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Price expectations play a critical role in commodity markets where producers must make input decisions well before output is realized. This paper brings together alternative expectations regimes, their estimation, and hypothesis tests for use in structural commodity models to determine their use by commodity producers. Extrapolative mechanisms and rational expectations are considered under risk neutrality and risk aversion. The assumptions implicit in the use of aggregate data in these models are made explicit. Structural models using individual survey data are discussed. While Muth's rational expectations hypothesis has found widespread acceptance in the macroeconomic literature, empirical results from industry studies indicate that commodity producers may have heterogeneous price expectations, with no single expectations hypothesis dominating. This is not surprising given that different producers possess different information and have different costs associated with information collection and processing.

Commodity production typically involves a time lag between input application and output realization. As a result, producers must base production decisions on known input prices and their output price forecasts. Economists have hypothesized alternative price expectation regimes, mainly naive expectations, adaptive expectations, quasi-rational, and rational expectations. However, since Muth's seminal paper (1961), economists have devoted increasing attention to developing econometric models compatible with his rational expectations hypothesis. Muth's hypothesis is appealing because it treats information like any other input in a firm's production process: producers are hypothesized to use available information efficiently in forming their predictions of future prices. This hypothesis has important implications. First, producers' predictions of future prices should be unbiased; otherwise, there would be a systematic prediction error, which at least some astute producers should be able to profitably exploit. Second, price prediction errors should be uncorrelated with the information set available at the time of the forecast. If the prediction error is correlated with any variable in the information set, the forecaster has not made efficient use of all available information.

While the rational expectations hypothesis has obvious appeal from an economist's perspective, it is not without fault. The rational expectations hypothesis implicitly assumes information is scarce yet costless to obtain and process (Feige and Pearce 1976; Arrow 1978; Grossman and Stiglitz 1976). When information collection and processing is costly, producers' optimal forecasts may involve simplistic rules, resulting in possibly biased and inefficient forecasts of future prices. With positive information costs, any number of expectations regimes may reflect the true underlying price forecasting model used by producers. Indeed, when the cost of forecasting is positive, rational utility maximizing agents will choose to use a simpler expectations mechanism, like naive expectations, if the losses incurred due to the inaccuracies of the expectations are less than the expected net benefit from a more accurate, but costly, expectations mechanism (Evans and Ramey 1992).
This paper reviews alternative expectations regimes, their estimation, and the hypothesis tests that have been applied to determine their acceptance among producers. We distinguish between the usual case where producers’ expectations are not directly observable and the case where expectations have been revealed through survey panel data. Empirical results from industry studies with implicit price expectations indicate commodity producers may have heterogeneous price expectations. This is not surprising given that different producers possess different information and have different costs associated with information collection and processing. To investigate the possibility of heterogeneous expectations, we turn to studies that use panel survey data of individual’s price expectations. Results from these studies are also inconclusive. For the most part, panel survey studies have focused exclusively on testing the rational expectations hypothesis. The limited focus of these studies results, in part, from the kinds of hypothesis tests that are available using only survey data. In the future, an alternative hypothesis test strategy that combines observed firm level data with survey data could be used to distinguish producers’ adherence to alternative price expectations mechanisms.

A Simple Aggregate Model with Unobserved Extrapolative Expectations Models

In this section, we specify a simple model of farm supply and processor demand for an agricultural commodity. While we model an agricultural commodity, the methods could be applied to any resource or product that involves a time lag between input decisions and output realization. We assume farmers’ supply decisions are made under uncertainty while processors decisions are not. In principle, any model should capture farmers’ attitudes toward risk regardless of the expectations mechanism. Early models, including Muth’s rational expectations model (1961), used a certainty-equivalent framework and implicitly assumed risk neutrality. For now, we will also utilize the certainty equivalent assumption.

Consider a market characterized by aggregate supply and demand equations. Stocks are assumed to be inconsequential, or unchanging from period to period, and hence to have no effect on equilibrium price. Competitive farmers are assumed to allocate inputs with price vector, \( w_{t-1} \) at time \( t-1 \) to produce output \( y_t \) at time \( t \). Aggregate supply is given by

\[
y_t = \alpha_0 + \alpha_1 t - 1 p_t + \alpha_2 w_{t-1} + v_t,
\]

where error term \( v_t \sim (0, \sigma_v^2) \), \( \alpha_0 \) and \( \alpha_1 \) are parameters, and \( \alpha_2 \) is a vector of parameters for input prices \( w_{t-1} \). Since output price \( p_t \) is unknown at time \( t-1 \), farmers must form price expectations. Expected price \( E_t(p_t) \) is formed at time \( t-1 \) for time \( t \), given available information at time \( t-1 \):

\[
t-1 p_t^e = E_t(p_t|\Omega_{t-1}),
\]

where \( E_t(\cdot) \) is the expectations operator conditional on the information set \( \Omega_{t-1} \) available at time \( t-1 \).

At time \( t \), processors of agricultural output base their demand \( y_t^D \) on known product price \( p_t \), a vector of known output prices \( z_t \) and a vector of known prices for other inputs into the production process \( u_t \):

\[
y_t^D = \beta_0 + \beta_1 p_t + \beta_2 z_t + \beta_3 u_t + v_t^D,
\]

where error term \( v_t^D \sim (0, \sigma_v^2) \), \( \beta_0 \) and \( \beta_1 \) are parameters, and \( \beta_2 \) and \( \beta_3 \) are parameter vectors corresponding to output price vector \( z_t \) and other processing costs vector \( u_t \), respectively. The supply and demand equations can be estimated as a system only if price expectations are directly observed. For now, we assume that producers’ expected output prices are not observed.

Extrapolative Expectations Models

Extrapolative price expectations models formulate expected price as a function of only past prices. There are a number of variations.

Naive Expectations

The earliest expectations models simply assumed that the best forecast of future price is current price: naive expectations

\[
t-1 p_t^e = p_{t-1}.
\]

Naive expectations implicitly assume that the underlying price series follows a random walk:

\[
p_t^e = p_{t-1} + \epsilon_t,
\]

where \( \epsilon_t \) is an error term. This simple model presumes that price at production planning time contains all the information from which astute producers could profit. It ignores possible producer knowledge of anticipated supply or demand shifts and their effects on price. In addition, in the presence of upward or downward price trends, the naive expectations mechanism will continuously under- or overpredict future price.

Econometric Estimation. Assuming the naive price expectation holds, unknown model parame-
ters can be estimated by substituting equation (4) into supply equation (1) and estimating the resulting supply equation using OLS. However, if supply and demand errors are correlated, OLS will result in biased and inconsistent estimates. In that case, consistent and asymptotically efficient parameter estimates can be obtained applying three-stage least squares or full-information maximum likelihood estimation to the supply and demand equations. Serially correlated errors in equation (5) can also result in biased and inconsistent OLS parameter estimates. Consistent estimates can be obtained using an instrumental variable estimator (see Johnston 1984, ch. 9).

**Econometric Estimation.** Supply, equation (7), is obtained by applying the Koyck transformation to expected price, equation (6), and substituting the result into equation (1), giving

\[
y_t' = \lambda y_{t-1}' + \alpha_0 (1 - \lambda) + \alpha_t (1 - \lambda) p_{t-1} + \alpha_2 w_{t-2} + \nu_t',
\]

where \(\nu_t' \sim \nu_t' - \lambda \nu_{t-1}'.\) If equation (7) is estimated OLS, resulting parameter estimates will be biased and inconsistent since \(y_{t-1}'\) is correlated with the autocorrelated disturbance \(\nu_t'.\) However, it is possible to obtain consistent, though not efficient, estimates using the instrumental variable method with \(w_{t-2}\) serving as an instrument for \(y_{t-1}'.\) (For estimation details, see Johnston 1984, ch. 9.)

A maximum likelihood (ML) estimator is also available, which gives consistent and efficient parameter estimates. To develop the ML estimator, rewrite equation (6) as the infinite geometrically decreasing series

\[
y_t' = (1 - \lambda) p_{t-1} + (1 - \lambda) \lambda p_{t-2} + \ldots.
\]

This series can be rewritten as

\[
y_t' = \sum_{i=0}^{t} (1 - \lambda) \lambda^i p_{t-i} + \sum_{i=t+1}^{\infty} (1 - \lambda) \lambda^i p_{t-i}.\]

The first right-hand term is historic prices and is observable. The second right-hand term represents expected price at \(t = 0.\) It involves data predating time period \(t = 0\) and, hence, is not observable. The second right-hand term can be rewritten as \(\lambda E(p_0 - \mu) = \lambda \delta\) where \(\mu\) is the mean of the price series \(p_t\) and \(\delta = E(p_0 - \mu).\) \(\delta\) can be treated as an additional parameter to be estimated.

The first right-hand term of equation (9) can be rewritten as an observable variable \(g_t:\)

\[
g_t = \sum_{i=0}^{t-1} (1 - \lambda) \lambda^i p_{t-i}.
\]

Given a value of \(\lambda,\) a data series for expected price can be built up recursively as:

\[
g_1 = (1 - \lambda) p_1
\]

\[
g_2 = (1 - \lambda) (p_2 + \lambda p_1)
\]

\[
g_3 = (1 - \lambda) (p_3 + \lambda p_2 + \lambda^2 p_1).
\]

This allows supply, equation (1), to be rewritten as

\[
y_t' = \alpha_0 + \alpha_t (g_t + \lambda \delta) + \alpha_2 w_{t-1} + \nu_t'.
\]

Assuming \(\nu_t' \sim N(0, \sigma_{\nu_t'}^2),\) maximum likelihood estimation proceeds with a grid search on \(\lambda\) over the

**Adaptive Expectations.**

The adaptive expectations model is well known but regarded as a rather ad hoc expectations process (Nerlove 1972). Expected price in the next period is formed by adjusting expected price by a proportion of the error made in predicting the current period's price (Hicks 1939; Koyck 1954; Cagan 1956; Muth 1960; Nerlove 1958):

\[
t - 1 \hat{v}_t - t - 1 \hat{v}_{t-1} = (1 - \lambda)(p_{t-1} - t - 2 \hat{v}_{t-1} - t),
\]

where \((1 - \lambda)\) is the forecast adjustment factor. If \(\lambda = 1,\) a price expectation never changes, regardless of past prediction error or any other information. If \(\lambda = 0,\) the adaptive expectation model is equivalent to the naive expectation model. If \(0 < \lambda < 1,\) price expectations are adjusted each period by some proportion of the discrepancy between the latest price and the price expectation formed for that period. If price is trending upward, the adaptive expectations model will continuously underpredict future prices. If price is trending downward, future prices will be overpredicted. Like the naive expectations mechanism, adaptive expectations do not account for the fountain of other information available to economic agents.

**Literature.** While widely criticized, naive expectations are often presumed when researchers require a simple price expectations mechanism to complete a model specification. Focused on other economic questions, many researchers ignore the potential effects of the chosen expectations mechanism on research results.

**Alternative Expectations Regimes**

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interval $0 \leq \lambda \leq 1$. For each specified value of $\lambda$, OLS is performed on equation (12) to obtain the supply parameters. Standard errors can be computed for $\lambda$ and all other estimated parameters from the information matrix (Johnston 1984, 359). Alternative estimation procedures are available if the supply error term $v_t^s$ is serially correlated or if the supply model contains lagged dependent variables (Johnston 1984, ch. 9; Doran 1988).

**Literature.** Adaptive expectations models have been widely used and were formalized by Nerlove (1958). Askari and Cummings (1977) provide an extensive survey of applications of adaptive expectations models in the literature. More recently, Shonkwiler and Hinckley (1985) have utilized adaptive expectations to model feeder cattle markets; Phillip and Abalu (1987) consider price expectations of Nigerian farmers; Doran (1988) provides an interesting specification test to distinguish between lag structures resulting from adaptive expectations and closely related partial adjustment supply processes.

**Other Extrapolative Expectations Models**

The naive and adaptive expectations models are members of a class known as extrapolative expectations models, which can be written in the general form

$$t-1p_i^e = \sum_{j=0}^{\infty} \theta_j p_{t-j},$$

where $\theta_j$ are fixed weights (Nerlove 1983). Other extrapolative expectations models include Almon distributed lag models (Almon 1965), rational distributed lag models (Jorgenson, 1966), and quasi-rational expectations models (Nerlove 1967; Nerlove, Grether, and Carvalho 1979; Nelson and Bessler 1992).

Almon distributed lag models, also known as polynomial distributed lag models, approximate the true distribution of lag coefficients with low-order polynomial functions of lagged variables. This reduces the number of lag parameters that must be estimated, reducing the problem of multicollinearity associated with estimating long lag functions (Almon 1965). Good review essays on distributed lag models are Almon (1965), Griliches (1967), and Nerlove (1972). The Almon lag structure is used in Schmidt and Waud (1973).

A rational distributed lag function is one that can be written as the ratio of two polynomials: one polynomial with a finite number of lags in the dependent variable and another for the independent variables. Estimation of rational distributed lag models is discussed in Jorgenson (1966) and in Maddala and Rao (1971).

Under the quasi-rational expectations hypothesis, agents form future value forecasts from an optimal statistical predictor such as an autoregressive integrated moving-average predictor or a simple vector autoregression. Agents are not required to know structural parameters for the entire economic model, as they would be under the full rational expectations hypothesis (Nelson and Bessler 1992). Friedman (1978), Wallis (1980), and Bessler (1980, 1982), show that the adaptive expectations model can be represented as an ARIMA $(0,1,1)$ model, and as such, the adaptive expectations model is a member of the quasi-rational expectations family.

Futures prices have also been used as direct measures of aggregate price expectations (Gardner 1976; Shonkwiler and Hinckley 1985; Chavas, Pope, and Kao 1983). Econometric estimation proceeds by substituting harvest time futures price observed at planting time for expected price and by estimating supply and demand directly.

**Rational Expectations Models**

Since Muth’s seminal paper, economists have devoted increasing attention to developing econometric models compatible with the rational expectations hypothesis. The rational expectations hypothesis asserts that “the economy generally does not waste information, and that expectations depend specifically on the structure of the entire system’’ (Muth 1961, 315). Specifically, the rational expectations hypothesis maintains that firms’ subjective expectations should be distributed about the objective predicted outcomes from economic theory. This implies that rational expectations are “model consistent” forward-looking projections of variables. In practice, empirical rational expectations models equate individual subjective expectations to the objective expectation generated from the model and, as a result, are not invariant to model specification.

Specification of a rational-expectations-based econometric model requires deriving a price expectation function that can be substituted into the supply equation. Given information available when production plans are implemented, a typical supplier formulates expectations of future prices such that his subjective expected price equals the price that equates demand and supply when output is realized.

Following Wallis (1980), Goodwin and Sheffrin (1982), and Huntzinger (1979), we can write the
supply and demand equations given in (1) and (3) matrix form as
\[ \begin{align*}
A &= \begin{bmatrix}
0 & -\alpha_1 \\
0 & 0
\end{bmatrix}, \\
B &= \begin{bmatrix}
1 & 0 \\
1 & -\beta_1
\end{bmatrix}, \\
\Gamma_1 &= \begin{bmatrix}
\alpha_2' \\
0
\end{bmatrix}, \\
\Gamma_2 &= \begin{bmatrix}
0 & 0 \\
\beta_2 & \beta_3
\end{bmatrix},
\end{align*} \]

where \( y' = (y_t, p_t) \) is a vector of the endogenous output and price variables, \( r - 1 \mathbf{y}_t^e \) is a vector of the expectation variables for output and price, \( \mathbf{w}_{t-1} \) is a vector of exogenous supply variables with values known at time \( t - 1 \), and \( \mathbf{x}_t' = (\mathbf{z}_t', \mathbf{u}'_t) \) is a vector of exogenous demand variables with future values not known at time \( t - 1 \). We will assume that errors across equations may be contemporaneously correlated, but that errors are not correlated across time periods. Thus, \( E(\mathbf{V}_t) = \mathbf{0} \) and \( E(\mathbf{V}_t \mathbf{V}'_t) = \Sigma \).

Solving equation (14) for \( \mathbf{y}_t \) gives
\[ \begin{align*}
y_t &= -\mathbf{B}^{-1} \mathbf{A}_{t-1} \mathbf{y}_t^e - \mathbf{B}^{-1} \Gamma_1 \mathbf{w}_{t-1} \\
& \quad - \mathbf{B}^{-1} \Gamma_2 \mathbf{x}_t + \mathbf{B}^{-1} \mathbf{V}_t.
\end{align*} \]

Equation (15) represents reduced form supply and demand equations. However, this formulation still involves unobserved expected prices and quantities for \( \mathbf{p}_t, \mathbf{y}_t, \) and \( \mathbf{x}_t \), at time \( t - 1 \). We can replace these unobserved variables by taking the conditional expectation as of time \( t - 1 \) \( (E_t) \) of both sides of equation (15) and substituting \( r - 1 \mathbf{y}_t^e \) for \( E_{t-1} \mathbf{p}_t \). This results in
\[ \begin{align*}
r - 1 \mathbf{y}_t^e &= - (\mathbf{A} + \mathbf{B})^{-1} \Gamma_1 \mathbf{w}_{t-1} \\
& \quad - (\mathbf{A} + \mathbf{B})^{-1} \Gamma_2 E_{t-1} \mathbf{x}_t.
\end{align*} \]

Equation (16) gives the expected price and quantity in period \( t \), given information available at time \( t - 1 \), as a function of the model’s structural parameters, known exogenous supply variables, and forecasts of exogenous demand variables for period \( t \). This equation represents producers’ rational price expectations function.

To cast the rational price expectations function entirely in terms of observable variables, it remains to specify producers’ expectations of exogenous demand variables, \( E_{t-1} \mathbf{x}_t \). A common procedure is to specify producers’ expectations of exogenous variables as low-order autoregressions:
\[ \mathbf{x}_{it} = \sum_{j=1}^{J} \phi_j \mathbf{x}_{it-j} + \nu_{it}, \]

where \( \phi_t = \{\phi_j\} \). To obtain estimatable equations, (17) can be substituted into price expectations function (16) and that result can be substituted into the structural supply and demand, equation (14).

Combining terms involving \( \mathbf{w}_{t-1} \) gives
\[ \begin{align*}
\mathbf{B}_t &= - (\mathbf{A} + \mathbf{B})^{-1} \Gamma_1 \mathbf{w}_{t-1} \\
& \quad - (\mathbf{A} + \mathbf{B})^{-1} \Gamma_2 E_{t-1} \mathbf{x}_t \\
& \quad + \Gamma_2 \mathbf{x}_t = \mathbf{V}_t.
\end{align*} \]

**Econometric Estimation**

Equation (18) represents a system of simultaneous equations. Wegge and Feldman (1983) have shown that econometric identification of parameters in rational expectations models of this form is guaranteed by the traditional rank and order conditions, so long as the number of imperfectly anticipated exogenous variables (elements in \( \mathbf{x}_t \)) is not less than the number of equations (elements in \( \mathbf{y}_t \)). Assuming the errors \( \nu_t \) are independent of the \( \nu_t \), unknown parameters can be jointly estimated using nonlinear full-information maximum-likelihood estimation procedures with cross-equation parameter restrictions on equations (17) and (18) (Wallis 1980; Taylor 1979; Fair and Taylor 1983; Revankar 1980).

While elements in \( \Phi_t \) can be estimated as additional parameters in (18), as just described, often parameters in the stochastic process used to forecast exogenous demand variables are estimated separately, with resulting predictions being used as instruments for \( E_{t-1} \mathbf{x}_t \). Nonlinear maximum-likelihood estimation is then applied to the simplified version of (18). Pagan (1984) develops a two-stage estimation procedure that can be used to obtain consistent estimates of both parameters and their estimated covariances. Step 1 of the two-step estimator consists of running regression (17) to get \( \hat{\mathbf{x}}_t \), then substituting \( \hat{\mathbf{y}}_t \) for \( r - 1 \mathbf{y}_t^e \) and \( \hat{\mathbf{y}}_t \) for \( E_{t-1} \mathbf{x}_t \) in (16) and using OLS fitted values to get \( r - 1 \mathbf{y}_t^e \).

Step 2 consists of substituting \( r - 1 \mathbf{y}_t^e \) for \( r - 1 \mathbf{y}_t^e \) for \( \hat{\mathbf{x}}_t \) in (15) and estimating (15) using two-stage least squares, including \( r - 1 \mathbf{y}_t^e \) among the instrumental variables. Although this approach sacrifices efficiency, it reduces the number of nonlinear parameters that must be simultaneously estimated and therefore simplifies estimation. Hoffman (1987) has extended Pagan’s estimation procedure to the multiple equation model. Gauger (1989) has expanded Pagan’s work to analyze the effects of generated regressors on inference in hypothesis testing. The two estimation approaches just described are often referred to as the “substitution method” because they substitute an expres-
Fair and Taylor (1983) present an iterative method for obtaining maximum-likelihood parameter estimates in nonlinear rational expectations models. Fair and Taylor's estimation procedure replaces the rational price expectation function with predicted values created from numerical model solutions. Starting with a set of consistent parameter estimates, the model is solved using an iterative solution method. Resulting price predictions are then substituted for expected price, and model parameters are reestimated using maximum-likelihood estimation procedures. The solution-estimation process continues until parameter convergence is achieved. In this way, Fair and Taylor's estimation procedure incorporates cross-equation restrictions imposed from rational price expectations, even when a closed-form solution may not exist for the rational price expectations function. In linear models, Fair and Taylor's estimation procedure yields the same results as those obtained through maximum likelihood estimation.

McCallum (1976, 1979) and later Wickens (1982) suggest an alternative "ERrors in Variables" (ERV) estimation method. To implement the ERV method, expected values for price, quantity, and exogenous demand shifters are replaced with their realized (observed) values. Since \( x_t \) is a random variable correlated with \( v_t \), equation (18) is now an incomplete model specification with more jointly dependent variables than equations. Estimation proceeds by augmenting the redefined version of (18) with

\[
(19) \quad x_t = \theta I_t + \xi_t,
\]

where \( I_t \) is a vector of instrumental variables, \( \theta \) is a vector of parameters and \( \xi_t \) is a random error term. The model is now completely specified. Equations (18) and (19) can be estimated using either the two-stage least squares or limited-information maximum-likelihood techniques. While ERV estimation is not asymptotically efficient, the efficiency loss may be small in small samples.

Pesaran (1987, ch. 6 and 7) considers the econometric identification and estimation of numerous alternative model specifications, including single-equation and simultaneous-equation specifications, with current and forward-looking expectations of endogenous and exogenous variables. Asymptotic distributions and consistent variance-covariance estimators are presented. Pesaran also discusses the relative asymptotic efficiency of the various estimators.

### Literature


### Unobserved Price Expectations and Producer Risk Aversion

In this section, the simple aggregate model with unobserved price expectations presented above is modified to include the influence of producers' risk preferences in determining aggregate supply. Since farmers do not know demand with certainty when the output decision is made, they cannot know expected price with certainty. If producers are risk averse, then measures of risk variables will have an important influence on production decisions (Sandmo 1971).

Including risk in farmers' supply equations means farmers must form expectations on both price and price-induced risk. The earliest models included risk variables in farmers' production decisions in an ad hoc way: they simply added an additional term to supply representing price risk (Behrman 1968; Just 1974, 1977; Traill 1978). In these models, supply is

\[
(20) \quad y_t = \alpha_0 + \alpha_1 y_{t-1} + \beta_1 p_t^e + \beta_2 w_t + \epsilon_t,
\]

where \( y_{t-1} p_t^e \) is the expected price variance at time \( t \), conditional on information at time \( t-1 \). While most authors do not present any justification for this, Antonovitz and Roe (1984) justify a similar
specification as a supply function resulting from a second-order Taylor-series approximation of indirect utility specified as a function of expected prices and price variances. While this justification is not necessarily incorrect, it seems more natural that producers maximize expected utility of profits. In this case, the mean and variance terms in equation (20) should be transformed through expected profit and variance of profit, since profit is the argument of utility. In the event input prices are known $\text{var}(\pi) = (y)^2 \text{var}(p)$ (Pope 1978). With these assumptions, the commonly used specification in (20) is incorrect.

### Extrapolative Expectations Models

**Adaptive Expectations.** Behrman (1968) was the first to adopt a supply specification like equation (20). He used a fixed-length moving average of squared deviations around a simple moving average of the same length as the measure of expected price variance $\text{var}(p_t)$, given the information at time $t - 1$. Just (1974) utilized the adaptive expectations hypothesis for both price and price risk to estimate supply equation (20). To operationalize price risk, Just used the standard adaptive expectations specification for expected price, as in equation (9). He then defined subjective price risk as

$$
\text{var}(p_t) = \sum_{i=0}^{\infty} (1 - \pi)\pi^i(p_{t-1} - p_{t-1}^e)^2,
$$

where $\pi$ is an unknown parameter. Just also considered the possibility that supply might be a function of multiple expected prices, possibly including own-price and substitute prices in production. Adaptive expectations of covariance can be operationalized through equation (21).

Just (1974, 1977) develops a maximum likelihood procedure for estimating (20) using the adaptive expectations hypothesis for expected price and expected price variance. His procedure transforms the unobserved expectational variables conditional on values of $\lambda$ and $\pi$ by building price series recursively as in equation (11). Model parameters are then estimated using OLS. A grid search is performed over $\lambda$ and $\pi$ to obtain their maximum likelihood values. Empirical studies by Just (1974, 1977), Traill (1978), Hurt and Garcia (1982), and Brorsen, Chavas, and Grant (1987) have found that risk terms are important in aggregate supply functions.

**Quasi-Rational Expectations with Risk Aversion.** The effects of price uncertainty in the quasi-rational expectations framework have been investigated by Antonovitz and Roe (1984, 1986), Antonovitz and Green (1990), Seale and Shonkwiler (1987), Schroeter and Azzam (1991), Holt and Aradhyula (1990), and Holt (1993). In these models, expected price and expected price variance are modeled as forecasts from auxiliary equations using time series methods. Antonovitz and Roe (1984, 1986), Antonovitz and Green (1990), and Seale and Shonkwiler (1987) generate price and price variance forecasts using ARIMA models. Price variances are created using lagged values of squared residuals from their respective ARIMA price models. These forecasts are then used as regressors for expected prices and expected price variances in structural supply and demand models like equations (20) and (3). Special estimation problems associated with this model formulation are considered in Pagan (1984) and in Pagan and Ullah (1988).

Seale and Shonkwiler (1987) use the substitution estimation procedure suggested by Wallis (1980) to estimate a quasi-rational expectations model including risk for the U.S. watermelon market. They estimate autoregressive models of degree one to produce forecasts of exogenous variables. Then they substitute these forecasts into a structural model to estimate the remaining parameters. Antonovitz and Green (1990) estimate a quasi-rational expectations model including price risk for fed-beef supply also using Wallis's method.

Holt and Aradhyula (1990) point out that using an ARIMA process to estimate expected price variance is inconsistent with the homoskedastic variance assumption of the ARIMA model. Instead, they suggest using Engle's ARCH (Autoregressive Conditional Heteroskedasticity) (1982) or Bollerslev's GARCH (Generalized ARCH) (1986) models to generate forecasts of expected price and expected price variance. A distinguishing feature of these models is that forecast variance of a series is allowed to vary over time. The ARCH process conditions variance forecasts on past realizations of the dependent variable, while the GARCH process extends the information set to include lagged variances of the dependent variable. Using a modified GARCH (1,1) model to form forecasts of price and price variance, Holt and Aradhyula estimate a supply model for the U.S. broiler industry. While their estimated supply equation compares favorably with one estimated using Just's adaptive expectations framework, the question of which model is better is left unanswered.

A number of authors have utilized price and price variance forecasts from ARCH or GARCH.

Rational Expectations Models

The effects of price uncertainty in the rational expectations framework have been investigated by Aradhyula and Holt (1989). The simple rational expectations model represented in equation (14) can be modified to include variance by adding the term $A_2 t^{-1}y^v_r$. Equation (14) then becomes

$$\begin{equation}
B_y + A_1 y_r + A_2 t^{-1}y^v_r + \Gamma_1 w_{t-1} + \Gamma_2 x_t = \sigma_{t}.
\end{equation}$$

Utilizing the rational expectations hypothesis, the price risk specification is derived from underlying model parameters. Following Aradhyula and Holt, the rational expectation of variance can be defined as

$$\begin{equation}
t^{-1}y^v_r = \text{diag} \{ E_{t-1}[(y_t - E_{t-1}(y_t|\Omega_{t-1}))^2] \}
\end{equation}$$

where $t^{-1}y^v_r$ is defined as a vector of expected variances for relevant endogenous variables conditional on information available at time $t-1$. An exact expression for equation (23) is formed in three steps. Step 1 is to find an expression for $t^{-1}y^v_r$ as in equation (16). Step 2 is to subtract $t^{-1}y^v_r$ from the reduced form representation of $y_t$ (a modified form of equation (15)):

$$\begin{equation}
(y_t - t^{-1}y^v_r) = -B^{-1}\Gamma_2(x_t - E_{t-1}x_t) + B^{-1}\sigma_{t-1}.
\end{equation}$$

Step 3 is to use the result from step 2 to compute $t^{-1}y^v_r$ from equation (23). These steps result in the rational expectations predictor of variances of endogenous variables:

$$\begin{equation}
t^{-1}y^v_r = \text{diag}(B^{-1}\Gamma_2\Psi_1\Gamma_2^{-1} + B^{-1}\Sigma B^{-1}),
\end{equation}$$

where $\Psi_1$ is the variance-covariance matrix associated with the predictions of demand shifts $x_t$. Equation (25) can now be substituted into the expression for $t^{-1}y^v_r$ to eliminate terms involving $t^{-1}y^v_r$. Together $t^{-1}y^v_r$ and $t^{-1}y^r$ can be substituted into equation (22) to obtain the following estimatable system of equations:

$$\begin{equation}
By_t - A_1(B + A_1)^{-1}\Gamma_1 w_{t-1} - A_2(B + A_1)^{-1}\Gamma_2 E_{t-1} x_t - A_1(B + A_1)^{-1}A_2 \text{diag}(B^{-1}\Gamma_2\Psi_1\Gamma_2^{-1} + B^{-1}\Sigma B^{-1}) + A_2 \text{diag}(B^{-1}\Gamma_2\Psi_1\Gamma_2^{-1} + B^{-1}\Sigma B^{-1}),
\end{equation}$$


Rational Expectations with Bounded Prices. Recently Maddala (1983), Shonkwiler and Maddala (1985), Holt and Johnson (1989), and Holt (1992) have considered rational expectations models where prices are bounded. Bounded prices often arise in agricultural markets where government intervenes to guarantee producers a minimum price. If market price is above the support level, producers get the prevailing market price and government intervenes to guarantee producers a minimum price. If market price falls below the support price, government intervenes in the market by purchasing commodities to raise price to support level. Hence, the market alternates between equilibrium and disequilibrium. Government intervention creates an effective lower bound on producers’ rational price expectations and creates a situation where no closed-form solution for the rational price expectations function, equation (16), exists.

Maddala (1983) proposes a two-stage tobit estimator to deal with the two price regimes. In the first stage, a tobit model is used to obtain price predictions. In the second stage, expected price is replaced with price predictions from the tobit equation, and two-stage least squares estimation is applied to an augmented structural supply and demand model. Additional terms are included in the estimating equation to correct for nonspherical disturbances. Maddala’s estimation procedure produces consistent but inefficient parameter estimates. Shonkwiler and Maddala (1985) assume the producers have perfect foresight with respect to periods when price supports are effective. This assumption allows them to use standard substitution estimation procedures. They compare perfect foresight results with Maddala’s two-step tobit estimator. Holt and Johnson (1989) and Holt (1992) both use Fair and Taylor’s estimation procedure described earlier, which is directly adapted to the
numeric formulation of the price expectations function.

Selecting Expectations Regimes When Expected Price Is Not Observed

When expectations are not observed, two categories of tests are available to evaluate a particular price expectations hypothesis with respect to actual market data. The first is for consistency of the expectations mechanism within the structural model. Tests in this category simply ascertain whether the expectations mechanism is consistent with observed behavior as constructed in the economic model. The second category consists of nonnested model selection tests designed to allow the researcher to select the expectations regime that best fits agent behavior in a particular market.

Testing for Within-Model Consistency

Extrapolative Expectations. Naive and adaptive price expectations mechanisms do not impose cross-equation parameter restrictions on supply and demand equations. Therefore, in contrast to rational expectations models, there are no direct tests of internal consistency that will reject or fail to reject naive expectations when expectations are not directly observed.

A weak test for within-model validity of adaptive expectations, in the absence of directly observed expectations, is whether \( \lambda \) lies on the unit interval.

Rational Expectations. The rational expectations hypothesis imposes cross-equation structural restrictions on model parameters. If the restrictions imposed by the rational expectations hypothesis cannot be rejected, the rational expectations hypothesis cannot be rejected. The statistical validity of cross-equation parameter restrictions imposed by the rational expectations hypothesis can be evaluated through standard hypothesis testing procedures (Hoffman and Schmidt 1981; Revankar 1980). A likelihood-ratio test can be computed by estimating the restricted model using the substitution method maximum-likelihood estimator and estimating the unrestricted model using standard FIML, where the unrestricted model is given by:

\[
By_t + G_1 w_{t-1} + G_2 E_{t-1} x_t + \Gamma_2 x_t = V_t.
\]

The number of restrictions is equal to the number of parameters in equation (27) minus the number of parameters in (18). Alternatively, a Wald test can be constructed by estimating the unrestricted model given in (27) and jointly testing the restrictions (Hoffman and Schmidt, 1981):

\[
(28) \quad H_0: G_1 = -A(A + B)^{-1} \Gamma_1 + \Gamma_1 \quad \text{and} \quad G_2 = -A(A + B)^{-1} \Gamma_2
\]

versus

\[
(29) \quad H_a: G_1 \neq -A(A + B)^{-1} \Gamma_1 + \Gamma_1 \quad \text{or} \quad G_2 \neq -A(A + B)^{-1} \Gamma_2.
\]

Goodwin and Sheffrin (1982) apply the likelihood ratio test to a model of the U.S. broiler industry and cannot reject the null hypothesis that price expectations are formed rationally. However, it should be noted that tests like the ones above are conditional on the correct specification of the underlying structural model.

Testing across Expectations Regimes Using Nonnested Tests

As discussed in the introduction, when information collection and processing are costly, any number of expectations mechanisms can potentially reflect a producer's actual expectations. Several authors have investigated this possibility in various commodity markets (Goodwin and Sheffrin 1982; Orazem and Miranowski 1986; Antonovitz and Green 1990). A number of nonnested hypothesis tests are available for distinguishing which price expectations regime producers are actually utilizing.

Alternative expectations models can be artificially nested to determine whether any hypothesized expectations regime dominates all other specifications. Care must be taken to maintain the same production and demand structures so that only the parameterization of each expectation regime differs between models. Selection of the expectation regime consistent with behavioral data can then be based on model specification tests, including Davidson and MacKinnon’s J-test (1981), Mizon and Richard’s encompassing principle (1986), and Pollak and Wales’s likelihood dominance criterion (1991).

J-test. Suppose there are two competing expectations hypotheses: \( t_{-1} \rho_{1,t} \) (adaptive expectations) and \( t_{-1} \rho_{2,t} \) (rational expectations). Assuming that the first price expectations hypothesis is true, \( t_{-1} \rho_{1}^{e} = t_{-1} \rho_{1,t} \), the null hypothesis can be written as

\[
(30) \quad H_0: y_t^{e} = \alpha_0 + \alpha_1 t_{-1} \rho_{1,t}^{e} + \alpha_2 w_{t-1}^{e} + v_{1,t}^{e} = XA_1 + v_{1,t}^{e},
\]

versus the alternative hypothesis that \( t_{-1} \rho_{1}^{e} = t_{-1} \rho_{2,t}^{e} \):

\[
(31) \quad H_a: y_t^{e} = \alpha_0 + \alpha_1 t_{-1} \rho_{2,t}^{e} + \alpha_2 w_{t-1}^{e} + v_{2,t}^{e} = ZA_2 + v_{2,t}^{e}.
\]
Since $H_0$ cannot be written as a restriction on $H_0$, nested hypothesis tests are not possible. However, Davidson and MacKinnon (1981) suggest combining the alternative models into a single compound model:

$$y_t^i = (1 - \delta)X A_i + \delta Z A_i + \nu_t^i.$$  

A test of $\delta = 0$ would be a test against $H_a$. However, $\delta$ cannot be estimated directly using equation (32). Davidson and MacKinnon suggest first estimating $A_k$ using a maximum likelihood estimator and then performing maximum likelihood estimation on the aggregate model:

$$y_t^i = (1 - \delta)X A_i + \delta \hat{A}_k + \nu_t^i.$$  

An asymptotically valid test of $t-\nu^i = t-\nu^f_i$ is $H_0$: $\delta = 0$ with the test statistic given by

$$\frac{\hat{\delta}}{SE(\hat{\delta})} \sim N(0,1).$$

This test is conditional on the truth of the hypothesis $t-\nu^f_i = t-\nu^f_{i,r}$; we cannot infer the truth of $H_a$ from $\delta$. However, the process can be reversed so that $t-\nu^f_i = t-\nu^f_{i,r}$ is taken as the truth. In total, four possibilities must be checked: reject both $H_0$ and $H_a$; reject neither $H_0$ nor $H_a$; reject $H_0$ but not $H_a$; reject $H_a$ but not $H_0$.

Orazem and Miranowski (1986), using Iowa county-level acreage data for corn, soybeans, hay, and oats, construct the J-test for three alternative expectations regimes: naive, perfect foresight ($t-\nu^f_i = p_i$), and quasi-rational expectations. They find that, while quasi-rational forecasts perform marginally better than the other regimes, they cannot accept any of the postulated expectations hypotheses.

Antonovitz and Green (1990) apply Davidson and MacKinnon’s J-test to six alternative expectations models for price and price variance: naive, two quasi-rational models, futures prices, adaptive, and rational expectations. Pair-wise test results do not consistently support any particular expectations specification over any other, with the least support for the adaptive expectations. Root mean-squared errors (RMSE) are also computed for each expectations mechanism. Interestingly, rational expectations generate the highest RMSE, while adaptive expectations generate the lowest.

Shideed and White (1989) also use the J-test to compare six alternative expectation formulations for prices of corn and soybeans: naive, futures prices, effective expected support prices, a combination of lagged cash and support prices, a combination of futures prices and lagged support prices, and a Koyck lag model. The results of pair-wise comparisons are, as in Antonovitz and Green, inconclusive and conflicting.

**Encompassing Principle.** Other nonnested test procedures are also available for testing alternative model specifications, including Mizon and Richard’s encompassing test (1986) and Pollak and Wales’s likelihood dominance criterion (1991). Mizon and Richard’s encompassing test is a joint test that compares $A_k$ and variance of the regression $\hat{\sigma}_k^2$ obtained under $H_0$ from equation (27) with the probability limits of these parameters under $H_0$. Comparing $A_k$ with plim $(A_k|H_0)$ gives the mean encompassing test; it is equivalent to an F-test for equality of model coefficients. Comparing $\hat{\sigma}_k^2$ with plim $(\hat{\sigma}_k^2|H_0)$ gives the variance encompassing test; equivalent to the J-test. Hence, as a joint test of mean and variance, the encompassing test is a more general test than the J-test alone.

**Likelihood Dominance Criterion.** Consider the two hypotheses $H_0$ and $H_a$ in equations (30) and (31), and a fictional composite hypothesis $H_c$ that nests $H_0$ and $H_a$. Let $L_0$, $L_a$, and $L_c$ denote the log likelihood values associated with each hypothesis. Let $C(v)$ denote the critical values of the chi-square distribution with $v$ degrees of freedom and a fixed significance level. Using a likelihood-ratio test, $H_i$ (i = 0 or a) cannot be rejected when tested against the composite if and only if $L_i - L_c < C(n_c - n) < C(n_c - n_a)$ where $n_c$ is the number of parameters in $H_c$. Using this relationship, Pollak and Wales (1991) show that $H_a$ dominates $H_0$ if and only if

$$L_a - L_0 > [C(n_c - n_a) - C(n_c - n_a)]/2 = C^*(n_c, n_0, n_a).$$

Since $n_c$ is not known, Pollak and Wales develop rules for establishing a model selection criterion, the likelihood dominance criterion (LDC):

(i) the LDC prefers $H_0$ to $H_a$ if

$$L_a - L_0 < [C(n_c + 1) - C(n_0 + 1)]/2,$$

(ii) the LDC is indecisive between $H_0$ and $H_a$ if

$$[C(n_c - n_0 + 1) - C(1)]/2 > L_a - L_0 > [C(n_c + 1) - C(n_0 + 1)]/2,$$

and, (iii) the LDC prefers $H_a$ to $H_0$ if

$$L_a - L_0 > [C(n_c - n_0 + 1) - C(1)]/2.$$

Since $L_a$, $L_0$, $n_c$, and $n_0$ are all known, the LDC test is easily implemented.

**Summary.** Neither Antonovitz and Green (1990) nor Shideed and White (1989) could support any particular expectations specification using the J-test. Likewise, Orazem and Miranowski’s results (1986) were inconclusive, finding only marginal support for the quasi-rational expectations hypotheses. However, nonnested tests have relatively low
power, and inconclusive results are not unusual (Judge et al. 1985, 885). Neither the encompassing principle nor the likelihood dominance criterion has yet been used to distinguish price expectations mechanisms.

**Forecast Performance Measures**

Goodwin and Sheffrin (1982) construct two tests to compare alternative expectations regimes: a prediction test and a futures market test. These tests are valid only for the maintained null hypothesis that expectations are formed rationally.

**Predictive \( R^2 \).** Goodwin and Sheffrin’s prediction test (1982) is based on the relative predictive efficiency of rationally formed expectations that use all available information versus less efficient extrapolative price expectations models that use information contained only in past prices. Pierce (1975) defines a measure of this efficiency called “the predictive \( R^2 \)”: 

\[
R^2 = \frac{MSE - MSE^*}{MSE},
\]

where \( MSE \) is the extrapolative mean squared error and \( MSE^* \) is the rational expectations mean squared error. \( R^2 \) measures the proportion of variation in a variable that is generated by utilizing structural information versus variation in the variable generated using only own-price history. Goodwin and Sheffrin estimate models of the U.S. broiler market using adaptive, quasi-rational, and rational price expectations regimes. Using the \( R^2 \) criterion, they find superior efficiency in the rational expectations hypothesis. However, they do not test for unbiasedness, another requirement of the rational expectations hypothesis.

Although widely used, forecast evaluation tests are valid only for a maintained null of rational expectations. Unbiasedness and efficiency are basic tenets of the rational expectations hypothesis. Therefore, if producers’ expectations are biased or inefficient, they are not rationally generated. However, there is no such accuracy or efficiency requirement for other expectations mechanisms. The predictive ability of an expectations mechanism might be the deciding factor when information is unlimited and costless. However, when information is costly, a farmer may choose a biased or less efficient mechanism.

**Futures Market.** Goodwin and Sheffrin (1982) also construct a rational expectations test based on futures market data. According to the rational expectations hypothesis, no information available at the time of the forecast should add to the predictive power of the rationally expected price. Goodwin and Sheffrin argue that futures market price data should include all available information in the marketplace. Hence, a test of the rational expectations hypothesis can be performed using regression model

\[
p_t = \alpha_0 + \alpha_1 \cdot \beta_t + \alpha_2 \cdot \gamma_t + \epsilon_t,
\]

where \( p_t \) is realized price, \( \cdot \beta_t \) is the price forecast through the rational price expectations mechanism for time \( t \) given the information at time \( t - i \), and \( \cdot \gamma_t \) is the future price at time \( t - i \) for delivery at time \( t \). If the rational expectations hypothesis is valid, \( H_0 \) will not be rejected: \( H_0: \alpha_0 = \alpha_2 = 0 \). \( \alpha_1 \) is not required to equal 1 because of transportation costs from a local market to a contract delivery point and because product form (e.g., iced broiler meat versus live chickens) may differ between cash price \( p_c \) and futures price \( p_f \).

Goodwin and Sheffrin (1982) estimate equation (40) using OLS and cannot reject the rational price expectations hypothesis. However, in general, \( \cdot \beta_t \) will estimate \( \cdot \beta_t^e \) with error. If \( \cdot \beta_t = \cdot \beta_t^e + \zeta_t \), and futures price \( \cdot p_f \) is not required to equal 1 because of transportation costs from a local market to a contract delivery point and because product form (e.g., iced broiler meat versus live chickens) may differ between cash price \( p_c \) and futures price \( p_f \).

Pagan (1984) suggests estimating the equation with generated regressors, like \( \cdot \beta_t^e \), using the two-step approach detailed above.

**Summary**

Not surprisingly, empirical results concerning which price expectations mechanisms are actually being used are inconclusive. Inability of these models to distinguish actual producer price expectations mechanisms may result from aggregation bias. When information is costly, information sets differ, and individuals begin the expectations process with different educational and analytic endowments, it is not reasonable to suppose an aggregate “representative farmer” price expectations model will be valid. If producers have heterogeneous expectations, then an aggregate model may not capture reality well enough to provide good statistical results. In particular, strong assumptions are made in the use of aggregate data. Tests of producer expectations mechanisms then incorporate these assumptions as part of the maintained hypothesis of model structure.

For example, to derive a simple structural model with stochastic output prices that admits the possibility of risk aversion similar to equation (20) requires numerous assumptions. First, assume that
price is normally distributed and that farmer i’s utility is characterized by constant absolute risk aversion. Maximizing expected utility is then equivalent to maximizing the certainty equivalent of profit \( E[U(\pi_i)] \) (Hildreth 1954; Freund 1956):

\[
E[U(\pi_i)] = E(\pi_i) - \frac{\lambda_i \sigma^2_{\pi_i}}{2},
\]

where \( E(\pi_i) \) is expected profit and \( \sigma^2_{\pi_i} \) is the profit variance perceived by farmer i. Alternatively, Farrar (1962) and others have justified equation (41) as a Taylor-series expansion of the utility of expected profit.

Equation (41) can be expanded to

\[
E[U(\pi_{it})] = t_{-1}p_t^e y_{it} - C_i(y_{it}, w_{t-1}; F) - \frac{\lambda_i}{2} y_{it}^2 t_{-1}p_t^e,
\]

where \( t_{-1}p_t^e \) is farmer i’s expected price, \( y_{it} \) is farmer i’s output, \( C_i(y_{it}, w_{t}; F) \) is his cost function, and \( F \) is fixed inputs. In order to aggregate across producers, cost functions must be either identical or of some aggregatable form such as the Gorman polar form, \( C_{it} = y_{it} c(\cdot, \cdot) + F \).

Each farmer chooses \( y_{it} \) to maximize expected utility of profits:

\[
\frac{\partial E[U(\pi_{it})]}{\partial y_{it}} = t_{-1}p_t^e - c_i(w_{t-1}) - y_{it}\lambda_i t_{-1}p_t^e = e_{it},
\]

where \( e_{it} \) is a small error with mean zero resulting from errors in specifying the true optimization problem (McElroy 1987). If firm-level data are available for input prices and output quantity, equation (43) can be estimated to recover the utility parameter \( \lambda_i \) and technology parameters in \( c_i(\cdot) \) by substituting in an agent’s expectation mechanism for price and price variance. Subtracting extrapolative price expectations mechanisms into (43) is straightforward. No other assumptions are necessary.

However, in the case of rational expectations, producers’ price expectations are the objective price expectations from the market given their information sets at production planning time. Though expectations are formed individually, an aggregate model is required. Rational expectations requires each individual to assess aggregate supply and demand conditions and to forecast price and price variance based on his perception of that aggregate. Agents must hypothesize aggregate models based on the information at hand and will likely consider their own technologies and utilities. Individuals may aggregate supply by presuming that technology, information, and risk attitudes of all other producers are the same as their own. Farmers might also believe that others in the market are forming their expectations rationally and will forecast the same price and price variance as they do. These or similar assumptions are necessary for rational expectations models.

Given these assumptions, \( c_i(w_{t-1}) = c(w_{t-1}) \), \( \lambda_i = \lambda, t_{-1}p_{t-1}^e = t_{-1}p_t^e \), and \( t_{-1}p_{t-1}^e = t_{-1}p_t^e \). As a result, aggregate supply is the sum over individual supplies

\[
y_t = \sum_{i=1}^{M} y_{it} = \sum_{i=1}^{M} t_{-1}p_t^e - c(w_{t-1}) - e_{it} = \lambda t_{-1}p_t^e + \bar{e},
\]

where \( M \) is the number of producers and \( \bar{e} \) is the mean error term with an expected value of zero.

To form rational expectations, producers must also know aggregate demand, equation (3). Aggregate supply can then be combined with aggregate demand to form model-consistent price and price variance expectations. While little is known about how producers might actually perform these computations, it is clear that numerous assumptions must be maintained in the estimation of rational expectations models.

To investigate the possibility that producers may have heterogenous price expectations, we now turn our attention to studies focused on discovering individuals’ price expectations mechanisms. To date, studies concerning individuals’ price expectations have been based solely on survey data. First, we review some test procedures used in panel data studies. Then we consider some new test procedures that could be implemented when firm-level data, including price expectations surveys, are available.

**Selecting Expectations Regimes When Expected Price Is Observed**

One of the pitfalls of hypothesis tests within structural models is that it is nearly impossible to separate errors in the expectations mechanism from errors that exist in the structural model (Jacobs and Jones 1980). Hypothesis testing within the context of a structural model is necessarily conditional on
model structure, presenting a serious limitation for testing the rational expectations hypothesis within the structural model context. As an alternative, several authors have turned to direct tests of underlying price expectations that use survey data. Some tests use aggregate (mean) survey measures of expectations, while others use individual panel data.

When expectations are observed, as in the case where survey data exist, direct tests to distinguish economic agents’ expectations mechanisms are possible. For instance, a direct test of naive price expectations is to obtain regression coefficients for the model

\[ t_{-1} P_t^e = \alpha_0 + \alpha_1 P_{t-1} + \nu_t, \]

where \( \nu_t \sim N(0, \sigma^2) \) and the expected price \( t_{-1} P_t^e \) is taken directly from the survey data. A hypothesis test for naive expectations is \( H_0: \alpha_0 = 0 \) and \( \alpha_1 = 1 \) versus \( H_1: \alpha_0 \neq 0 \) or \( \alpha_1 \neq 1 \). Failure to reject \( H_0 \) implies that the surveyed price expectations are consistent with the naive price expectations mechanism.

Adaptive expectations are a special case of extrapolative expectations

\[ t_{-1} P_t^e = \sum_{j=0}^{\infty} \omega_j P_{t-j} \]

where the \( \omega_j \)'s are geometrically declining weights. There is no direct test for adaptive expectations since no hypothesis can be constructed from survey data alone to test whether the data are consistent with the adaptive expectations hypothesis.

Rational Expectations

Several tests are available to determine whether the rational expectations mechanism is consistent with survey data. First, the rational expectations hypothesis implies that expected prices should be unbiased. The restriction can be checked using the regression equation

\[ P_t = \tau_0 + \tau_1 t_{-1} P_t^e + \epsilon_t, \]

where \( \tau_0 \) and \( \tau_1 \) are parameters and \( \epsilon_t \) is an error term with \( E(\epsilon_t) = 0 \). Testing for unbiased expectations means testing \( H_0: \tau_0 = 0, \tau_1 = 1 \) (Friedman 1980).

Second, for Muth rational expectations, \( \epsilon_t \) must be uncorrelated with the expected price \( t_{-1} P_t^e \). Since the error term is correlated with the realized price, \( P_t \), the realized price variance should be larger than the expected price variance. This suggests a second test: \( \text{var}(P_t) > \text{var}(t_{-1} P_t^e) \).

Third, the “weak rationality” concept (Nelson and Bessler 1992; Sargent 1982; Lovell 1986) implies that the error term \( \epsilon_t \) should be uncorrelated with the information available in historic price levels at the time the forecast is made. This hypothesis can be tested using the regression equation

\[ (48) \quad P_t = \eta_0 + \eta_1 t_{-1} P_t^e + \eta_2 P_{t-1} + \nu_t, \]

where \( \eta_0, \eta_1 \) and \( \eta_2 \) are parameters and \( \nu_t \) is an error term. A third hypothesis test for rational expectations is thus \( H_0: \eta_2 = 0 \).

A variation of the weak rationality hypothesis test is presented by Pesando (1975). He suggests running the following two regressions:

\[ (49) \quad P_t = \alpha_1 P_{t-1} + \alpha_2 P_{t-2} + \ldots + \alpha_n P_{t-n} + \nu_{1,t} \]

and

\[ (50) \quad t_{-1} P_t^e = \beta_1 P_{t-1} + \beta_2 P_{t-2} + \ldots + \beta_n P_{t-n} + \nu_{2,t}, \]

where \( \alpha_i \) and \( \beta_i \) are parameters and \( \nu_t \) are error terms. If expectations are weakly rational, then an F-test will not reject the null hypothesis \( H_0: \alpha_1 = \beta_1, \alpha_2 = \beta_2, \ldots, \alpha_n = \beta_n \). All information used in the expected price and the realization is captured in the historic price. Carlson (1977) suggests a similar test.

The F-test requires that errors \( \nu_1 \) and \( \nu_2 \) be identically and independently distributed. Pesando (1975) acknowledges, and Mullineaux (1978) further argues, that there are reasons to believe the errors in equations (49) and (50) will not be homoskedastic. Mullineaux suggests that Bartlett’s statistic testing variance homogeneity is a necessary companion to Pesando’s and Carlson’s techniques.

Recently, the problem of possible nonstationarity of \( p_t \) and \( t_{-1} P_t^e \) has received attention. Fischer (1989) develops several necessary conditions for survey data to be generated from rational forecasts. He argues that preliminary tests for unit roots and cointegration are crucial before rationality of survey data can be established and recommends first testing the order of integration for \( t_{-1} P_t^e \) and \( p_t \). If \( \nu_t \sim I(1) \), \( t_{-1} P_t^e \sim I(1) \) and \( c \neq d \), where \( I(d) \) indicates order of integration, then \( t_{-1} P_t^e \) cannot be a rational forecast of \( p_t \). If \( t_{-1} P_t^e, P_t \sim I(d) \) and \( d > 0 \), then cointegration between \( t_{-1} P_t^e \) and \( P_t \) can be tested. If the data series are cointegrated, tests for weak-form rational expectations can be performed using residuals of the constrained cointegration regression. Failure to pretest for unit roots may result

Mullineaux (1978) further suggests another test for weak rationality. Equations (49) and (50) can be combined through subtraction:

$$ (51) \quad (p_t - p_{t-1}) = (\alpha_0 - \beta_0) + (\alpha_1 - \beta_1)p_{t-1} + (\alpha_2 - \beta_2)p_{t-2} + \ldots + (\alpha_n - \beta_n)p_{t-n} + (v_1 - v_2), $$

or

$$ (52) \quad \Delta p_t = \delta_0 + \delta_1 p_{t-1} + \delta_2 p_{t-2} + \ldots + \delta_n p_{t-n} + \phi_t. $$

Weak rationality implies that $\delta_0 = \delta_1 = \delta_2 = \ldots = \delta_n = 0$. Mullineaux’s formulation specifically allows a constant term and allows nested hypothesis testing.

Fourth, “strong rationality” (Lovell 1986) requires that the error term be uncorrelated with any piece of information available at the time of the forecast whether or not this information has been captured in past prices. This rationality concept is also called “sufficient expectations” (Lovell 1986) since expectations must be based on all information available at the time the expected price is formed. One hypothesis test for this involves estimating

$$ (53) \quad t - t p_t^e = f(X_t, \beta_1) + \epsilon_t^e $$

and

$$ (54) \quad p_t = f(X_t, \beta_2) + \epsilon_t, $$

where $\beta_1$ and $\beta_2$ are parameter sets, $X_t$ is an information set, and $\epsilon_t^e, \epsilon_t$ are error terms. For expectations to be rational in a structural model, the expected price must be related to the exogenous variables in the same way as the realized price. In other words, the two variables must follow the same autoregressive process (Turnovsky 1970). The test is thus $H_0^r: \beta_1 = \beta_2$. While Turnovsky performs this test using time series analysis, a similar test could be implemented for structural models where equations (53) and (54) are taken as price-dependent supply equations. Abel and Mishkin (1983) present an integrated view of tests for rational expectations and show the equivalence of many of the above procedures.

Engsted (1991) tests for strong rationality using aggregate U.S. sow farrowing data. He estimates the model

$$ (55) \quad F_{t+k} = \alpha_0 + \alpha_1 t + \kappa e_t + \beta x_t + \epsilon_t, $$

where $F_{t+k}$ are actual farrowings at $t + k$, $t + k e_t$ are farmers’ planned farrowings for $t + k$ at time $t$, and $x_t$ is a vector of variables containing forecast information at time $t$. Parameters $\alpha_0, \alpha_1, \beta$ can be consistently estimated using Hansen’s generalized-method-of-moments estimator (1982).

Runkle’s strong rationality test requires orthogonality of estimated regression residuals to each regressor in the model. Results indicate that neither the one-quarter-ahead nor the two-quarters-ahead farrowing intentions are rational forecasts of actual farrowings.

Eales et al. (1990) use survey data from farmers and grain merchandisers to investigate the extent to which subjective respondent’s price distributions reflect actual futures and options price distributions. Not surprisingly, they find agreement between respondent’s expected price and futures price. However, they also find strong evidence of systematic disagreement between respondents and the market with respect to variance forecasts.

Panel Data Studies and Some Pitfalls

There are a number of possible difficulties in testing for rational expectations using survey data. Keane and Runkle (1990) maintain that most tests in the literature are incorrect. First, they assert that the use of average survey responses rather than individual micro data can bias hypothesis tests by leading to false rejection of the rational expectations hypothesis. Individual forecasts are necessarily predicated on different information sets. Thus, tests based on aggregated data, such as an average survey response, are conditional on a set of information sets, not a single shared information set. Aggregation nullifies the single information set assumption required for expectations to be strongly rational. As a result, standard tests for expectations’ unbiasedness might be falsely accepted. Alternatively, aggregation may lead to a failure to reject the rational expectations hypothesis because aggregation may mask individual forecaster’s biases.

Aggregation bias may also result from using pooled cross-sectional time-series expectations survey data. Goodfriend (1992) points out that agents have randomly heterogeneous and imperfectly informed expectations. In pooled panel data studies, this may result in a stochastic regressor problem arising from correlation between the surveyed price expectation and error term. Adapting aggregate supply equation (1) to pooled panel data gives farmer $i$’s supply equation:
where $a_{0i}^c$, $a_{i}^{d}$, and $a_{2}^{c}$ are parameters for agent $i$, $t-1\rho_{t}^{e}$ is the price expected by agent $i$, $w_{t-1}$ is a vector of input prices, $\alpha_{2}$ is a vector of parameters common for all agents, and $\varepsilon_{i}$ is an error term. If the number of time-series observations is large, and conditions $\text{Cov}(t-1\rho_{t}^{e}, \varepsilon_{i}) = 0$ and $\text{Cov}(w_{t-1}, \varepsilon_{i}) = 0$ hold, OLS will produce consistent parameter estimates for $a_{1}$, $a_{2}$, and $\alpha_{2}$ and $\alpha_{3}$ for each individual. However, in the usual case where the observations per individual are few, consistent parameter estimates require the $\varepsilon_{i}$’s be uncorrelated across individuals as well. In the strong rational expectations case, the $\varepsilon_{i}$’s are uncorrelated across time for a particular individual, but there is no condition on errors across agents. In fact, continual small shocks to the economy make it highly likely that the $\varepsilon_{i}$’s will be correlated across agents, which will cause information-aggregation bias in models using pooled panel data.

One possible method to deal with information-aggregation bias is suggested by Zeldes (1989). Wave dummies can be used to capture an aggregate price expectations component. For example, Goodfriend (1992) suggests using individual’s variation from aggregate expectation in disaggregated supply equations like (56). Using this technique, all commonality in individual expectations is purged. However, the purging process may result in the false inference that agents do not make efficient use of information, a condition that could lead to false rejection of the rational expectations hypothesis (Goodfriend 1992).

Using revised data in model estimation may also bias hypothesis tests and parameter estimates (Keane and Runkle 1990). Often, data are revised after producer’s planning decisions. At the time producers form price expectations, their information sets contain the original data release. When testing for rational expectations, care should be taken to use the exact information set available at the forecast time, without revisions that have been made to information in the intervening period between forecast and academic research.

Survey instruments purport to measure what an individual is thinking, but responses are generally without consequences to the respondent. This is in contrast to market data, which are gathered from observing what individuals actually do. In surveys, there is no incentive for respondents to correctly reveal their expectations nor any way to mitigate such problems as interviewer-induced bias, survey instrument-induced bias, etc. Hence, any conclusions about individuals’ expectations resulting from survey evidence are conditional on the survey quality (Keane and Runkle 1990).

Keane and Runkle (1990), Lovell (1986), Pessando (1975), Carlson (1977), Mullineaux (1978), Knobl (1974), Nerlove (1983), Colling, Irwin, and Zulauf (1992), Frankel and Froot (1987), Runkle (1991), and Zarnowitz (1992, ch. 16), review numerous studies that test expectations formation of macroeconomic variables including inflation and interest rates using survey data. Most studies narrowly focus on testing the rational expectations hypothesis, though Knobl (1974), Nerlove (1983), and Frankel and Froot (1987) do consider other mechanisms. Generally, survey-based tests of the rational expectations hypothesis reject unbiasedness: the weakest requirement for rationality. Other studies fail to reject stronger forms of the rational expectations hypothesis. This ambiguity of results has led Lovell (1986) to suggest that alternative expectations mechanisms be evaluated against each other. However, to date, few studies have sought to distinguish alternative expectations regimes using survey data.

Summary

A major reason survey-based studies have focused almost exclusively on testing the rational expectations hypothesis is their inability to distinguish alternative price expectations mechanisms using only survey data for expected price and price realization data. Given only price information, researchers are restricted to testing unbiasedness and efficiency of respondents’ price forecasts when compared with actual price realizations. These are characteristics of the rational expectations hypothesis, but not necessarily of extrapolative expectations mechanisms. To distinguish among the myriad of expectations mechanisms, it is necessary to add more economic structure.

To test a broad spectrum of expectations mechanisms, including those that do not require unbiasedness or efficiency, it is necessary to cast hypothesis tests concerning price expectations mechanisms in terms of producers’ choice variables. The price expectation mechanism resulting in model predictions that most closely correspond to observed behavior can be accepted as the one to which producers are actually adhering. Since price expectations in commodity markets affect production, it is reasonable to cast hypotheses concerning price expectations mechanisms in terms of supply. This can be achieved through the nonnested model.
specification tests discussed in the previous section.

When survey data are available, a benchmark model comprised of supply (56) and demand (3) can be estimated directly using the survey data for expected price and expected price variance. The benchmark model can then be tested against each alternative price expectations mechanism by (1) substituting the assumed price expectation function into supply, (2) estimating the model as if price expectations were not observed, and (3) using non-nested model specification tests to determine whether the benchmark model dominates the hypothesized model. In addition, forecast tests can be used to test the validity of each price expectation mechanism against surveyed expected values.

Conclusions

This paper reviews alternative expectations regimes, their estimation, and the hypothesis tests that have been applied to determine their acceptance among producers. Muth (1961) introduced the rational expectations hypothesis over thirty years ago. While his idea has found widespread acceptance in the macroeconomic literature, there have been relatively few applications of the rational expectations hypothesis in commodity markets. This is surprising given the lengthy production lags typical in most commodity markets. Indeed, in his original paper, Muth demonstrated his "model consistent" expectations mechanism using a typical commodity market structure. Economists assign a preeminent role to expected price in determining supply, yet there are few empirical industry studies that test the nature of producers' price expectations mechanisms. Existing studies must make strong assumptions about producer risk attitudes, production technology, and limits on the number of stochastic variables in order to work with aggregate data.

The empirical results from industry studies that are available indicate that commodity producers may have heterogeneous price expectations. This is not surprising given that different producers possess different information and have different costs associated with information collection and processing. To investigate the possibility of heterogeneous expectations, we turned our attention to studies that use panel survey data of individual's price expectations. Results from these studies are also inconclusive. For the most part, panel survey studies have focused exclusively on testing the rational expectations hypothesis. The limited focus of these studies results, in part, from the kinds of hypothesis tests that are available using only survey data. In the future, a hypothesis test strategy that combines observed firm-level data with survey data of producer price expectations may be used to distinguish each producer's adherence to a particular price expectations mechanism. In the final analysis, each market participant has to make his own guess about what the future will bring. Some will take great care in making their projections; others will not. It seems unlikely that all individuals will follow the same rule.

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Alternative Expectations Regimes

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