footnotes

1. The case when area subsidies remove the full credit constraint can be analogously derived.

2. To simplify the derivations we assume one representative farm. This assumption does not affect the results.

3. If the initial value of $\alpha_s$ is zero or not large, then with decreasing return to scale it also holds that

$$\frac{\alpha_s A^T}{k} \left( p f_{AK} + p f_{KK} \frac{\alpha_s s}{k} \right) \geq 0.$$

4. The case when area subsidies remove all credit constraints can be analogously derived.

5. The case when area subsidies remove all credit constraints can be analogously derived.

6. The case when area subsidies remove all credit constraints can be analogously derived.

7. The intuition is the same as shown in the proof of proposition 1 in Appendix A1.

8. The case when area subsidies remove the full credit constraint can be analogously derived. To simplify the derivations we assume one representative farm. This assumption does not affect the results.

9. We consider the case when $\alpha_w < 1$. If this is not the case then this would imply that farm is not credit constrained. Banks would be willing to give sufficient credit to farms.