Stochastic efficiency optimisation of alternative agricultural water use strategies

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Abstract

Mathematical programming was used to optimise utility efficient deficit irrigation schedules for maize in Vaalharts, based on certainty equivalence assuming an exponential utility function. Total gross margin risk resulting from production risk of alternative deficit irrigation practices was quantified using an irrigation simulation model and stochastic budgeting procedures. Results showed that the portfolio of irrigation schedules for a risk averse farmer may include schedules with high production risk, due to the interaction of resource use between alternatives when water is limited. Owing to the difficulty of implementing the best portfolio of irrigation schedules, the optimised schedules may best be used to benchmark the efficiency of second best alternatives that are easier to implement. Ignoring risk may underestimate the value decision-makers attach to the security of water supply and policy-makers should take cognisance of this result.

1. Introduction

The National Water Resources Strategy (NWRS) (DWAF, 2004) is a requirement of the National Water Act (Act 36 of 1998) and describes how the water of South Africa will be protected, used, developed, conserved and managed. The NWRS seeks to identify opportunities where water can be made available for productive livelihoods and also to give the support and assistance needed to use the water efficiently. Furthermore, South Africa is going through water allocation reform in order to promote equity, to address poverty, to generate economic growth and to create jobs (DWAF, 2005). Politicians recognise that the allocation process should allow for the sustainable use of water and that it must promote the efficient and non-wasteful use of water. Agriculture utilises about 62% of all South Africa's water with losses ranging between 30%-40% (DWAF, 2005). Many people reason that water savings in irrigated agriculture will make a considerable amount of water available for other users. The National Water Conservation and Demand Management Strategy form an integral part of the NWRS. The

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agricultural sector strategy was recently finalised and endeavours to provide a supportive and enabling framework to improve irrigation efficiency. Thus, there is a clear need to optimise water use in the agricultural sector with the aim of conserving water.

According to Weinberg *et al* (1993), irrigated agriculture may conserve water in three ways: a) improved efficiency of water applications; b) alternative crops; and c) deficit irrigation. Deficit irrigation is defined as the deliberate under-irrigation of a crop with the aim of conserving water or of increasing the profitability of the farming enterprise over the long term (Dent *et al*, 1988:19). Potential benefits include increased irrigation efficiencies, cost savings and gains by acknowledging the opportunity cost of water (English and Raja, 1996). However, reduced water application levels due to deficit irrigation will increase yield variability (Willis, 1993).

Much research has been done in South Africa to evaluate the profitability of deficit irrigation. Early research aimed at showing that deficit irrigation is a viable option to optimise water use under conditions of limited water supply (Virag, 1988; De Jager and Mottram, 1995; Mottram *et al*, 1995; Grové and Oosthuizen, 2002). None of these researchers included the impact of deficit irrigation schedules on production risk. Botes (1990) evaluated the risk associated with deficit irrigating wheat using stochastic dominance with respect to a function (SDRF). Grové *et al* (2006) used a more robust variation of the technique called stochastic efficiency with respect to a function (SERF) (Hardaker and Lien, 2003; Richardson, 2004) to discriminate between alternative full irrigation and deficit irrigation schedules based on certainty equivalence. A shortcoming of these research efforts is that predefined irrigation areas and irrigation schedules are assumed. Thus, the interaction between water availability, irrigation schedule, the area planted and the risk associated with these decisions, is not optimised. In the meanwhile, farmers have to make decisions with respect to these variables each production season. The importance to develop procedures that will enable better decision support increase if one considers that many irrigation schemes are operated at low levels of assurance of water supply, which makes quota reductions common (Breedt *et al*, 2003; Scott *et al*, 2004).

The main objective of this paper is to develop an optimisation model that will enable decision makers with specific risk preferences to optimise the decisions on how much to plant and how much water to apply, taking limited water supplies into account.

In Section 2 a short background is given on how certainty equivalence is used to discriminate between risky alternatives. Section 3 describes the mathematical programming model and the data used for the analyses. The production risk of alternative irrigation schedules and the optimised interaction between water availability, irrigation schedule and the area planted, are presented in Section 5. Finally, some conclusions and recommendations for further research are given.

2. Discriminating amongst risky alternatives based on certainty equivalence

The principal theory of choice underlying risky decision making is the subjective expected utility theory which is based on the existence of an ordinal utility function by which alternatives can be ranked (Boisvert and McCarl, 1990; Hardaker *et al*, 1997). Given the existence of the subjective expected utility theory, the optimal decision (X_j*) is defined as one which maximises expected utility:

$$EU\left(X_j^*\right)=\underset{j}{Max}\ U\left(X_j\right)=\underset{j}{Max}\left[\sum_i p_i U\left(y_{ij}X_j\right)\right] \tag{1}$$

Where

X_j decision to plant specific area with irrigation schedule j

p_i probability that state of nature i will occur

y_{ij} gross margin of X_j given state of nature i occurs

U defines utility

From the latter part of Equation 1 it is clear that expected utility is a function of the probability distribution of alternative outcomes and the decision maker's preference for these outcomes as presented by the utility function $U(y)$. The shape of the utility function reflects an individual's attitude towards risk (Hardaker *et al*, 1997).

Owing to the ordinal scale used for utility, it is not trivial to go from the shape of the utility function to a measure of the level of risk aversion (Hardaker *et al*, 1997). This difficulty is resolved by a measure that is constant for any positive linear transformation of utility, known as the coefficient of absolute risk aversion (RAC) (Hardaker *et al*, 1997:96-97). RAC is defined as (Pratt, 1964; Arrow, 1965):

$$RAC = -\frac{U''(y)}{U'(y)} \tag{2}$$

The property of constant absolute risk aversion means that the preferred option in a risky situation is unaffected by the addition or subtraction of a constant amount to all payoffs. The negative exponential utility function exhibits the property of constant absolute risk aversion and is widely used in the literature to rank risky alternatives. The expected utility of one hectare planted with irrigation schedule *j* when constant absolute risk aversion is assumed, may be calculated as follows:

$$EU\left(X_j\right) = \sum_i p_i \left(-e^{-RAC\ y_{ij} X_j}\right) \tag{3}$$

However, the interpretation of utility remains difficult due to its ordinal nature. A measure that is directly related to utility and is easier to interpret is the concept of certainty equivalence. Certainty equivalence is defined as the minimum amount of money a decision maker would require as a lump sum payment to make him/her indifferent between the certainty equivalent (CE) and the future payment of a risky alternative (Richardson, 2004). A one-to-one correspondence exists between CE and utility and the maximisation of utility implies maximisation of CE (Hardaker *et al*, 1997). CE of the negative exponential utility function may be calculated as follows:

$$CE = \frac{\ln\left(-EU\left(X_j\right)\right)}{-RAC} \tag{4}$$

Larger negative values for RAC are associated with increasing risk-seeking behaviour, whereas larger positive values are associated with increasing risk-averse behaviour. A RAC of zero implies risk neutrality and alternatives will be ranked based on average outcomes.

3. Procedures

3.1 Mathematical programming model

To evaluate the impact of limited water supply conditions on optimal land and water allocation decisions, taking production risk of deficit irrigation into account, a non-linear mathematical programming model was developed. The model follows the Direct Expected Maximisation Non-linear Programming (DEMP) specification as presented by Boisvert and McCarl (1990) with the exception that CE is maximised. The complete specification of the model is as follows:

$$\text{Maximise} \quad CE = \frac{\ln\left(-\sum_i p_i\left(-e^{-RAC\, y_{ij} X_j}\right)\right)}{-RAC} \tag{5}$$

s.t.

$$\sum_j X_j \le land \tag{6}$$

$$\sum_j gir_{jt} X_j \le water_t \tag{7}$$

Where:

land maximum land availability
$water_t$ water availability in period *t*
gir_t gross irrigation requirement using irrigation schedule *j*

The objective of the model is to maximise the CE and therefore utility, based on the negative exponential utility function subject to a maximum land and water constraint in each time period. Parameterisation of the RAC will yield the set of irrigation schedule combinations that is stochastically efficient over all the other irrigation schedules over a given range of RACs. A priori knowledge of the decision-maker's RAC is not necessary to apply this model, since the optimised cumulative distributions of outcomes can be presented to the decision maker for decision-making purpose. The benefit of the model is that it can generate stochastically efficient distributions taking farmer-specific resource constraints into account.

To demonstrate the applicability of the model, two water supply conditions are optimised. The first water supply scenario (U) supplies enough water to the irrigator to irrigate 50 ha of maize with a full irrigation schedule. With the second water supply scenario (L), water availability in each time period is reduced proportionally by approximately 50%. The procedures used to develop the data sets for the model are now discussed.

3.2 Data development

Bernardo (1985) cautions against the use of neoclassical production functions which presuppose technical efficiency to optimise intraseasonal water allocation problems. When water allocation between multiple crops is of concern and intraseasonal water supply is constrained, economic theory suggests that water allocation does not need to be technically efficient. A multi-period model where the impact of decisions in previous periods is linked to current period decisions is necessary to optimise intraseasonal water use (Bernardo, 1985). The implication is that one needs to include a large number of technically inefficient irrigation schedules to optimise water use when water is limited during specific time periods.

The SAPWAT (Crosby *et al*, 2000) simulation model was used to determine the gross water applications of 455 alternative deficit irrigation schedules for maize in Vaalharts. SAPWAT uses readily available data on crop coefficients, soil water- holding capacities, irrigation technology and weather to model a daily cascading water budget, taking all appropriate water contributions and losses into account. To develop the alternative irrigation schedules it is assumed that the managerial ability of the decision maker is at a high level and that the irrigator is able to measure the soil moisture status accurately. SAPWAT identifies four crop growth stages with different yield responses to water stress. The model was set up so that it would trigger irrigation when a specified consumptive use deficit was reached during a specified crop growth stage. However, the amount of water applied was limited to a minimum of weekly irrigation capacity or the amount necessary to refill the soil to field capacity. Table 1 portrays some of the characteristics of the irrigation schedules which were combined to form the optimised stochastic efficiency frontier.

Table 1: Key characteristics of selected maize deficit irrigation schedules simulated with SAPWAT based on normal weather conditions in Vaalharts

Irrigation Strategy	Irrigation trigger Fraction	1-Eta/Etm Fraction	ky(1-Eta/Etm)[1] Fraction	Applied water mm.ha	Crop Yield Ton/ha
30-N-N-75					
Stage 1	0.30	0.30	0.12	32.51	
Stage 2	0.00	0.08	0.05	50.40	
Stage 3	0.00	0.03	0.04	201.60	
Stage 4	0.75	0.00	0.00	0.00	9.65
45-N-N-75					
Stage 1	0.45	0.30	0.12	28.80	
Stage 2	0.00	0.09	0.05	50.40	
Stage 3	0.00	0.04	0.04	201.60	
Stage 4	0.75	0.00	0.00	0.00	9.54
60-N-N-75					
Stage 1	0.60	0.30	0.12	25.10	
Stage 2	0.00	0.01	0.01	50.40	
Stage 3	0.00	0.03	0.04	201.60	
Stage 4	0.75	0.00	0.00	50.40	10.05
N-N-30-N					
Stage 1	0.00	0.07	0.03	62.84	
Stage 2	0.00	0.06	0.04	50.40	
Stage 3	0.30	0.23	0.29	100.80	
Stage 4	0.00	0.01	0.01	50.40	7.94
N-15-N-75					
Stage 1	0.00	0.07	0.03	62.84	
Stage 2	0.15	0.06	0.04	50.40	
Stage 3	0.00	0.05	0.06	201.60	
Stage 4	0.75	0.00	0.00	0.00	10.58

1- Eta: Actual evapotranspiration Etm: maximum evapotranspiration ky: sensitivity of crop for water stress

The acronym used to identify the alternative irrigation schedules in Table 1 corresponds with the trigger levels used to initiate irrigation. For instance, 30-N-N-75 implies a trigger level of 0.3 in Stage 1 and 0.75 in Stage 4 with no stress in Stages 2 and 3.

From Table 1 it is evident that the specified consumptive use deficit that triggers irrigation does not correspond with the cumulative deficit in a specific growth stage (*1-ETa/ETm*) and the deficits may even carry forward into the next stages. This is a direct result of the continuous water budget and the weekly capacity constraints. The calculated evapotranspiration deficits from each irrigation schedule were used to determine crop yields based on crop yield response factors (*ky*), which relate relative yield decrease (*1-Ya/Ym*) to relative evapotranspiration deficit (*1-ETa/ETm*). More specifically the Stewart multiplicative (De Jager, 1994) relative evapotranspiration formula was used to calculate crop yield, taking the effect of water deficits in different crop growth stages into account. Irrigation schedule N-N-30-N has the lowest crop yield due to the fact that maize is very sensitive to water stress during Stage 3 of production.

A triangular distribution was used to characterise the production risk for each irrigation schedule based on crop yields calculated for the three states of nature (severe drought, normal weather and wet) included in SAPWAT. The cumulative probability distribution of the triangular distribution is completely defined in terms of the minimum (severe drought), maximum (wet) and most probable (normal) outcome (Hardaker *et al*, 1997). Stochastic budgeting (Hardaker *et al*, 1997) was used to link information on the irrigation schedules with economic parameters obtained from an agricultural cooperative to calculate the total gross margin for each irrigation schedule based on the irrigation cost of a 50 ha centre pivot. The assumption was made that the water tariff is paid only if water is utilised for irrigation. Total water cost (application and tariff) was calculated to be R1,82/mm.ha. Output prices were kept constant at R900/ton for maize.

4. Results

4.1 Production risk of alternative deficit irrigation schedules

Figure 1 shows the crop yield variability for irrigated maize in Vaalharts associated with selected deficit irrigation schedules.

Irrigation schedule N-15-N-75 has the lowest crop yield risk associated with it and is classified as the full irrigation schedule. As expected, the highest crop yield risk is associated with deficit irrigating maize during its most sensitive crop growth stage (N-N-30-N). The other irrigation schedules show similar crop yield risk. However, the cumulative probability distributions overlap, which may result in different rankings by decision makers with varying risk preferences. The profitability of an irrigation schedule is determined not only by the variability of crop yields, but also by its water use and therefore by the total area that can be irrigated.

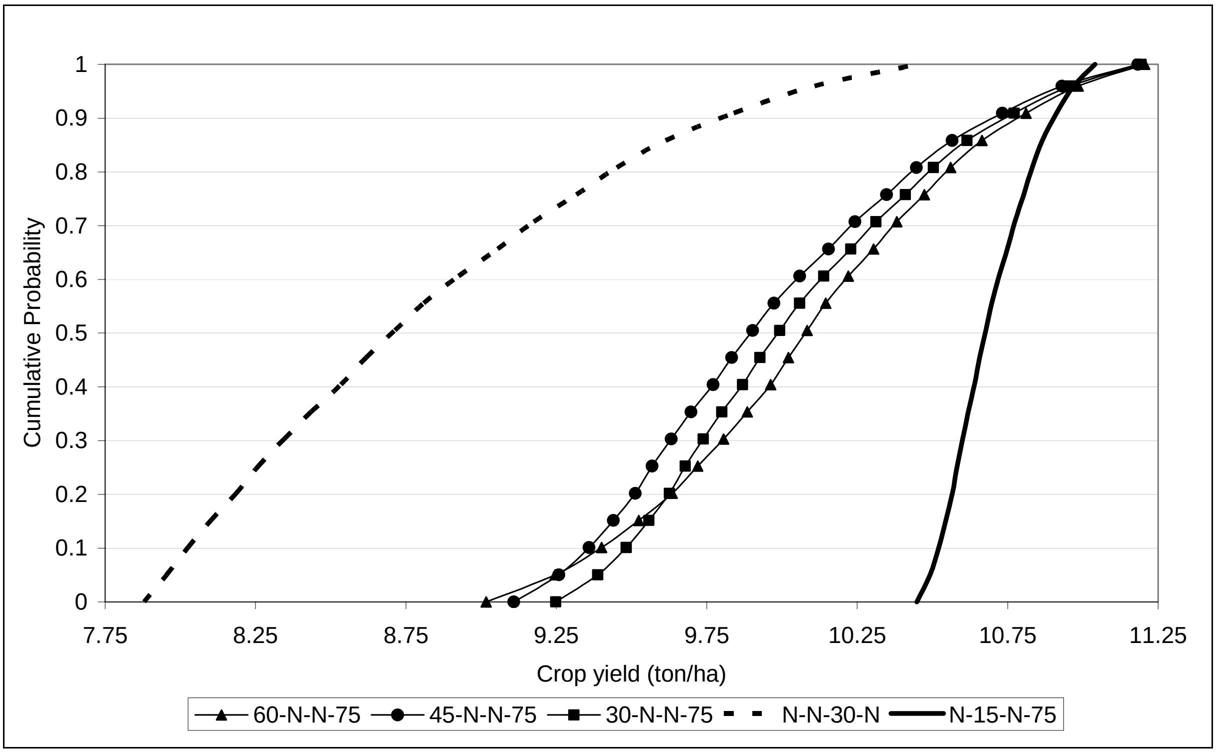

Figure 1: Cumulative probability distribution of maize yields calculated from SAPWAT outputs for selected maize deficit irrigation schedules in Vaalharts

In the next section the mathematical programming model was used to optimise the combination of the 455 irrigation schedules that will maximise the CE for a decision maker with a specific risk preference.

4.2 Stochastic efficiency of optimised irrigation schedule combinations

Table 2 shows the irrigation area and adopted irrigation schedules for a range of RACs which maximised certainty equivalent. Within the specified range the combination of irrigation schedules did not change when the RACs were parameterised.

Under unlimited water supply conditions it is optimal to adopt a single irrigation schedule given a specific range of risk aversion. Irrigators who are risk-seeking (-0.00005 to 0) or risk- averse (0 to 0.0004) will adopt the full irrigation strategy (N-15-N-75), irrigating the full 50 ha. Irrigators who are relatively more risk-seeking will adopt irrigation strategy 60-N-N-75 on 50 ha, since it produces higher gross margins at higher probability levels, even though it has a 20% chance of obtaining crop yields which are less than that of strategy 30-N-N-75.

Table 2: Optimised irrigation area (ha) and deficit irrigation schedule combination under limited (L) and unlimited (U) water supply conditions for four absolute risk- aversion ranges (A,B,C,D)

Irrigation Strategy	**Absolute Risk-Aversion Coefficients (RAC)**			
	A **-0.0004** **<** **-0.00015**	**B** **-0.00015** **<** **-0.00005**	**C** **-0.00005** **<** **0**	**D** **0** **<** **0.0004**
Unlimited water supply (U)				
60-N-N-75	50	50		
N-15-N-75			50	50
Total area irrigated	50	50	50	50
Limited water supply (L)				
30-N-N-75	17.857	14.286	14.286	
45-N-N-75	32.143	32.143	32.143	25
60-N-N-75		3.571	3.571	
N-N-30-N				7.143
N-15-N-75				17.857
Total irrigated area	50	50	50	50

When water supplies were reduced proportionally, it is optimal to adopt more than one irrigation schedule while irrigating a total area of 50ha. In total, three distinct combinations of irrigation schedules (A, BC, D) were adopted over the range of risk-aversion parameters; two combinations (A, BC) for RACs smaller than zero and one for RACs greater than zero. It is interesting that risk-averse decision makers should adopt 7 ha of strategy N-N-30-N which has the lowest crop yield and the highest risk. This result highlights the importance of taking the opportunity cost of water into account when evaluating deficit irrigation strategies. The total gross margin is not only a function of applied water per hectare and the associated variability of crop yields, but how different irrigation schedules are combined to produce the desired outcome in accordance with the decision-maker's preferences.

To gain a better understanding of the gross margin risk associated with each of the optimised irrigation schedule combinations, Figure 2 shows the cumulative probability distributions of gross margins for the adopted irrigation schedules.

From Figure 2 it is clear that irrigation schedule combination L_D has an 80% chance of obtaining larger gross margins than any of the other irrigation schedule combinations under limited water supply conditions. Thus, the added area irrigated (7 ha) through the adoption of a relative risky irrigation schedule (N-N-30-N) proved to be of benefit when combined with relatively less risky irrigation schedules (N-15-N-75). When U_AB and U_CD are compared there is only a 4% chance that U_AB will realise a larger gross margin.

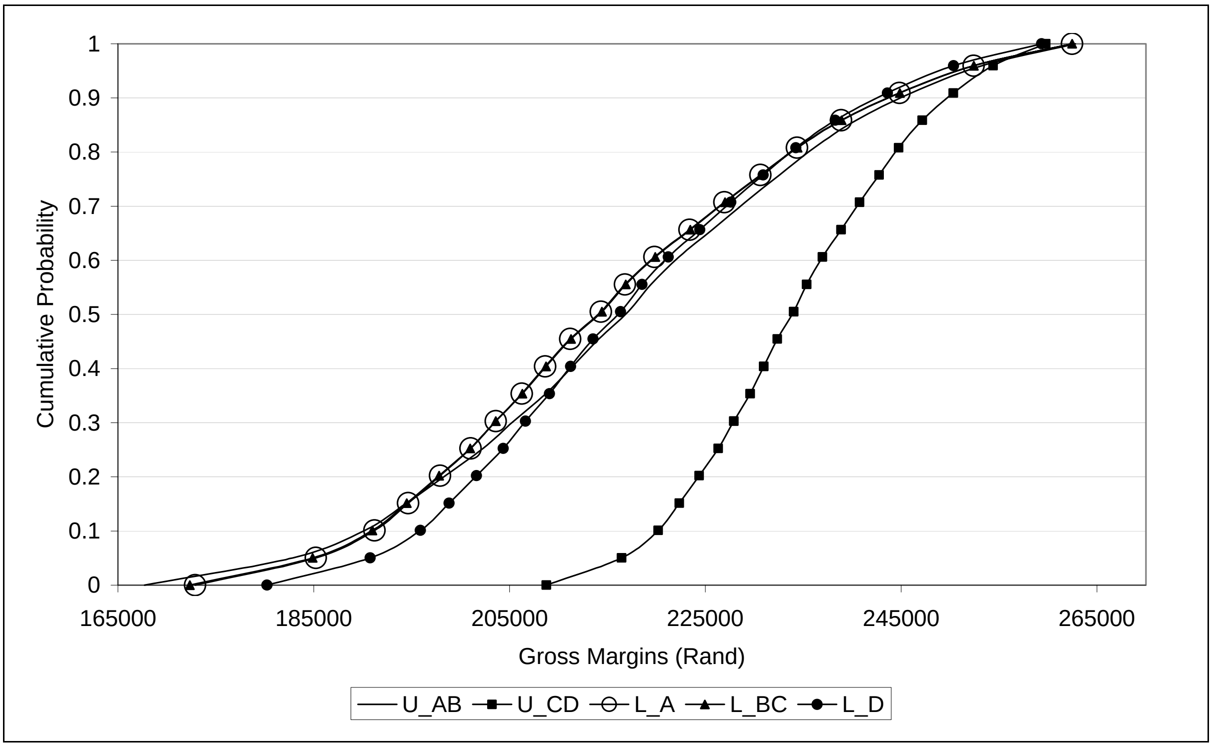

Figure 2: Cumulative probability distribution of gross margins for the optimised irrigation schedules

The fact that the cumulative probability distributions of gross margins overlap, implies that decision makers with varying risk preferences will choose differently between the alternatives and therefore will attach different values to security of supply. To gain insight into the amount of money (premium) a person with a specific risk attitude is willing to pay for a more secure water supply, the results of the stochastic efficiency with respect to function analyses are given in Figure 3. One should recall that the CE specifies the minimum amount of money a decision maker would require as a lump-sum payment to make him/her indifferent between the CE and the future payment of a risky alternative. Thus, the difference between two CEs will determine the premium for a specific decision maker. For instance, the premium attached to unlimited water supply conditions compared with limited water supply conditions for decision makers that are risk-averse is the difference in the CEs of U_CD and L_D at a specific RAC. At a RAC of 0.0001 the premium is approximately R25000. From Figure 3 it is clear that the premium increases as the level risk-averseness increases. More risk-averse people would thus be willing to pay more for security of water supply.

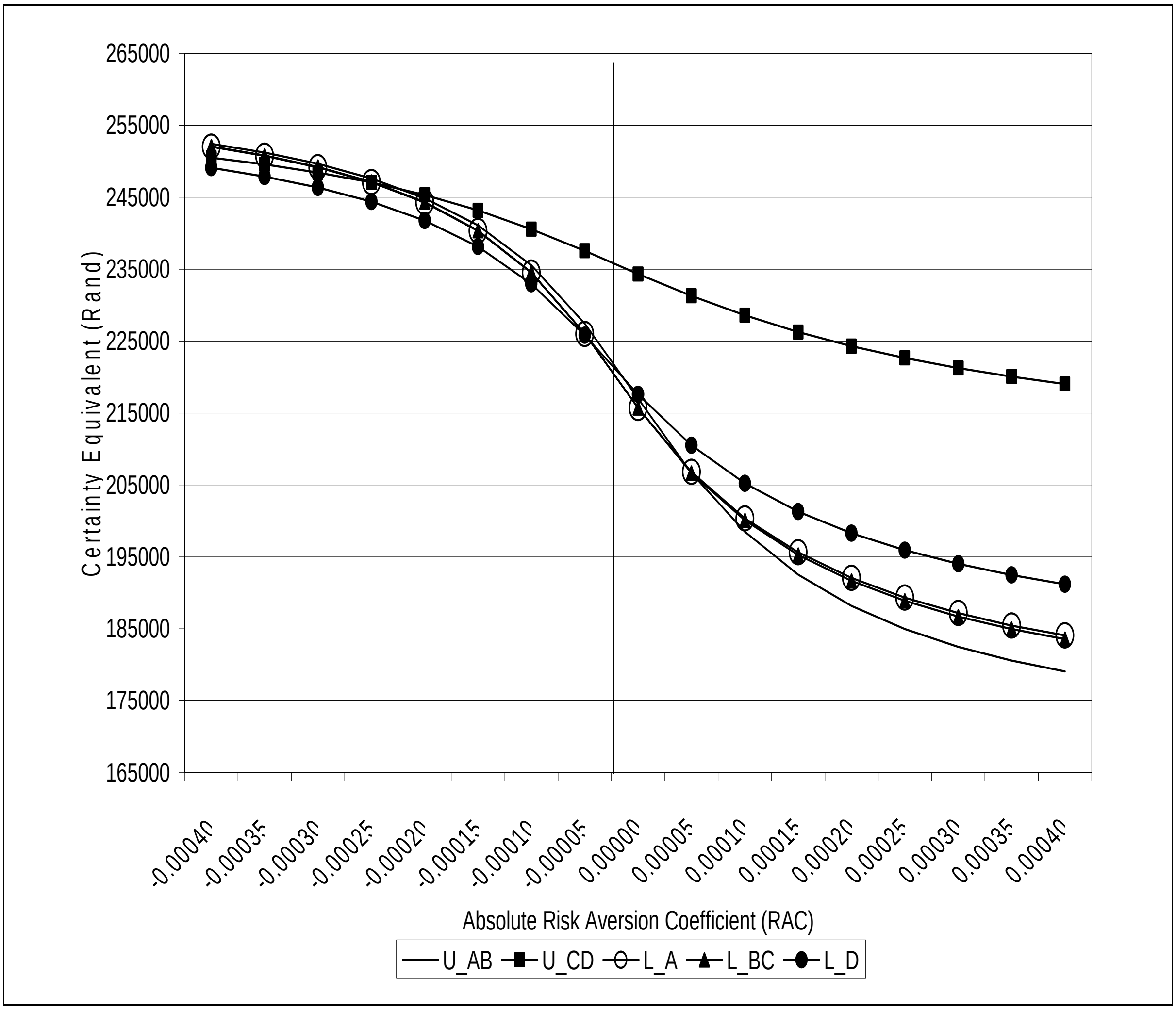

Figure 3: Certainty equivalents of the optimised combinations of alternative deficit irrigation schedules

5. Conclusions

The mathematical programming model proved to be valuable in optimising the relationship between areas irrigated, water supply conditions and the amount and timing of water applications. The results clearly showed the importance of resource availability with the adoption of irrigation schedules. Given limited water supply conditions during specified periods, the results showed that a risk-averse farmer should adopt irrigation schedule N-N-30-N which has a relatively high production risk associated with it. The conclusion is that risk should be managed by looking at the portfolio of available strategies and how resource availability influences the choice of the strategies in the portfolio. Implementing the first best combination of irrigation schedules may be troublesome under a single centre pivot system because the results imply irrigating different areas under the same pivot with different water applications at different times. First best strategies may, however, be used to benchmark the efficiency of more practical strategies that are easier to implement. Further research is necessary to determine the efficiency of second best alternative irrigation schedules.

The use of an economic instrument such as increased water use charges are allowed for, in the National Water Act (Act 36 of 1998) to achieve more efficient use of water. Ignoring risk (RAP = 0) may underestimate the value decision makers attach to the security of water supply.

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