

## THE EFFECT OF STOCHASTIC STORMS ON OPTIMAL BEACH MANAGEMENT

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## THE EFFECT OF STOCHASTIC STORMS ON OPTIMAL BEACH MANAGENENT

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### Abstract

Adaptation to climate change is a serious concern for coastal communities because rapidly eroding beaches and storms threaten both oceanfront property and coastal infrastructure. As the climate changes, coastal environments are likely to experience accelerated sea level rise and more frequent, intense storms increasing the need for policy intervention to protect coastal development. The effects of climate change vary across regions, and physical scientists are still grappling with the relative impacts of sea level rise and increased storm risks on shoreline dynamics. Furthermore, as communities respond through active shoreline stabilization driven by economic factors, economics become coupled with shoreline dynamics. In this paper we model beach nourishment in a stochastic dynamic framework to incorporate the effect of discrete storm events in optimal nourishment decisions. We find that the expectation of increased storm risk due to climate change is likely to intensify feedbacks in the human-natural system, increasing nourishment frequency and decreasing coastal property values in the long run. We also find that optimal nourishment response is more sensitive to changes in storm probability than to changing sea level rise. While scientists and policy makers are primarily concerned with sea level rise, our findings suggest that storm risks could be a greater concern.

## I. INTRODUCTION

Adaptation to climate change is a serious concern for coastal communities because beach erosion and frequent storms threaten both oceanfront property and coastal infrastructure. As the climate changes, coastal environments are likely to experience accelerated sea level rise and more frequent, intense storms (Komar and Allen, 2007; Bender *et al.*, 2010) increasing the need for policy intervention to protect coastal development. Storm damages have already grown to substantial levels in the United States with estimated losses of 198 billion USD in the decade 1995-2005 (NOAA). Shoreline stabilization policies, which attempt to mitigate or adapt to erosion and storms, include the construction of hardened structures such as sea walls, beach nourishment, and strategic retreat from coastal property. Beach nourishment – the process of periodically rebuilding an eroding beach by adding sand dredged from other locations – is prevalent in many parts of the US Atlantic coast and is likely to expand in the coming decades. With continually increasing coastal populations, accounting for the possibility of major storm events is particularly relevant to coastal planners in determining short and long-term adaptation strategies.

Shoreline dynamics are influenced by cross-shore and alongshore movement of beach sand caused by local wave action (Fredsoe and Deigaard, 1992; Ashton *et al.*, 2001). The effects of climate change vary across regions, and physical scientists are still grappling with the relative impacts of sea level rise and increased storm risks on shoreline dynamics. Recent research has shown that changes in storm patterns, which influence local wave climate, can have as large an impact on the amount of erosion or accretion on the coast as sea level rise, and changes in shoreline positions and shapes within local and regional geographic scales depend primarily on wave-driven alongshore sediment transport (Slott *et al.*, 2006). As communities respond through

active shoreline stabilization driven by economic factors, economics become coupled with shoreline dynamics. A natural question that follows is: When policy incorporates the expectation of storm events, does it lead to faster or slower nourishment? Is the economic impact of increased likelihood of storms on coastal communities greater or lesser relative to higher rates of sea-level rise?

Empirical economic analyses have focused primarily on measuring the impact of storms on coastal communities to provide reliable estimates for cost-benefit analyses. Previous studies have estimated the economic impact of storm risk capitalized in coastal property values (Hallstrom and Smith, 2005), and the value of protection against storm risk afforded naturally by coastal wetlands (Farber, 1987; Carbone *et al.*, 2006) or due to damage mitigation measures undertaken by communities (XXX REF). While reliable estimates of the impacts of storms on coastal economies are an essential first step, coastal managers are ultimately interested in long-term adaptation strategies. To our knowledge, this is the first analysis that incorporates the dynamic effects of human intervention to protect coastal property from storm damage and beach erosion in determining optimal shoreline stabilization policy.

In this paper we model beach nourishment in a stochastic dynamic framework to incorporate the effect of discrete storm events in optimal nourishment decisions. In addition to the effect on alongshore and cross-shore sediment flux, storm events also have discrete, short-term impacts on the flow of beach amenities, instantaneously causing severe erosion and significant damage to oceanfront property. We build on previous work (Smith *et al.*, 2009; Gopalakrishnan *et al.*, 2011), and develop a model for optimal re-nourishment in a representative community, incorporating stochastic storm events that occur within a nourishment interval. As in Smith *et al.* (2009), beach nourishment is a periodically recurring event and the beach manager

chooses a nourishment interval ( $T$ ) that maximizes discounted net benefits. Benefits from wide beaches are capitalized into coastal property values (Pompe and Rinehart, 1995; Bin *et al.*, 2008; Landry and Hindsley, 2010; Gopalakrishnan *et al.*, 2011) and the costs of beach nourishment include fixed capital costs and variable costs of nourishment sand. We introduce stochastic storm events that follow a Poisson distribution and affect the expected benefits over the nourishment interval. Our approach is similar to previous work on the impact of forest fires on optimal forest rotation (Reed, 1984). An optimal nourishment interval is chosen to maximize the expected net benefits over time. Results suggest that the expectation of increased storm risk due to climate change is likely to intensify feedbacks in the human-natural system, increasing nourishment frequency and decreasing coastal property values in the long run. We also find that optimal nourishment response is more sensitive to changes in storm probability than to changing sea level rise. While scientists and policy makers are primarily concerned with sea level rise, our findings suggest that storm risks could be a greater concern.

## **II. OPTIMAL BEACH NOURISHMENT IN THE PRESENCE OF STOCHASTIC STORMS**

Beach nourishment is modeled as an optimal rotation problem in which coastal managers choose a constant time interval between nourishment events to maximize net benefits (Smith *et al.*, 2009). By choosing an optimal nourishment interval, the beach manager determines how much the beach erodes before it is restored. Beach width ( $x$ ) depends on linear background erosion ( $\gamma$ ) and exponential decay ( $\theta$ ) of the nourished portion ( $\mu$ ) of the beach as it returns to the equilibrium profile. Every time beach nourishment is undertaken, the beach returns to its initial width ( $x_0$ ). Beach width at any given time ( $t$ ) is characterized by:

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$$(1) \quad x(t) = x_0 - \gamma t - \mu x_0 (1 - e^{-\theta t})$$

Benefits from wide beaches are capitalized in to coastal property values and the total benefits for an interval T between two nourishment projects are:

$$(2) \quad B(T) = \int_0^T e^{-\delta t} \delta A(x(t)^\beta) dt$$

where A is the baseline value that captures the value of all property attributes except beach width,  $\beta$  is the hedonic coefficient on beach width (derived from previous empirical studies), and  $\delta$  is the discount rate for future benefits and costs. We assume that the discount rate is the same as the capitalization rate to convert the stock value of the property in to amenity flows.

Nourishment costs include fixed costs ( $c$ ) and variable costs ( $\phi$ ) that depend on the amount of nourishment sand required to return the beach to its initial width.

$$(3) \quad \begin{aligned} C(T) &= c + \phi(x_0 - x(T)) \\ \Rightarrow C(T) &= c + \phi(x_0 - \mu x_0 e^{-\theta T} - (1 - \mu)x_0 + \gamma T) \\ \Rightarrow C(T) &= c + \phi(\mu x_0 (1 - e^{-\theta T}) + \gamma T) \end{aligned}$$

We assume that storms are Poisson distributed and if a storm occurs at some time ( $\tau$ ) before the next nourishment period (T), then the benefits from replenishment accrue only to the moment the storm occurs. Let  $\tau_1, \tau_2, \tau_3,$  represent times between loss of nourished beach width (times between nourish events or storms). If storms are Poisson distributed and the planned nourishment interval is denoted as T then the cumulative distribution function for the independent random variable  $\tau_i$  is written as:

$$(4) \quad \Pr(\tau \leq t) = F(t) = \begin{cases} 1 - e^{-\lambda t} & \text{if } t < T \\ 1 & \text{if } t \geq T \end{cases}$$

This leads to a probability distribution function given by

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$$(5) \quad \frac{d}{dt}F(t) = f(t) = \left\{ \begin{array}{l} \lambda e^{-\lambda t} \\ e^{-\lambda t} \end{array} \right\}$$

With the possibility of a major storm, the net benefits  $\pi(\tau_k)$  from the  $k^{\text{th}}$  nourishment episode are:

$$(6) \quad \pi(\tau_k) = \left( \begin{array}{l} B(T) - C(T) \quad \text{if } \tau_k = T \\ B(\tau) + B_2(T - \tau) - C(T) \quad \text{if } \tau_k < T \end{array} \right)$$

where  $B(T)$  is the cumulative benefits over the nourishment interval ( $T$ ) and  $C(T)$  is the cost of nourishing every  $T$  years. If a storm occurs before the next nourishment ( $T$ ), then the benefits from replenishment (nourished portion of the beach) accrue only to the moment the storm occurs ( $\tau_k$ ). After a storm, the nourished portion of the beach is lost and benefits accrue only from the remaining beach, which faces background linear erosion. In the real world, this assumption may be realistic as a significant departure from a nourishment schedule would require mobilizing funding and capital equipment as well as securing sand resources and permitting. Exceptions would occur for emergency stabilizations, such as filling new inlets in barrier islands.

$$(7) \quad B(T) = \int_0^T A \delta e^{-\delta t} [\mu x_0 e^{-\theta t} + (1 - \mu)x_0 - \gamma t]^\beta dt$$

$$(7') \quad B_2(T - \tau) = \int_\tau^T A \delta e^{-\delta t} [(1 - \mu)x_0 - \gamma t]^\beta dt$$

The total expected present value of all future cost and revenue is:

$$(8) \quad J(T) = E \left\{ \sum_{k=0}^{\infty} e^{-\delta(\tau_0 + \tau_1 + \tau_2 + \dots + \tau_k)} \pi(\tau_{k+1}) \right\}$$

Because the random variables  $\tau_k$  are independent, the expectation of the inner product in the summation can be split as

$$(9) \quad E\left\{e^{-\delta(\tau_0+\tau_1+\tau_2+\dots+\tau_k)}\pi(\tau_{k+1})\right\} = \left(E\left\{e^{-\delta\tau}\right\}\right)^k E\{\pi(\tau)\}$$

Taking the first expectation in the above expression:

$$\begin{aligned} E\left\{e^{-\delta\tau}\right\} &= \int_0^{\infty} e^{-\delta\tau} f(\tau) d\tau \\ &= \int_0^T e^{-\delta\tau} \lambda e^{-\lambda\tau} d\tau + e^{-\delta T} e^{-\lambda T} \\ &= \frac{\lambda + \delta e^{-(\lambda+\delta)T}}{\lambda + \delta} \end{aligned}$$

Similarly for the second expectation from Eq. (9):

$$\begin{aligned} E\{\pi(\tau)\} &= \int_0^T [B(t) + B_2(T-t) - C(T)] \lambda e^{-\lambda t} dt + [B(T) - C(T)] e^{-\lambda T} \\ &= \int_0^T [B(t) + B_2(T-t)] \lambda e^{-\lambda t} dt + C(T)(e^{-\lambda T} - 1) + [B(T) - C(T)] e^{-\lambda T} \end{aligned}$$

Putting these results back into Eq. (8) gives

$$(10) \quad J(T) = \sum_{k=0}^{\infty} \left( \frac{\lambda + \delta e^{-(\lambda+\delta)T}}{\lambda + \delta} \right)^k \left[ \begin{array}{l} C(T)(e^{-\lambda T} - 1) + (B(T) - C(T))e^{-\lambda T} \\ + \int_0^T [B(t) + B_2(T-t)] \lambda e^{-\lambda t} dt \end{array} \right]$$

Summing the infinite geometric series and canceling common terms results in



$$(11) \quad J(T) = \left( \frac{\lambda + \delta}{\delta(1 - e^{-(\lambda+\delta)T})} \right) \left[ B(T)e^{-\lambda T} - C(T) + \int_0^T [B(t) + B_2(T-t)] \lambda e^{-\lambda t} dt \right]$$

Now we take the derivative of Eq. (11) and set it to zero to give an expression that can be solved for the T that maximizes the expected net benefits:

$$(12) \quad \frac{[B'(T)e^{-\lambda T} - C'(T)](\lambda + \delta)(1 - e^{-(\lambda+\delta)T}) - [B(T)e^{-\lambda T} - C(T)](\lambda + \delta)^2 e^{-(\lambda+\delta)T} - \lambda(\lambda + \delta)^2 e^{-(\lambda+\delta)T} \int_0^T B(t)e^{-\lambda t} dt}{\delta(1 - e^{-(\lambda+\delta)T})^2} = 0$$

The interval  $T^*$  that solves equation (12) is the optimal time between nourishment intervals, accounting for the expectation of storm events. Note that if  $\lambda$  is set to 0, the above expression matches Smith et al. 2009 (Eq 10). In the next section we discuss the parameter values used to calibrate the numerical model and then show the result of numerically determining the optimum nourishment interval T.

### III. MODEL CALIBRATION

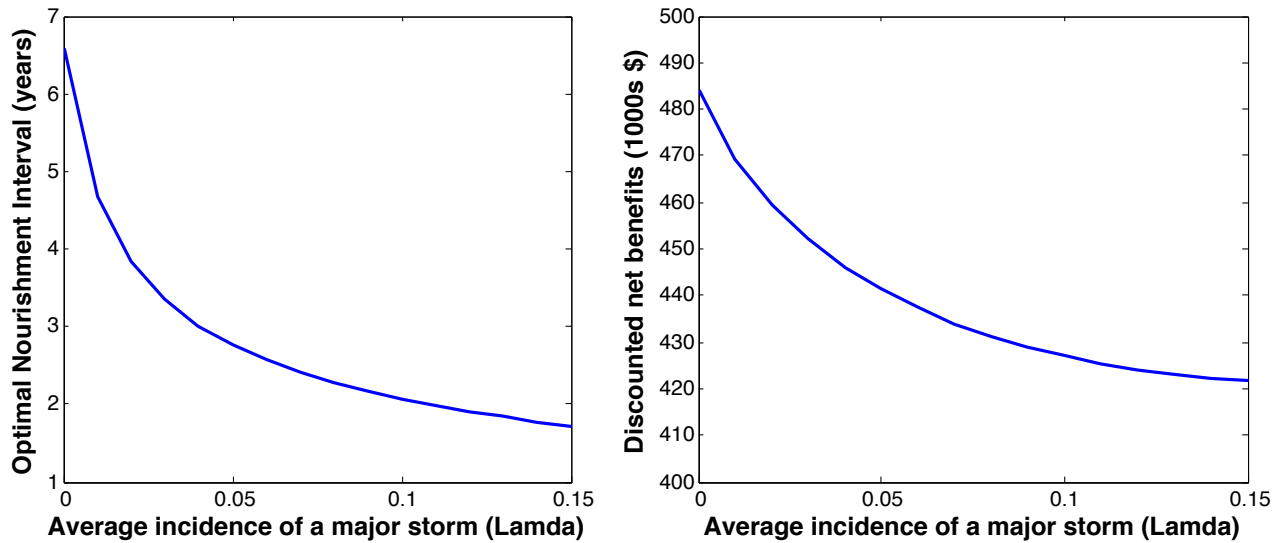
Parameter values used in the numerical analysis are based on previous studies (Smith *et al.*, 2009) in order to enable comparison of optimal nourishment when storm risk is incorporated. Initial beach width is 100ft, which is similar to the average width observed in an empirical study of ten beach towns in North Carolina (Gopalakrishnan *et al.*, 2011). Baseline erosion is 2 ft/year, 35% of the beach represents the nourished portion of the beach that decays exponentially to return to equilibrium profile, and the nourishment decay rate is 0.10 (roughly half of the nourishment sand lost in 7 years). Erosion events in the model are represented by hurricanes with return intervals modeled as a Poisson process (Parisi and Lund, 2008). We use mean return intervals from the NOAA hurricane center (<http://www.nhc.noaa.gov/climo/>) to establish upper

bounds on the Poisson distribution parameter  $\lambda$ , the mean number of occurrences in a given time interval. Specifically, along the North Carolina Outer Banks and in portions of South Florida, mean return intervals of 5 years suggest a  $\lambda$  value of 0.2. Analysis of LIDAR data regarding the fate of beach nourishment sand after Hurricane Fran suggest a weak category 2 hurricane can remove nearly the entire portion of a newly nourished beach (Gares *et al.*, 2006).

The discount rate for the economic analysis is 0.06. Nourishment costs consist of fixed and variable costs. We assume fixed costs of capital equipment to be \$1 million normalized over 100 ocean front properties. Variable costs are assumed to be \$1000 per cross-shore foot of beach build-out. We assume a baseline property value that captures the value of all housing attributes excluding beach width to be \$200,000. The hedonic value of beach width is 0.25, which is similar to estimates in previous studies (Pompe and Rinehart, 1995; Smith *et al.*, 2009).

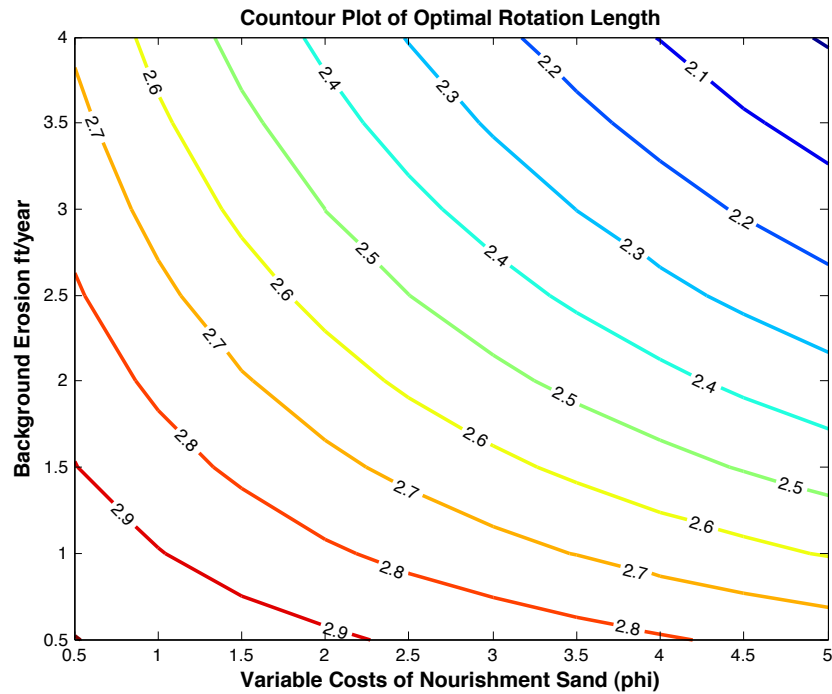
#### **IV. RESULTS**

Optimal nourishment interval is calculated by numerically solving equation (12) to determine  $T^*$ . We explore the effect of uncertain storm events on the optimal beach nourishment in a representative coastal community by solving the model for a range of storm parameter values. We find that the expectation of a storm increases the nourishment frequency and decreases the discounted value of net benefits over time (Figure 1). As in the forestry model with possible forest fires (Reed 1984), stochastic storms have the effect of increasing discounting and coastal communities nourish sooner than they would in the absence of storms. Storm risk essentially adds to the discount rate.



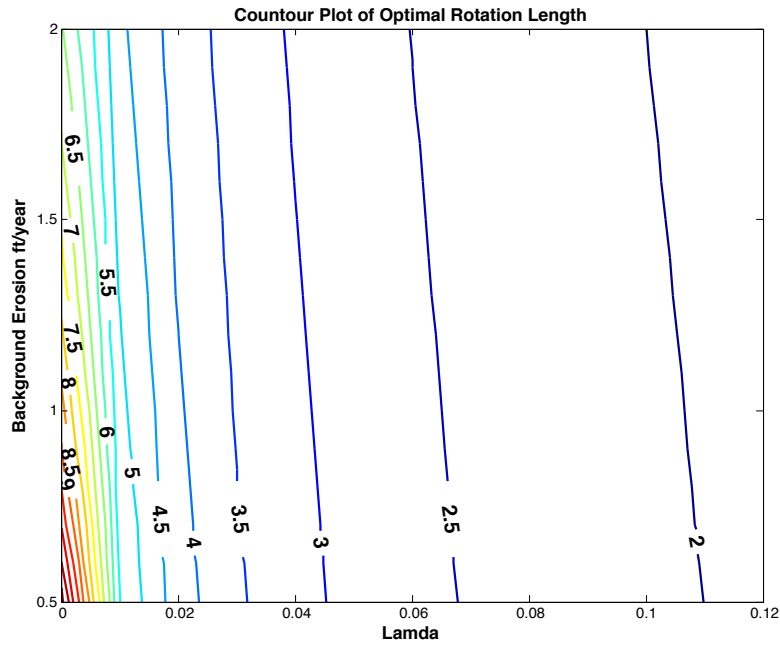
**Figure 1: Effect of Stochastic Storms on Optimal Beach Nourishment: As the likelihood of a major storm increases, the optimal response is for communities to shorten the nourishment interval (Left Panel). Cumulative discounted long-run value also decreases with higher probability of storms (Right Panel).**

To understand the optimal response to changes in economic and physical parameters of the system in the presence of storms, we ran simulations with a range of values for background erosion rates and variable nourishment costs. Smith *et al.* demonstrate that as variable costs of nourishment increase, the coastal community may nourish more frequently for some regions of the parameter space as the importance of variable sand costs increase relative to the fixed costs of nourishment. We find that incorporating storms in the model intensifies this impact of an increase in variable costs, leading to shorter optimal nourishment intervals. The effect is also more persistent when beach managers are uncertain about future benefits due to possible storm damage such that optimal economic response further accelerates feedbacks in the coupled system.

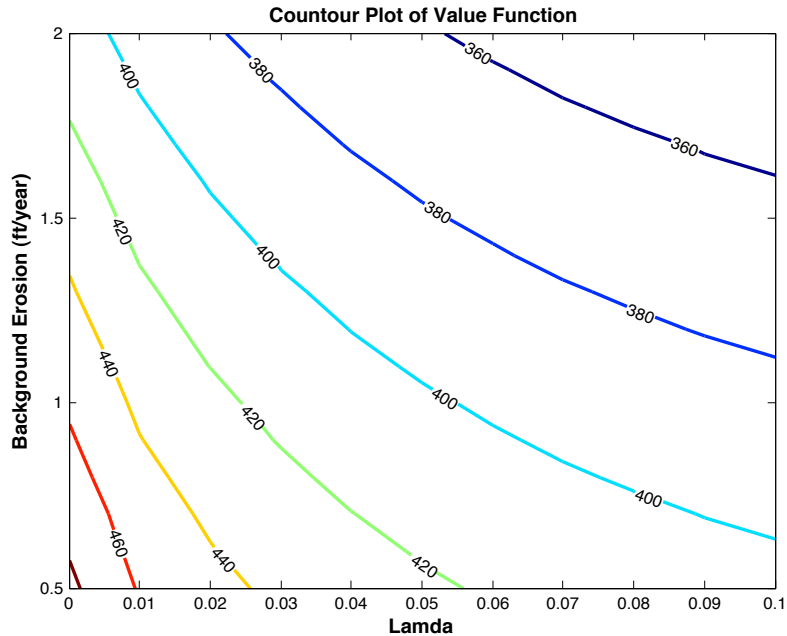


**Figure 2: Increasing variable costs accelerate nourishment: The simulations were conducted using a storm parameter value of 0.05. Increase in background erosion and variable sand costs of nourishment lead to shorter nourishment intervals.**

Finally, we compare the effect of changes in the likelihood of storms and changing background erosion that can be attributed to accelerated sea-level rise. Optimal economic response is more sensitive to changes in storm likelihood relative to changes in the background erosion rate (Figure 3). The expectation of a major storm that can potentially cause severe damage to oceanfront property leads to more frequent nourishment to increase current benefits because of the uncertainty of future benefits. The capitalized value of net benefits from wider beaches decreases as the likelihood of major storms increase and as the background erosion rate increases. We find that, at low levels of storm probability, property values are more sensitive to a small change in the likelihood of storms and as the average expectation of storms increases, property values decrease less rapidly to changes in storm likelihood.



**Figure 3: Optimal nourishment response is more sensitive to changes in the likelihood of storms. Increase in the likelihood of major storm events leads to shorter nourishment interval. Steep contour plots show that nourishment intervals respond more rapidly with changes in the likelihood of storms relative to changes in background erosion.**



**Figure 4: Coastal property values decline as the likelihood of major storms and background erosion increase. When storm probability is low (low values of  $\lambda$ ) property values are more sensitive to a change in the likelihood of storms.**

## V. DISCUSSION

Coastal communities are especially vulnerable to the impacts of climate change with oceanfront property and development being threatened by rapid beach erosion. Whereas scientists and policy makers have been primarily concerned with the consequences of increased sea-level rise in a changing climate, recent research shows that storm patterns are likely to change dramatically as we face warmer climates, and the number of high intensity storms (category 3 and above) could double by the end of the century (Bender *et al.*, 2010). Communities planning adaptation strategies need to consider the potential impact of more frequent storms and how their policy responses feed back into coastal dynamics.

In this paper, we model optimal shoreline stabilization through periodic beach re-nourishment incorporating the expectation of increased storm risk. We find that nourishment frequency increases as the likelihood of severe storms increases. Our results are consistent with similar work in the forestry literature on the effect of stochastic forest fires in optimal harvest patterns (Reed 1984). As in the case of forest fires, we find that stochastic storms effectively increase the discount rate and shift nourishment benefits towards current time periods, placing a lower value on future benefits. Increased likelihood of storms also reduces the total discounted value of long-run net benefits. Shorter nourishment intervals that account for the expectation of storm risk will lead to wider (average) beaches in the long run, which in turn may also mitigate some impacts of storm damage.

As accelerated sea level rise and increased storminess together cause demand for nourishment sand to increase, we can expect higher nourishment costs in the future, which will feed back on optimal nourishment patterns. Previous work has shown that higher variable costs of nourishment may lead to more frequent nourishment for some regions of the parameter space (Smith et al. 2009). We find that incorporating the expectation of storm risk tends to intensify this impact, leading to even shorter optimal rotation lengths as variable costs increase (Figure 2). This counter-intuitive result raises a greater concern about the sustainability of beach nourishment as a long-term shoreline stabilization policy.

When we compare the effects of increasing probability of storm damage and higher rates of sea level rise, we find that the optimal nourishment response is more sensitive to an increase in the storm likelihood (Figure 3). Optimal nourishment intervals decrease rapidly as the likelihood

of storms increase compared to an increase in the background erosion resulting from increased sea level rise. This suggests that if communities optimally respond to a changing coastal climate, their nourishment strategies will be affected more by small changes in storm risks relative to incremental changes in background erosion. This economic finding complements and extends results in the geomorphology literature that suggest that changing storm patterns could dominate sea level rise as a driver of coastline change (Slott et al. 2006). In a coupled system, communities respond to the changing coastal environment through active shoreline stabilization, and in turn influence the coastal system. Storm risk can further accelerate dynamic feedbacks in the coastal-economic system leading to more frequent nourishment.

In this first attempt to incorporate storm risk in a dynamic model of optimal beach nourishment, we made some simplifying assumptions about the physical environment. Our model takes a single community as the unit of analysis and assumes a constant nourishment interval. Though the assumption of stationarity (the state of the world returns to time 0 at the end of a nourishment interval) is unrealistic, it provides qualitative insights that are relevant to policy-makers. Beach nourishment plans often span several decades with coastal managers choosing how often to nourish their beaches. Furthermore, beach communities do not exist in isolation and are often affected by management strategies in neighboring communities. We focus here on a single representative community, but insights from this model motivate further research on the impact of future sand availability and a potential “race to dredge” as multiple communities compete for the limited sand resource.



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