Journal of Agricultural and Resource Economics 45(3):549–570 ISSN: 1068-5502 (Print); 2327-8285 (Online)
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doi: 10.22004/ag.econ.302496

Firm Decision Making under Both Input and Output Price Uncertainty

Ardian Harri, Joshua G. Maples, John Michael Riley, and Jesse B. Tack

Theory of the firm suggests that optimal production levels decrease as output price becomes random. Firms operating in industries with long production lags are also exposed to input price uncertainty. This paper provides a novel decision-theoretic model in the presence of both input and output price uncertainty and uses U.S. beef sector data to test theoretical propositions concerning firm behavior. Our findings confirm that, in a two-stage production, the introduction of input price uncertainty leads to increased use of the input and an increased level of output in stage one and a decreased level of output in stage two.

Key words: firm, firm behavior, input, output, price, uncertainty

Introduction

The majority of previous research on the topic of price uncertainty focuses on output price uncertainty. However, output price uncertainty is not the only source of uncertainty faced by producers. Recent increased price volatility for most agricultural commodities (Cashin and McDermott, 2002; Roache, 2010; Jacks, O'Rourke, and Williamson, 2011), as well as established linkages between agricultural and energy markets, expose producers to additional types of systematic price risk related to input prices (Irwin and Good, 2009; Schweikhardt, 2009; Harri, Nalley, and Hudson, 2009). This market interdependence has added to the increased volatility—and hence uncertainty—in the agricultural cash and futures markets as commodity markets potentially import volatility from the energy sector Schweikhardt (2009). While this imported volatility flows to most agricultural commodities—either directly or indirectly—it is especially true for biofuel feedstocks (Nazlioglu, Erdem, and Soytas, 2013; Hertel and Beckman, 2011).

Profit-maximizing producers form expectations over output price when decisions on input levels are made. Temporal variation of this price naturally induces exposure to uncertainty—especially in industries facing long production lags. Long production lags, which incorporate multiple input decisions/purchases throughout the production period, are inherent to agricultural sectors that rely on feedstocks as inputs (e.g., livestock production), which are subjected to the increased volatility. As a result, profits can be greatly affected by divergences of actual from expected outcomes. Importantly, anecdotal evidence through personal contacts with feedlot managers points to the increased attention by the managers to the volatility of feedstock prices. In this case, producers must form expectations for costs of variable inputs that enter production after initial decisions are made and as a result are exposed to both output and input price uncertainty. Understanding the effects of input price uncertainty on firm decision making would prove helpful to both industry participants

Ardian Harri (corresponding author) is a professor and Joshua Maples is an assistant professor in the Department of Agricultural Economics at Mississippi State University. John Michael Riley is an assistant professor in the Department of Agricultural Economics at Oklahoma State University. Jesse Tack is an associate professor in the Department of Agricultural Economics at Kansas State University.

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Review coordinated by Anton Bekkerman.

and researchers studying firm behavior under price uncertainty. Thus, the focus of this research is to study the decision making of the firm under *both* input and output price uncertainty.

With this in mind, the contributions of this research are as follows: First, we provide a novel expected utility maximizing model of firm behavior in the presence of input price uncertainty. Second, we extend the theoretical model to the all-encompassing case in which the firm faces both input and output price uncertainty. Third, we empirically explore several propositions derived from the theoretical model using data for the livestock sector. Our theoretical (using a two-stage model) and empirical results show that introduction of input price uncertainty leads to an increased use of the input and increased production in the first stage and decreased production in the second stage.

Cattle finishers can use results from this research to better understand and cope with the dynamics of the input and output price uncertainty to which they are exposed. Our findings suggest that the increase in corn price uncertainty since the mid-2000s has become a significant factor that cattle finishers consider when making purchasing and production decisions. More broadly, our results suggest that input price uncertainty has a significant effect on the firm's optimal level of output. According to our results, increasing input price uncertainty is associated with a decline in the level of output in the second stage in a two-stage production. The implication is that the exit of less efficient firms from the industry in periods of high price volatility could further exacerbate the negative impact of input price uncertainty on industry supply.

Literature Review

Sandmo (1971), Leland (1972), Batra and Ullah (1974), and Ishii (1977) introduced uncertainty into decision-theoretic models of firm behavior by relaxing the traditional assumption that demand for the firm's output is known with certainty when production decisions are made. Batra and Ullah and Hartman (1975) extended the theory by examining the effect of output price uncertainty on factor demands. Turnovsky (1973) further extended the model by allowing the firm to adjust its initial production, at additional costs, after the output price is realized. Hartman (1976) also allowed for one input to be chosen after the output price is observed, relaxing the assumption that all inputs are chosen before the output price is observed. Epstein (1978) generalized the analysis of the behavior of the firm under price uncertainty modeled by Sandmo, Batra and Ullah, Turnovsky, and Hartman in the case of both no production flexibility and production flexibility. Unlike Sandmo and Batra and Ullah, regarding the *ex post* (after uncertainty is resolved) decision, Epstein showed that the firm might increase rather than decrease output level under production flexibility, even with decreasing relative risk aversion (DARA) preferences.

Further, regarding the *ex ante* (before uncertainty is resolved) decision, Epstein Epstein (1978) showed that, under production flexibility, a shift in the output price, while input prices are nonstochastic, could either increase or decrease the use of input x compared to the case of a nonstochastic output price. The use of the *ex ante* chosen input will increase if the decrease in the absolute risk aversion is large enough to offset the price risk affinity. Epstein further derives conditions under which a marginal increase in the uncertainty of output price can lead to a determinate sign on the use of the *ex ante* chosen input.

These theoretical studies have in common the fact that the only source of price uncertainty is related to output price. In turn, several studies have emphasized the importance of input prices in firm decisions under uncertainty. Feder, Just, and Schmitz (1980) showed that when firms have access to forward pricing instruments (e.g., futures contracts), optimal production levels depend only on the futures price and input costs. Stewart (1978), White (1986), and Lächler (1984) studied the decision of a competitive firm when choosing the optimal level of a fixed input (capital) and one or more variable inputs. However, in each of these studies, the variable inputs are chosen after the uncertainty is resolved. As such, these models would not be appropriate for studying the behavior of the firms in industries like the livestock industry.

Devadoss and Choi (1991) investigated the effect of uncertainty related to the input price through changes in the minimum and maximum prices of the random input price. Using a two-stage model, they investigated the effect of a change in the maximum and minimum level of the random price of a variable input on the use of the quasi-fixed capital input. The capital input is chosen *ex ante*, before the price of the variable input is known with certainty. Additionally, they investigated the effect of a mean-preserving contraction (MPC), implemented through simultaneous changes in both the minimum and maximum prices of the random input price, on the use of the capital input and output level. They found that, for a risk-averse decision maker, an increase in the maximum price of the variable input increases (decreases) the optimal level of the capital input if the two inputs are substitutes (complements). The effect of a change in the minimum price of the variable input on the optimal level of the capital input is generally indeterminate. With respect to an MPC, they showed that, for a risk-averse decision maker, an MPC in the random input price of the variable input decreases (increases) the optimal use of the capital input if the two inputs are substitutes (complements) and if the demand curve for the variable input becomes flatter (steeper) as the capital input increases. Finally, they derived conditions under which a risk-averse decision maker facing an MPC in the random input price of the variable input would increase or decrease expected output.

Greenwald and Stiglitz (1993) incorporated exposure to financial risk into the overall measure of total uncertainty faced by the firm. More recently, Alghalith (2007, 2008, 2010) studied the effect of input price uncertainty on a firm's decision but provided no detail of the firm's decision and, more importantly, of the timing of the input and output choices. In summary, we current theoretical developments lack a model that allows for the study of firm behavior in cases (i) when the variable inputs need to be chosen before the uncertainty is resolved, (ii) under input price uncertainty related to changes that effect the whole of the price density rather than only minimum and maximum price, and—more importantly— (iii) under *both* input and output price uncertainty.

Given these theoretical shortcomings, even fewer empirical studies investigate the effect of price uncertainty on input and output decisions. As an example, Bellemare, Lee, and Just (2020) studied the effects of output price uncertainty on output using both laboratory and field experiments. Tonsor (2018) studied cattle producers' decision making under uncertainty by using alternative formats of presenting uncertainty to producers and multiple alternative reference points. Other empirical studies include Mattos and Zinn (2016), who studied producers' marketing decisions under price uncertainty, and Engle Warnick et al. (2011), who examined the relationship between price uncertainty and crop diversification.

This research addresses the theoretical gap in modeling the firm decision making under both input and output price uncertainty. Additionally, this research adds to the empirical literature on the effect of price uncertainty on input and output decisions by empirically testing several propositions derived from the theoretical model using data from the livestock sector.

Theoretical Model

We develop a model that shows that both input and output price uncertainty affect firm's production decisions. We assume that the firm operates in competitive markets and that the decision makers form rational expectations of price and price risk (Holt, 1993).[1] We also assume that the firm is a price taker in both output and input markets. The firm employs a two-stage flexible technology described by the function $F(x, z_1, z_2, y)$, where x is an input whose level is determined in stage one. The level of input z is determined in both stages. The level of input z_1 in the first stage is determined

[1] Under imperfect competition, firms need to be strategic and consider the actions/strategies of their rivals in addition to their own. Using game-theory models to study firm behavior under imperfect competition and price uncertainty, previous works have found a similar result to that of perfect competition studied in Sandmo (1971), Leland (1972), Turnovsky (1973), Batra and Ullah (1974), Hartman (1976), Ishii (1977), and Epstein (1978), namely that the optimal production level decreases for a risk-averse decision maker under price uncertainty. A few examples include Fishelson (1989), Chevalier-Roignant et al. (2011), and Ryu and Kim (2011).

while its price is known with certainty but before the random output price and the random price for input z_2 are determined with certainty. We refer to the decisions in stage one as *ex ante* decisions. Let $F_1(x,z_1,y_1)$ denote the technology for the initial production level determined in stage one and $F_2(z_2,y_2,F_1(x,z_1,y_1))$ denote the technology for the second stage of production when the level of input z_2 is also chosen, after the random price for input z_2 is determined but before the random output price is determined. We refer to the decision in stage two as an *ex post* decision.

An example would be a livestock producer who chooses feed level at the initial stage and at a later stage of production or a row crop producer who chooses fertilizer level at planting and at some time after planting. A broader example would be a firm that uses energy inputs in a continuous production, and the energy input decision does not necessarily coincide with the output level decision. Transportation firms and airlines in particular (Morrell and Swan, 2006) are good examples. For the case of the livestock producer, the producer in stage one, or *ex ante*, would choose the level of input x, the number of feeder cattle to place on feed, and the level of input z_1, the initial feed level. In the second stage of production, or *ex post*, the producer may choose to harvest the cattle, at this point, as fed cattle. Harvesting cattle at the beginning of the second stage would imply a 0 level of feed input, z_2. Alternatively, the producer may choose to continue to feed the cattle, which implies a positive level of feed input, z_2.

The cost function $C\left(y,x,z_1,z_2,r,l,\tilde{l}\right)$ describes the total cost of producing quantity y (Turnovsky, 1973), where $\tilde{p},r,l,$ and $\tilde{l}$ are the prices associated with y,x,z_1 and z_2, respectively. Prices r and l are known with certainty while tildes denote that $\tilde{p}$ and $\tilde{l}$ are random variables. In other words, $\tilde{p}$, and $\tilde{l}$ are random variables reflecting uncertainty about discounted future input and output prices. Then, $C(y_1,x,z_1,r,l;0)$ denotes the cost of producing y_1 during the first stage and the difference, $C(y_1,x,z_1,r,l;y_2,z_2,y_1,l)-C(y_1,x,z_1,r,l;0)$, denotes the cost of producing y_2 during the second stage. The $C(y_1,x,z_1,r,l;0)$ cost function would denote the case when a livestock producer chooses to harvest cattle at the onset of the second stage of production rather than continue to feed them or a row crop producer who chooses not to fertilize, irrigate, or apply a herbicide/insecticide in the middle of the growing season after planting but not abandon the crop. We employ the same assumptions about the costs function as in Turnovsky:

1. When planned output is 0, costs are 0,
2. The marginal cost of increasing the initial output and the marginal cost of any additional changes are both positive;
3. The cost function has the usual convexity properties; and
4. Production in the second stage is more costly than production in the first stage.

We further assume (assumption 5) that the marginal products of the inputs are positive. Decision makers maximize the expected utility of the firm's profit so that the objective function is given by

$$(1) \qquad \max_{y,x,z_1,z_2} EU\left[\tilde{p}y-rx-lz_1-\tilde{l}z_2|F(x,y,z_1,z_2)=0\right],$$

where x and z_1 are chosen *ex ante* while y and z_2 are chosen *ex post*. We assume (assumption 6) that marginal utility of profit is positive (i.e., $U'>0$); however, we allow for risk-averse, neutral, or risk-loving behavior as the sign for U'' is left unrestricted. We assume (assumption 7) that r and l are known with certainty at the time of the *ex ante* production decision in stage one while $\tilde{p}$ and $\tilde{l}$ are random variables. We also assume (assumption 8) that $\tilde{p}$ and $\tilde{l}$ are defined by their probability density functions $f(p)$ and $f(l)$ with means $E(p)=\mu_p$ and $E(l)=\mu_l$ and σ_p and σ_l as the second moments for $\tilde{p}$, and $\tilde{l}$, respectively.

Breaking the maximization into two stages, we can express equation (1) as[2]

$$\max_{x\geq 0, z_1\geq 0} EU\left[g\left(\tilde{p},\tilde{l},x,z_1\right) - rx - lz_1\right]; \tag{2}$$

$$g\left(p,l,x,z_1\right) = \max_{y,z_2}\left[py - lz_2 | F\left(x,y,z_1,z_2\right) = 0\right]; \tag{3}$$

where g is the variable profit function dual to F. Given that z_2 is chosen *ex post* and optimally subject to the *ex ante* chosen x and z_1, the random profit as a function of x, z_1, $\tilde{p}$, and $\tilde{l}$ is[3]

$$\tilde{\pi} = g\left(\tilde{p},\tilde{l},x,z_1\right) - rx - lz_1, \tag{4}$$

where the function g is nondecreasing in p and nonincreasing in l, convex and (for given x and z_1) linear homogeneous in (p,l), and increasing and concave in x and z_1.

Let $y^* = y\left(p,l;x,z_1\right)$ and $z_2{}^* = z\left(p,l;x,z_1\right)$, the solutions to equation (3), denote the short-run supply and demand functions. Then, using Hotelling's lemma, $y^* = g_p\left(p,l;x,z_1\right)$ and $z_2{}^* = -g_l\left(p,l;x,z_1\right)$.

We next discuss the effect of uncertainty related to the input price, $\tilde{l}$, in the form of a marginal shift in the expected input price distribution. First, we assume (assumption 9) that output price, p, is nonstochastic and later allow for both input price $\tilde{l}$ and output price $\tilde{p}$ to be random.

Determining the Effect of Input Price Uncertainty on the Use of Input x

Two propositions are derived in this case. The details of this section are provided in the Appendix, while proofs of the propositions are included in the Online Supplement (www.jareonline.org).

PROPOSITION 1. *Under assumptions 1–9 and*

1. *if inputs x and z are complements and z is used in both stages, or*
2. *if inputs x and z are substitutes and z is used in the second stage only,*

then decreasing absolute risk aversion preferences are sufficient to ensure that the firm will, in stage one, use more of the input x and increase production under uncertain price for input z.

PROPOSITION 2. *Under assumptions 1–9 and*

1. *if inputs x and z are complements and z is used in the second stage only, or*
2. *if inputs x and z are substitutes and z is used in both stages,*

then decreasing (increasing) absolute risk aversion preferences are sufficient to ensure that the firm will, in stage one, use more (less) of the input x and increase (decrease) production under uncertain price for input z.

Input and Output Price Uncertainty

Up to this point, the source of uncertainty faced by the firm has been either the output price or the input price. We now consider the case in which both prices are uncertain. In this case, according to

[2] Epstein (1975) showed that the consumer choice problem under uncertainty is made mathematically tractable by breaking the maximization problem into two stages using the duality between the direct and indirect utility functions and reformulating the problem in terms of the indirect utility function. See Bellemare, Barrett, and Just (2013) for an application to the consumer choice problem under multiple price uncertainties. Epstein (1978) applies a similar approach to the firm choice problem under uncertainty.

[3] Farm programs and crop insurance would reduce the uncertainty faced by the firm. In the presence of these programs, the findings of this research would be applicable for the remaining level of uncertainty.

Epstein (1978), the direction of the impact of uncertainty on the use of x depends on the form of the relative risk aversion for multiple sources of uncertainty (in our case, two sources of uncertainty related to output and input prices). We employ the multivariate risk aversion measure developed by Karni (1979). Let $\boldsymbol{\mu} = (\mu_p, \mu_l)$ be the vector of means $E(p)$ and $E(l)$. Also, let $\lambda_U(\boldsymbol{\omega}, \boldsymbol{Z})$ be a risk premium function (Kihlstrom and Mirman, 1974), defined by

$$(5) \qquad U(\boldsymbol{\omega} - \lambda_U) = E[U(\boldsymbol{\omega} + \boldsymbol{Z})],$$

where $\boldsymbol{\omega} = \left(w, \tilde{p}, \tilde{l}\right)$, $\boldsymbol{Z} = \left(w - \bar{w}, \tilde{p} - \mu_p, \tilde{l} - \mu_l\right)$, and $E(w) = \bar{w}$. Note that risks are actuarially neutral (i.e., $E(\boldsymbol{Z}) = \boldsymbol{0}$). Finally, let $\tilde{Z}$ denote the joint distribution of $\boldsymbol{Z}$. Following Karni (1979), the solution to equation (5) is

$$(6) \qquad \lambda_{IJ}(\bar{\boldsymbol{\omega}}, \tilde{\boldsymbol{Z}}) = -\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}\frac{\psi_{ij}}{\psi_1}(\bar{\boldsymbol{\omega}}),$$

where $\psi(\cdot)$ denotes the indirect utility function; ψ_1 and ψ_{ij} denote the first and second partial derivatives, respectively, of $\psi(\cdot)$ with respect to its i and j arguments; and σ_{ij} is the covariance between the ith and jth elements of $\boldsymbol{Z}$.

Based on equation (7), Karni (1979) defined the matrix measure of local risk aversion, $\boldsymbol{R}$, as follows:

$$(7) \qquad \boldsymbol{R} = [r_{ij}] = \left[\frac{-\psi_{ij}}{\psi_1}\right].$$

The diagonal elements of $\boldsymbol{R}$, $r_{22} = -\psi_{22}/\psi_1$ and $r_{33} = -\psi_{33}/\psi_1$, are proportional to the risk premium per variance of output price (σ_{pp}), and input price (σ_{ll}), respectively. The off-diagonal elements, $-\psi_{23}/\psi_1$ and $-\psi_{32}/\psi_1$, represent the additional risk premium when both p and l are random, over the sum of risk premium when only p and only l are random, per unit of the covariance between p and l (σ_{pl}). Thus, these off-diagonal elements capture the firm's local aversion to the interaction between the risks related to p and l after the firm is compensated for bearing the risks related to p and l separately. In a study of the consumer choice under multiple price uncertainties, Bellemare, Barrett, and Just (2013) found that neglecting the covariance terms leads to biased estimates of welfare effects.

PROPOSITION 3. *Under assumptions 1–9 and*

1. *if inputs x and z are complements and z is used in both stages, or*
2. *if inputs x and z are substitutes and z is used in the second stage only, then*
   (a) *if covariance, σ_{pl}, between the uncertain input price l and output price p is 0 or positive, then the firm will use more of the input x and increase production under uncertain price for input z and output y.*
   (b) *if covariance, σ_{pl}, between the uncertain input price l and output price p is negative, then the firm's use of the input x under uncertain price for input z and output y is ambiguous.*

Proof. See the Online Supplement. ■

PROPOSITION 4. *Under assumptions 1–9 and*

1. *if inputs x and z are complements and z is used in the second stage only, or*
2. *if inputs x and z are substitutes and z is used in both stages,*

then the firm's use of the input x under uncertain price for input z and output y is ambiguous.

Proof. See the Online Supplement. ■

Empirical Application

We use the beef cattle finishing industry to explore some of the propositions generated by the theoretical model. Propositions 1 and 3 are particularly appropriate for this application.[4] We first provide a brief description of this industry. Next we describe of data and econometric specification.

Producers in the beef cattle finishing industry face both output and input price uncertainty. Beef cattle finishers typically purchase 700 to 850 lb cattle (referred to as feeder cattle) and use predominantly grain-based feeds to produce fed cattle, which are then sold to meat processers (Anderson and Trapp, 2000). The range 700–850 lb includes the largest proportion of cattle placed on feed for the U.S. total (U.S. Department of Agriculture, 2017b). Often when corn prices increase, placement weights per animal increase (more weight is added outside of the feedlot via stockering/backgrounding) and when corn prices decline, placement weight per animal declines. The cost of feed is directly linked to the price of corn because most feed used is corn based (Anderson and Trapp, 2000; Dhuyvetter, Schroeder, and Prevatt, 2001). Further, Schroeder et al. (1993) showed that 60%–72% of the variability of feeding cost can be attributed to the variability of corn prices. This implies that increased volatility of corn prices generates increased uncertainty of feed costs and, thus, potentially increased overall uncertainty for finishers.

Econometric Specification

The econometric specification consists of a system of several equations. From the theoretical model, the three decision variables are x, y, and z. For the beef cattle finishing industry these variables are, respectively, the quantity of cattle placed on feed (placements), which is determined at the beginning of the production cycle; the quantity of beef production (harvest) which is determined at the end of the production cycle; and the quantity of corn, which is determined at the beginning and during the production cycle. The equation for placements follows from the solution to equation (A2), x^* as stated in equation (A3), and is specified as follows:

$$
\begin{aligned}
Placement_t = {} & a_0 + \sum_{i=1}^{11} a_i Placement_{t-i} + a_{12} Harvest_t + a_{13} P\_Feeder_t + a_{14} LC\_Fut_t^{\tau} \\
& + a_{15} P\_Corn_t + a_{16} FWR_t + a_{17} FUEL_t + a_{18}\sigma_t^{LC} + a_{19}\sigma_t^{C} \\
& + a_{20}\left(I \times \sigma_t^{C}\right) + a_{21}\sigma_t^{C,LC} + a_{22}\left(I \times \sigma_t^{C,LC}\right) + a_{23} SIN1_t + a_{24} COS1_t \\
& + a_{25} SIN2_t + a_{26} COS2_t + \varepsilon_{1t},
\end{aligned} \tag{8}
$$

where $Harvest$ is the quantity of cattle harvested at time t; P_Feeder is the cash price for 700–850 lb feeder cattle; $LC\_Fut_t^{\tau}$ is the live cattle futures price for the nearby contract at the end of the production cycle, τ, and represents the expected output price at time t; P_Corn is the Chicago cash price for corn; FWR is an index for the farm wage rate; $FUEL$ is an index for fuel prices; σ^{LC} is the implied volatility obtained using options on $LC\_Fut_t^{\tau}$ and is used to capture fed cattle price uncertainty; σ^{C} is the implied volatility obtained using options on the nearby corn futures contract at the end of the production cycle $C\_Fut_t^{\tau}$ and is used to capture feed price uncertainty; I is an indicator variable that takes a value of 1 starting in January 2006 and a value of 0 otherwise to

[4] Russell, Breunig, and Chiu (1998) showed that no restrictions, and in particular the convexity of technology sets, are required in the aggregation of individual supplies of price-taking producers. In other words, "there is no loss of generality in positing the existence of a 'representative producer', which generates aggregate net-supply functions by maximizing aggregate profit subject to the constraint that the aggregate net-supply vector be contained in the aggregate technology set. . . As a result, the Jacobian of the system of aggregate net supply functions has the same properties as those of individual producers" (Russell, Breunig, and Chiu, 1998, p. 178). Therefore, the aggregate supply for the industry has the same properties as the firm supply.

capture the almost 50% increase (Table A1) in corn price implied volatility after 2006 following the implementation of the Renewable Fuel Standard based on the Energy Policy Act of 2005 (U.S. Congress, 2005); $\sigma^{C,LC}$ is the historical covariance between corn and live cattle cash prices; $SIN1$–$COS2$ are harmonic variables for 6- ($SIN1$ and $COS1$) and 12- ($SIN2$ and $COS2$) month cycles to account for potential intrayear seasonality (calculated, using $SIN1$ as an example, as $SIN1 = sin(2\pi t/6)$, where SIN is the sine function, and $t = 1,\dots,T$); and $\alpha_0,\dots,\alpha_{26}$ are parameters to be estimated.

The equation for harvest follows from the solution to equation (3), y^*, and is specified as follows:

$$\begin{aligned}(9)\quad Harvest_t = {} & \beta_0 + \sum_{i=1}^{11} \beta_i Harvest_{t-1} + \beta_{12} P\_Feeder_t + \beta_{13} P\_Fed_t + \beta_{14} COF_{t-1} \\ & + \beta_{15} Placement_{t-5} + \beta_{16} P\_Corn_t + \beta_{17}\sigma_t^{LC} + \beta_{18}\sigma_t^{C} + \beta_{19}(I \times \sigma_t^{C}) + \beta_{20}\sigma_t^{C,LC} \\ & + \beta_{21}(I \times \sigma_t^{C,LC}) + \beta_{22} DIS_{t-1} + \beta_{23} SIN1_t + \beta_{24} COS1_t + \beta_{25} SIN2_t + \beta_{26} COS2_t + \varepsilon_{2t},\end{aligned}$$

where P_Fed is the cash price for fed cattle (cattle harvested for beef), COF is the total number of cattle on feed and represents the total inventory of cattle in the finishing stage, and DIS is referred to as "disappearance" and is an indirect measure of beef consumptions derived as a combination of current and previous months production, storage, exports, and imports. The other variables are as previously defined, and $\beta_0,\dots,\beta_{26}$ are parameters to be estimated.

The equation for the quantity of corn follows from the solution to equation (3), z^*, and is specified as follows:

$$\begin{aligned}(10)\quad Corn_t = {} & \gamma_0 + \sum_{i=1}^{11} \gamma_i Corn_{t-1} + \gamma_{12} COF_{t-1} + \gamma_{13} P\_Corn_t + \gamma_{14} FWR_t + \gamma_{15} FUEL_t + \gamma_{16}\sigma_t^{LC} \\ & + \gamma_{17}\sigma_t^{C} + \gamma_{18}(I \times \sigma_t^{C}) + \gamma_{19}\sigma_t^{C,LC} + \gamma_{20}(I \times \sigma_t^{C,LC}) + \gamma_{21} P\_HAY_t + \gamma_{22} SIN1_t \\ & + \gamma_{23} COS1_t + \gamma_{24} SIN2_t + \gamma_{25} COS2_t + \varepsilon_{3t},\end{aligned}$$

where P_HAY is the price of alfalfa hay while all other variables are as previously defined, and $\gamma_0,\dots,\gamma_{25}$ are parameters to be estimated.

The system of equations described in equations (8)–(13) consists of three jointly dependent (endogenous) dependent variables. Additionally, the independent variable COF, the total number of cattle on feed, is also jointly determined with the other three dependent variables. Therefore, an additional equation is added to the system with COF as a dependent variable. There is an additional reason for specifying a separate equation for COF, which represents the available cattle inventory from across production cycles. From this inventory, producers choose the level of harvest depending on market conditions. This available inventory from multiple cycles allow for the harvest at the end of the cycle to be lower and, more importantly, higher than placements (in terms of head of cattle) determined at the beginning of the cycle. Similarly, P_Fed, the cash price for fed cattle, is also jointly determined, leading as a result to the specification of an additional equation for P_Fed.

The equation for the total cattle on fed is specified as

$$\begin{aligned}(11)\quad COF_t = {} & \delta_0 + \sum_{i=1}^{11} \delta_i COF_{t-i} + \sum_{j=1}^{5} \delta_{11+j} Placement_{t-j} + \delta_{17} P\_Fed_t + \delta_{18} P\_Corn_t \\ & + \delta_{19} FWR_t + \delta_{20} FUEL_t + \delta_{21}\sigma_t^{LC} + \delta_{22}\sigma_t^{C} + \delta_{23}(I \times \sigma_t^{C}) + \delta_{24}\sigma_t^{C,LC} \\ & + \delta_{25}(I \times \sigma_t^{C,LC}) + \delta_{26} WEATHER_t + \delta_{27} SIN1_t + \delta_{28} COS1_t + \delta_{29} SIN2_t \\ & + \delta_{30} COS2_t + \delta_{31} D1_t + \delta_{32} D2_t + \varepsilon_{4t},\end{aligned}$$

where $WEATHER$ is a measure of weather shocks, all other variables are as previously defined, and $\delta_0,\ldots,\delta_{32}$ are parameters to be estimated. Prior to January 1994, total cattle on feed was based on reporting from only seven states (Arizona, California, Colorado, Iowa, Kansas, Nebraska, and Texas). Starting in January 1994, reporting from four additional states was added (Idaho, Oklahoma, South Dakota, and Washington). However, from January 1994 until January 1996, reporting from these four new states was only conducted for the months of January, April, July, and October. Starting in January 1996, reporting of cattle on feed for all states was done on a monthly basis. To account for these differences in reporting one dummy variable, $D1$ was added for the months January, April, July, and October for the period from January 1994 to December 1995. A second dummy variable, $D2$ was added for the 8 months remaining after excluding January, April, July, and October for the period from January 1994 to December 1995 and for all 12 months for the period from January 1990 to December 1993.

The equation for P_Fed is specified in first-difference form as preliminary analysis indicated the presence of a unit root in P_Fed:

$$\begin{aligned} FD\_P\_Fed_t = {} & \theta_0 + \sum_{1=1}^{11} \theta_1 FD\_P\_Fed_{t-1} + \theta_{12} FD\_COF_t + \theta_{13} FD\_PP_t + \theta_{14} FD\_PDI_t \\ & + \theta_{15} SIN1_t + \theta_{16} COS1_t + \theta_{17} SIN2_t + \theta_{18} COS2_t + \varepsilon_{5t}, \end{aligned} \tag{12}$$

where $FD\_$ in front of the variable name represent first difference, PP is cash price of pork, PDI is per capita personal disposable income in dollars, other variables are as previously defined, and $\theta_0,\ldots,\theta_{18}$ are parameters to be estimated.

It is clear that the dependent variables in the five-equation system specified in equations (8)–(12) are jointly dependent. Identification of the parameter estimates in equations (8)–(12) requires that "the number of exogenous variables that appear elsewhere in the system must be at least as large as the number of endogenous variables" (Greene, 2012, p. 325). Greene further states, "A simple sufficient order condition for an equation system is that each equation must contain "its own" exogenous variable that does not appear elsewhere in the system" (p. 325). To achieve identification of our system, we include the following exogenous variables in equations (8)–(12), respectively, live cattle futures price for the nearby contract at the end of the production cycle, one-period lagged disappearance, price of alfalfa hay, weather shocks, and both cash price of pork and per capita personal disposable income.

Table A1 in the Appendix shows the symbol and definition of each variable along with summary statistics. Further, the conditional variance dynamics for each equation are specified using GARCH processes. Previous work has shown that cattle price dynamics include movements of the higher order moments (Holt, 1993). Thus, we utilize GARCH(p,q) processes to specify the conditional variance dynamics for each of the equations above. The conditional variances are specified as

$$\begin{aligned} & h_{iit} = \kappa_{i0} + \eta_{i1}\varepsilon_{it-1}^2 + \psi_{i1} h_{iit-1}, \\ & h_{ijt} = \rho_{ij}(h_{iit} h_{jjt})^{\frac{1}{2}} \\ & i,j = 1\,(Placement), 2\,(Harvest), 3\,(Corn), 4\,(COF), 5\,(P\_Fed)\; i \neq j. \end{aligned} \tag{13}$$

We use the full information maximum likelihood (FIML) esti6remation procedure to estimate the 10-equation system consisting of equations (8)–(13). Thus, we assume that $\boldsymbol{\varepsilon}_t = \sqrt{\boldsymbol{h}_t}\boldsymbol{e}_t$, where $\boldsymbol{e}_t \sim \mathrm{N}(\mathbf{0},\boldsymbol{\Sigma})$ and Σ is the contemporaneous cross-equation covariance matrix. The log-likelihood for observations $t = 1,\ldots,T$ and equations $n = 1,\ldots,N$ is specified as

$$\ln L(\boldsymbol{\Theta}) = -\frac{TN}{2}\ln(2\pi) - \frac{T}{2}\ln(|\boldsymbol{H}|) - \frac{1}{2}\mathrm{tr}(\boldsymbol{H}^{-1}\sum_{t=1}^{T}\boldsymbol{\varepsilon}_t'\boldsymbol{\varepsilon}_t), \tag{14}$$

where $\boldsymbol{H}$ is specified in equation (13), $\boldsymbol{\Theta} = (\alpha, \beta, \gamma, \delta, \theta, \kappa, \eta, \psi, \Sigma)$ is the matrix of the parameters in equations (8)–(13), and $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t}, \varepsilon_{4t}, \varepsilon_{5t})$.

Data

The data are monthly time-series observations from January 1990 through April 2017. Cash prices for feeder cattle are obtained from the *Oklahoma National Stockyards, Feeder Cattle Weighted Average* report of cash prices (U.S. Department of Agriculture, 2017c). Cash fed cattle prices are obtained from the *Five Area Daily Weighted Average Direct Slaughter Cattle* report (U.S. Department of Agriculture, 2017a). The Chicago cash price for corn is from the *Weekly Feedstuff Wholesale Prices* report (U.S. Department of Agriculture, 2017d). Wholesale price of pork is a monthly average of daily data from the *National Daily Direct Hog Prior Day Report - Slaughtered Swine* (U.S. Department of Agriculture, 2017b).

Quantity of beef production is federally inspected beef production in pounds from the *Livestock Slaughter* monthly report (U.S. Department of Agriculture, 2019b). The quantity of cattle placed on feed (placements) and the total supply of cattle in the finishing stage are from monthly *Cattle on Feed* reports (U.S. Department of Agriculture, 2017b). Monthly disappearance values were calculated using data for beef stocks (U.S. Department of Agriculture, 2019a), production (U.S. Department of Agriculture, 2019b), and trade (U.S. Department of Agriculture, 2019c). The quantity of corn is from the quarterly *Feed Outlook* report (U.S. Department of Agriculture, 2017d). The farm wage rate, fuel price index, and alfalfa hay prices are each obtained from the *Agricultural Prices* report (U.S. Department of Agriculture, 2017a). The consumer price index (CPI) is obtained from the *Price Indices for Personal Consumption Expenditures by Major Type of Product, Monthly* report (Bureau of Economic Analysis, 2017b). Personal disposable income is obtained from the *Personal Income and Its Disposition, Monthly* report (Bureau of Economic Analysis, 2017a).

Implied price volatilities for live cattle and corn and futures prices for live cattle and feeder cattle are obtained from the Commodity Research Bureau (2017). All prices are deflated by the CPI. Temperature data were obtained from the PRISM model (PRISM Climate Group, Oregon State Univeristy, 2019).

Results

Parameter estimates, standard errors, t-values, and short- and long-run elasticities for placements, harvest, and cattle on feed inventory are reported in Tables 1, 2, and 3, respectively. Table 4 summarizes the effects of live cattle and corn price uncertainties on the level of the feeder cattle input use, the level of production, and the overall level of cattle on feed. Table 5 reports results for the corn use equation. while results for fed cattle price are reported in Table S1 in the Online Supplement. The elasticities/flexibilities were calculated using the mean values reported in Table A1 and the reduced-form equations following Chavas, Hassan, and Johnson (1981); standard errors were calculated using the delta method as described in Casella and Berger (2001). Here we discuss the findings regarding the effects of input and output price uncertainty. We discuss other results in the Online Supplement.

Results of Tables 1–4 show that a 1% increase (0.137, calculated using the mean value reported in Table A1 in live cattle price volatility is associated with an increase in placements of 0.14 million lb, a decrease in harvest of 0.23 million lb, and an increase in total cattle on feed of 0.27 million head. Note that the decrease in harvest of 0.23 million lb represents the effect of live cattle price volatility in the second stage of production only. Live cattle price volatility also affects harvest in the second stage through increased placements in the first stage. Using the increase in placements by 0.14 million lb and an approximate factor of 2 (the average increase in weight from feeder to fed cattle) results in an increase in harvest by 0.28 million lb. Therefore, the second-stage decrease in

Table 1. Maximum Likelihood Estimates and Elasticities for Beef Cattle Placements

Variable	Coeff.	*t*-Value	Short-Run Elasticity	Long-Run Elasticity
Placement				
Constant	−0.351	−0.25		
	(1.430)			
Placement (lagged) (hundred million lb)	0.563***	8.64		
	(0.065)			
Harvest (hundred million lb)	0.426***	8.53	0.672***	0.803***
	(0.050)			
Price of feeder cattle ($/lb)	−3.289***	−4.40	−0.293***	−0.350***
	(0.748)			
Live cattle futures price ($/lb)	0.065***	5.39	0.431***	0.515***
	(0.012)			
Price of corn ($/bu)	−0.525***	−5.92	−0.140***	−0.167***
	(0.089)			
Farm wage rate (index - base 2012)	−1.749**	−2.12	−0.228**	−0.273**
	(0.824)			
Fuel index (index - base 2012)	−0.214*	−1.69	−0.033*	−0.039*
	(0.127)			
Price volatility of live cattle	0.010**	2.14	0.010**	0.012**
	(0.005)			
Price volatility of corn prior to 1996	0.023**	1.98	0.038**	0.045**
	(0.011)			
Price volatility of corn after 1996	−0.017*	−1.71	−0.009*	−0.011*
	(0.010)			
Live cattle/corn covariance prior to 1996	−0.238*	−1.93	−0.003*	−0.004*
	(0.123)			
Live cattle/corn covariance after 1996	0.259**	2.05	0.019**	0.022*
	(0.127)			
*SIN*1	−0.301**	−2.00		
	(0.151)			
*COS*1	−1.376***	−11.23		
	(0.123)			
*SIN*2	−0.443***	−2.96		
	(0.150)			
*COS*2	0.138	1.40		
	(0.099)			
Placement conditional variance				
Constant	0.534***	7.99		
	(0.067)			
ε_{1t-1}	0.231***	2.24		
	(0.103)			
	$T \times N = 1{,}570$	Log likelihood: −1,655		

Notes: Single, double, and triple asterisks (*, **, ***) indicate significance at the 10%, 5%, and 1% level. Numbers in parentheses are standard errors. *SIN*1–*COS*2, are harmonic variables for 6- (*SIN*1 and *COS*1) and 12- (*SIN*2 and *COS*2) month cycles to account for potential intrayear seasonality, calculated, using *SIN*1 as an example, as $SIN1 = \sin(2 \times \pi \times t/6)$, where sin is the sine function, and $t = 1, \ldots, T$.

Table 2. Maximum Likelihood Estimates and Elasticities for Beef Cattle Harvest

Variable	Coeff.	t-Value	Short-Run Elasticity	Long-Run Elasticity
Harvest				
Constant	-1.705^{***} (0.599)	-2.84		
Harvest (lagged) (hundred million lb)	0.235^{***} (0.039)	6.1		
Price of feeder cattle ($/lb)	-2.203^{***} (0.258)	-8.54	-0.124^{***}	-0.163^{***}
Price of fed cattle ($/lb)	0.021^{***} (0.003)	6.35	0.093^{***}	0.122^{***}
Cattle on feed (million head)	0.006^{**} (0.003)	2.08	0.028^{**}	-0.037^{**}
Placements (hundred million lb)	0.037^{***} (0.011)	3.45	0.024^{***}	0.031^{***}
Price of corn ($/bu)	0.116^{***} (0.028)	4.19	0.020^{***}	0.026^{***}
Price volatility of live cattle	-0.017^{**} (0.007)	-2.44	-0.011^{**}	-0.014^{**}
Price volatility of corn prior to 1996	-0.015^{**} (0.008)	-1.97	-0.016^{**}	-0.020^{**}
Price volatility of corn after 1996	0.023^{***} (0.007)	3.25	0.008^{**}	0.011^{**}
Live cattle/corn covariance prior to 1996	-0.102^{**} (0.048)	-2.11	0.001^{**}	0.002^{**}
Live cattle/corn covariance after 1996	0.101^{**} (0.05)	2.03	0.005^{**}	0.006^{**}
Disappearance (hundred million lb)	0.008^{***} (0.001)	39.04	0.790^{***}	1.033^{***}
*SIN*1	-0.122^{***} (0.026)	-4.79		
*COS*1	0.120^{***} (0.035)	3.45		
*SIN*2	-0.656^{***} (0.057)	-11.55		
*COS*2	-0.166^{***} (0.056)	-2.99		
Harvest conditional variance				
Constant	0.052^{***} (0.009)	5.74		
ε_{2t-1}	1.012^{***} (0.153)	6.63		
	$T \times N = 1{,}570$	Log likelihood: $-1{,}655$		

Notes: Single, double, and triple asterisks (*, **, ***) indicate significance at the 10%, 5%, and 1% level. Numbers in parentheses are standard errors. *SIN*1–*COS*2, are harmonic variables for 6- (*SIN*1 and *COS*1) and 12- (*SIN*2 and *COS*2) month cycles to account for potential intrayear seasonality, calculated, using *SIN*1 as an example, as $SIN1 = \sin(2 \times \pi \times t/6)$, where sin is the sine function, π is the constant π, and $t = 1, \ldots, T$. Disappearance is an indirect measure of beef consumptions derived as a combination of current and previous months production, storage, exports, and imports.

Table 3. Maximum Likelihood Estimates and Elasticities for Cattle on Feed

Variable	Coeff.	*t*-Value	Short-Run Elasticity	Long-Run Elasticity
Cattle on feed (COF)				
Constant	−9.601***	−5.09		
	(2.336)			
Cattle on feed (lagged) (million head)	0.960***	97.39		
	(0.011)			
Placements (hundred million lb)	0.926***	12.04	0.121***	0.357***
	(0.076)			
Price of fed cattle ($/lb)	2.149**	2.34	0.021**	0.062**
	(1.025)			
Price of corn ($/bu)	−0.159**	−2.44	−0.006**	−0.016**
	(0.062)			
Farm wage rate (index - base 2012)	−1.439	−1.88	−0.025	−0.072
	(0.938)			
Fuel index (index - base 2012)	−0.293**	−2.72	−0.006**	−0.017**
	(0.124)			
Price volatility of live cattle	0.019**	2.34	0.003**	0.008**
	(0.008)			
Price volatility of corn prior to 1996	0.033**	2.60	0.007**	0.021**
	(0.016)			
Price volatility of corn after 1996	0.012	1.54	0.001	0.003
	(0.008)			
Live cattle/corn covariance prior to 1996	0.032**	1.98	0.001**	0.001**
	(0.016)			
Live cattle/corn covariance after 1996	0.078	1.65	0.000	0.000
	(0.122)			
Weather shock (temperature change)	0.062**	2.45	0.003**	0.008**
	(0.025)			
$SIN1$	−0.811***	−8.53		
	(0.095)			
$SIN2$	0.136	0.86		
	(0.158)			
$COS2$	1.503***	7.26		
	(0.207)			
$D1$	−0.993**	−2.29		
	(0.434)			
$D2$	7.672***	12.30		
	(0.623)			
Conditional variance				
Constant	0.390***	6.37		
	(0.055)			
ε_{4t-1}	0.683***	5.43		
	(0.126)			
	$T \times N = 1,570$	Log likelihood: $-1,655$		

Notes: Single, double, and triple asterisks (*, **, ***) indicate significance at the 10%, 5%, and 1% level. Numbers in parentheses are standard errors. $SIN1$–$COS2$, are harmonic variables for 6- ($SIN1$ and $COS1$) and 12- ($SIN2$ and $COS2$) month cycles to account for potential intrayear seasonality, calculated, using $SIN1$ as an example, as $SIN1 = \sin(2 \times \pi \times t/6)$, where sin is the sine function, π is the constant π, and $t = 1, \ldots, T$. $D1$ is a dummy variable for January, April, July, and October from January 1994 to December 1995 and $D2$ is a dummy variable for the 8 months remaining for the period from January 1994 to December 1995 and for all 12 months for the period from January 1990 to December 1993.

Table 4. Summary of Effects of Input and Output Price Uncertainty on Placements, Harvest, and Cattle on Feed

	Source of Uncertainty					
	Input (Corn) Price Uncertainty			Output (Live Cattle) Price Uncertainty		
	Jan 1990–Dec 1995	Jan 1996–Apr 2017		Jan 1990–Dec 1995	Jan 1996–Apr 2017	
Effect	**Overall Effect**	**Additional Effect**	**Overall Effect**	**Overall Effect**	**Additional Effect**	**Overall Effect**
Placements	0.499**	−0.124*	0.375**	0.135**	0.000	0.135**
Harvest	−0.322*	0.171***	−0.151**	−0.227**	0.000	−0.227**
Fed cattle supply	0.730**	0.000	0.730**	0.267**	0.000	0.267**
	Covariance between Input and Output Price			**Combined Input and Output Price Uncertainty**		
	Jan 1990–Dec 1995	**Jan 1996–Apr 2017**		**Jan 1990–Dec 1995**	**Jan 1996–Apr 2017**	
Effect	**Overall Effect**	**Additional Effect**	**Overall Effect**	**Overall Effect**	**Additional Effect**	**Overall Effect**
Placements	0.046*	0.245**	0.291**	0.680**	0.121**	0.802**
Harvest	0.020**	0.096**	0.116**	−0.529**	0.267**	−0.262**
Fed cattle supply	0.007**	0.000	0.007**	1.004**	0.000	1.004**

Notes: Only statistically significant effects are reported here. Placements and harvest are measured in million lb and fed cattle supply is measured in million head. Single, double, and triple asterisks (*, **, ***) indicate significance at the 10%, 5%, and 1% level. Additional Effects are the the additional effect for the period from January 1996 to April 2017 compared to the period from January 1990 to December 1995.

harvest is a decrease from elevated levels from the first stage. Thus, the overall effect of live cattle price volatility on harvest for both stages of production is an increase in harvest by 0.05 million lb.

With regard to corn price volatility, we measure the effect during two periods. The first period is January 1990 to December 1995 and the second is January 1996 to April 2017. Results of Tables 1–4 show that, a 1% increase (0.22) in corn price volatility in the first period is associated with an increase in placements of 0.50 million lb, a decrease in harvest of 0.32 million lb, and an increase in total cattle on feed of 0.73 million head. Similarly, for the live cattle price volatility, accounting for the indirect effect through placements in the first stage results in an increase in harvest of 1.00 million lb in the first stage and an overall increase in harvest of 0.68 million lb. For the second period, a 1% increase (0.29) in corn price volatility is associated with an additional decrease in placements compared to the first period of 0.12 million lb, resulting in an overall increase of 0.38 million lb in the second period. On the other hand, the effect of a 1% increase in corn price volatility on harvest in the second period is an additional increase in harvest of 0.17 million lb, resulting in an overall decrease in harvest of 0.15 million lb. Taking the indirect effect into account results in an increase in harvest of 0.76 million lb in the first stage and an overall increase in harvest of 0.61 million lb. Finally, the effect of the corn price volatility on the level of cattle on feed in the second period is the same as in the first period.

To summarize the effect of live cattle (output) and corn (input) price volatility, our findings indicate that increases in both volatilities are positively associated with the use of the *ex ante* feeder cattle input (placements) and increased production in the first stage, negatively associated with the level of production as measured by cattle harvest in the second stage, and positively associated with the overall level of cattle on feed. Further, periods of increased levels of volatility related to the *ex post* input (corn) price are associated with a lower increase in the level of the feeder cattle input, a lower decrease in the level of production in the second stage, and the same increase in the level of the overall cattle supply. The implication of these findings, according to Proposition 1, is that producers in the beef cattle finishing industry, under flexible production and corn price uncertainty, exhibit decreasing absolute risk aversion behavior with regard to placements of cattle on feed and increased production.

Tables 1–3 also present the effect of the covariance between live cattle and corn prices on cattle placements, harvest, and overall level of cattle on feed as discussed in Proposition 3. The covariance between live cattle and corn prices is negatively (positively) associated with cattle placements, harvest, and overall level of cattle on feed during the earlier (later) period of January 1990 to December 1995 (January 1996 to April 2017) when the mean covariance is negative (positive) (see Table A1). Thus, for the earlier (later) period, an increase in the covariance, or the covariance becoming more negative (positive), is associated with a decrease (increase) in placements, harvest, and overall level of cattle on feed. However, the effect of the covariance during the January 1990 to December 1995 period is negligible. Similarly, the effect the covariance on the overall level of cattle on feed is also negligible.

Table 4 also summarizes the combined effects of live cattle and corn price volatilities and the covariance between live cattle and corn prices on the level of feeder cattle placements, harvest, and overall level of cattle on feed. Similar patterns of the combined effects are observed as in the case of the effects of live cattle and corn price volatilities.

Table 5 presents the results regarding the effects of live cattle and corn price volatilities and the covariance between live cattle and corn prices on the use of the corn input. Live cattle price volatility is positively associated with corn use while corn price volatility is negatively associated with corn use. The covariance between live cattle and corn prices has no significant effect on the use of the corn input. Results of several diagnostics tests regarding normality and autocorrelation are presented in Table S2 in the Online Supplement.

Table 5. Maximum Likelihood Estimates and Elasticities for Corn Use by Beef Cattle Finishers

Variable	Coeff.	*t*-Value	Short-Run Easticity	Long-Run Elasticity
Corn use				
Constant	156.544***	4.36		
	(35.867)			
Corn use (lagged) (million bu)	0.610***	9.09		
	(0.067)			
Cattle on feed (million head)	0.629***	3.02	0.019***	0.048***
	(0.209)			
Price of corn ($/bu)	−9.311***	−4.26	−0.074***	−0.190***
	(2.183)			
Farm wage rate (index - base 2012)	−6.504	−0.31	−0.025	−0.065
	(20.717)			
Fuel index (index - base 2012)	15.476***	4.13	0.071***	0.182***
	(3.747)			
Price volatility of live cattle	0.787*	1.77	0.025*	0.063*
	(0.46)			
Price volatility of corn prior to 1996	−0.702**	−2.23	−0.035**	−0.090**
	(0.314)			
Price volatility of corn after 1996	−0.186	−0.88	−0.003	−0.008
	(0.211)			
Live cattle/corn covariance prior to 1996	−2.725	−1.04	−0.001	−0.003
	(2.684)			
Price of hay ($/ton)	−0.224**	−2.51	−0.061**	−0.156**
	(0.089)			
*SIN*1	2.216	0.83		
	(2.684)			
*COS*1	−32.807***	−4.96		
	(6.617)			
*SIN*2	−30.700***	−4.09		
	(7.503)			
*COS*2	14.194*	1.72		
	(8.236)			
Corn use conditional variance				
Constant	349.45***	5.99		
	(58.384)			
ε_{3t-1}	0.228*	1.71		
	(0.115)			
	$T \times N = 1,570$	Log likelihood: −1,655		

Notes: Single, double, and triple asterisks (*, **, ***) indicate significance at the 10%, 5%, and 1% level. Numbers in parentheses are standard errors. *SIN*1–*COS*2, are harmonic variables for 6- (*SIN*1 and *COS*1) and 12- (*SIN*2 and *COS*2) month cycles to account for potential intrayear seasonality, calculated, using *SIN*1 as an example, as $SIN1 = \sin(2 \times \pi \times t/6)$, where sin is the sine function, π is the constant π, and $t = 1, \ldots, T$.

Conclusions

This research provides theoretical and empirical frameworks for evaluating firm behavior in the presence of both input and output price uncertainty. Previous work in the literature has focused primarily on output price uncertainty. We use beef cattle finishing operations as a motivating example for the importance of input price uncertainty, where long feeding periods naturally introduce uncertainty around both input price (feed) and output price (fed cattle). Other industries in which input price uncertainty is relevant include row crop production, other animal production systems, and many nonagricultural industries that use a variable input such as fuel.

The proposed two-stage economic model demonstrates that a firm will increase its use of the input and output in the first stage and decrease output in the second stage under a flexible production function when faced with input price uncertainty. We empirically explore the propositions derived in the theoretical sections that are applicable to the beef cattle finishing industry using results obtained from a multiple-equation econometric specification for cattle-finishing operations. Empirical results indicate that for the beef cattle-finishing industry, the introduction of input price uncertainty leads to increased use of the input in stage one and a decreased level of output in stage two. In particular, a 1% increase in corn price volatility is associated with an $585,000 ($444,600) increase in placements (0.50 (0.38) million lb multiplied by mean feeder cattle cash price $1.17/lb) and a $313,600 ($147,000) decrease in harvest (0.32 (0.15) million lb multiplied by mean live cattle cash price $0.98/lb) during the period from January 1990 to December 1995 (January 1996 to April 2017.)

Our empirical findings suggest some policy implications for the cattle industry. First, cattle finishers can use results from this research to better understand and cope with the dynamics of the input and output price uncertainty to which they are exposed. As evidenced from the increased volatility in agricultural (specifically, corn) markets since the mid-2000s, this understanding has taken on even greater importance. Second, our findings suggest that the increase in corn price uncertainty since the mid-2000s has become a significant factor that cattle finishers consider when making purchasing and production decisions. In more recent years, corn price volatility has continued to be a key concern for cattle finishers. For example, during the spring and summer of 2019, cattle finishers faced a spike in corn price volatility driven by uncertainty of an extremely wet planting season in the Midwest.

More broadly, our results suggest that input price uncertainty has a significant effect on the firm's optimal level of output. According to our results, increasing input price uncertainty is associated with a decline in the level of output in the second stage in a two-stage production. The implication is that the exit of less efficient firms from the industry in periods of high price volatility could further exacerbate the negative impact of input price uncertainty on industry supply.

Future theoretical research on this topic includes the development of a game-theoretic model of firm behavior under imperfect competition and both input and output price uncertainty. Further empirical research could utilize different data sources and also analyze the different business structures for finishers. We used national-level data to show the effect of input price uncertainty on the finishing industry. Researchers with access to firm-level data could apply this research to different operation sizes. Depending on data availability, future research may also investigate potential regional differences in the effect of price uncertainty in the beef industry studied here or in other industries.

[First submitted May 2019; accepted for publication March 2020.]

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Appendix

Determining the Effect of Input Price Uncertainty on the Use of Input x

With output price, p, nonstochastic, the only source of uncertainty is related to the input price, $\tilde{l}$. Thus, equation (2) becomes

(A1) $$\max_{x \geq 0, z_1 \geq 0} EU\left[g\left(p, \tilde{l}, x, z_1\right) - rx - lz_1\right].$$

Given our focus on the input x, the first-order condition corresponding to equation (A1) is

(A2) $$E\left[U'\left(g_x - r\right)\right] = 0,$$

where g_x is the partial derivative of g with respect to x. The second-order condition requires concavity in x of $EU\left[g\left(p, \tilde{l}, x\right) - rx\right]$. The assumption $U'' \leq 0$ provides a sufficient, though not a necessary, condition to satisfy the concavity. Denote by x^* the optimal solution for equation (A1). This is the risk-responsive input demand function, and it is a function of p, r, and l as well as the random input price, $\tilde{l}$, defined by the probability distribution with parameters μ_l and σ_l. Thus, the general form for the optimal solution is

(A3) $$x^* = x\left(p, r, l, \mu_l, \sigma_l\right).$$

Denote by $\partial x^* / \partial \text{Shift}\left(\tilde{l}\right)$ the impact on x^* of a marginal shift in expected input price distribution. Replacing $\tilde{l} + a$ for $\tilde{l}$ in equation (A2), totally differentiating and evaluating the derivative at $a = 0$, we obtain

(A4) $$\partial x^* / \partial \text{Shift}\left(\tilde{l}\right) = -E\left[U' g_{xl} + g_l U''\left(g_x - r\right)\right] / D,$$

where D is the second-order derivative of equation (A1) with respect to x and is negative in sign. The sign of equation (A4) depends on the signs of g_{xl} and U''. The sign of g_{xl} depends on whether x and z are complements or substitutes and whether $z_1 > 0$ and $z_2 > 0$ (i.e., input z is used in both stages) or $z_1 = 0$ and $z_2 > 0$ (i.e., input z is used only in the second stage).

PROPOSITION 5. *Under assumptions 1–9 and*

1. *if inputs x and z are complements and z is used in both stages, or*
2. *if inputs x and z are substitutes and z is used in the second stage only,*

then decreasing absolute risk aversion preferences are sufficient to ensure that the firm will, in stage one, use more of input x and increase production under uncertain price for input z.

Proof. See Online Supplement. ■

PROPOSITION 6. *Under assumptions 1–9 and*

1. *if inputs x and z are complements and z is used in the second stage only, or*
2. *if inputs x and z are substitutes and z is used in both stages,*

then decreasing (increasing) absolute risk aversion preferences are sufficient to ensure that the firm will, in stage one, use more (less) of input x and increase (decrease) production under uncertain price for input z.

Proof. See Online Supplement. ■

Table A1. Descriptive Statistics (N = 314)

Variable	Description (units)	Mean	Std. Dev.	Min.	Max.
Placement	Quantity of cattle placed on feed (hundred million lb)	13.13	2.54	7.58	21.09
Harvest	Quantity of fed cattle slaughtered (hundred million lb)	20.71	1.68	16.40	24.74
Corn	Quantity of corn used (million bu)	437.24	188.54	6.07	865.35
COF	Cattle on feed (million head)	100.11	15.23	61.86	121.10
P_Fed	Fed (live) cattle cash price ($/lb)	0.98	0.17	0.72	1.55
P_Feeder	Feeder cattle cash price ($/lb)	1.17	0.27	0.70	2.21
LC_Fut	Fed (live) cattle futures price ($/lb)	0.89	0.25	0.61	1.68
P_Corn	Corn cash price ($/bu)	3.49	1.32	1.58	7.66
FWR	Farm wage rate (index, base 2012)	1.71	0.17	1.37	2.04
FUEL	Fuel price index (index, base 2012)	2.01	0.82	0.96	4.24
WPP	Wholesale pork cash price ($/lb)	0.75	0.16	0.24	1.28
PDI	Personal disposable income ($billions)	95.16	19.74	63.53	128.51
DIS	Beef consumption proxy (million lb)	2,164.50	166.41	1,761.97	2,608.85
WEATHER	Mean temperature (°C)	11.63	8.33	−1.95	24.80
P_HAY	Price of alfalfa hay ($/ton)	119.03	40.68	71.00	227.00
σ^{LC}	Fed (live) cattle price volatility	13.74	3.49	7.35	24.50
$\sigma^{C}(90--05)$	Corn price volatility 1990–2005	21.84	3.48	2.73	30.33
$\sigma^{C}(06--17)$	Corn price volatility 1906–2017	29.28	5.79	20.55	45.60
$\sigma^{C}(90--17)$	Corn price volatility 1990–2017	24.97	5.88	2.73	45.60
$\sigma^{C,LC}(90--05)$	Corn/live cattle price covariance 1990–2005	(0.19)	0.63	−2.19	1.22
$\sigma^{C,LC}(06--17)$	Corn/live cattle price covariance 1906–2017	0.95	9.35	−22.27	17.98
$\sigma^{C,LC}(90--17)$	Corn/live cattle price covariance 1990–2017	0.23	5.74	−22.27	17.98

Journal of Agricultural and Resource Economics 45(3):S1–S4 ISSN 1068-5502
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doi: 10.22004/ag.econ.302496

Online Supplement: Firm Decision Making under Both Input and Output Price Uncertainty

Ardian Harri, Joshua G. Maples, John Michael Riley, and Jesse B. Tack

PROPOSITION 1. *Under assumptions 1–9 and*

1. *if inputs x and z are complements and z is used in both stages, or*
2. *if inputs x and z are substitutes and z is used in the second stage only, then decreasing absolute risk aversion preferences are sufficient to ensure that the firm will, in stage one, use more of the input x and increase production under uncertain price for input z.*

Proof. If x and z are complements and z is used in both stages, then a second stage input price $\tilde{l}$ increase would result in increased costs in the second stage for z_2. Then the firm's response is to decrease the use of z_2 in stage two and increase production and therefore increase the use of inputs z_1 and x in stage one. Therefore, $g_{xl} \geq 0$. For the case of the livestock producer, given the possibility of higher costs of the feed input in the second stage, the producer would choose to increase production in stage one by increasing the total pounds of feeder cattle placed on feed, either through increased number of cattle or increased pounds per animal placed on feed and also increased corn use. The increased placements in the first stage allows for the possibility of earlier harvest (shorter time on feed) at the beginning of stage two to avoid possible increased feeding costs in the second stage.

Thus, with $g_{xl} \geq 0$, then $\partial x^* / \partial \text{Shift}\left(\tilde{l}\right) \geq 0$ if $E\left[g_l U''\left(g_x - r\right)\right] = -E\left[A g_l U'\left(g_x - r\right)\right] \geq 0$. A sufficient condition for this is that $\partial\left(g_l A\right)/\partial l) \leq 0$ or $\left(dA/d\tilde{\pi}\right)/A \leq -g_{ll}/g_l$. It is clear that $\left(dA/d\tilde{\pi}\right)/A \leq 0$ whether under no production flexibility $(g_{ll} = 0)$ or with production flexibility and price risk affinity $(g_{ll} \geq 0)$, given that $g_l < 0$. Thus, decreasing absolute risk aversion is sufficient to ensure that the firm will, in the first stage, use more of the input x and increase production under uncertain input price $\tilde{l}$ when x and z are complements and z is used in both stages. Further, this effect is the same under production flexibility as it is under no production flexibility. The effect of input price uncertainty on the use of input x is thus more determinate than the effect of output price uncertainty discussed above (see also Epstein, 1978).

Similarly, $g_{xl} \geq 0$ when x and z are substitutes and z is used in the second stage only. In this case, a second stage input price $\tilde{l}$ increase would cause the firm to decrease the use of z_2 in stage two, while increasing production and the use of input x in stage one. An example here would be a crop producer who uses two different types of nitrogen fertilizers, one that is more appropriate (slow releasing) for fall (before planting) application (input x in this case) and another that is more appropriate (faster releasing) for spring (after planting) application (input z in this case). Given the possibility of higher costs of the input z_2 in the second stage, the producer would choose to increase the level of the input x in stage one.

Again, decreasing absolute risk aversion is sufficient to ensure that the firm will, in the first stage, use more of the input x and increase production under uncertain input price $\tilde{l}$ when x and z are substitutes and z is used in the second stage only. The case when z is used in the second stage only is the case considered in Devadoss and Choi (1991). Our finding here is similar to their finding in the case of a change in the maximum price only or the mean-preserving contraction (MPC) simultaneous changes in both the minimum and maximum prices of the random input price. ■

PROPOSITION 2. *Under assumptions 1–9 and*

1. *if inputs x and z are complements and z is used in the second stage only, or*
2. *if inputs x and z are substitutes and z is used in both stages, then decreasing (increasing) absolute risk aversion preferences are sufficient to ensure that the firm will, in stage one, use more (less) of the input x and increase (decrease) production under uncertain price for input z.*

Proof. If x and z are complements and z is used in the second stage only or when x and z are substitutes and z is used in both stages, $g_{xl} < 0$. In the first case, a second stage input price $\tilde{l}$ increase would cause the firm to decrease the use of z_2 in stage two and decrease output and the use of input x in stage one. An example here would be a crop producer who uses nitrogen fertilizers only in the spring (after planting) application (input z in this case). Given the possibility of higher costs of nitrogen fertilizer in the second stage, the producer would choose to decrease the level of the input x (seed) in stage one by planting fewer acres to the nitrogen dependent crop. Again, this finding is similar to the finding in Devadoss and Choi (1991). In the second case, a second stage input price $\tilde{l}$ increase would cause the firm to decrease the use of z_2 in stage two, and increase the use of input z_1 and increase the use of input x in stage one. An example here would be a crop producer who uses two different types of nitrogen fertilizers, one that is more appropriate (slow releasing) for fall (before planting) application (z_1) and another that is more appropriate (faster releasing) for spring (after planting) application (z_2). Given the possibility of higher costs of the input, z_2 in the second stage, the producer would choose to increase the level of the fall (before planting) application (z_1) and the seed input (x) (to take advantage of the increased level of z_1) in stage one. Under decreasing absolute risk aversion, the two terms inside the expectation in (A4), $U'g_{xl}$ and $g_l U''(g_x - r)$ have opposite signs and the firm will use more of the input x if $U'g_{xl} < g_l U''(g_x - r)$ and less x if $U'g_{xl} \geq g_l U''(g_x - r)$. Under increasing absolute risk aversion, the firm will use less of the input x. ■

Proof. Proof of Proposition 3. Proposition 1 covers the cases when x and z are complements and z is used in both stages or when x and z are substitutes and z is used in the second stage only. If covariance σ_{pl} is zero, then the combined effect of uncertainty related to both p and l will be determined by the combined effect of r_{22} and r_{33}. It was shown earlier, that for these two cases, decreasing absolute risk aversion is sufficient to ensure that the firm will use more of the input x under uncertain input price $\tilde{l}$. Additionally, if the decrease in the absolute risk aversion is large enough to offset the price risk affinity the firm will use more of the input x under uncertain output price $\tilde{p}$. Given the similar directional effects of the separate uncertainties related to $\tilde{l}$ and $\tilde{p}$ in these cases, then the combined effect with both $\tilde{l}$ and $\tilde{p}$ uncertain, is that under decreasing absolute risk aversion the firm will increase the use of input x. Similarly, for the first two cases, under decreasing absolute risk aversion the firm will also increase the use of input x if covariance $\sigma_{pl} > 0$. ■

Finally, if covariance $\sigma_{pl} < 0$, the direction of the impact of the uncertainty is ambiguous. The direction depends on the magnitudes of both of r_{22} and r_{33}, firm's local aversion to the risks related to p and l, as well as the magnitudes of r_{23} and r_{32}, firm's local aversion to the interaction between the risks related to p and l.

Proof. Proof of Proposition 4. Proposition 2 covers the cases when x and z are complements and z is used in the second stage only or when x and z are substitutes and z is used in both stages. For these cases, given that uncertainties related to both $\tilde{l}$ and $\tilde{p}$ have opposite effects on the use of input x, the direction of the combined effect will be determined by the sum of the two separate effects and the effect of the covariance σ_{pl}. ■

Table S1. Maximum Likelihood Estimates and Elasticities for Fed Cattle Price

Variable	Coeff.	Std. Err.	*t*-Value	Short-Run Elasticity	Long-Run Elasticity
Fed Cattle Demand					
Constant	0.001	0.002	0.78		
Price of Fed Cattle (lagged) ($/lb.)	0.386***	0.059	6.55		
Cattle on Feed (Million head)	−0.001**	0.000	−2.89	−0.080**	−0.131**
Price of Pork ($/lb.)	0.052**	0.028	2.24	0.040**	0.065**
Personal Disposable Income (Billion $)	0.002*	0.000	1.67	0.160*	0.260*
$SIN1$	−0.012***	0.002	−5.74		
$COS1$	0.012***	0.002	5.03		
$SIN2$	0.091***	0.013	7.02		
$COS2$	−0.097***	0.009	−10.61		
FC Supply Conditional Variance					
Constant	0.0001	0.000	0.95		
ε_{5t-1}	0.096**	0.044	2.10		
h_{5t-1}	0.867***	0.112	7.84		
	$T \times N = 1{,}570$			Log likelihood: −1,655	

Notes: Single, double, and triple asterisks (*, **, ***) indicate significance at the 10%, 5%, and 1% level. $SIN1$–$COS2$, are harmonic variables for 6- ($SIN1$ and $COS1$) and 12- ($SIN2$ and $COS2$) month cycles to account for potential intrayear seasonality, calculated, using SIN1 as an example, as $SIN1 = sin(2 \times \pi \times t/6)$, where sin is the sine function, π is the constant π, and $t = 1, \ldots, T$.

Additional Regression Results

Table S1 reports results for the fed cattle price equation. Based on the results of Table S1, a one percent increase in the quantity of cattle (1 million head) is associated with a decrease in the price of fed cattle of $0.08/lb. On the other hand, one percent increases in the price of pork ($0.075/lb) and personal disposable income ($0.95 billion) are associated with increases in the price of fed cattle of $0.04/lb and $0.16/lb, respectively. All these effects are in the expected direction.

Results of Several Diagnostics Tests

Table S2 presents results of several diagnostics tests. Panel A of Table S2 presents skewness and kurtosis measures, the Shapiro-Wilk (1965) normality test and its probability value for residuals of each equation. Panel A of Table S2 also reports the results of Mardia skewness, Mardia kurtosis (Mardia, 1970) and Henze and Zirkler (1990) tests of a multivariate normal distribution. Based on the results of Table S2 we fail to reject the hypothesis that the residuals follow a joint multivariate normal distribution with marginal densities also following a normal distribution. Panel B of Table S2 presents the probability values for the Ljung-Box (1978) test. The hypothesis of no autocorrelation in the standardized residuals and squared standardized residuals is tested for a length of up to six lags. Results of Table S2 indicate that there is some remaining autocorrelation in the residuals for the corn use equation. Additionally, results indicate no signs of autocorrelation in the squared residuals. Finally, Panel C of Table S2 presents the cross correlation between equations. The estimated correlations between equations are small.

[Received May 2019; final revision received March 2020.]

Table S2. Diagnostic Tests of Residuals

Panel A. Tests for Normality of Residuals

Test: Equation	Skewness	Kurtosis	Shapiro-Wilk test	Probability
Placement (Hundred Million lbs.)	0.06	−0.33	0.996	0.54
Harvest (Hundred Million lbs.)	−0.03	−0.16	0.996	0.51
Corn Use (Million bu.)	−0.07	−0.25	0.988	0.24
Cattle on Feed (Million head)	0.22	0.04	0.993	0.30
Fed Cattle Demand ($/lb.)	−0.02	−0.06	0.997	0.83
Mardia Skewness:System			35.80	0.43
Mardia Kurtosis: System			−0.30	0.76
Henze-Zirkler T: System			0.92	0.49

Panel B. Test of Correlations of Residuals and Squared Residuals

Test Equation	Residuals Q(Lag 6)	Squared Residuals Q(Lag 6)
Placement (Hundred Million lbs.)	0.51	0.60
Harvest (Hundred Million lbs.)	0.35	0.90
Corn Use (Million head)	0.01	0.19
Cattle on Feed (Million head)	0.11	0.39
Fed Cattle Demand ($/lb.)	0.52	0.67

Panel C. Cross-Equations Correlations of Residuals

	Placement	Harvest	Corn Use	COF	FC Demand
Placement (Hundred Million lbs.)	1	−0.02	−0.07	0.03	0.13
Harvest (Hundred Million lbs.)		1	−0.02	−0.11	−0.04
Corn Use (Million head)			1	0.01	−0.14
COF (Million head)				1	−0.14
FC Demand ($/lb.)					1

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