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A Linear Inverse Demand System

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We present an inverse demand system that can be estimated in a linear form. The model is derived from a specification of the distance function which is parametrically similar to the cost function underlying the Almost Ideal Demand System. Simulation results suggest that this linear inverse demand system has good approximation properties.

Key words: Almost Ideal Demand System, demand analysis, distance function, duality.

Introduction

The Almost Ideal Demand System (ALIDS) of Deaton and Muellbauer is one of the most commonly used in applied demand analysis. While the ideal connotation of this model stems from its aggregation properties, it is arguable that one of the main reasons for its popularity is the availability of an approximate version of this system that is linear in the parameters; in fact, it is this linear version of the ALIDS model that is typically estimated (Heien and Wessells; Gould, Cox, and Perali; Moschini and Meilke). The purpose of this article is to illustrate how a linear system for inverse demand equations that resembles the ALIDS model can be derived, and we term this system the Linear Inverse Demand System (LIDS).[1]

Inverse demand functions, where prices are functions of quantities, provide an alternative and fully dual approach to the standard analysis of consumer demand (Anderson), and may be more appropriate when quantities are exogenously given and it is the price that must adjust to clear the market (Barten and Bettendorf). This situation is likely to be of relevance to modeling agricultural demand using data based on frequent time series observations (say monthly or quarterly). The chief advantage of using LIDS to model inverse demands is linearity, which may be useful for some applications (say large demand systems or systems involving dynamic adjustment). Although the parametric structure of the model that we present is similar to that of ALIDS, it does not claim the same aggregation properties. Nonetheless, its simplicity and its approximation abilities, documented in this article, are likely to make the LIDS model suitable for empirical studies.

Duality and the Linear Inverse Demand System

Commonly used demand systems typically are derived from parameterizations of dual representations of preferences through the derivative properties. This approach ensures integrability of the resulting demand equations by construction. To derive an inverse demand system, one can start either from the direct utility function and exploit Wold's identity (which yields ordinary inverse demands), or start from the distance (transformation) function and exploit Shephard's theorem (which yields compensated inverse

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demand functions) (Weymark). As will be clear in what follows, for our purposes it is better to start with the distance function, an alternative representation of preferences which has proved convenient in related contexts (Deaton).

If $U(q)$ is the direct utility function, where q denotes the vector of quantities, the transformation or distance function $F(u, q)$ is implicitly defined by $U[q/F(u, q)] \equiv u$, where u is the reference utility level. Under standard regularity conditions, $F(u, q)$ is continuous in (u, q), decreasing in u, and nondecreasing, concave, and homogeneous of degree one in q. These properties establish a useful parallel between the distance function and the cost function $C(u, p)$ derived from the utility-constrained expenditure minimization problem (where p is the price vector corresponding to q). As Blackorby, Primont, and Russell put it (p. 27), "... except for the direction of monotonicity of the utility variable, these conditions suggest that C could be interpreted as a transformation function and F as a cost function."

The parallel features of cost and distance functions are useful because, as emphasized by Hanoch, they imply that any standard functional form for the cost function can be applied also to the distance function.[2] The preceding discussion is pertinent to the problem at hand because the useful linear form of the approximate ALIDS model is made possible by the specific functional form chosen for the cost function. Exchanging the role of the variables (u, p) in the PIGLOG cost function of the ALIDS model with the variables $(-u, q)$ of the distance function, where the negative sign on u emphasizes the opposite monotonicity direction of F and C relative to the utility index, one obtains the following parametric specification for $F(u, q)$:

$$\ln(F) = a(q) - ub(q), \tag{1}$$

where $a(q)$ and $b(q)$ are quantity aggregator functions defined as:

$$a(q) = \alpha_0 + \sum_i \alpha_i \ln(q_i) + \frac{1}{2}\sum_i \sum_j \gamma_{ij}\ln(q_i)\ln(q_j), \tag{2}$$

$$b(q) = \beta_0 \prod_i q_i^{\beta_i}. \tag{3}$$

Because $F(u, q)$ is homogeneous of degree one in q, the following restrictions apply: $\Sigma_i \alpha_i = 1$, $\Sigma_j \gamma_{ij} = \Sigma_i \gamma_{ij} = 0$, and $\Sigma_i \beta_i = 0$. Also, without loss of generality, $\gamma_{ij} = \gamma_{ji}$ (the symmetry property).

From Shephard's theorem, the first derivatives of the distance function yield compensated inverse demands as $\pi_i = \partial F/\partial q_i \equiv h_i(u, q)$, where $\pi_i \equiv p_i/x$ is the normalized price of the ith good (the nominal price divided by total expenditure x). Because at $F = 1$ the distance function is an implicit form of the direct utility function, then (1) implies the utility function $U(q) = a(q)/b(q)$. This, together with the derivative property, implies that the uncompensated inverse demand functions associated with (1)–(3) can be written in share form as:

$$w_i = \alpha_i + \sum_j \gamma_{ij}\ln(q_j) - \beta_i \ln(Q), \tag{4}$$

where $w_i \equiv \pi_i q_i$ is the ith budget share, and $\ln(Q)$ is a quantity index defined as $\ln(Q) \equiv a(q)$.

The distance function in (1)–(3) has the same parametric structure of the PIGLOG cost function of the ALIDS model. It should be clear, however, that this distance function is not dual to the PIGLOG cost function of the ALIDS model. It follows that the aggregation properties of the ALIDS are not shared by the inverse demand system in (4). Hence, the attribute "Almost Ideal," used by Eales and Unnevehr and by Barten and Bettendorf to label (4), is somewhat misleading and does not appear warranted for this inverse demand model.

Equations (2) and (4) together entail a nonlinear structure for the inverse demand model.

In practice, however, $\ln(Q)$ can be replaced by an index $\ln(Q^*)$ constructed prior to estimation of the share system to yield:

$$w_i = \alpha_i + \sum_j \gamma_{ij}\ln(q_j) - \beta_i\ln(Q^*). \tag{5}$$

The resulting set of equations (5) is a linear system of inverse demands, the LIDS model. Many index formulae for $\ln(Q^*)$ may be considered here. Similar to the original suggestion of Deaton and Muellbauer, one may use the geometric aggregator $\ln(Q^*) = \Sigma_i\, w_i\ln(q_i)$, although other indices (say Diewert's superlative indices) may have better approximation properties. It should be understood, however, that in general quantities must be properly scaled for the geometric aggregator to be admissible. This point also applies to the equivalent price aggregator of direct ALIDS models, typically referred to as the Stone price index.[3]

The inverse demand system presented here satisfies standard flexibility properties. It can be verified that the distance function (2)–(4) has enough parameters to be a flexible functional form for an arbitrary distance function once it is realized that the ordinality of utility always allows one to put $\partial^2\ln(F)/\partial u^2 = 0$ at a point.[4]

The notion of flexible functional form in demand perhaps is defined more usefully in terms of demand functions (which are ultimately estimated) rather than in terms of the function representing preferences (which are unobservable). Hence, a flexible inverse demand system must have enough parameters to approximate, at a point, an arbitrary set of quantity elasticities and of normalized price levels (i.e., it must provide a first order local approximation to an arbitrary inverse demand system). If n is the number of goods, it is verified that (after imposing homogeneity, adding-up, and symmetry) the demand system (5) has $\frac{1}{2}(n - 1)(n + 4)$ free parameters [$(n - 1)$ parameters α_i, $(n - 1)$ parameters β_i, and $\frac{1}{2}n(n - 1)$ parameters γ_{ij}]. These constants could be chosen to represent at a point an arbitrary set of quantity elasticities [of which $\frac{1}{2}n(n + 1) - 1$ are independent after accounting for homogeneity, adding-up, and symmetry] and an arbitrary set of left-hand-side shares [of which $(n - 1)$ are independent after accounting for adding-up].

Simulation Results

To illustrate the approximation properties of the LIDS model, we report the results of a small simulation exercise. Specifically, we generate repeated stochastic realizations from a known structure and then look at how close the elasticity estimates from LIDS are to the true ones. Following similar studies by Kiefer and MacKinnon, and Wales, the data generating model chosen is a Linear Expenditure System (LES). Specifically, shares for a three-good system are generated using the inverse share equations of LES; that is,

$$w_i = \frac{\alpha_i[q_i/(q_i - \gamma_i)]}{\sum_j \alpha_j[q_j/(q_j - \gamma_j)]}, \tag{6}$$

where $\Sigma_i\, \alpha_i = 1$. The quantity data that we use for q_1, q_2, and q_3 are U.S. per-capita demand of beef, pork, and chicken, respectively, for the period 1960–89. These data, normalized to equal one at the mean of the sample period, are reported in the appendix.[5] The parameters used are: $\alpha_1 = .5$, $\alpha_2 = .3$, $\alpha_3 = .2$, $\gamma_1 = .2$, $\gamma_2 = .3$, and $\gamma_3 = -.3$. From this structure we generated 250 samples, each with 30 observations, by appending multinormal disturbances to the shares. The (full) covariance matrix used to generate the multinormal errors is the same as that used by Kiefer and MacKinnon, and Wales; that is,

$$\begin{bmatrix} .000036 & -.000025 & -.000011 \\ -.000025 & .000049 & -.000024 \\ -.000011 & -.000024 & .000035 \end{bmatrix}. \tag{7}$$

With these data, we estimate five different models 250 times. First, we estimate the nonlinear inverse demand system of equation (4), and we label this system NLIDS. Similar to the case of ALIDS discussed by Deaton and Muellbauer, the parameter α_0 is virtually impossible to estimate, so we set $\alpha_0 = 0$.[6] Second, we estimate the LIDS model, that is equation (5) with the geometric index $\ln(Q^*) = \Sigma_j w_j \ln(q_j)$. Third, as a benchmark, we estimate the true LES model of equation (6). Note that while LES has five free parameters, both NLIDS and LIDS have seven free parameters. Finally, for comparison, we estimate two versions of the inverse translog (ITL) system introduced by Christensen, Jorgenson, and Lau, and applied by Christensen and Manser, which, after an arbitrary normalization of parameters, can be written as:

(8)
$$w_i = \frac{\alpha_i + \sum_j \beta_{ij}\ln(q_j)}{1 + \sum_j \sum_i \beta_{ij}\ln(q_j)},$$

where $\Sigma_i \alpha_i = 1$ and $\beta_{ij} = \beta_{ji}$.

It can be verified that the ITL system has eight parameters, one more parameter than the LIDS model. Hence, ITL has one more parameter than is needed to make it a flexible (local) approximation to an arbitrary utility, which means that (8) could be suitably restricted without affecting its flexibility properties. Specifically, one can always find a monotonic transformation of utility such that $\Sigma_i \Sigma_j \partial^2 U/\partial\ln(q_i)\partial\ln(q_j) = 0$ at a point. To make this argument more explicit, let $\bar{U}(q)$ denote an arbitrary utility function for which, at a point q^0, $\partial\bar{U}/\partial\ln(q_i) = a_i$ and $\partial^2\bar{U}/\partial\ln(q_i)\partial\ln(q_j) = a_{ij}$. Because $\bar{U}(q)$ is ordinal, one can put $\Sigma_i a_i = 1$ without loss of generality. Now consider the monotonic transformation $U = G(\bar{U}(q))$. Then, at the point q^0, $\partial^2 U/\partial\ln(q_i)\partial\ln(q_j) = (G'' a_i a_j + G' a_{ij})$, where the derivatives G' and G'' are evaluated at $\bar{U}(q^0)$. Hence, at the point q^0, $\Sigma_i \Sigma_j \partial^2 U/\partial\ln(q_i)\partial\ln(q_j) = G'' + G'(\Sigma_i \Sigma_j a_{ij})$. If one chooses the transformation $G(\cdot)$ such that, at the point q^0, $G' = 1$ and $G'' = -(\Sigma_i \Sigma_j a_{ij})$, then at this point $\Sigma_i \Sigma_j \partial^2 U/\partial\ln(q_i)\partial\ln(q_j) = 0$. Because in the translog utility underlying (8), $\beta_{ij} = \partial^2 U/\ln(q_i)\partial\ln(q_j)$, it follows that we can set $\Sigma_i \Sigma_j \beta_{ij} = 0$ and still have a local approximation to an arbitrary utility function.[7] Given that the translog model (8) with the normalization $\Sigma_i \Sigma_j \beta_{ij} = 0$ achieves what Barnett and Lee called the "minimality" property, the resulting model is termed here the minimal inverse translog (MITL). Like LIDS and NLIDS (with $\alpha_0 = 0$), MITL has seven free parameters.

The approximation properties of the models considered are illustrated in terms of "how close" the estimated elasticities are to the true elasticities. We consider uncompensated quantity elasticities (flexibilities) and scale elasticities (in inverse demand analysis the concept of scale effect, discussed by Anderson, plays a role similar to that of the income effect of direct demands). Quantity elasticities are defined as $\epsilon_{ij} \equiv \partial\ln(p_i)/\partial\ln(q_j)$, and scale elasticities are defined as $\epsilon_i \equiv \partial\ln[p_i(\theta q)]/\partial\ln(\theta)$. Quantity elasticities for LES are computed as:

(9)
$$\epsilon_{ij} = w_j\left(\frac{\gamma_j}{q_j - \gamma_j}\right) - \delta_{ij}\left(\frac{q_i}{q_i - \gamma_i}\right),$$

whereas for LIDS and NLIDS they are computed as:

(10)
$$\epsilon_{ij} = \frac{\gamma_{ij}}{w_i} - \frac{\beta_i}{w_i}\left(\alpha_j + \sum_k \gamma_{jk}\ln(q_k)\right) - \delta_{ij},$$

and for ITL and MITL they are computed as:

(11)
$$\epsilon_{ij} = \frac{\beta_{ij} - w_i\left(\sum_k \beta_{kj}\right)}{\alpha_i + \sum_k \beta_{ik}\ln(q_k)} - \delta_{ij},$$

where δ_{ij} is the Kronecker delta ($\delta_{ij} = 1$ for $i = j$ and $\delta_{ij} = 0$ otherwise). Scale elasticities are readily computed using (9)–(11) because $\epsilon_i = \Sigma_j \epsilon_{ij}$.

A possible issue, in light of the arguments presented in Green and Alston, is whether (10) is an appropriate formula for LIDS. It is verified easily that under procedures we have followed (that is, scaling the right-hand-side variables, and setting $\alpha_0 = 0$), each parameter of LIDS will approximate the corresponding parameter of NLIDS. Thus, formula (10), which is derived from NLIDS, is appropriate for LIDS as well. An alternative for LIDS, which is consistent with taking $\ln(Q^*)$ as given in estimation, is to use:

$$\epsilon_{ij} = \frac{\gamma_{ij}}{w_i} - \beta_i \frac{w_j}{w_i} - \delta_{ij}. \tag{12}$$

To investigate what may be called the "local" approximation properties of the model, elasticities were computed at the mean point (at which $q_i = 1 \ \forall \ i$), and summary statistics are reported in table 1.[8] The first column of table 1 reports the elasticities, at the mean point, of the true LES model used to generate the data. Then, for each of LES, ITL, MITL, NLIDS, and LIDS we report the mean, computed over the 250 replications, of the estimated elasticities at the mean point, and the root mean square error (RMSE) of each of these estimated elasticities. Also, for each model we report the average RMSE for the 12 elasticities involved.

All models seem to provide a reasonable approximation. As expected, the best results are obtained by estimating the true LES model. The performances of LIDS, NLIDS, and MITL are similar, with an average RMSE roughly double that of the true model. The fact that MITL does better than ITL perhaps may seem surprising. The reason is that the restriction ($\Sigma_i \Sigma_j \beta_{ij} = 0$) is not rejected; when estimating ITL, the empirical distribution of the quantity ($\Sigma_i \Sigma_j \beta_{ij}$) over the 250 replications had a mean of .4 and a standard deviation of 1.7. Maintaining the restriction ($\Sigma_i \Sigma_j \beta_{ij} = 0$) in MITL results in a considerable efficiency gain (the average absolute t-ratio for the five independent β_{ij} in MITL over the 250 replications was about 3, whereas the average absolute t-ratio for the six independent β_{ij}s in ITL was about 1.4).

Table 1 makes it clear that the linear approximation made possible by the use of $\ln(Q^*)$ instead of $\ln(Q)$ is very good, as LIDS and NLIDS produce virtually identical results. In the context of ALIDS for direct demands, it is believed that the use of the Stone index is likely to produce good approximations because prices typically are highly correlated (Deaton and Muellbauer). In our application, however, the data are not very correlated: the coefficient of correlation between q_1 and q_2 is $-.25$, between q_1 and q_3 is .05, and between q_2 and q_3 is .27. Yet the approximation made possible by the use of $\ln(Q^*)$ appears quite good, suggesting that it is robust to the design matrix of the exogenous variables.

Although the results of table 1 are encouraging as to the approximation properties of LIDS, and consistent with the notion that all the models considered (apart from the true model) are capable of providing a local approximation to an arbitrary demand system, the question arises as to "how local" these results are. If the inverse demand system is to be used for forecasting or welfare analysis, one would want to be reassured that the approximation abilities of the model extend to a reasonably wide range of the data. To investigate this issue, we consider what we term the "extended" approximation properties of the models. Specifically, we evaluate true and estimated elasticities at each of the 30 sample points, and for each of the 12 elasticities we compute the mean square error over the resulting 7,500 estimates (30 sample points for 250 replications).

The square roots of such mean square errors, and their average over all 12 elasticities, are reported in table 2.[9] The approximation abilities of MITL, NLIDS, and LIDS hold up very well in this extended analysis, with the average RMSE increasing only by .004 relative to the approximation at the mean point (up 6.6%). For these models the average RMSE is still roughly twice the RMSE of the true LES model. ITL, on the other hand, shows a much larger increase (up .02 or 30%) in the average RMSE relative to the result at the mean. Again, the restriction ($\Sigma_i \Sigma_j \beta_{ij} = 0$) seems very fruitful in terms of improving the efficiency of the translog inverse demand system.

Table 1. Local Approximation Properties

Elasticity	True Value	LES Mean	LES RMSE	ITL Mean	ITL RMSE	MITL Mean	MITL RMSE	NLIDS Mean	NLIDS RMSE	LIDS Mean	LIDS RMSE
ϵ_{11}	−1.121	−1.120	.023	−1.123	.029	−1.130	.025	−1.132	.027	−1.132	.027
ϵ_{12}	.152	.150	.029	.152	.035	.150	.031	.148	.031	.147	.031
ϵ_{13}	−.029	−.030	.005	−.028	.009	−.030	.008	−.030	.008	−.030	.008
ϵ_{21}	.129	.129	.025	.133	.037	.135	.037	.134	.038	.133	.039
ϵ_{22}	−1.276	−1.273	.052	−1.276	.064	−1.278	.063	−1.278	.063	−1.278	.063
ϵ_{23}	−.029	−.030	.005	−.028	.015	−.028	.015	−.027	.015	−.028	.015
ϵ_{31}	.129	.129	.025	.130	.115	.151	.093	.161	.099	.166	.104
ϵ_{32}	.152	.150	.029	.150	.111	.167	.101	.175	.105	.176	.107
ϵ_{33}	−.799	−.798	.032	−.805	.043	−.799	.035	−.800	.035	−.799	.035
ϵ_1	−.998	−.999	.039	−.999	.055	−1.010	.043	−1.014	.044	−1.015	.045
ϵ_2	−1.176	−1.174	.062	−1.171	.078	−1.171	.078	−1.171	.079	−1.172	.080
ϵ_3	−.517	−.519	.046	−.525	.214	−.481	.165	−.464	.175	−.456	.184
Avg. RMSE			.031		.067		.058		.060		.062
Avg. log-likelihood[a]		227.73		229.40		228.68		228.66		228.60	

[a] Average maximized log-likelihood computed assuming that each model in turn is the true model.

Table 2. Extended Approximation Properties

Elasticity	LES	ITL	MITL	NLIDS	LIDS
			RMSE		
ϵ_{11}	.024	.045	.029	.030	.030
ϵ_{12}	.030	.043	.036	.036	.036
ϵ_{13}	.005	.013	.009	.009	.009
ϵ_{21}	.026	.051	.040	.041	.042
ϵ_{22}	.053	.086	.071	.069	.069
ϵ_{23}	.005	.017	.015	.016	.016
ϵ_{31}	.026	.140	.096	.101	.106
ϵ_{32}	.030	.132	.104	.107	.110
ϵ_{33}	.033	.062	.043	.048	.049
ϵ_1	.041	.069	.050	.051	.052
ϵ_2	.063	.104	.087	.087	.089
ϵ_3	.047	.277	.169	.178	.186
Avg.	.032	.087	.062	.064	.066

Note: Entries are RMSEs over all 30 sample points.

Conclusion

In this article we have illustrated a linear specification for an inverse demand system. This specification is based on a distance function which has a parametric structure similar to the PIGLOG cost function underlying the ALIDS model commonly used for direct demand models. The approximation properties of the new model were illustrated with a simulation exercise. Of course, although the results presented are useful in terms of ranking the models used relative to the performance of the true model, the actual size of the approximation error (say, the average RMSE) cannot be generalized because it depends, among other things, on the design matrix of right-hand-side variables, on the structure and parameters of the true model, and on the signal-to-noise ratio of the stochastic terms.

The simulation results show that the new (nonlinear) inverse demand system derived from the chosen parametric specification of the distance function performs well relative to the true model, and very similar to that of an (appropriately restricted) inverse translog demand system. Moreover, the linear version of the new inverse demand system, which we have termed LIDS, results in a good approximation to the nonlinear model. The simplicity of LIDS is likely to make it a useful specification for empirical analysis, especially in applications where linearity is appealing (for example, in dynamic demand systems). Because the derivation of LIDS parallels that of ALIDS for direct demand systems, the simulation results reported in this article are somewhat more general and could be interpreted, with minor modifications, as evidence of the approximation properties of ALIDS models, and as supporting the linear version of ALIDS as a good approximation to the nonlinear ALIDS.

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Notes

[1] After the first draft of this article was completed, a paper by Eales and Unnevehr, giving a similar derivation of the linear inverse demand system, came to our attention. They call this system the "Inverse Almost Ideal Demand System," and use it to model U.S. quarterly meat demand. Barten and Bettendorf also allude to an "Almost Ideal Inverse Demand System," but they do not provide an explicit derivation.

[2] Hanoch formalizes this parallel further by developing the concept of "symmetric" duality, which in our case requires defining the distance function in terms of $(1/u, q)$ rather than (u, q). In Hanoch's words, this approach allows ". . . 'getting two for the price of one' in the search for useful functional forms" (p. 111).

[3] The Stone index fails what Diewert calls the "commensurability test," which defines a fundamental property of index numbers. This property requires that the index should be invariant to the choice of units of measurement.

It is clear that the Stone index, or equivalently the geometric aggregator $\ln(Q^*) = \Sigma_i w_i \ln(q_i)$, is not invariant to the choice of units of measurement. This problem arises when one uses natural units (i.e., pounds or metric tons). In such a situation, an easy way to get around the problem is to scale prices (or quantities for LIDS) by dividing through by the mean. When one aggregate indices with a common base, such as in Deaton and Muellbauer, the problem clearly does not arise.

[4] A similar argument applies to the ALIDS model (Deaton and Muellbauer, p. 313).

[5] These data are from U.S. Department of Agriculture sources. The sample means were 78.417 lbs./capita (retail cut equivalent) for beef, 60.037 lbs./capita (retail cut equivalent) for pork, and 44.283 lbs./capita (ready-to-cook weight) for chicken.

[6] Fixing α_0 basically entails a local normalization of the utility function at the point $q_i = 1$ (the mean point in our case).

[7] The direct demand system derived from an indirect translog utility function also has one more parameter than the linear ALIDS model (or the nonlinear ALIDS with α_0 set to some constant). In this context, imposing the restriction $\Sigma_i \Sigma_j \beta_{ij} = 0$ reduces the indirect translog utility function to be a member of the PIGLOG family of preferences, thereby giving it desirable aggregation properties (Lewbel).

[8] Given that we are evaluating elasticities at the point $q_i = 1$, the distinction between formulae (10) and (12) for LIDS is immaterial, as the two formulae reduce to the same expression at this point.

[9] When evaluating elasticities away from the point $q_i = 1$, formulae (10) and (12) for LIDS are not identical. However, for the three-digit rounding reported in table 2, formulae (10) and (12) give the same results, whereas at a five-digit rounding level formula (10) gives slightly smaller RMSEs.

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Appendix

Table A1. Normalized U.S. Per-Capita Consumption of Beef, Pork, and Chicken

Year	q_1	q_2	q_3
1960	.81870	1.00439	.62778
1961	.83911	.96108	.67520
1962	.84421	.98440	.67294
1963	.89139	1.01605	.69552
1964	.94240	1.01605	.70455
1965	.93858	.91111	.75198
1966	.99596	.90944	.80166
1967	1.01764	.99939	.81972
1968	1.04570	1.02937	.82424
1969	1.05207	1.00938	.86037
1970	1.07630	1.03104	.90553
1971	1.06738	1.13098	.90553
1972	1.09033	1.03936	.93715
1973	1.02657	.94942	.90779
1974	1.08905	1.02271	.91457
1975	1.12221	.84115	.90102
1976	1.20255	.89279	.95973
1977	1.16557	.92943	.98909
1978	1.11201	.92943	1.04780
1979	.99469	1.06102	1.13587
1980	.97428	1.13431	1.12458
1981	.98321	1.08101	1.15845
1982	.97938	.97440	1.19006
1983	.99724	1.03104	1.20587
1984	.99596	1.02437	1.24652
1985	1.00489	1.03270	1.30072
1986	.99979	.97607	1.32556
1987	.93603	.98440	1.41588
1988	.91945	1.05102	1.45653
1989	.87736	1.04270	1.53782