Looking for Rational Bubbles in Agricultural Commodity Markets

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Looking for Rational Bubbles in Agricultural Commodity Markets

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The timing is difficult to call, as always, but...it is not so hard to guess where bubbles might be lurking.
K. Rogoff

Abstract

In this paper, we use a bootstrap methodology to helps us to compute the finite sample probability distribution of the asymptotic tests recently proposed in Phillips et al. (2009b) and Phillips and Yu (2009c). Simulation shows that the bootstrap methodology works well and allows us to identify explosive processes and collapsing bubbles. We apply the bootstrap procedure to the wheat and rough rice commodity prices. We find some evidence of price exuberance for both prices in the 2007-2008 period.

Keywords: Rational Speculative Bubbles, Bootstrap, Unit Root Tests, Commodity Prices.
JEL classification: G14, Q14, C12, C15

1Spotting the tell-tale signs of bubbles approaching, Financial Times, April 8th, 2010.
1 Introduction

Although they later fell back markedly, there was an upward trend in the international price of agricultural commodities in the years 2007-2008, something that had not been seen since the 1970s. For example, in February 2008 the price of wheat was three times that of January 2007. In February-March 2008 the wheat price started to decrease and now it is approximately at the same level as it was in January 2007. What caused the boom and burst in commodity prices, stock prices and house prices are currently highly debated issues. In addition, it is relevant to analyse the timeline of the origin and subsequent collapse of asset prices in order to validate, for example, the sequential hypotheses proposed by Caballero et al. (2008). In a recent paper, they argue that the Internet bubble in the 1990s, the asset bubbles in 2005-2006, the subprime crisis in 2007, and finally the commodity prices exuberance of 2008 are all closely related. Most economists and media commentators have expressed similar views of the interconnectedness of the crisis during the recent price explosions.

Among the potential explanations of explosive behavior in economic variables, the most prominent are, perhaps, models with rational bubbles. In this paper our analysis of explosive behavior is related to the “rational bubble” literature, where standard econometric tests have encountered difficulties in identifying rational asset bubbles, see Evans (1991). The econometric methods used here were originally proposed in Phillips et al. (2009b). Their methods rely on forward recursive and rolling regressions coupled with sequential right-sided unit root tests. Banerjee et al. (1992) have shown that recursive and rolling regression-based DF statistics are important tools for the detection of the unit root properties of the time series. The sequential tests allow us to assess period by period the possible nonstationarity behavior of prices against mildly explosive alternatives, and recently Phillips and Magdalinos (2009a) have provided the large sample asymptotic theory of mildly explosive process.

While the previous strategy is based on asymptotic theory, we propose a bootstrap econometric methodology to test whether or not, and when, bubbles emerged and collapsed in the commodity markets. Advances in computing have made the bootstrap methodology increasingly attractive. The approach generates a large number of simulated values of a generic test statistic and compare it with the empirical distribution function of the simulated ones. Thus the bootstrap approach replaces the asymptotic sampling distribution with an exact distribution that acts as if the empirical distribution of the sample is the population distribution. In synthesis, an analysis of a large amount of literature shows that first bootstrap, if applied appropriately, helps to compute the critical values of asymptotic tests more accurately especially in finite samples, and secondly that the tests based on the bootstrap critical values generally have actual finite sample rejection probabilities closer to their asymptotic nominal values, see for example Hall (1992) and Horowitz (2001).

We apply our econometric approach to two daily agricultural commodity prices observed over the sample period from 1985 to 2010. To be more precise, we investigate the bubble characteristics of the wheat and rough rice commodity prices. Using the forward recursive and rolling regression techniques, we are able to date the origin and conclusion of the explosive behavior. The statistical evidence from these methods indicates that the rational bubble of wheat prices started in September 2007 and continued until March 2008 when the wheat bubble burst. Similar signs of price exuberance are noted for the rough rice commodity price. In this case, the bubble started in February, 2008 and the collapse in May, 2008.

The paper is organized as follows. Section 2 presents the model. Section 3 describes the bootstrap procedure and the Monte-Carlo experiment used to check for the robustness of the procedure. Section 4 presents the dataset and empirical results for wheat and rough rice commodity prices. Finally, section 5 concludes.

2 The model

The main aim of the paper is to detect periods of exuberance in agricultural commodity prices, i.e. periods in which commodity prices rose far higher than could be easily explained by the fundamental values.
These periods are fully consistent with the present value model of rational commodity pricing, where the real commodity price $P_t$ equals the sum of the expected future price $P_{t+1}$ and expected future payoff $\psi_{t+1}$, or benefit, from ownership of the asset, both discounted at the constant rate $R$, i.e.

$$P_t = \frac{E_t(P_{t+1} + \psi_{t+1})}{1 + R}.$$  (1)

For a storable commodity, the variable $\psi_t$ is usually defined as convenience yield. The convenience yield is a well known concept. Early authors [Kaldor(1939); Working(1949)], in an effort to explain the often observed phenomenon of spot prices being higher than futures prices (see Garcia and Leuthold (2004) for a survey on this and other themes connected to the agricultural future markets), defined the convenience yield as a negative component of the carrying charges. In this case the convenience yield is a benefit which accrues to inventory holders from the increased utility associated with availability in periods when supplies are scarce. Pindyck (1993) used equation (1), and the above mentioned convenience yield definition, to explain the pricing of storable commodities. If aggregate storage is always positive, as is usually for the agricultural commodities, equation (1) holds and the present value model provides a simple explanation for changes in the price of a commodity and price exuberance. In order to explain how the present value model can be usefully used to explain price exuberance, we follow Campbell et al. (1998) and log-linearize (1). Using the law of iterated expectations, we end up with the following solution of the difference equation (1)

$$p_t = p_t^f + b_t,$$  (2)

where

$$p_t^f = \frac{\kappa - \gamma}{1 - \rho} + (1 - \rho)\sum_{i=0}^{\infty} \rho^i E_t\psi_{t+1+i},$$  (3)

$$b_t = \lim_{i \to \infty} \rho^i E_t p_{t+i},$$

$$E_t(b_{t+1}) = \frac{b_t}{\rho} = (1 + \exp(\psi - p)) b_t$$  (4)

and $p_t = \ln(P_t), \ \psi_t = \ln(\psi_t), \ \gamma = \ln(1 + R), \ \rho = 1/(1 + \exp(\psi - p))$, with $\psi - p$ being the average convenience yield-price ratio, $0 < \rho < 1$, and finally

$$\kappa = -\ln(\rho) - (1 - \rho) \ln\left(\frac{1}{\rho} - 1\right).$$  (5)

Note from (2) that when $b_t = 0$, i.e. $b_t = \lim_{i \to \infty} \rho^i E_t p_{t+i} = 0$, the price $p_t$ is fully determined by the fundamental price $p_t^f$ or, in other words, by the discounted expected future convenience yield $\psi_t$, see (3). If $b_t \neq 0$, there are an infinite number of solutions to (1) and any solution can be written as (2). Campbell et al. (1998) call the term $b_t$ "rational" bubble. The adjective "rational" is used because the presence of $b_t$ in (2) is fully consistent with rational expectations and constant expected returns. The word "bubble" recalls financial exuberance episodes in which investors appeared to be betting that other investors drive prices even higher in the future, far higher than explained by fundamentals. In this case from (4) we can always write

\footnote{That is imposing the transversality condition.}
\[ b_t = \frac{1}{\rho} b_{t-1} + \varepsilon_t = (1 + g) b_{t-1} + \varepsilon_t \]

where \( g = \frac{1}{\rho} - 1 = \exp(\psi - p) > 0 \) is the growth rate of the natural logarithm of the bubble and \( \varepsilon_t \) is a random error. Thus as \( \exp(\psi - p) > 0 \), \( b_t \) will be explosive as well as the price \( p_t \), irrespective of whether the convenience yield \( \psi_t \) is or is not a stationary variable. Moreover, also \( \Delta p_t \) will be explosive. This finding explains the way followed by Diba and Grossman (1988) to test for explosive rational bubbles. They propose investigating the stationary properties of the prices series \( \Delta p_t \). Specifically, they use standard unit root tests, and find that \( \Delta p_t \) is a stationary variable, i.e. \( \text{I}(0) \), and thus \( p_t \) is an \( \text{I}(1) \) variable, ruling out the presence of possible explosive rational bubbles. Unfortunately Evans (1991) has shown that when applied to periodically collapsing rational bubbles, Diba and Grossman (1988) procedure may, and probability will, lead to the incorrect conclusion that bubbles are not present. This is because, unless the probability that the bubble does not collapse is very high, \( b_t \) there will appear to be a stable linear autoregressive process. As a result, the unit root test procedure will erroneously come to the conclusion that bubbles are not present. In synthesis, in case of periodically collapsing bubbles, the unit root test statistics will incorrectly reject the null hypothesis of nonstationarity of \( \Delta p_t \).

### 3 A bootstrap procedure

Recently in a series of seminal papers Phillips and Yu (2009b) and Phillips et al. (2009c) have suggested an interesting methodology that permits first to test for explosive bubbles and secondly to date the origin and collapse of bubbles. The method is mainly based on the recursive and rolling implementation of right-side unit root tests. In practice, the method requires one to estimate the following augmented Dickey-Fuller (ADF) regression

\[ y_t = c + \rho y_{t-1} + \sum_{i=1}^{k-1} \beta_i \Delta y_{t-i} + \varepsilon_t, \quad t = 1, \ldots, T \]

where in our case \( y_t \) is the log of the agricultural commodity price \( p_t \) (or as we will see the log of the convenience yield \( \psi_t \)), \( c \) is a constant, \( k \) is a lag parameter, and \( \varepsilon_t \) is an identically and independently distributed error term. Equation (7) is estimated repeatedly, using subsets of the sample data. Recursive ADF test statistics are computed using subsamples \( t = 1, \ldots, m \) for \( m = m_0, \ldots, T \), where \( m_0 \) is a start-up value and \( T \) is the previously defined size of the full sample. For the rolling method, the ADF statistics are computed using subsamples that are a constant fraction \( \delta_0 \) of the full sample, rolling through the sample. Both recursive and rolling statistics are aggregate in order to construct the following single test statistic

\[ SADF_1 = \sup_{m \in \delta_0, \delta \leq 1} ADF_m, \quad 0 < \delta_0 \leq \delta \leq 1 \]

for the recursive case, and the following test statistic for the rolling case

\[ SADF_2 = \sup_{m \in T(\delta - \delta_0) + 1, \delta \leq 1} ADF_m, \quad 0 < \delta_0 \leq \delta \leq 1. \]

Phillips and Yu (2009b) and Phillips et al. (2009c) suggest to compare the estimates of (8), for the recursive method, or (9) if we are using the rolling method, with the right tailed critical values of their distributions to
test for a unit root against explosive bubbles. Thus, if the estimated test statistics \( \hat{SADF}_1 \) or \( \hat{SADF}_2 \) are higher than the right-side 100\( \alpha \)% critical value \( c_{v_{ADF}}^\alpha \) they reject the null hypothesis of a unit root \( H_0 : \rho = 1 \) for the alternative hypothesis an explosive process \( H_A : \rho > 1 \).

Moreover, if we are able from the previous tests to reject the null hypothesis for the alternative hypothesis of a mildly explosive process, Phillips and Yu (2009b) and Phillips et al. (2009c) also propose a method that allows to date the origin, \( \tau^o_s \), and the collapse, \( \tau^c_s \), of a bubble. The origin is estimated as

\[
\hat{\tau}^o_s = \inf_{s \geq m} s : ADF_m > c_{v_{ADF}}^\alpha
\]  

and the collapse of the bubble is estimated as

\[
\hat{\tau}^c_s = \inf_{s \geq m} s : ADF_m < c_{v_{ADF}}^\alpha
\]  

where \( c_{v_{ADF}}^\alpha \) is the right-side 100\( \alpha \)% critical value of the ADF statistic based on \( \tau = [T \delta] \) observations in the case of recursive case and \( \tau = [T \delta_o] \) observations in the case of rolling method.

While Phillips and Yu (2009b) and Phillips et al. (2009c) derive the asymptotic distribution of (8) and (9) and obtain the asymptotic critical values by a Monte-Carlo simulation, we use bootstrap methods to make inference about the small sample distribution of the test statistics. The basic bootstrap idea is to consider the residuals \( \varepsilon_t \) from fitted model (7) as "approximately independent" and then resample the residuals (with a suitable centering adjustment) to define the bootstrap observations through the structural equation (7). The bootstrap observations can then be used to compute the distribution of (8) and (9) and clearly the ADF test as well as their quantiles.

If the process is unstable, i.e. we have an autoregressive process that shows unit roots, it is now well known that the OLS estimator \( \hat{\rho} \) of \( \rho \) is a function of the Wiener process and the ADF statistic has the following limit distribution for \( T \to \infty \)

\[
I_{\hat{\rho}} \Rightarrow \frac{\int_1^T WdW}{(\int_1^T W^2)^{1/2}},
\]  

where \( \Rightarrow \) indicates converge in distribution (or in law) and \( W \) is a standard Wiener process (see for example Hamilton, 1994) and \( r_1 \) and \( r_2 \) are here the two extremes. In the case of recursive method \( r_1 = 0 \) and \( r_2 = \delta \) and for the rolling method \( r_1 = (\delta - \delta_0) \) and \( r_2 = \delta \).

By the way the asymptotic distribution of (8) and (9) will be given by

\[
SADF_i \Rightarrow \sup_{r_1, r_2} \frac{\int_1^T WdW}{(\int_1^T W^2)^{1/2}}, \quad i = 1, 2.
\]  

To construct the bootstrap tests corresponding to (12) and (13), we use a sieve bootstrap method for two reasons. The first is that the sieve bootstrap approximates the general linear process by a finite autoregressive process of order \( k \) increasing with the sample size, and resampling from the approximated autoregressions. It is called as such by Buhlmann (1997), since this method is based on \( T \to \infty \) an approximation of an infinite dimensional and nonparametric model by a sequence of finite dimensional parametric models. The second one is that the sieve bootstrap has proved to perform better than other
bootstrap methods, such as block-bootstrap methods, see Palm et al. (2008).

In order to implement the bootstrap methodology, we follow Chang and Park (2003) and assume that the time series $y_t$ is generated by the following DGP

$$y_t = c + v_t, \quad v_t = \rho v_{t-1} + u_t$$  \hspace{1cm} (14)$$

where the process $u_t$ to follow a stable ARMA process. The bootstrap sample can be obtained from the following steps

- **Step 1:** Fit the following autoregressive process by OLS,

$$\Delta y_t = \alpha_0 + \sum_{i=1}^{k} \alpha_i \Delta y_{t-i} + \varepsilon_t$$  \hspace{1cm} (15)$$

- **Step 2:** Denote by $\hat{\alpha}_k$ the OLS estimator of $\alpha_k$ and $\hat{\varepsilon}_t$ the OLS residuals in regression (14). Now resample $\hat{\varepsilon}_t^*$ from the centered residuals $\hat{\varepsilon}_t - \bar{\varepsilon}$, where $\bar{\varepsilon} = \sum_{t=1}^{T} \hat{\varepsilon}_t / T$.

- **Step 3:** Generate $y_t^*$ using the OLS estimates $\hat{\alpha}_k$ and the residuals $\hat{\varepsilon}_t^*$, i.e.

$$y_t^* = y_{t-1}^* + \sum_{i=1}^{k} \alpha_i \Delta y_{t-i}^* + \varepsilon_t^*.$$  \hspace{1cm} (16)$$

- **Step 4:** Using $y_t^*$ compute the statistics (8) or (9) and repeat $B$ times from Step 2 to Step 4 in order to compute the bootstrap distribution of the test statistics. Moreover, $y_t^*$ can be also used to compute bootstrap distribution of the ADF test and the quantiles of (10) and (11).

The above mentioned bootstrap procedure needs some remarks. First Chang and Park (2003) and Palm et al. (2008) show that the asymptotic distribution of the ADF bootstrap statistics under the null is the same as the asymptotic distribution of the original ADF statistics. Thus given that $\sup \cdot$ in (13) is a continuous functional and applying the continuous mapping theorem, the previous result intuitively will hold for the bootstrap $SADF_t$ test statistics, Hamilton (1994). Second, we impose the null hypothesis of a unit root in (14) because Basawa et al. (1991) have shown that the unit root must be imposed for the generation of bootstrap sample to achieve consistency. Third, the autoregression (17) must be initialized to obtain the bootstrap sample. We fix the first $k+1$ of $y_t^*$ to zero in order to avoid possible nonstationary autoregressive process. Finally, in order to determine the lag order $k$ in (16) following Assumptions 2 and 3 we fix $k = 5T^{1/4}$.

### 3.1 A Monte-Carlo simulation

Before presenting the empirical results it is useful to check how the bootstrap procedure performs compared to the asymptotic analysis of Phillips et al. (2009c). We present the same Monte-Carlo experiment as in Phillips et al. (2009c) in order to compare the two methods. They propose the following model to capture both exuberance and collapse of the time series

$$y_t = y_{t-1}^1 \quad t < \tau^o + \rho y_{t-1} \quad \tau^o \leq t \leq \tau^c + \left( \sum_{k=\tau^c+1}^{T} \varepsilon_k + y_{\tau^c}^* \right) \quad 1 \quad t > \tau^c + \varepsilon_t \quad t < \tau^c$$  \hspace{1cm} (17)$$
where $1\{\cdot\}$ is the indicator function, $\tau^o$ and $\tau^c$ are respectively the origin of the exuberance and the collapse date, $y^*_c$ is the new initial value of the variable after the collapse and $\varepsilon_i$ is a pseudorandom normal variate generated using the GAUSS subroutine RNDN (the simulation program is available on request). As in Phillips et al. (2009c), we compute 1,000 sample path replications, setting $\tau^o = 0.4$ and $\tau^c = 0.6$ and $\rho = 1.035, 1.040, 1.045, 1.050$ and the number of bootstrap replications have set to $B = 999$.  For reasons of space, we present the results only for $T = 1000$ and the rolling method, further results can be found in Gutierrez (2010). The asymptotic theory required to obtain the critical values of Phillips et al. (2009c) test is very complicated. In their paper, they propose to use an (arbitrary) expansion rule and fix critical value for the ADF test to $(2/3) \log(\log^2(i))$ which increases slowly with the sample size $T$. Phillips et al. (2009c) suggest to use the previous expansion because it produce a significance level close to the 1% level of the ADF test statistic and they say that a conservative 1% rule seems acceptable because reduces the risk of choosing the explosive alternative when it is not true. Thus we use the 1% quantile (on the right-side of the bootstrapped distribution of the ADF test) in the Monte-Carlo simulation. Finally, as in Phillips et al. (2009c), we impose the (small infinity) duration condition that $\hat{\tau}^c - \hat{\tau}^o \geq \ln(t)/t$. In the following table we report the mean, standard errors and root mean square errors (RMSE) computed using the bootstrap and Phillips et al. (2009c) methodologies.

Some interesting results emerge. First both methods seem to work fine in detecting $\tau^o$ and $\tau^c$ and the true values are inside the 2 standard deviations of the estimated values. Second, the $\tau^c$ is estimated better than $\tau^o$. Third, when the explosive behavior is stronger, it is easier to estimate $\tau^o$ and in this case both the bias and the standard error become smaller while minor changes are seen for $\tau^c$. Finally the bootstrap methodology seems to work slightly better than the asymptotic methodology for this number of observations. However the bootstrap procedure provides better estimates and standard deviations for smaller values of $T$ while the differences between the two methodologies become negligible for larger values of $T$, see Gutierrez (2010).

| Table 1: Monte-Carlo estimates of $\hat{\tau}^o$ and $\hat{\tau}^c$ based on ADF test critical values. Rolling method with 1,000 observations. (a) |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| Bootstrap simulation (b) |                |                |                |                |
| $\rho = 1.035$ | $\rho = 1.040$ | $\rho = 1.045$ | $\rho = 1.050$ |
| $\hat{\tau}^o$ | 0.4346          | 0.4307          | 0.4280          | 0.4248          |
| $\hat{\tau}^c$ | 0.6004          | 0.6011          | 0.6010          | 0.6006          |
| Mean            | 0.4346          | 0.4307          | 0.4280          | 0.4248          |
| Std             | 0.0237          | 0.0040          | 0.0197          | 0.0180          |
| RMSE            | 0.0419          | 0.0040          | 0.0342          | 0.0307          |
| Asymptotic simulation |
| $\rho = 1.035$ | $\rho = 1.040$ | $\rho = 1.045$ | $\rho = 1.050$ |
| $\hat{\tau}^o$ | 0.0152          | 0.0035          | 0.0035          | 0.0080          |
| $\hat{\tau}^c$ | 0.0152          | 0.0040          | 0.0035          | 0.0080          |
| Mean            | 0.0152          | 0.0035          | 0.0035          | 0.0080          |
| Std             | 0.0040          | 0.0035          | 0.0035          | 0.0080          |

3 Setting $B=999$ is a common choice in bootstrap procedures. The results are similar for a larger number of iterations as $B=1999$.
4 These are approximately the same number of observations used in the empirical analysis.
5 We do not report for brevity the same statistics for the recursive case. They are similar to the rolling case.
6 Of course the bootstrap procedure is more time-consuming than Phillips et al. (2009c) method. However, the additional computation requirements are quite modest.
<table>
<thead>
<tr>
<th></th>
<th>0.4381</th>
<th>0.6006</th>
<th>0.4335</th>
<th>0.6008</th>
<th>0.4298</th>
<th>0.6007</th>
<th>0.4262</th>
<th>0.6007</th>
</tr>
</thead>
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<tr>
<td>Mean</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Std</td>
<td>0.0268</td>
<td>0.0122</td>
<td>0.0245</td>
<td>0.0104</td>
<td>0.0218</td>
<td>0.0103</td>
<td>0.0186</td>
<td>0.0104</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.0465</td>
<td>0.0122</td>
<td>0.0415</td>
<td>0.0104</td>
<td>0.0369</td>
<td>0.0104</td>
<td>0.0321</td>
<td>0.0104</td>
</tr>
</tbody>
</table>

(a) Based on 1,000 iterations with \( \tau^* = 0.4 \) and \( \tau^* = 0.6 \) as true values.
(b) Based on 999 iterations.

4 Testing and dating bubbles

Our data are collected from Bloomberg database. We focus on wheat and rough rice futures prices. For wheat, we collect the generic commodity contracts (Bloomberg ticker symbols: W1 and W3) for which we have daily data from March 1985 until April 2010. The US consumer price index (CPI) has been used to convert nominal series to real series (the source is the IMF database). We use the first nearby future price as a proxy for cash prices in commodity markets. This choices is not new and many authors, see Fama and French (1987), Gibson and Schwartz (1990) and Bessembinder et al. (1995) among others, caution that there is no true spot market for some commodity markets because of the delay in delivery. As a result, we measure the spot price as the nearby future price (in our case the price with ticker symbol W1) and the convenience yield as the difference between this price and the actualized futures contract prices, which mature three months apart. Specifically we measure the convenience yield as

\[
\Psi_t = P_t - F_t e^{-i(t,T)(T-t)/365}
\]

where the second term on the right of (19) is the discounted futures price, \( t \) is time of observation, \( T \) here is the time of expiration, \( i(t,T) \) is the value of the 3-months US Treasury Bond rate over the interval and \( (T-t) \) the number of days until expiration. This discounting procedure is similar to that proposed in Brennan (1986).

Table 2 summarizes the descriptive statistics for the time series analyzed, including sample size, sample frequency, sample minimum, date of the minimum, sample maximum, date of the maximum.

### Table 2: Summary statistics

<table>
<thead>
<tr>
<th>Data</th>
<th>Ticker symbol (a)</th>
<th>Sample Size</th>
<th>Freq</th>
<th>Min</th>
<th>Date (min)</th>
<th>Max</th>
<th>Date (max)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheat (US cents/bu)</td>
<td>W1</td>
<td>6321</td>
<td>Daily</td>
<td>280.0</td>
<td>13/12/1999</td>
<td>1280.0</td>
<td>27/02/2008</td>
</tr>
<tr>
<td>Rough Rice (US cents/cwt)</td>
<td>RR1</td>
<td>5366</td>
<td>Daily</td>
<td>3.43</td>
<td>06/05/2002</td>
<td>24.5</td>
<td>23/04/2008</td>
</tr>
</tbody>
</table>

(a) Bloomberg ticker symbol.

The maximum value for the wheat and rough rice prices was found in the first semester of 2008. The wheat price reached its maximum of 1280 US cents per bushel at end of February 2008 and the rough rice price reached its maximum of 24.5 cents per hundredweight in April 2008. This means that these commodity prices rose considerably in 2008 relatively to the minimum values observed during the period of analysis. Wheat prices which are free from existing carrying costs between period \( t \) and \( T \).
increased by 357% and rice of 614%. Looking at the last value available in the dataset (April, 12 2010), the commodity prices are now well below the values picked in the first half of 2008. Wheat are priced 467 US cents per bushel, which means a decrease of 64% relatively to the maximum value, and rough rice 13.12 US cents per hundredweight (-46%).

In Table 3 we present the test values and the right-side critical values obtained from the boostrapped distribution of the test statistics. They all refer to the forward rolling regression method. For the wheat prices, the initial start-up sample for the rolling regression covers the period from March 1985 to March 1990 for a total amount of 1265 daily observations. For the rough rice prices, the start-up sample for the rolling regression covers the period December 1988 to June 1993, with 1060 daily observations. We fix \( B = 999 \) in STEP 4 of the bootstrap procedure. In order to compare the bootstrap and asymptotic quantiles, the latter have been also included in Table 3.

Several conclusions emerge from the table. First, if we were to follow the convention and apply the \( ADF \) test to the full sample (March 1985 to April 2010 for wheat price and December 1988 to April 2010 for rough rice), the tests could not reject the null hypothesis \( H_0 : \rho = 1 \) in favor of the right-tailed alternative hypothesis \( H_A : \rho > 1 \), and therefore one would conclude that there was no significant evidence of exuberance in the agricultural commodity prices. This result is consistent with Diba and Grossman (1988), as well as with the recent findings of Phillips and Yu (2009b) and Phillips et al. (2009c) for a set of stock price indexes, house price indexes and commodity prices. Moreover, the results are subject to the criticism leveled by Evans (1991) because standard unit root tests for the full sample naturally have difficulty in detecting periodically collapsing bubble. Second, and differently from the previous results, the \( SADF_2 \) tests provides some evidences of explosiveness in the price data but only at the 10 percent level for wheat and 5 percent level for rough rice.

![Table 3: Test statistics and bootstrapped critical values: Commodity Prices](image)

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Statistics(a)</th>
<th>Test value</th>
<th>Critical values (b)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ADF</td>
<td></td>
<td>1%</td>
</tr>
<tr>
<td>Wheat</td>
<td>-2.560</td>
<td>-3.475</td>
<td>-2.875</td>
</tr>
<tr>
<td></td>
<td>(-3.458)</td>
<td>(-2.868)</td>
<td>(-2.577)</td>
</tr>
<tr>
<td></td>
<td>SADF_2</td>
<td>1.725</td>
<td>-0.567</td>
</tr>
<tr>
<td></td>
<td>(-0.624)</td>
<td>(-0.251)</td>
<td>(-0.067)</td>
</tr>
<tr>
<td>Rough rice</td>
<td>ADF</td>
<td>-2.069</td>
<td>-3.562</td>
</tr>
<tr>
<td></td>
<td>(-3.458)</td>
<td>(-2.868)</td>
<td>(-2.577)</td>
</tr>
<tr>
<td></td>
<td>SADF_2</td>
<td>2.172</td>
<td>-0.431</td>
</tr>
<tr>
<td></td>
<td>(-0.624)</td>
<td>(-0.251)</td>
<td>(-0.067)</td>
</tr>
</tbody>
</table>

(a) Forward rolling regression method.

---

10 All the previous dates refers to the business day contracts as reported in Bloomberg commodity prices database.

11 The asymptotic critical values are obtained by Monte-Carlo simulation with 10,000 replications.

12 Similar values, not reported for brevity, have been obtained using the recursive method.
As we have seen the $SADF_2$ tests cannot reveal the location of the exuberance. To locate the possible origin and the conclusion of exuberance we use the procedure highlighted in (10) for origin and (10) for the conclusion of exuberance. The wheat price shows two periods of exuberance. The test detects the origin of the bubble on September 27 2007, with a peak on February 27 2008 (this date coincides with the maximum value of the wheat price). The bubble finally collapsed on March 27 2008. The total duration of the wheat price exuberance was 142 trading days. \(^\text{13}\) For rough rice the origin of price exuberance can be dated to February 25 2008, thus later than the previous bubble, reaching a peak on April 23 2008, and than collapsing on May 28 2008 for a total number of 66 trading days exuberance of rice prices. \(^\text{14}\) These results are summarized in the first half of Table 4. In the second half of Table 4, we locate the origin and collapse of price exuberance using the asymptotic method proposed in Phillips et al. (2009c). For wheat price this method provide approximatively the same location of the exuberance as the bootstrap methodology. However for rough rice the exuberance period is much more larger, and embraces 103 trading days.

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Origin</th>
<th>Peak</th>
<th>Collapse</th>
<th>Duration (b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheat</td>
<td>September 4 2007</td>
<td>February 27 2008</td>
<td>March 27 2008</td>
<td>142</td>
</tr>
<tr>
<td>Rough Rice</td>
<td>February 25 2008</td>
<td>April 23 2008</td>
<td>May 28 2008</td>
<td>66</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Origin</th>
<th>Peak</th>
<th>Collapse</th>
<th>Duration (b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheat</td>
<td>August 30 2007</td>
<td>February 27 2008</td>
<td>April 4 2008</td>
<td>150</td>
</tr>
<tr>
<td>Rough Rice</td>
<td>February 2 2008</td>
<td>April 23 2008</td>
<td>July 7 2008</td>
<td>103</td>
</tr>
</tbody>
</table>

(a) Based on 1% critical values.
(b) Number of trading days

Of course, explosive characteristics in $p_t$ could in principle arise from explosive behavior in the convenience yield $\psi_t$. This is why we analyze if the convenience yield, measured using (19), has shown exuberance during the period of analysis. Table 5 presents the results.

Interestingly, the results are quite different from the previous price statistics. First of all, looking at the ADF test statistics and their critical values, we note that the convenience yield for both price series is a stationary variable (the values of the test statistics are below the 1% critical values of the ADF test). This means the the difference between the spot and the discounted future price is a stationary, i.e. I(0), variable or, in other words, that the two prices may be cointegrated. This result is agree with those of many authors, see for

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\(^\text{13}\) Actually from the figure can be noted a collapse of bubble prior March 27th, 2008, precisely on October 23th, 2007 and a new exuberance period starting from December 7th, 2007.

\(^\text{14}\) As before, the previous dates refers to the business day contracts as reported in Bloomberg commodity prices database.
example Pindyck (1993) and more recently Gospodinov and Ng (2010) who report stationarity of convenience yield for a large set of storable commodity markets. Moreover, from Table 5 it also emerges that the convenience yield series are non-explosive, which is different from the results obtained for the price series. In fact the $SADF_2$ tests statistics strongly reject the alternative hypothesis of $H_A: \rho > 1$. Thus, because the convenience yield is not explosive, from (3) we can conclude that the fundamental price is also not explosive and therefore, from (2), the rational bubble $h_t$ must be explosive. Thus, under the assumption of constant discount rate, the results give evidences of the existence of exuberance or bubble activity for wheat and rough-rice commodity prices.

<table>
<thead>
<tr>
<th>Commodity</th>
<th>Statistics(a)</th>
<th>Test value</th>
<th>Critical values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$ADF$</td>
<td>-5.028</td>
<td>-3.343</td>
</tr>
<tr>
<td></td>
<td>$SADF_2$</td>
<td>0.774</td>
<td>-0.553</td>
</tr>
<tr>
<td>Wheat</td>
<td>$ADF$</td>
<td>-6.345</td>
<td>-3.634</td>
</tr>
<tr>
<td></td>
<td>$SADF_2$</td>
<td>-0.684</td>
<td>-0.622</td>
</tr>
<tr>
<td>Rough rice</td>
<td>$ADF$</td>
<td>-6.345</td>
<td>-3.634</td>
</tr>
<tr>
<td></td>
<td>$SADF_2$</td>
<td>-0.684</td>
<td>-0.622</td>
</tr>
</tbody>
</table>

(a) Forward rolling regression method.

5 Conclusion
In this paper we analyze the recently boom and burst of agricultural commodity prices. We propose a bootstrap method which helps to compute the finite sample probability distribution of the asymptotic tests recently proposed in Phillips and Yu (2009b) and Phillips et al. (2009c). Simulation shows that the bootstrap methodology works well and allows one to identify explosive processes and collapsing bubbles. We apply the methodology to the wheat commodity prices observed daily from March 1985 to April 2010 and the rough rice commodity prices observed daily from December 1988 to April 2010. Using the bootstrap method we are able to provide some evidence of explosiveness in the price data for both prices. For wheat price, the test detects the origin of the bubble in September 27, 2007, with a peak in February 27, 2008. The bubble finally collapsed in March 27, 2008 and the total duration of the wheat price exuberance was 142 trading days. For the rough rice price, the price exuberance started on February 25, 2008, thus later than that of wheat, reached its peak on April 23, 2008, and then collapsed on May 28, 2008, for a total number of 66 trading days of price exuberance. All the results in this paper have been obtained under the hypothesis of a constant discount rate. Further research will be directed to the analysis of models that allow for a time-varying discount rate.

References


