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ONGOING RELATIONS, MONEY, AND CREDIT
by
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Ongoing Relations, Money, and Credit

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Abstract

A model which explains, at a primitive level, the coexistence of money and credit, even though buyers prefer credit, and which allows the study of the interaction of money and credit is introduced. This is done in a setting with ongoing relations between sellers and untrustworthy buyers in which the choice of the medium of exchange is endogenous. The introduction of money results in less credit availability but helps to overcome the trading frictions so that the volume of trade increases. It is possible for there to be a monetary equilibrium which is Pareto dominated by the nonmonetary equilibrium. The mix of money and credit in equilibrium is a function of the cost of using money and primitives such as production cost and discount factors. A change in the cost of using money leads, generally, to an ambiguous change in welfare.
1. INTRODUCTION AND LITERATURE REVIEW

Fundamental challenges of monetary economics include developing models that explain, at a primitive level, the coexistence of money and credit, even though buyers prefer the use of credit, and allow the study of the interaction of money and credit. These issues are addressed in this paper by presenting a model of ongoing relations between buyers and sellers in which the choice of the medium of exchange is endogenous and the trading friction which underlies the usefulness of money is explicitly modeled.¹

This paper represents the first step in the development of a micro-based model of money and credit that centrally incorporates realistic frictions to credit trade. The model is relatively primitive in the sense of allowing the explicit consideration of buyers' decisions with respect to repayment of credit and the use of money, allows one to consider the choice of the means of payment and price-setting behavior by sellers, and is flexible enough to address a range of monetary and credit issues.

The trading friction here stems from the assumptions that buyers are not trustworthy in the sense that failure to repay a debt causes no intrinsic loss of utility and that the enforcement of contracts is either impossible or too costly to pursue via legal channels. In this setting, the quality of a buyer's credit will vary across circumstances.

That frictions are needed for money to be useful follows from observing smoothly-functioning model economies. It has been argued that models which ignore these basic frictions may be misleading with respect to money's role in the economy.² An interesting and important question is to what extent the implications of monetary models that explicitly incorporate the basic frictions of economic life differ from those of models that do not.

Perhaps the most commonly-used model today is the cash-in-advance (CIA) model which exogenously imposes, in a frictionless environment, a constraint that

¹ This is a version of a challenge issued by Hicks (1935): that economists explicitly model the frictions that lead to the use of money despite the fact that it offers a relatively low yield.

² See, for example, Laidler (1988).
requires the use of money to purchase at least some commodities. Although the CIA framework has yielded important contributions, it has been criticized because it imposes money exogenously. For example, Wallace (1990) suggests that it would be better to "discuss the properties of outside money that allow it to yield utility or to be used for goods purchases and that prevent inside money from playing these roles. Perhaps it is because outside money is trusted and inside money would not be" (p.19, emphasis added). The present paper focuses on the issue of trust and a framework is proposed in which there can be substitution—for any given commodity—between inside and outside money.

Because the model of this paper is a matching model, it may at first glance be reminiscent of a recent model which has gained prominence—the Kiyotaki-Wright model (1989,1991,1993). The models are, however, quite different. Kiyotaki and Wright use a search framework to exploit another trade friction to develop a model wherein money helps to overcome the double-coincidence of wants problem. They study money and barter while the model of this paper studies the more natural trade-off in modern societies—money and credit. In structure and results, their model seems to be much more closely aligned with the models of, for example, Starr (1972), Ostroy (1973), and Ostroy and Starr (1974).

The model here is related to and its development aided by some other models. In the spirit of attempting to understand monetary economies by considering models of spatial separation, this model is most closely related to some of the work of Robert Townsend (1980,1983,1987,1989). It also uses the friction of the lack of trustworthiness or creditworthiness as used by Douglas Gale in his insightful book *Money: in equilibrium* (1982) which culminates in the examination

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3 For example, Lucas and Stokey (1983,1987) exogenously impose a constraint that allows certain commodities to be purchased on credit while other commodities require the use of currency. See Aiyagari and Eckstein (1994) and Lacker and Schreft (1992) for extensions where the means of payment is modeled more flexibly. They assume that credit difficulties can automatically be overcome by paying a real resource cost.

4 See, for example, Kareken and Wallace (1980). See also the discussion on pp.166-167 of Blanchard and Fischer (1989).

5 Prescott (1985) considers the use of currency versus checks. Unlike the present model, his model does not allow for the use of credit and sellers are assumed always to be indifferent as to the medium of exchange. Also, see Chapter 1 of the *Handbook of Monetary Economics* for a useful classification of models of the medium of exchange.
of the core of a monetary economy. This friction appears to be widely shared as an intuitive explanation for the use of money. (See, for example, Lucas (1980) p.144, Lucas and Stokey (1983) pp.78-79, Friedman (1960) p.6, Tobin (1980) p.88, Kohn (1991) chapter 1). The intent of this paper is to develop a model which explores the implications of this friction.

The basic structure of this model is meant to capture the fact that most trade occurs in a decentralized manner and that an acceptable medium of exchange may depend on the structure of this trade. For instance, direct credit may be available to well-known, regular customers but not to relative strangers. In the model, buyers are randomly (but not uniformly so) matched with sellers who must undertake costly production if they choose to trade with a given buyer. If a seller chooses to trade, a medium of exchange must be agreed upon. A seller will demand cash when the apparent quality of a buyer's credit is too low.

The model is developed initially in a world with no currency or checks. (Suppose no one has thought of them yet.) No intermediated credit exists either--only direct credit from seller to buyer is possible. Then currency is introduced.  

The model generates an endogenous Lucas-Stokey type of cash-in-advance constraint. In so doing, it matches the observation that "credit is often used, particularly when there are ongoing relations." (Prescott, p.43) Sellers grant credit to their "best" customers and demand cash from the rest. The model gives conditions under which money will be useful in overcoming the trading frictions and under which its introduction is Pareto improving. By assuming a given matching technology, it is able to address some issues and obtain results not available in models which assume, for example, that credit difficulties are automatically solved. The structure makes the granting of credit problematic. Studying the repayment decisions of buyers and consequent credit-granting or cash-demanding behavior of sellers in a dynamic game setting provides a more basic foundation and understanding of money's role in the economy. Money in the

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6 It is possible to introduce checks and intermediation, though doing so complicates the model and will not be included here for ease of exposition.
model in effect anonymously links markets across the economy together. Moreover, by considering the behavior of buyers and sellers in this fashion, one obtains a cash-in-advance constraint that depends on basic factors such as production costs and discount factors. Thus, one is able to study how the mix of cash and credit changes with different basics.

The novelty of the model stems from the merging of this particular matching technology with a pricing mechanism whereby an incumbent seller and an entrant post prices in their market. This model thus allows movement away from the competitive price-taking behavior assumed in cash-in-advance models and the Nash bargaining behavior assumed in some other matching models of money toward the analysis of price-setting behavior in an endogenous monetary model.

In summary, the model produces results that suggest that the friction employed here to derive the usefulness of money does perhaps mirror the essence of actual problems in exchange and that there is promise for this line of research. Fiat money clearly serves as a medium of exchange even though it is second-rate--observed credit transactions are preferred to cash transactions. Sellers in the model play a prime role in the determination of an acceptable medium of exchange, as they do in fact. The endogenous coexistence of money and credit allows the study of the interaction of and potential substitution of these media of exchange as conditions in the economy change and the corresponding welfare effects. Because the mix of cash and credit changes with changes in the opportunity cost of using money, welfare may change in unexpected ways when the opportunity cost is changed.

The remainder of the paper is organized as follows. Section 2 presents the basic structure of the model in the absence of money. Section 3 gives the structure of competition and applies it to consider, as a benchmark, the nonmonetary, or pure credit, economy. Section 4 introduces money, studies its effect on the amount of credit, and derives a condition that must be met for money to be used. Section 5 presents and discusses a monetary equilibrium. Section 6 studies changes in the mix of cash and credit while Section 7 contains a welfare analysis. The paper concludes with Section 8.
2. BASIC STRUCTURE OF THE MODEL

The details of the model will now be presented. The basic structure involves matchings of buyers and sellers. In each period, each buyer visits a market for a good. In the nonmonetary economy, sellers may decide to trade by granting credit. In the following period, any buyer who was granted credit chooses to either bear the cost of repayment or not. If not, trade possibilities in that market for the buyer may be altered. A seller's credit terms are constrained by the possibility that the buyer may be able to obtain credit from another seller in the market. The basics of the economy described below are summarized in Table 1 at the end of this section. Details of the monetary economy are deferred until Section 4.

This is a matching model with an infinite horizon. Each period consists of two subperiods, the AM and the PM. Let (t,AM) [(t,PM)] denote the AM [PM] subperiod of period t. Time begins at (0,PM).7

There are two basic types of agents--buyers and sellers. Each buyer lives an infinite number of periods. Sellers' periods of activity within their market vary and will be discussed below.

Suppose that there is a market for each of N commodities, where N is a large, finite number. In equilibrium there will be one active seller who transacts per market. The number of buyers in the economy can be finite or infinite. For concreteness, it may be helpful to think of there being a large, finite number of buyers, though the analysis does not hinge on this.

Sellers (also identified as "firms") are able, if they choose, to produce instantaneously, at a disutility of c per unit, an indivisible, perishable-within-a-subperiod good in the PM when buyers shop. Buyers are able to produce only in the AM. ("Buyers" are called such because that is their role in credit transactions, but they are also producers.) A buyer's good is also perishable-

7 The timing convention is adopted to study credit. The AM-PM convention is used simply because it allows somewhat cleaner mathematical expressions than does merely assuming alternating periods.
within-a-subperiod but is perfectly divisible. A buyer incurs disutility of \( D(x) \) when producing \( x \) units.

Buyers obtain utility only from the sellers' PM good; sellers obtain utility only from the buyers' AM good. Beginning at \((0, PM)\), and in each succeeding PM, each buyer is matched according to a stationary probability distribution to a market for a particular good. That is, buyers "go shopping" in each PM seeking the PM good. Each buyer is matched to one market per period though a market may be visited by more than one buyer. The probability that Buyer \( j \) (\( B_j \)) will visit the market for some good \( i \) is denoted by \( \pi(i, B_j) \) or, where the market and the buyer under consideration are clear or arbitrary, by \( \pi \). For the purposes here, it is assumed here that \( \pi(i, B_j) \) is common knowledge to \( B_j \) and any seller of the good \( i \) with whom the buyer transacts, as are all of the buyers' characteristics.

No matching occurs in the AM. A buyer may at that time produce to repay a seller who has granted credit in the previous PM. Given the structure of production and trade and the absence of money, when a buyer visits a seller at \((t, PM)\) all that he can offer in exchange for a unit of the \((t, PM)\) good is a promise to pay some amount of the \((t+1, AM)\) good.

Let \( S_i \) be a seller in market \( i \). Assuming that \( S_i \) and \( B_j \) meet at \((t, PM)\) and agree to trade, \( S_i \)'s utility from the trade via credit is given by
\[
\begin{align*}
    u^i_t &= \beta R_t - C & \text{if } B_j \text{ repays } S_i \text{ the amount } R_t \text{ at } (t+1, AM) \text{ and} \\
    u^i_t &= -C & \text{if } B_j \text{ does not repay.}
\end{align*}
\]

Future utility is discounted using the discount factor \( \beta \); \( 0 < \beta < 1 \). \( R_t \) is the gross interest rate or price between \( t \) and \( t+1 \). It is measured in terms of the quantity of the buyer's AM good that is paid per unit of the seller's PM good.

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8 The reasons that lie behind the differences in divisibility are as follows. Allowing the seller's good to be indivisible simplifies the model in that a seller's decision to grant credit is a 0-1 decision. Any buyer's good is assumed to be divisible in order to allow for flexibility of payment for the seller's good.

9 For simplicity, it is assumed that buyers differ only in that their matching probabilities may differ across sellers. It is clear that one could allow buyers to differ in other dimensions as well for the analysis.
To distinguish this utility function from that of the buyer, the seller's per trade utility function will frequently be referred to as profit.

Bj's utility function per trade is given by

\[ u_t^j = u(1) - \beta D(R_t) \]
if Bj repays Si at \((t+1, AM)\) and
\[ u_t^j = u(1) \]
if Bj does not repay.

The instantaneous utility from consuming a unit of the PM good is given by
\[ u(1) = MU(1) \]. Consuming two units of a given good in a subperiod yields
\[ u(2) = u(1) + MU(2) \], where \( MU(2) < MU(1) \). Similarly, it is assumed that
\[ u(N) = u(N-1) + MU(N) \], where \( MU(N) < MU(N-1) < \ldots < MU(1) \) so that consumption of the PM good yields diminishing marginal utility.

The function \( D \), which gives the instantaneous disutility of producing a buyer's AM good, is differentiable with \( D' > 0 \) and \( D'' > 0 \). Further, \( u(1) > D(c/Z) \) and \( SD(c/Z) > MU(2) \) so that a buyer would be willing to trade \( c/Z \) units of the AM good to obtain a unit of the PM good, if necessary, but would not be willing to produce another \( c/Z \) units of the AM good to obtain a second unit of the PM good.

(In equilibrium, sellers charge the zero profit price of \( c/Z \) units of their AM good for a unit of the PM good.)

It is assumed that all buyers are equally untrustworthy--refusing to repay a debt causes no intrinsic loss of utility. Hence, buyers will choose to not repay if doing so increases lifetime utility. It is also assumed that sellers have no means to enforce contracts--they are unable to find and "prosecute" any buyer or it is too costly to do so. If a buyer fails to repay, the seller's only recourse is to alter the trade relationship. This is consistent with the finding of Macauley (1963) that, even in modern economies, businesses "seldom use legal sanctions to adjust (exchange) relationships or to settle disputes" (p.55).

In summary, at each \((t, PM)\), each buyer is matched into a market for a good. With no money in the economy, each seller must decide whether or not to grant credit to any buyer who visits at \((t, PM)\). That is, sellers must decide whether to give a buyer a unit of the indivisible PM good in exchange for a promise of
some amount of that buyer's (t+1, AM) good.

(While I will refer to the AM good and the PM good, it is easiest in the context of the matching model to think of buyers and sellers as producing different goods and that the problem of double coincidence of wants has been solved. Then one may think of a relatively high $\pi$ as reflecting trade between a buyer and a corner grocer or a baker or between traders on a commodity exchange, lower $\pi$'s as reflecting visits to purchase a television set or an appliance, and a $\pi$ close to or equal to zero may represent a tourist visiting a store. However, one should not identify a particular $\pi$ with a particular type of good or service. Wholesalers and retailers may have relatively high $\pi$'s for items for which most persons would have relatively low $\pi$'s.)

### TABLE 1

| Buyer $j$ ($B_j$) is matched to market $i$ with probability $\pi(i, B_j)$.
| Seller $i$ in PM; buyer $j$ in AM.
| Sellers know all inherent characteristics of buyers and their repayment histories in their market but not in other markets.
| $c/\beta$ is the zero profit price or gross interest rate charged by sellers.
| Large, finite number of sellers
| Large, finite number of buyers

Buyers characteristics differ only in that they may have different matching probabilities across markets.

Sellers credit policies may differ only with respect to a buyer's matching probability and repayment history.

**($t, PM$)**

Incumbent (I) publicly posts credit schedule $R^I_t(\pi, h_t)$ for the PM good.

Next, entrant (E) observes the incumbent's credit schedule and publicly posts credit schedule $R^E_t(\pi, h_t)$.

Next, each buyer is matched to one market to shop for the PM good and observes the credit schedules of I and E.

Next, a seller produces a unit at a utility cost of $c$ for any buyer who meets the terms of the credit schedule.

**($t+1, AM$)**

No matching occurs.

Any buyer who was granted credit at ($t, PM$) is able to produce the AM good to repay.
3. NONMONETARY EQUILIBRIUM

Structure of Competition

This section gives the structure of competition sellers face and, as a benchmark, discusses equilibrium in the absence of money. For each time period, at $(t,PM)$, in each market there will be two sellers (firms): a seller who is in the market in every period from the beginning called "the incumbent" (denoted by I) and another seller called "the entrant." The period $t$ entrant (denoted by $E_t$) exists only from $(t,PM)$ through $(t+1,AM)$. The entrants are used in the model to reflect the fact that buyers usually have a number of sellers with whom they can shop and to constrain the incumbent's pricing.

The information structure is as follows. All sellers in a given market know all of the inherent characteristics of buyers who visit their market and their repayment histories within that market but not their repayment histories in other markets. The key feature is that sellers can condition their credit policies on buyer behavior only in their particular market, not across the economy.

The incumbent moves first and posts a price schedule, $R^{I}(\pi,h)$, which is the amount of the $(t+1,AM)$ good which a buyer who is matched into the market with probability $\pi$ (is of type $\pi$) and whose repayment history in the market is given by $h$ should repay in exchange for a unit of the $(t,PM)$ good. Next, the entrant posts a price schedule $R^{E}(\pi,h)$ after observing the incumbent's price schedule. A price schedule also indicates a seller's credit policy--i.e., conditions that must be met by a buyer in order to be granted credit. These price schedules are publicly-posted or advertised. That is, sellers post prices that refer to generic repayment histories. For example, a seller may list a price of 1.03 for buyers with flawless repayment histories who are their "best customers" (i.e., have relatively high matching probabilities) and not offer credit otherwise.

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10 It is possible to obtain the same results as in this paper in a version of the model where the credit history of a buyer and a given firm is private information and where incumbents face entry by potentially long-lived entrants. (The existing incumbent exits the market and an entrant becomes the ongoing incumbent if the entrant ever successfully undercuts and is rationally repaid by buyers.) This version is presented since it incorporates the key features, yields the same outcome, and is less complicated and messy to work with.

11 The sellers are in fixed, known locations and the quoted prices are adhered to.
Finally, buyers are matched into the market to make purchases. A given buyer of type \( \pi \), \( B_j \), is matched into a market at time \( t \) and observes whether he is offered credit by the incumbent, \( I \), and, if so, the price he should pay at \((t+1, AM)\) in exchange for a unit of the \((t, PM)\) good. \( B_j \) then observes the price charged by the current entrant, \( E_t \). In the following AM, \( B_j \) chooses to either repay or not repay any seller who has given credit.

No seller is in more than one market and buyers' strategies in one market will be independent of their strategies in any other market. Thus equilibrium will be considered for a given market. Sellers are allowed to discriminate and offer different credit terms based on a buyer's \( \pi \) for their market. Thus, the pricing and credit availability strategies will be given for each matching probability within a given market.

The incumbent's strategy at the component game beginning at \((t, PM)\) and going through \((t+1, AM)\) is \( S_t^I = R_t^I \) which sends a buyer's characteristics into the extended reals, \( R' \). \( E_t \) chooses a strategy \( S_t^E = R_t^E \) which depends on a buyer's characteristics and the price schedule of \( I \).

Let \( U^I \), \( U^E_t \), and \( U^{B_j} \) denote the lifetime utility of the initial incumbent, the period \( t \) entrant, and Buyer \( j \), respectively. Further, let \( S^I_t \), \( S^E_t \), \( b^j \) represent the strategies for the repeated game of the initial incumbent, period \( t \) potential entrant, and Buyer \( j \), respectively. Let \( S^E = (S^E_0, S^E_1, S^E_2, \ldots) \). Then let \( S^{E-j} \) be the same vector but without \( E_t \)’s strategy; e.g., \( S^{E-1} = (S^E_0, S^E_1, S^E_2, \ldots) \). Similarly, let \( b = (b^1, b^2, \ldots, b^N) \) be the vector of strategies for the \( N \) buyers who may be matched into this market. Then \( b^j = (b^1, b^2, \ldots, b^{j-1}, b^{j+1}, \ldots, b^N) \). Let \( S^I_t \), \( S^E_t \), \( B^j \) denote the strategy sets for \( I \), \( E_t \), and \( B_j \), respectively.

**Nash Equilibrium**

It is now possible to define a Nash equilibrium (NE) for the game.

\((s_{I^0}, s^E, b)\) is a Nash equilibrium iff

\[ U^I(s^I_{I^0}, s^E, b) \geq U^I(s_{I^0}, s^E, b) \text{ for all } s_{I^0} \in S_{I^0} \]

\[ U^E_t(s_{I^0}, s^E_t, s^{E-t}, b) \geq U^E_t(s_{I^0}, s^{E_t}, s^{E-t}, b) \text{ for all } t, \text{ for all } s_{I^0} \in S_{I^0} \]

\[ U^{B_j}(s_{I^0}, s^E, b^j, b^{-j}) \geq U^{B_j}(s_{I^0}, s^E, b^{j'}, b^{-j'}) \text{ for all } j, \text{ for all } b^{j'} \in B^j \]
One equilibrium is that buyers choose to fail to repay any credit that is offered and no firm offers credit. As described in the following result, there is also a Nash equilibrium in which credit is offered to some buyers so that some, but not all, possible credit transactions are consummated.

**Result:** For any given market, there exists a Nash equilibrium in which 1) the incumbent offers credit at the zero profit price \( R^I_t = C/\beta \) for all \( t \) to buyers with matching probabilities no less than \( \bar{\pi} \), but buyers with lower matching probabilities are not offered credit, 2) no entrant offers credit, 3) all buyers who are granted credit repay. \( \bar{\pi} \) is defined to be the matching probability \( \pi \) that leaves a buyer indifferent between repaying and not at the zero profit price.

To save space, this equilibrium will not be derived here. The proof that it is an equilibrium follows that presented for buyers who are granted credit in the monetary section of the paper. In the equilibrium, the incumbent uses the trigger strategy of cutting off credit to any buyer who ever fails to repay him. The entrants use a modified trigger strategy: They cut off credit to those buyers who ever failed to repay a seller who offered the "best" credit price that was rational to repay. That is, they forgive any buyers who failed to repay an entrant who charged a credit price that was at least as great as the incumbent’s or failed to repay the incumbent when his price was no greater than the entrant’s. This strategy captures that the entrants are independent firms who accept that firms that offer relatively poor credit terms are vulnerable to buyers’ incentives to fail to repay. Should a buyer ever fail to repay the incumbent when the incumbent offers a price no greater than the entrant, that buyer is cut off from future credit in this market.

Buyers adopt the strategy of repaying the seller with the lowest price but exhibit "loyalty" to the incumbent (and thereby support the equilibrium with credit) by repaying the incumbent if he charges a price that is no greater than

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It also mimics the behavior in the imperfect information version of the model where buyers will anonymously fail to repay high-priced firms and confer incumbency status on an entrant who successfully undercuts.
the entrant's. If an entrant should offer credit at a price no less than the incumbent's price, buyers obtain the good from, but do not repay, the entrant.

The logic of the equilibrium is made clear by examining the repayment decision of a buyer who faces a constant price of $c/\beta$ over time in a given market and whose credit is cut off in that market should he fail to repay, as is true in the equilibrium. The buyer's expected lifetime utility from choosing to always repay is

$$u(1) - \beta D(R_0) + \sum_{t=1}^{\infty} \pi \beta^t [u(1) - \beta D(R_t)] = \frac{1 - \beta + \pi \beta}{1 - \beta} [u(1) - \beta D(c/\beta)].$$

A buyer obtains $u(1)$ if he fails to repay and is cut off from future trade in this market. Given the constancy of the interest rate, if a buyer decides to not repay, he will do so immediately. A buyer will choose to repay if

$$D(c/\beta) = \frac{\pi u(1)}{1 - \beta + \pi \beta}.$$

That is, once a buyer has received credit (the PM good) from the seller, it is best to repay if the cost of doing so, $D(c/\beta)$, is no greater than the expected present value of benefits from being granted credit in the future. This condition can be restated as follows: Repay if

$$\pi \geq \frac{(1 - \beta) D(c/\beta)}{u(1) - \beta D(c/\beta)} \equiv \bar{\pi}.$$

In the equilibrium, the incumbent in any market takes the strategy of not offering credit to buyers with no incentive to repay ($\pi < \bar{\pi}$), granting credit to buyers with incentive to repay ($\pi \geq \bar{\pi}$), and cutting off credit to any buyer who ever fails to repay. The zero profit price is offered by I since entrants are able to profitably undercut the incumbent and "steal the market" should I post a price above $c/\beta$ to buyers with flawless credit records. Buyers with $\pi \geq \bar{\pi}$ in

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13 This trigger strategy captures the oft-cited central importance of a person's repayment history in a firm's decisions to grant credit.
a market rationally repay the incumbent.

Further, $0 < n < 1$ so that some types of buyers are granted credit in a given market while others are not. That is, less trade occurs than would under full trust. By full trust is meant the situation where all agents repay all debts, even if doing so yields less lifetime utility than not repaying does.

4. INTRODUCTION OF MONEY

At this point, "money" is introduced. A central issue is whether money "works" in this economy. That is, will sellers accept it, buyers use it, and will it help to overcome the problem of the lack of trustworthiness?

As the economy is currently structured, the good in any market is produced at the same cost, $c$. With this structure and resultant identical zero profit prices across the economy, money will, for simplicity, take the form of simple, indivisible tokens. These tokens are easy to carry, non-counterfeitable, easily recognizable, and so forth.

Suppose, then, that each buyer in the economy is endowed with a single token (call it a dollar). It is assumed that there is associated with its use a fixed cost, $f$, which represents the foregone interest earnings and "shoe-leather" costs of using currency. If a buyer is able to use credit and produces $c/S$ units in the following AM, his utility from the trade is $u(1) - SE(c/S)$. If a buyer is required to use cash and produces $c/S$ units in the following AM, his utility from the trade is $u(1) - f - SE(c/S)$.

One can think of this as resulting from the following sort of situation.

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14 This specification should not be confused with a money in the utility function set-up. No intrinsic utility is derived from cash. In this model, because of the reduced-form disutility $f$, a buyer's utility is higher if he is able to avoid using cash in a period. This disutility works against rather than for this economy becoming a monetary economy.
In the PM, buyers learn their matchings. One can imagine, without formally modeling a financial sector, that credit-using buyers are able to invest their dollars and earn interest while cash-using buyers are unable to do so. One can also think of the inconvenience costs involved in obtaining currency from its place of safe-keeping.

**Institutional Structure**

Each buyer is, as in the nonmonetary economy, matched to a market for a good in the PM of each period. When a buyer is matched in the PM to a market for which his matching probability is \( \pi \), a firm has several options. It can 1) offer credit as the medium of exchange; or 2) demand payment in cash; or 3) refuse to trade with the buyer.

In each market, sellers now post schedules that indicate the medium of exchange they will accept as well as the price they charge for each matching probability and credit history. Again, if seller \( i \) offers credit with \( R^i_t(\pi, h_t) = b \), a buyer of type \( \pi \) with history \( h_t \) in this market should pay \( b \) units of the AM good in exchange for a unit of the PM good.

Seller \( i \) may demand cash. This means that the seller accepts a dollar for a unit of the PM good. At \((t+1,AM)\), sellers that obtained cash at \((t,PM)\) are able to trade their dollars for the AM good with buyers who paid cash. A seller who accepts cash posts a price, \( Q^i_t \), which is the amount of the AM good seller \( i \) will accept in exchange for a dollar at \((t+1,AM)\). This price is posted by firm \( i \) at \((t,PM)\) for monetary trade at \((t+1,AM)\).

 Buyers who exchange their dollars for a unit of the PM good are able, if they choose, to produce goods in the following AM to sell for cash. At \((t+1,AM)\), buyers and sellers who transacted in cash can take part in an economy-wide matching "money market." Sellers trade dollars obtained at \((t,PM)\) for the AM
good produced by buyers who spent their dollars at \((t, PM)\). These buyers are randomly matched in uniform fashion to these sellers. Each seller must adhere to the price of dollars in terms of the AM good he quoted at \((t, PM)\). Buyers either accept or reject this take-it-or-leave-it price. Since there is a finite number of sellers, there is a positive, though perhaps very small, probability that a buyer will meet the seller he transacted with at \((t, PM)\). Buyers thus have incentive to pick a seller who offers the lowest price when the buyer shops at \((t, PM)\). This structure allows AM prices to be determined by the competition incumbents face in the PM without resorting to entry of some sort in the AM in the model. In the meantime, buyers who were granted credit may produce and repay the sellers. Table 2 summarizes the timing of events.

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<td>((t, PM))</td>
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Incumbent publicly posts a schedule that indicates conditions under which credit will be granted and at what price, conditions under which cash will be demanded and the price, and conditions, if any, under which no sale will be offered. Next, entrant observes the incumbent’s schedule and publicly posts a schedule with the same options. Next, each buyer is matched to one market and observes the payment options and prices posted by the sellers. Next, a seller may produce a unit at a utility cost of \(c\) for a buyer who meets the terms presented.

| \((t+1, AM)\) |
Any buyer who was granted credit at \((t, PM)\) is able to produce the AM good to repay \(R_t\) and deliver the good to a seller who granted credit. Any buyer who used cash at \((t, PM)\) can produce the AM good to sell for cash to sellers who accepted cash at \((t, PM)\). Such buyers and sellers meet in an AM market where buyers are uniformly randomly matched to sellers.

It will now be possible for firms to safely compete with a demand that buyers pay cash if money works—buyers who use cash choose to produce and sell AM goods for cash. Safe competition is possible in the sense that a firm that
demands cash is not vulnerable to a failure to repay as it is when it offers credit. If a buyer refuses to give the firm his token in the PM, the firm can simply refuse to give the buyer the PM good. If the buyer gives the firm his token, it will be able to exchange the token for goods in the following AM if money works in this economy.

In the next few sections it will be verified that money can help overcome the lack of creditworthiness and the resultant lack of trade that occurs when the matching parameter between a buyer and a market is low. First, the next subsection derives a condition under which it is in a buyer's interest to produce goods to sell in order to regain possession of a token in the AM if he was required to pay cash for goods in the preceding PM.

The second subsection discusses a perverse feature of introducing money into the economy--it makes some buyers bad credit risks who were perfectly good credit risks prior to its introduction. That is, there are markets in which certain buyers would have repaid debt in the absence of money but will not do so when money is used.

**Decision to Produce to Re-obtain a Dollar**

First, will a buyer who pays cash in the PM produce goods in the following AM to sell in exchange for tokens from the PM sellers? Suppose, as will be the case in equilibrium, that a dollar trades for \(c/B\) units of the AM goods. If the buyer does not produce for sale ever in order to re-obtain a token (given the stationary structure of prices, a buyer decides once and for all whether it is worth producing for sale or not), he obtains \(u(1) - f\) in utility from using his token in the PM. If he does produce for sale and obtains a token by selling goods in the following AM market in each period which follows his use of cash in the PM, his expected lifetime utility from cash trade is
(4) \[ u(1) - f - \beta D\left(\frac{\mathcal{C}}{\beta}\right) \] 
+ \pi^c \sum_{t=1}^{\infty} \beta^t [u(1) - f - \beta D\left(\frac{\mathcal{C}}{\beta}\right)],

where \( \pi^c \) is the probability that, in a given period, he will be required to pay cash in transactions; that is, \( \pi^c \) is the sum of the buyer's matching probabilities for all markets in which cash will be demanded. If this utility exceeds \( u(1) - f \), a buyer will produce to obtain a token in the AM following the use of cash. This condition can be written as, do produce to re-obtain money if

(5) \[ \pi^c \geq \frac{(1 - \beta) D\left(\frac{\mathcal{C}}{\beta}\right)}{u(1) - f - \beta D\left(\frac{\mathcal{C}}{\beta}\right)}. \]

Inequality (5) says that if there is enough trade between buyers and sellers who have low enough matching probabilities that the sellers demand cash, cash works. An interpretation of this is that money will be useful if society is large and complex enough so that enough potential trading opportunities exist where reputation effects are not large enough to rule out failure of repayment. As Gale notes (p.195), "The only non-monetary economies which exist are very primitive indeed."

Money's Effect on Repayment Incentives

The second point has to do with the perverse effect the introduction of money has on some buyers' incentives to repay in some markets. With money in the economy, if a firm offers a buyer credit and it is possible to engage in cash trade, it may be best for the buyer to fail to repay the firm that grants credit and to pay cash from then on.

To see this, suppose, without loss of generality, that as of \( (0, \text{PM}) \) I offers credit at \( R_t = \frac{c}{\beta} \) for all \( t \) and that another firm will accept cash for

\[ 15 \text{ In what follows, except where noted in Section 7, (5) will be assumed to hold for all buyers. If this condition holds for some, but not all buyers, price would have to rise to provide zero expected profit to firms.} \]
the PM good, assuming that a dollar can be exchanged for \( c/\beta \) units of the AM good in the money market. Before continuing, it is useful to note the following

**Lemma:** If a buyer chooses to not repay I's credit, it is better for the buyer to not use cash in the period.

**Proof:** Obtaining the good on credit from I but not repaying and not using cash with another firm yields a buyer \( u(1) \) from the \((0,PM)\) and \((1,AM)\) transactions. Failing to repay I's credit and also using a token with the other firm yields utility from these transactions of \( u(1) - f + MU(2) - \beta D\left( \frac{C}{\beta} \right) \). The first option is better if \( 0 > MU(2) - f - \beta D\left( \frac{C}{\beta} \right) \), which is true by assumption.

**Proposition 1:** Given that the buyers produce \( c/\beta \) units of the AM good to repay credit or to re-obtain a dollar, buyers prefer to fail to repay credit offered by a given firm and to pay cash in the future if

\[
(6) \quad \pi < \frac{(1-\beta)D\left( \frac{C}{\beta} \right)}{f} \equiv \pi'.
\]

Moreover, \( \pi' > \tilde{\pi} \) so that credit is less available than in the nonmonetary economy.

**Proof:** The lifetime utilities of the two choices must be compared. Repaying credit continually in a market yields

\[
(7) \quad u(1) - \beta D\left( \frac{C}{\beta} \right) + \pi \sum_{t=1}^{\infty} \beta^t [u(1) - \beta D\left( \frac{C}{\beta} \right)].
\]

Failing to repay credit and using cash henceforth yields

\[
(8) \quad u(1) + \pi \sum_{t=1}^{\infty} \beta^t [u(1) - f - \beta D\left( \frac{C}{\beta} \right)].
\]

Using cash is preferred if

\[
(9) \quad D\left( \frac{C}{\beta} \right) > \frac{\pi f}{1-\beta} \quad \text{or} \quad \pi < \frac{(1-\beta)D\left( \frac{C}{\beta} \right)}{f} \equiv \pi'.
\]

Since

\[
\tilde{\pi} = \frac{(1-\beta)D\left( \frac{C}{\beta} \right)}{u(1) - \beta D\left( \frac{C}{\beta} \right)}, \quad \pi' > \tilde{\pi}
\]

as long as \( u(1) - \beta D\left( \frac{C}{\beta} \right) > f \), which it must for money
to be viable.\[11\]

Now \( \pi' \) denotes the matching parameter such that buyers are indifferent between repaying credit and not repaying and using cash henceforth.

Before the introduction of money, buyers in markets for which they had matching probabilities less than \( \bar{\pi} \) failed to repay sellers who had granted credit. Now with \( \pi' > \bar{\pi} \) the matching parameter at which buyers are indifferent between repaying and not repaying a debt when money is in the economy is greater than the parameter when there is no money. That is, in markets where a buyer's \( \pi \) is such that \( \pi \in (\bar{\pi}, \pi') \), buyers would repay credit before cash was introduced but will not do so after.

The intuition is that when there is no money, should a buyer fail to repay the incumbent's zero-profit price, the buyer forgoes future trade in that market since other firms will not offer credit, anticipating that such a buyer will fail to repay them as well. With money in the economy, firms will accept cash from such buyers since cash will assure that a firm will not take a loss as long as the AM money market works. When buyers have the chance to use money in future transactions in a given market, they are more willing to fail to repay credit than when failure to repay credit means no future trade in that market. That is, the introduction of money lowers the opportunity cost of failing to repay a debt. This means that sellers will not grant credit to buyers whose matching probability \( \pi \) is less than \( \pi' \). Thus, \( \pi^c \), the probability a buyer will have to use cash in a period, is given by \( \pi^c = \sum_{\pi_i \in [0, \pi')} \pi_i \), where \( \pi_i \) is the probability
that the buyer is matched into market i.

5. MONETARY EQUILIBRIUM

To be listed now are the strategies that yield a Nash equilibrium outcome in which credit is granted to buyers with \( r \geq r' \) and cash is demanded from buyers with lower matching parameters. Then follows a discussion of these strategies. A proof that they give the equilibrium described is in the appendix.

The trade and payment options the incumbent may post at \((t, PM)\) consist of:

1) Offer credit and state the amount of the \((t+1, AM)\) good the buyer should pay depending on the buyer's matching probability and repayment history. (Denoted by Credit\((R_{t'}\)).)

2) Demand payment in cash and charge \( Q_{t'} \) of the AM good for a dollar in the following AM market. (Note that sellers' prices here are independent of buyers' characteristics. It will be shown that money works anonymously, as would be expected.) (Denoted by Cash\((Q_{t'})\)).

3) Refuse to trade. (No Sale)

\( E_t \) has the same set of options. These constitute the set of medium-of-exchange policies a firm may adopt.

Buyers observe the options offered by the sellers and can choose to shop with any of the sellers who offer a sale or to not shop and go home. For each transaction they make, they can decide whether to produce in the following AM to repay credit or re-obtain cash, as appropriate.

To be listed now are the strategies for an arbitrary market. For a given market, let \( F_{t}^i \) be the set of buyers who have ever failed to repay I prior to \((t, PM)\) and \( F_{t}^k \) be the set of buyers who have ever failed to repay an entrant and are not forgiven by the entrants as explained in Section 3. Let \( s_{t,i} \) be a firm's
(i = I or E) strategy for the (t,PM)-(t+1,AM) trading round. Also, let \( R(\pi) \)
be the real number such that \( u(1) - \beta D(R(\pi)) + \frac{n\beta}{1-\beta} [u(1) - \beta D\left(\frac{C}{\beta}\right)] = u(1); \) \( \bar{R}(\pi) \)
is the highest credit price a buyer will rationally repay in a period assuming
future prices return to the zero profit level of \( \frac{c}{S} \). Similarly, let \( \bar{Q} \) be the
largest amount of the AM good a buyer will rationally exchange for a dollar in
the AM assuming he will be able to do so in the future for \( \frac{c}{S} \) units. Finally,
define \( \bar{R}_t^I \), a function of \( Q_t^I \), by \( D(\bar{R}_t^I) = [D(Q_t^I) + f] \).

\[ s_t^I = \text{Credit}\left(\frac{C}{\beta}\right) \text{ if } \pi \geq \pi' \text{ and } B_j \in F_t^I \]
\[ = \text{Cash}\left(\frac{C}{\beta}\right) \text{ otherwise} \]

For \( \pi \geq \pi' \) and \( B_j \in F_t^E \),

\[ s_t^E = \text{Credit}(R_t^E), \text{ if } R_t^I > \frac{C}{\beta} \]
\[ \text{where } R_t^E = \min\left\{\frac{1}{2}(R_t^I + \frac{C}{\beta}), \frac{1}{2}(\bar{R}_t^I + \frac{C}{\beta}), \bar{R}(\pi)\right\} \]
\[ = \text{Cash}\left(\frac{C}{\beta}\right), \text{ if } R_t^I \leq \frac{C}{\beta} \]

For all other buyers,

\[ s_t^E = \text{Cash}(Q_t^E), \text{ where } Q_t^E = \min\left\{\frac{1}{2}(Q_t^I + \frac{C}{\beta}), \bar{Q}\right\} \]

The strategy for a buyer \( B_j \) depends upon his matching probability in a market and
can be most easily described as follows:

For \( \pi \geq \pi' \), use the medium of exchange that yields the greatest payoff for a
(t,PM)-(t+1,AM) trading round (forecasting that credit will be available at the
price of \( \frac{c}{S} \) in the future) and produce the required amount in the following AM
unless i) \( E \), offers credit with \( R_t^E \geq R_t^I \) or ii) it is irrational to produce in the
AM at the price quoted (i.e., \( \min\{Q_t^I, Q_t^E\} > \bar{Q} \) and \( \min\{R_t^I, R_t^E\} > \bar{R}(\pi) \)). In case

\[ ^{16} \text{As is the case in the usual Stackelberg game, it is not possible to optimally undercut the incumbent since the set of prices strictly less than } R_t^I \text{ is open.} \]
i), obtain the good only from E, but do not repay. If case i) does not hold but case ii) does, do not shop.

For \( \pi < \pi' \), if a seller offers the good on credit at any finite price, obtain the good but do not repay. Otherwise use cash with the seller that offers the lowest price, \( Q_t^* \), unless \( Q_t^* > \bar{Q}_t \). If so, do not shop.\(^{17}\)

**Proposition 2:** These strategies constitute a Nash Equilibrium which has the following outcome.

I chooses

- \( \text{Credit}(\frac{C}{\beta}) \) if \( \pi \geq \pi' \), for all \( t \)
- \( \text{Cash}(\frac{C}{\beta}) \) if \( \pi < \pi' \), for all \( t \)

\( E_c \) chooses \( \text{Cash}(\frac{C}{\beta}) \) for all \( \pi \) for all \( t \)

\( B_j \) chooses

- Use I's credit and repay if \( \pi \geq \pi' \), for all \( t \)
- Use cash with I and produce to re-obtain cash if \( \pi < \pi' \), for all \( t \)

The proof is contained in the appendix. Zero profit prices result from the competition between I and the entrants. Frequent buyers in a market are granted direct credit; infrequent buyers must use cash.

6. CHANGES IN THE MIX OF MONEY AND CREDIT

Consider, briefly, the effect of a change in the distribution of matching probabilities. Cagan (1958) finds that currency is used more often when more transactions occur between strangers, for example, during wartime. This common sense result is matched in this model. If the matching distribution were changed

\(^{17}\) As noted previously, if the sellers offer equally good deals, the buyer gives preference to the incumbent.
so that buyers have less sellers to whom they are matched with probability at least \( \pi' \), cash would be used more frequently. That is, \( \pi^c = \sum_{\{i \in [0, \pi']\}} \pi_i \) increases. Conversely, if the distribution is changed so that buyers have more sellers to whom they are matched with probability at least \( \pi' \) (that is, they transact more often with sellers they "know well"), credit is used more frequently and cash is used less frequently.

Addressed next is the effect of changes in basic features of the model, given a fixed matching distribution.

**Proposition 3:** The proportion of trade conducted via credit (relative to that conducted with money) moves inversely to the costs of production and directly with \( f \) and \( \beta \).

**Proof:** This is clear when one recalls that \( \pi' = \frac{(1-\beta)D\left(\frac{C}{\beta}\right)}{f} \). A lower \( \pi' \) means credit is more available.

An issue of interest is how changes in production cost affect the usage of money versus trade credit. Valerie Ramey (1992) studies a real business cycle model and finds that it predicts that money and trade credit will move together when costs of production change. She finds empirically, however, that money and trade credit move inversely—as in this model.

If either the firms' cost of production is higher or the buyers' disutility of production is greater, the immediate repayment of credit (with cost \( D(c/\beta) \)) looks less attractive relative to the use of cash in the future (with a present value cost of \( f/(1-\beta) \)). The resulting lesser use of direct credit conforms roughly with actual practice. For example, doctors require payment in cash or evidence of insurance that guarantees payment prior to major procedures but will grant direct credit for, for example, office visits. Local bakeries may grant direct credit up to a certain amount but require cash for sales beyond that
credit limit.

A higher $S$ lowers the current disutility of production while raising the present value of the cost of using cash so repayment of credit is more likely. Greater patience in the economy, not surprisingly, implies greater availability of credit.

If the opportunity cost of using money, $f$, is higher, the cash-in-advance constraint is relaxed and credit is used more often in this economy. This economy is one example which provides an affirmative answer to the speculation of Blanchard and Fischer (pp. 166-167) that payment arrangements are sensitive to basic changes in the economy.

In this model, buyers purchase more on credit when the opportunity cost of money is higher. That is, suppose that the opportunity cost of using money is $f_1$ rather than $f_0$ where $f_1 > f_0$. Then $\pi'$ decreases from $\pi'_0$ to $\pi'_1$. Buyers with $\pi \in (\pi'_1, \pi'_0)$ are granted credit where before cash would be demanded. Sellers are now willing to grant credit because, with the higher opportunity cost of using money, they realize that buyers with $\pi \in (\pi'_1, \pi'_0)$ now prefer using credit to having to use cash with the higher opportunity cost.

Also note that buyers obtain greater utility from trade in markets for which their matching probabilities fall in $[\pi'_1, \pi'_0)$ because they are able to use credit and avoid the opportunity cost of money in these markets. On the other hand, buyers obtain less utility from trade in markets with lower matching probabilities because they still must use money and its opportunity cost has increased. This observation naturally leads one to consider welfare in this economy.
In this model, the introduction of money does not unambiguously increase utility, because of the opportunity cost associated with the use of money and because money must be used in some circumstances in which credit would have otherwise been granted. In the pure credit economy, buyers with matching probabilities lower than \( \bar{\pi} \) for a given market are not granted credit in that market. Without money in the economy, expected utility for a buyer from \((t,PM)\) to \((t+1,AM)\) trade is given by

\[
(10) \quad (1 - \bar{\pi}) [u(1) - \beta D(\frac{C}{\beta})], \text{ where } \bar{\pi} = \sum_{n_i \in [0, \bar{\pi}]} \pi_i.
\]

That is, the expected utility is the product of the probability that a buyer is matched in the period to a market for which the buyer's matching probability is at least \( \bar{\pi} \) and the trade surplus obtained in such a market.

Money helps to overcome the difficulties that result from the buyers' lack of trustworthiness and the sellers' lack of information about repayment across the economy but not perfectly so. That is, first, the possibility of monetary trade with other sellers makes buyers less likely to repay when credit is extended; i.e., the matching probability for a single market at which buyers begin to repay credit rises from \( \bar{\pi} \) to \( \pi' \). As noted, there is a monetary equilibrium as long as \( \bar{\pi} \), the probability that a buyer is matched to a seller who would not grant credit to the buyer when money is in the economy, is large enough.

Secondly, the use of money entails an opportunity cost not associated with the use of direct credit. A buyer's expected utility from \((t,PM)\) to \((t+1,AM)\) trade with money in the economy is
For a representative buyer, the introduction of money yields the following change in expected utility for \((t, \text{PM})\) to \((t+1, \text{AM})\) trade.

\[
(12) \quad -n^c f + \pi \left( u(1) - \beta D\left( \frac{c}{\beta} \right) \right)
\]

That is, the monetary economy yields buyers greater expected utility as long as the expected foregone trade surplus in the pure credit economy outweighs the expected opportunity cost of monetary trade.

The introduction of money has two opposite effects on expected utility. In (13), the sum on the right reflects the fact that it allows buyers to trade whose matching probabilities were too low for trade to take place on a credit basis. This obviously increases utility. The other effect is that the matching probability at which credit is granted rises from \(\bar{\pi}\) to \(\bar{\pi}'\). This effect, represented in (13) by the sum on the left, decreases utility because buyers must now pay the opportunity cost of using money for trade in markets with matching probabilities from \(\bar{\pi}\) to \(\bar{\pi}'\), whereas previously they were able to use credit.

Now, let \(A\) denote the change in expected utility from the introduction of money, given by (12) and (13). If all buyers' matching distributions are the same, money increases expected utility if, and only if, expression \(A\) is positive. (Sellers are indifferent as to the introduction of money--they receive zero profit whether or not money is in the economy.)

Now recall that money works in the sense that buyers will choose, after having spent their dollars in the previous PM, to produce \(c/\beta\) units in the AM to re-obtain cash if
\[ u(1) - f - \beta D \left( \frac{C}{\beta} \right) + \pi^c \left[ 1 - \beta \left( u(1) - f - \beta D \left( \frac{C}{\beta} \right) \right) \right] \geq u(1) - f \] or, rewriting,

\[ \pi^c (-f) + \sum_{n_i \in [\pi, \pi']} \pi_i \left[ u(1) - \beta D \left( \frac{C}{\beta} \right) \right] + \bar{n}[u(1) - \beta D \left( \frac{C}{\beta} \right)] \geq (1-\beta)[D \left( \frac{C}{\beta} \right)] \text{ or} \]

\[ (14) \bar{n}[u(1) - \beta D \left( \frac{C}{\beta} \right)] + \pi^c (-f) \geq - \sum_{n_i \in [\pi, \pi']} \pi_i \left[ u(1) - \beta D \left( \frac{C}{\beta} \right) \right] + (1-\beta)[D \left( \frac{C}{\beta} \right)]. \]

The left hand side of inequality (14) has already been identified as (A). Let the right hand side be identified as (B). Now, for the purposes of considering further whether or not money would be adopted by a society and its effects, there are four possibilities of interest:

1) \( A > 0, A > B \); 2) \( A > 0, A < B \); 3) \( A < 0, A > B \); 4) \( A < 0, A < B \)

Case 1) Money increases expected utility and buyers will produce to re-obtain the money they have spent. The monetary equilibrium is Pareto improving.

Case 2) Money increases expected utility relative to what it would be in a nonmonetary economy but buyers will not produce in order to re-obtain money they have spent. That is, the AM market for money trade does not work and there is no monetary equilibrium. In this case, although buyers would be better-off with money than without, they would, given the chance, be even better-off from obtaining the good for money and not producing to re-obtain it.

Case 3) The introduction of money would decrease utility but once it was introduced, buyers would be better-off producing AM goods to re-obtain goods. This may be the case if \( f \) is relatively large and much trade occurs with firms for which \( \pi \) is between \( \pi \) and \( \pi' \). Sellers are indifferent to the introduction of money; buyers would prefer that it not be introduced in this case. It is thus possible for there to be a monetary equilibrium which is Pareto dominated by the nonmonetary equilibrium.

Case 4) The introduction of money decreases welfare and buyers
would not produce AM goods to re-obtain money in any case so there is no monetary equilibrium.

Note also that for small changes in $f$, the effect on welfare is unclear. For example, a decrease in $f$ lowers the cost of using money but raises $\pi'$ and, thus, the volume of trade for which money must be used and the (now lower) opportunity cost must be borne.

If $f$ is increased, then $\pi'$ decreases, at least initially. However, once $f$ gets large enough that $\pi' = \bar{\pi}$, or $u(1)-f-\beta D(c/\beta)=0$, $\pi'$ will not drop any further--money trade ceases to provide any value to the buyers. Moreover, at some $\pi' > \bar{\pi}$, buyers will cease to produce in the AM to re-obtain money that they have spent in the previous PM. (This is so since for a buyer to be willing to produce to re-obtain a dollar it must be that $\frac{\pi'}{1-\beta} [u(1)-f-\beta D(c/\beta)] > D(\frac{c}{\beta})$
while $\bar{\pi}$ satisfies $\frac{\bar{\pi}}{1-\beta} [u(1)-\beta D(c/\beta)] = D(\frac{c}{\beta})$. For $f>0$, $\frac{\bar{\pi}}{1-\beta} [u(1)-f-\beta D(c/\beta)] < D(\frac{c}{\beta})$.) This is the analog in this model of the repudiation of money that may take place when inflation is too high.

All of this assumes that the change in $f$ is once-and-for-all. One would, of course, need to rework the model if there were expectations of changes in $f$ over time.

It is possible to show (though it will not be pursued here) that in a full information economy, where buyers' repayment histories in all markets are costlessly available to all sellers across the economy, there may be an equilibrium (depending on the matching distribution) in which credit is extended to all buyers and all buyers repay. (It can also be shown, for a given matching distribution, that such an equilibrium is less likely than a monetary equilibrium.) This full information equilibrium is supported by the sellers conditioning their granting of credit on whether or not a buyer has ever failed
to repay in any market. In this full information economy, a buyer’s expected utility from \((t, PM)\) to \((t+1, AM)\) trade is greater than in the nonmonetary economy and is simply

\[
(15) \quad u(1) - \beta D \left( \frac{C}{\beta} \right).
\]

Less utility is obtained in the monetary economy than in the full information economy, so one may ponder the role of money as compared to information sharing. As noted above, a monetary equilibrium is more likely to occur than is the universal credit equilibrium in the full information economy. Beyond that, when one considers that a buyer may visit a large number of sellers who are widely geographically dispersed and that a given seller may not know which other sellers a given buyer visits, it may be very difficult and costly to share repayment information. As communications technology has developed, credit card companies and credit bureaus have arisen in response to problems of trust. However, keeping and processing records on individuals’ credit histories is still costly and fraught with error. Money may often be the best mechanism to help overcome the lack of trustworthiness.\(^{18}\)

This points to the key feature of money in this model. It effectively anonymously transmits information across the economy by linking markets together in a buyer’s decision-making. In the monetary equilibrium, a firm can safely exchange its good without needing to know anything about a buyer’s repayment history. In the absence of money when the information in a buyer’s repayment history remains private information within a market, failure by a buyer to

\(^{18}\) It is possible to incorporate instruments such as checks and intermediated credit in an expanded and more complicated version of the model. As long as information processing is costly, money continues to be used in equilibrium.
produce to repay affects future transactions only within that market. This leads low frequency buyers in a market to default if granted credit. In the monetary economy, a buyer's failure to produce to re-obtain money reverberates across the economy—he is unable to transact in the many markets where money is demanded.

8. CONCLUSION

As Blanchard and Fischer (1989) note, "Explaining why and when money is used instead of credit is difficult" (p.165). This paper presented an explanation by using a model with ongoing relations between buyers and price-setting sellers in which a satisfactory medium of exchange must be agreed upon. Without money in the economy, not all possible trades take place because buyers are not trustworthy. An equilibrium with money and credit exists (even though money is second-rate) in which the use of money helps to overcome trade frictions. The possibility of a Pareto inferior monetary equilibrium appears. The introduction of money allows all possible trades to be consummated but, due to money's perverse effect on repayment incentives, credit is less available.

By explicitly modeling the trading friction that brings about the usefulness of money, it was shown that the mix of money and credit in the equilibrium varies with primitives such as production cost and discount factors. Moreover, the position of the endogenous cash-in-advance constraint varies with the opportunity cost of money so that the welfare effects of, say, a decrease in the cost of using money are ambiguous—it is less costly to use money but credit is used less.

Stripped to its essentials, the model is simple yet seems to capture the essence of actual problems in exchange that lead to a usefulness of currency. The model is quite clearly primitive but there is room for further research.
Several extensions immediately come to mind. The structure naturally lends itself to the consideration of issues such as costly information pooling and the interaction of currency and other means of payment such as intermediated credit and checks. It is possible to consider sellers with degrees of market power and multimarket contact and their effect on the mix of cash and credit, though the framework becomes more complex. It is hoped that extensions of the model will allow a greater analysis of the interaction of money and credit and a range of monetary issues within an endogenous monetary model with price-setting sellers.
APPENDIX

Proof of Proposition 2: It will now be confirmed that these strategies do yield the NE claimed. The proof makes use of the "unimprovability in a single step" principle from dynamic programming. See Kreps (1990) for a discussion of the principle. The proof considers an arbitrary buyer with a matching probability of \( \pi_0 \) for a given market and assumes that \( \pi^c > \bar{\pi} \) for all buyers.

Case A: \( \pi = \pi_0 \geq \pi' \)

Can I benefit by deviating, given the others' strategies? If I follows the prescribed strategy, his expected payoff is zero.

If I deviates by offering credit at \( R_I > c/\beta \), \( E_t \) undercutts I, buyers shop with \( E_t \), not I. This leads to zero profit for I, so I may as well charge \( c/\beta \).

If I deviates by offering credit at \( R_I = c/\beta - a < c/\beta \) and then reverts to \( R_s = c/\beta \) for all \( s > t \), \( E_t \) does not offer credit, buyers shop with and repay I. I makes a loss from trade at \( t \). Given the strategies of later entrants and buyers, this strategy yields a loss for I of \( \mu_t(\pi_0) \alpha \), where \( \mu_t(\pi_0) \) is the number of buyers who visit the market at \((t, PM)\).

If I deviates by not offering credit, \( E_t \) offers credit, buyers shop with and repay \( E_t \). I earns zero profit from this. Thus, I may as well charge \( R_I = c/\beta \) for all \( t \).

Next, can \( E_t \) benefit by deviating, given the others' strategies? If \( E_t \) follows the equilibrium strategy, \( E_t \) does not offer credit but demands Cash\((c/\beta)\) and obtains a payoff of zero.

If \( E_t \) offers credit with \( R_S > R_I = c/\beta \), buyers do not shop with I but obtain goods from \( E_t \) and do not repay so that \( E_t \) loses \( c\mu_t(\pi_0) \). If \( E_t \) enters with \( R_S > R_I = c/\beta \), buyers repay \( E_t \), do not shop with I, but \( E_t \) makes a loss. Thus, it
is better not to offer credit at $R_t^F < c/S$. Thus, $E_t$ may as well choose $Cash(c/S)$.

Lastly, can $B_j$ benefit by deviating, given the others’ strategies? Given that $E_t$ demands $Cash(c/S)$, and that $R_t^I = c/S$, it is clearly best for $B_j$ with $\pi = \pi_t \geq \pi'$ to repay $I$. If $B_j$ fails to repay $I$, $B_j$ can use cash in the future but, since $\pi > \pi'$, it is better to repay credit.

Case $B$: $\pi = \pi_0 < \pi'$

First, can $I$ benefit by deviating, given the others’ strategies? If $I$ follows his strategy of demanding $Cash(c/S)$, his expected payoff is zero. That is, if $I$ gives a unit of the PM good in exchange for a dollar and charges $Q_t^I = c/S$ units of the AM good for a dollar in the AM market, then $E_t$ chooses $Cash(c/S)$, and $B_j$ produces $c/S$ units of the $(t+1,AM)$ good. Thus, $I$ will be able to obtain $c/S$ units of the AM good at $(t+1,AM)$ and earn zero profit.

If $I$ deviates by demanding cash at $Q_t^I > c/S$, buyers shop with $E_t$ using cash and do not shop with $I$. This yields $I$ zero profit so $I$ may as well follow the putative equilibrium strategy.

If $I$ deviates by demanding $Cash(c/S-a)$ and then reverts to $Cash(c/S)$, buyers shop with $I$ using cash. $I$ loses $\delta t(\pi_0)$ by doing so.

If $I$ deviates by offering the PM good on credit, buyers obtain the good and do not repay; $I$ loses $c\mu_t(\pi_0)$. If $I$ deviates by offering No Sale, $I$ obtains zero.

Given the incumbent has dollars at $(t+1,AM)$, it is clearly best for the incumbent to trade the dollar for $c/S$ AM goods at $(t+1,AM)$ than not. Thus, $I$ may as well trade goods for dollars.

Next, can $E_t$ benefit by deviating given the others’ strategies? If $E_t$ offers credit, buyers obtain goods from him and do not repay; $E_t$ loses $c\mu_t(\pi_0)$. If $E_t$ demands $Cash(Q_t^F)$, $Q_t^F < \frac{c}{\beta}$, buyers shop with $E_t$. This, however, leads to
a loss on each trade. If \( E_t \) demands \( \text{Cash}(Q_t^E) \), \( Q_t^E > \frac{C}{\beta} \), buyers do not shop with \( E_t \). If \( E_t \) chooses No Sale, buyers do not shop with \( E_t \) and he earns zero. Thus, \( E_t \) may as well follow the equilibrium strategy.

Finally, can \( B_j \) benefit by deviating, given others' strategies? That is, given \( I \) demands \( \text{Cash}(c/\mathcal{E}) \), \( E_t \) demands \( \text{Cash}(c/\mathcal{E}) \), can \( B_j \) do better by deviating from the strategy of using cash with \( I \) at \((t, PM)\) and then producing \( c/\mathcal{E} \) units of the \((t+1, AM)\) good in exchange for the dollar?

If \( B_j \) chooses not to pay the dollar for the \((t, PM)\) good, he does not obtain goods at \((t, PM)\) and forgoes consumption of the \( PM \) good. Further, since \( \pi^o > \pi \), it is best to produce to re-obtain a dollar. Thus, it is a best response for \( B_j \) to follow the prescribed equilibrium strategy.
References


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