U.S. Wheat Storage Control Under Joint Criteria of Mean Benefits and Price Variation

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A method based on stochastic dynamic programming is developed to derive efficiency frontiers for the trade-off between long-run average social benefits and price variation. The method is used to quantify the importance of price variation *per se* as a criterion in U.S. wheat storage policy. The results suggest that a single criterion of maximum expected social benefits, calculated by the traditional surplus measures, is satisfactory because price variation is incidentally reduced enough that further reductions can be attained only at considerable opportunity cost.

Empirical stochastic commodity storage models have been dominated by a criterion of expected social benefits, where benefits are measured by the sum of consumers' and producers' surpluses. [Burt, Koo, and Dudley; Just and Hallam; Taylor and Talpaz]. Others have taken supply as exogenous and focused on consumers' surplus as the primary criterion. [Gustafson, Johnson and Sumner; Kennedy; Reutlinger; Konandreas and Schmitz]. Konandreas and Schmitz and Just and Hallam made empirical attempts to use theoretical results deduced about the benefits of storage where the criterion was the sum of consumers' and producers' surpluses. [Hueth and Schmitz; Just *et al.*; Turnovsky]. The Kennedy and Reutlinger studies also considered producer benefits measured as market revenues.

There are several reasons for questioning the applicability of these surplus measures of social value, which are associated with static economic theory, in the context of dynamic stochastic models. Rogerson has recently analyzed the conditions under which results of static theory can be applied to stochastic storage problems. He concluded that stochastic variation emanating from the supply side of the market can be reconciled in a consistent manner so that consumers' surplus is a reasonable approximation to consumer utility under comparable assumptions made for the static case, but variation arising from the demand side causes serious conceptual problems.

In the context of world markets, most of the variation in commodities stems from the supply side, either directly or indirectly. Although demand for U.S. exports of wheat is subject to large disturbances, the disturbances are dominated by variations in supply instead of demand conditions in foreign nations. However, tariffs and other governmental restrictions on imports in world trade raise serious doubts about the validity of using the area under an empirical demand equation for U.S. exports as an estimate of foreign consumers' surplus (see Carter and Schmitz for an analysis of the market power of major world wheat importers).

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Just and Hallam have examined the consequences of price variation entering directly in supply response and the likely effects on empirical social value measurement. They conclude that social benefits of commodity storage programs will be underestimated if the influence of price variability on supply is ignored. Just found that price variation is a significant factor in California field crops supply response [Just, 1974]. Hazell and Scandizzo have analyzed risk response in a programming model and their results illustrate the importance of price variation in supply response.

Although distributed lags have been used in consumer demand studies for over 20 years [Nerlove], little theoretical research has been done to reconcile the apparent inconsistency between empirical reality and classical consumer preference theory. If consumers respond to changing prices with a distributed lag, there are costs (psychological or otherwise) associated with rapid changes in consumption, and these costs would not be reflected by consumers' surplus calculated from traditional static demand curves containing only mean prices.

The possible influence of price variation on both the supply and demand sides suggests that there are social costs associated with price variation not measured by the traditional surplus calculations used in commodity storage models. Ideally, demand and supply equations in commodity storage models should incorporate price variability, but the realities of empirical econometrics are not very encouraging in this regard. In this study, traditional demand and supply curves without terms to account for risk through price variation are used as the basis for surplus measurement, but a penalty term on price variation is introduced into the criterion function of the storage model. The rationale for this method is that the simplified expected surplus measures are useful approximations to more ideal measurement of expected net social benefits, and the price variation penalty term allows computation of an efficiency frontier between expected benefits and price variation. The penalty term and associated parameter are not associated with a social welfare function. The objective of the research is to quantify the trade-off between price variability and an admittedly quite crude measure of expected net social benefits.

Empirical Economic Relationships

The wheat industry is described by three economic relationships, domestic supply and demand plus foreign demand for U.S. exports. Domestic supply was partitioned into separate acreage and yield equations while demand was estimated separately for food and feed. The various equations estimated in Burt *et al.* used data through 1976 and were tested for prediction on more recent data for this study. They appeared to stand up quite well for time series estimates with one exception, the autoregressive price equation for foreign demand. Considerable computational costs could be saved by salvaging the earlier equations, and there is also an advantage in making comparisons with results reported in Burt *et al.* A compromise was made by reestimating the autoregressive price equation for foreign demand with data from 1959-79.

An oversight in the earlier study was not recognizing short-run/long-run implications of an autoregressive price equation when an *a priori* estimate of price elasticity of demand was introduced into the equation. Since a linear instead of log-linear autoregressive equation performed better on the extended sample used here, let the equation be

$$(1) \qquad p_t = \alpha + \beta p_{t-1} + \gamma x_t,$$

where p and x denote price and exports, respectively. Then the short-run elasticity is $1/(1-\beta)$ times that for the long-run, $0<\beta<1$. Since the *a priori* estimated elasticity from results in Konandreas (equal to -2.50) was a static concept, it would be preferable to introduce another term into (1) to remove the distributed lag response,

$$(2) \quad p_t = \alpha + \beta p_{t-1} + \gamma x_t - \beta\gamma x_{t-1}.$$

However, this formulation would call for another state variable, lagged exports, in the dynamic optimization model for storage control.

The extra state variable was avoided by choosing γ such that long-run price elasticity equaled the static *a priori* estimate. The final empirical demand equation for U.S. exports which was used in the storage control model was

$$(3) \qquad p_t = 0.722 + 0.5959p_{t-1} - 0.388(10)^{-3}(x_t - \bar{x}) + \varepsilon_t,$$

where p is average price per bushel at the Gulf Ports and x is U.S. exports in million bushels; $\bar{x}$ = 813 which is mean exports during 1960-79; average price for the period is \$1.97 in 1967 dollars, and the estimated standard deviation of ε_t is 0.229. The t-ratio for the coefficient on p_{t-1} is 6.2, but no precision is available for the coefficient on exports because it was derived by aggregation of world regional demand elasticities from Konandreas. The coefficient on exports implies a price elasticity of -2.50 and -6.25 in the long- and short-run, respectively, at mean price and exports.

The Storage Model

Only the simplified asymptotic storage model from Burt *et al.* is used here. This model simplifies the higher order time dependencies in the supply and demand equations so that only lagged price enters as a state variable to describe the dynamics of the domestic market. The asymptotic approximation will provide good results if the system is not "too far" from the long-run equilibrium, and should be adequate for evaluation of a joint mean benefit and price variability criterion.

The partial equilibrium model is specified to satisfy competitive conditions in the wheat industry except that exports and storage are completely under governmental control. The foreign demand equation, (3), made exports a natural choice for the decision variable. Since no disposal of wheat outside of the market is allowed, and the demand equation for exports is linear, long-run mean price is independent of the governmental decision rule on exports. Independence of long-run mean price from the dynamic decision rule is an important simplification in the model, otherwise price variance *per se* could not be used in the objective function. The point around which expected squared deviations are defined as a measure of price variation must be independent of the decision rule to meet the Markovian dependence requirement of dynamic programming.

The Gulf Ports price is used in the decision model and an auxiliary equation relates it to average price received by farmers which enters the domestic demand and supply equations. Therefore, p denotes Gulf price in all the relationships below.

Net production after deduction of domestic consumption is denoted $w_t(x_t, p_{t-1})$ which is a random variable with the arguments x_t and p_{t-1} indicating that these variables appear as parameters in the probability density function of the random variable w_t. Lagged price p_{t-1} enters because it is a variable in the supply function and x_t enters because it affects current price through (3) and consequently domestic demand. Stocks are defined to include both old crop carryover and current year production so that the year is defined to begin shortly after harvest, say the first of October. With these definitions, and letting s denote wheat stocks, dynamic behavior of stocks is given by

$$(4) \qquad s_{t+1} = s_t - x_t + w_t(x_t, p_{t-1}).$$

The discrete time model is formulated such that all exports occur at the beginning of the year, stocks are measured at the beginning of the year, and price is an average for the crop year. Conditional expected net social benefits (exclusive of any costs associated with price variability *per se*) during year t are denoted $G_t(x_t, s_t, p_{t-1})$.

The model is quite analogous to the joint mean/variance criterion of portfolio theory or farm diversification at the micro-level [see

S. R. Johnson]. The primary differences are (1) mean and variance are calculated for different random variables instead of the same variable, and (2) the model is basically dynamic with discounting instead of static.

The optimization problem can be stated as

$$(5)\quad \sum_{t=1}^{\infty} [G_t(x_t, s_t, p_{t-1}) - \lambda E(p_t - \mu)^2]/(1+r)^t$$

a maximum with respect to $x_1, x_2 \ldots$, subject to (3) and (4) together with the inequality

$$(6)\quad 0 \leq s_t - x_t \leq C,\ t = 1, 2, \ldots,$$

and nonnegativity constraints on the variables, where E is the expectation operator, r is the discount rate, C is a storage capacity limit, λ is a penalty parameter, and μ is long-run mean price. Note that price p_t appearing in the variance formula of the criterion function is given by the right hand side of (3) and contains the state variable p_{t-1} and decision variable x_t. The assumptions which justify the inequality constraint in (6) are explained in Burt *et al.* (p. 181).

Writing (3) in an abbreviated form as

$$(3)'\quad p_t = \psi(x_t, p_{t-1}) + \varepsilon_t,$$

the functional equation of dynamic programming associated with the above optimization problem is

$$(7)\quad f_n(s,p) = \underset{x}{\text{Max}}\ [G_n(x,s,p) - \lambda E\{\psi(x,p) + \varepsilon - \mu\}^2 + \alpha E\{f_{n-1}(s - x + w_n(x,p), \psi(x,p) + \varepsilon)\}],$$

$$x \geq s - C,$$

where n denotes number of stages remaining in the planning horizon and α is a discount factor. This is the counterpart of (12) in Burt *et al.* Numerical solutions are obtained from a discrete variable approximation of (7) which yields a Markov process with a finite number of states. Additional details and assumptions required to make the model operational are given in Burt *et al.*, e.g., enforcement of the inequality constraint on storage given in (6).

There is no straightforward economic justification for discounting the variance terms in (5) and (7). It is possible to reformulate (7) so that the variance terms are excluded from discounting, but computations are considerably easier with (7) as given above. Discounting the variance terms also provides a convergent criterion under the infinite planning horizon specification used in (5).

Discounting the variance component in the dynamic programming formulation in (7) could be avoided by partitioning $f_n(s,p)$ into two terms, one each for expected benefits and price variance which are denoted $g_n(s,p)$ and $h_n(s,p)$, respectively. Then (7) is replaced by

$$(8)\quad f_n(s,p) = g_n(s,p) + h_n(s,p)$$
$$= \underset{x}{\text{Max}}\ [G_n(x,s,p) - \lambda E\{\psi(x,p) + \varepsilon - \mu\}^2 + \alpha E\{g_{n-1}(s - x + w_n(x,p), \psi(x,p) + \varepsilon)\} + E\{h_{n-1}(s - x + w_n(x,p), \psi(x,p) + \varepsilon)\}],$$

$$x \geq s - C.$$

The functions $g_n(\cdot)$ and $h_n(\cdot)$ are defined recursively by adding only the periodic mean benefit and variance terms, respectively. For example, the right hand side of the recursion for $h_n(\cdot)$ would be the sum of the price variance term and the term for $E\{h_{n-1}(\cdot)\}$ in (8).

The price variance component of (5) can be written as

$$(9)\quad -\lambda \sum_{t=1}^{\infty} [E\{\psi(x_t, p_{t-1}) - \mu\}^2 + \text{Var}(\varepsilon_t)]/(1+r)^t$$

by using (3)′ and the specification that $E(\varepsilon_t) = 0$. If a storage agency had unlimited stocks of wheat and no constraints on funds to buy wheat, the absolute minimum on price variance would be $Var(\varepsilon_t) = \sigma_\varepsilon^2$. The associated decision rule would be to choose x_t each period such that $\psi(x_t, p_{t-1}) = \mu$ for any given lagged price, p_{t-1}.

The Efficiency Frontier

Conceptualization of the efficiency frontier tradeoffs between mean benefits and price variance are probably best understood in the context of the optimization problem as stated in (5) and (6). The expression in (5) can be viewed as a Lagrangean form of constrained optimization where λ is the multiplier associated with a fixed storage capacity, C. Taking storage capacity, C, as fixed, each solution to the optimization problem for a given value of λ can be interpreted as a point on the short-run efficiency frontier because of the one-to-one correspondence between λ and the implicit fixed price variance.

The long-run efficiency frontier is defined as the envelope of the short-run curves determined by expected benefits being a maximum with respect to storage capacity for any given level of price variance. But the monotonic relationship between λ and price variance for a short-run curve lets the long-run curve be defined as being associated with maximum expected benefits with respect to capacity for any given level of λ, instead of a given level of price variance. This observation is very important for numerical calculations used to approximate long-run efficiency frontiers because it is far easier to obtain the long-run curve directly rather than indirectly through many short-run curves.

The efficiency frontiers, both short- and long-run, are dependent on the initial state of the decision process because discounted expected benefits are dependent on the initial state. When the discount rate is set equal to zero, expected benefits and price variance must be defined as asymptotic annual measures which are independent of the initial state. In the empirical work reported below, the same discount rate is used on both the mean and variance terms of the criterion function. Discounting price variance terms is likely to appeal to policy makers because of the greater latitude provided for controlling price dispersion in the near future.

Empirical Results

Two criteria were analyzed in Burt *et al.*, but only their domestic criterion is used here which is the sum of consumers' and producers' surpluses minus storage costs. Foreign consumers' surplus is excluded because foreign demand for U.S. exports is at least as much a result of arbitrary government trade policies as it is a reflection of foreign consumers' preferences [D. Gale Johnson].

Discrete values are in intervals of 200 million bushels for wheat exports and stocks, and 50¢ per bushel for the price state variable. The stock state variable is the sum of old crop carryover and production for the immediately completed harvest with a range of possible values between 1.0 and 3.8 billion bushels. Exports can vary between .05 and 2.05 billion bushels. The price state variable has a range of \$1.00 to \$5.50 and all dollar measures are in 1976 dollars. Discounting was at a real rate of six percent. For additional details on the model, the reader is referred to Burt *et al.*

Fortunately, the lower limit on price variability which could be achieved with unlimited storage capacity and continuous variation on exports is known. The variance of the error term in the autoregressive price equation, (3), is this lower bound, 0.15 measured in 1976 dollars; the standard deviation is 0.39. The latter figure should be kept in mind with respect to the results reported below which use standard deviation as the measure of price variability.

Since mean price, μ, in (7) is not dependent on the export decision rule, an arbitrary rule was used to calculate asymptotic mean price from the resulting Markov chain associated with the discrete variable approximation to the model. The calculated value was 3.08.

The optimal decision rule for given values of λ and C specifies the level of exports for each possible state which might be experienced, and is not dependent on any initial state specified in the computational algorithm, but the optimal rule does depend on the discount rate. Amortized (time weighted) measures of expected benefits and price variance for construction of efficiency frontiers are dependent on the initial state. Since storage capacity, C, is held fixed in a short-run efficiency curve, the same optimal decision rules can be applied over various values of λ for any arbitrary choice of the initial state, even though the curve itself will change with the initial state chosen. The same simplicity does not hold for long-run efficiency curves because the optimal storage capacity, for a given value of λ, changes with the initial state chosen for making the evaluation across capacity levels. Therefore, an entirely new set of computations for deriving optimal decision rules is required for each initial state associated with a long-run curve.

The long-run efficiency frontier requires heavy computational expense because a search must be made over storage capacity for each value of λ, and the dynamic programming problem of (7) must be solved for each pair of values (λ,C). Each optimization over C for given λ yields only one point on the long-run efficiency curve. A quadratic approximation method in the neighborhood of optimal capacity for given λ was used in the search procedure. Because of the heavy computational burden, the long-run curve was estimated for only one initial state, 3.0 billion bushels of wheat stocks and a price of $3.00 per bushel. This state is near that prevailing during recent years. An arbitrary discount rate of 6 percent was used.

Optimal decision rules for an initial stock of 3.0 billion bushels were used to construct two sets of long- and short-run curves, one set for an initial state associated with the optimal rules and the other for an initial stock of 2.0 instead 3.0 billion bushels. In addition, optimal decision rules were computed for the short-run under a zero interest rate which provided asymptotic results independent of the initial state. All the short-run curves were for a fixed capacity of 2.6 billion bushels.

Some preliminary exploration was required to find an order of magnitude value for λ which is in units of dollars. These preliminary computer runs suggested increments in λ at intervals of about 250 million. Optimal capacity was then explored with λ going from 0 to 2.0 billion in discrete jumps of 250 million. The results are graphed in Figure 1 which is necessarily rather crude because of the number of points to which a free-hand curve was fitted. Two adjustments are made in the optimization as λ increase: (1) the decision rule changes to reduce price variance for a given capacity, and (2) capacity is increased to permit more flexibility in the decision rule. The graph suggests that the first adjustment jointly with modest changes in capacity prevail until λ reaches about 1.25 billion. Then, between 1.25 and 1.75 billion substantial increases in capacity are optimal, but for $\lambda > 1.75$, extra capacity helps very little in reducing price variance and the cost in reduced expected benefits is large.

Problems in numerical accuracy occur when the capacity constraint does not coincide with the discrete intervals on the wheat stocks state variable. Therefore, only points for which λ equaled 0.25, 0.75, 1.25, 1.50, and 2.00 billion were used to construct the long-run efficiency curve with corresponding optimal capacity levels of 2.0, 2.2, 2.4, 2.6, and 3.0 billion bushels. This long-run curve and the short-run curve with capacity fixed at 2.6 billion bushels are graphed in Figure 2 where λ is decreasing from left to right and the dots are points at which calculations were made. The data used to construct the graphs are given in the first four columns of Table 1. The horizontal axis was translated by subtracting 27 billion dollars from expected amortized net benefits; benefits have amortized fixed costs of storage deducted which were estimated at 16¢ per bushel of capacity measured in 1976 dollars.

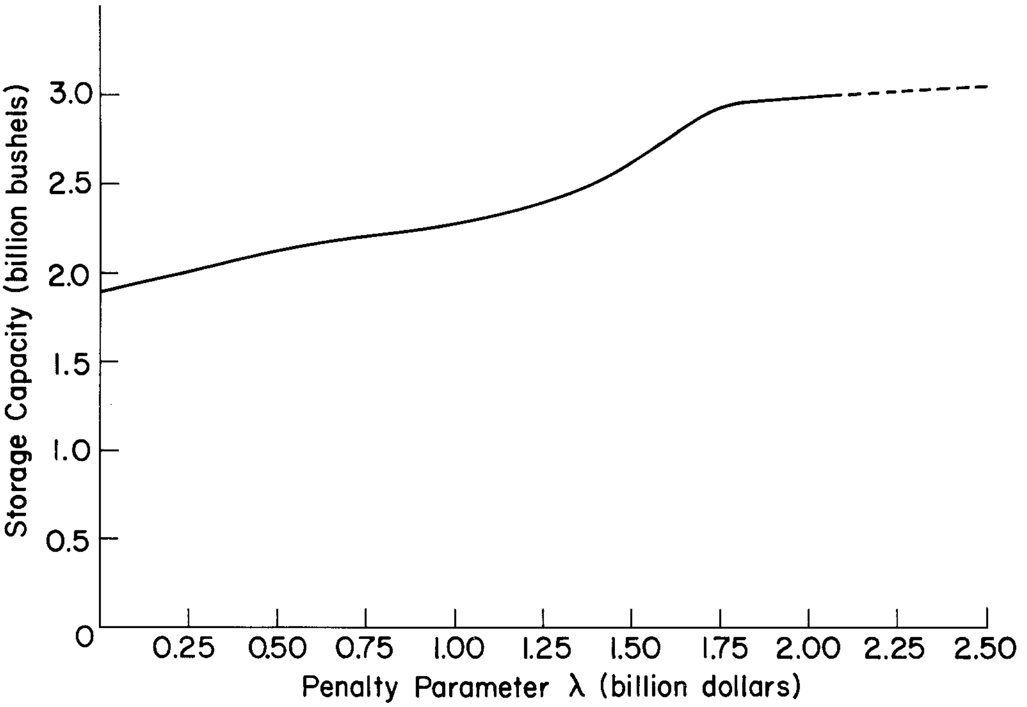

Figure 1. Optimal Storage Capacity.

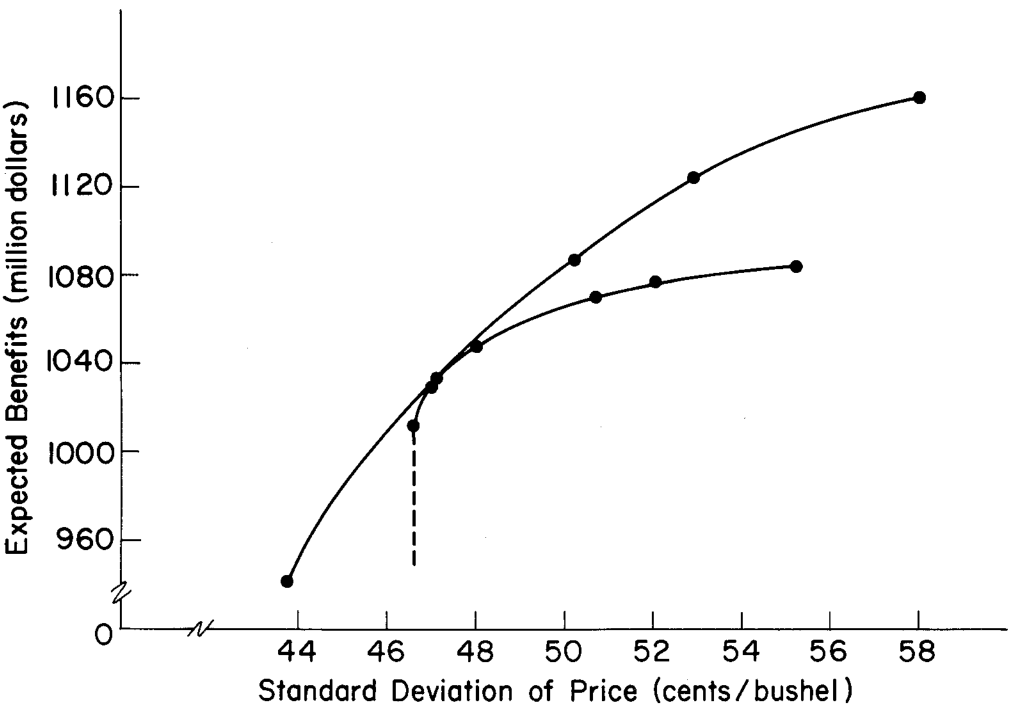

Figure 2. Efficiency Frontiers for Initial Stock of 3.0 Billion Bushels.

TABLE 1. Amortized Expected Benefits and Standard Deviation of Price.

	3.0 Billion Bushels Initial Capacity[a]				2.0 Billion Bushels Initial Capacity				Asymptotic Results[b]	
	Short-Run[c]		Long-Run[d]		Short-Run		Long-Run		Short-Run	
Penalty Parameter	Expected Annual Benefits	Standard Deviation Price	Expected Annual Benefits	Standard Deviation Price	Expected Annual Benefits	Standard Deviation Price	Expected Annual Benefits	Standard Deviation Price	Expected Annual Benefits	Standard Deviation Price
($ bil.)	($ mil.)	(¢/bu.)	($ mil.)	(¢/bu.)	($ mil.)	(¢/bu.)	($ mil.)	(¢/bu.)	($ mil.)	(¢/bu.)
0	28,084	55.21	NC[e]	NC	27,954	52.90	NC	NC	26,366	48.61
0.25	28,077	52.05	28,160	58.03	27,947	50.54	28,043	51.51	26,364	48.04
0.50	28,070	50.68	NC	NC	27,942	49.54	NC	NC	26,362	47.61
0.75	NC[e]	NC	28,127	52.69	NC	NC	28,003	49.73	NC	NC
1.00	28,048	47.95	NC	NC	27,931	48.55	NC	NC	26,360	47.37
1.25	NC	NC	28,087	50.20	NC	NC	27,964	48.86	NC	NC
1.50	28,034	47.11	28,034	47.11	27,914	47.36	27,914	47.36	26,356	47.16
2.00	28,031	46.97	27,943	43.77	27,910	47.19	27,835	46.36	26,354	47.06
∞	28,012	46.57	NC	NC	27,888	46.85	NC	NC	26,352	46.85

[a]The decision rule used was optimal for an initial wheat stock of 3.0 billion bushels and price of $3; other initial capacity results assume an initial price of $3. Discounting was at 6 percent.

[b]Optimal decision rule with no discounting and fixed capacity of 2.6 billion bushels.

[c]Storage capacity equal to 2.6 billion bushels.

[d]Optimal storage capacities of 2.0, 2.2, 2.4, 2.6, and 3.0 billion bushels in association with the penalty parameter equal to 0.25, 0.75, 1.25, 1.50, and 2.00 billion dollars, respectively.

[e]Not computed.

These mean-benefit/price-standard-deviation curves are crude approximations because of the few data points used and other discrete approximations in the model, particularly the discrete intervals on exports. The short-run curve falls vertically when the standard deviation of price approaches 46.6¢/bushel from the left. This short-run lower limit would be somewhat less if exports were varied continuously and the state variables were treated as continuous in the optimization model. Additional points on the long-run curve for $\lambda > 2.0$ billion could not be used because of problems associated with wheat stocks being truncated at 3.8 billion bushels and an associated bias.[1] Points to the right of the truncated curves in Figure 2 are of no interest because they would imply $\lambda < 0$.

The absolute extremes portrayed by the long-run efficiency curve lend some perspective to the tradeoffs involved. A decrease in the standard deviation of price by 14¢ per bushel is achieved at a sacrifice of $218 million in expected benefits and requires an increase in optimal capacity from 2.0 to 3.0 billion bushelss. These figures imply an average cost in benefits per penny reduction in standard deviation in price equal to $15.5 million. The smallest marginal change from the graph is at $\lambda = 0$ where a one cent reduction in the standard deviation of price is achieved at a cost of $4.0 million in expected benefits. The smallest marginal sacrifice in benefits per penny reduction in standard deviation of price on the short-run curve is $1.6 million, and the standard deviation can be reduced from 55.2¢ to 50.7¢ at a cost of $14 million in benefits which is $3.1 million per penny reduction in standard deviation of price. These results suggest a more economical tradeoff on the short-run curve where the reduction in price variation is achieved without additional storage capacity.

The mean benefit/price variance tradeoff from the same decision rule when the initial state is 2.0 instead of 3.0 billion bushels is portrayed in Figure 3 and the supporting data are in the middle four columns of Table 1. The short-run curve corresponds to an optimal decision rule, but the long-run curve is only an approximation because optimal storage capacity is dependent on the initial state. Notice that the long-run curve does not fit smoothly through the five data points as it did in Figure 2, and the tangency drawn between the two curves probably does not exist. A larger initial stock of wheat would tend to increase optimal capacity associated with the long-run curve, which would tend to make the long-run curve in Figure 3 steeper than under the optimal decision rule for an initial stock of 2.0 instead of 3.0 billion bushels.

Since the short-run curve in Figure 3 is completely valid for comparison with the same curve in Figure 2, the absolute limits on controlling the standard deviation of price are compared. The length of intervals on which controls can be effected are 8.6 and 6.1 cents in Figures 2 and 3, respectively. Thus, there is greater flexibility available in controlling price variability when initial stocks are high, a result which emanates from price variance being a weighted measure using the discount factor.

Asymptotic results for the optimal decision rule under a zero discount rate and fixed capacity of 2.6 billion bushels for a short-run efficiency frontier are given in the last two columns of Table 1. The asymptotic probability distribution can be interpreted as being associated with a randomly chosen year in the decision process many years after its initiation. The absolute limit on controlling the asymptotic standard deviation of price is an interval length of 1.8¢. Mean price and exports do not change with λ, but mean stocks increase with λ, being equal to 2.22

[1]Numerical solution of stochastic dynamic programming models is effected by truncation of the state variable where the probability of an outcome in the truncated portion is arbitrarily small. The storage model here is particuarly vulnerable to errors caused by too severe a truncation on the upper end of stocks because the result is a defacto disposal activity for taking wheat out of the world market.

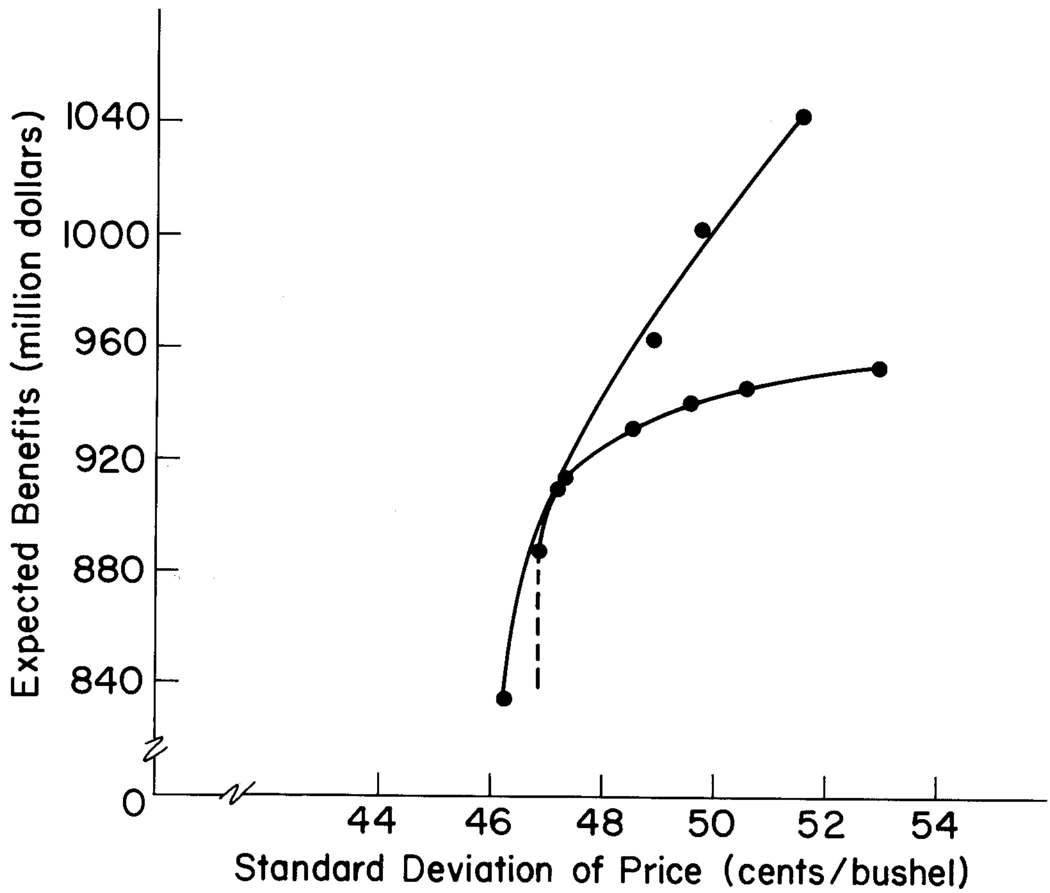

Figure 3. Efficiency Frontiers for Initial Stock of 2.0 Billion Bushels.

and 2.49 billion bushels with λ at zero and 2.0 billion, respectively.

When the price variability measure is not discounted in conjunction with some initial state of the decision process, there is little latitude for controlling price variability under the joint criteria function which includes both expected benefits and price variance. Part of this apparent limitation on control of price variation can be explained by the tendency of the single criterion of maximum expected benefits to reduce price variation without any explicit introduction into the criterion function. When $\lambda = 0$, which gives no weight to reduction in price variance, asymptotic standard deviation of price is 50¢ under the optimal decision rule for discounting at six percent, and it is 49¢ without discounting. The smallest attainable asymptotic standard deviation which can be achieved in the empirical model is 47¢ when expected benefits are entirely excluded from the criterion function. These results from the model are for 2.6 billion bushels storage capacity.

A single criterion of maximum expected benefits in conjunction with an autoregressive price in the foreign demand equation for U.S. exports reduces price variation to such an extent that there is little need to explicitly introduce price variability into the criterion function. The only case where there is much advantage in doing so is in starting from some initial state, particularly with large wheat stocks, and using a discounted measure of price variance. Discounting the variance terms, together with large initial wheat stocks, provides enough control on price in the near future to substantially reduce this measure of price variance. Even in this case, the reduction in price variability is achieved at a rather high cost in reduced expected benefits.

Conclusions

Methodologically, it is feasible to use a joint criterion of expected net social benefits and price variability in a stochastic dynamic programming model if asymptotic mean price is independent of the decision rule used. Although the computational burdens are large, useful approximations to the mean-benefit/price-variation efficiency frontier can be estimated.

Empirically, the main conclusion is that an expected value measure of net social benefits for the U.S. is an adequate criterion to evaluate wheat storage decision rules, and any considerations with respect to explicit recognition of price variability in the storage model are relatively unimportant. However, this should be a somewhat tentative conclusion because of limitations in the social benefit measure and other aspects of the model which are of necessity somewhat arbitrary. With this caveat in mind, some additional interpretation of the results is given below.

Price variability is thought to be relatively unimportant because a large sacrifice in expected benefits was required to achieve a marginal reduction in price variance. In fact, there was a strong propensity for the pure mean benefit criterion to greatly reduce price variation, leaving little room for further reductions when the price variance component of the criterion was introduced.

The autoregressive price equation imbedded in the demand equation for U.S. exports [equation (3)] does contribute to the phenomenon described above. Conceptually, the lagged price in this equation is viewed as a parsimonious representation of many state variables which would summarize the world supply, demand, and stocks situation. A conditionally optimal decision rule for U.S. storage control which uses this kind of information under a pure expected net benefit criterion will substantially reduce price variation compared to a conditionally optimal decision rule under the same criterion but which ignores this information. The term containing lagged price in (3) acts as a demand shifter, and an expected net benefit criterion yields a policy which increases exports when the shift in demand is positive and vice versa, thus providing a stabilizing influence on price.

Most commodity storage models explicitly or tacitly assume that the outcome of any random components in supply and demand equations are known at the moment carryover decisions are made [Cochrane and Danin]. This assumption is usually not addressed directly, and in the "theoretical" models such as used by Turnovsky it is implied by the assumption that price variability can be reduced to zero. The comparable assumption in this study would be that ε_t in (3) is a known datum when export decisions for period t are made. Such an assumption would accentuate the propensity of an expected net benefit criterion to reduce price variation. Therefore, the empirical results reported here would tend to hold under the kinds of assumptions which are common in other commodity storage models. Nevertheless, additional research to evaluate the consequences of other specifications on the demand equation for U.S. exports would be desirable. In particular, an extreme departure from the structure of (3) with the lagged price term deleted and an assumption that ε_t was observed before the annual storage decision was made would be most informative. This model would not be particularly realistic, but it would provide a valuable contrast.

Price variability is important in commodity storage modeling, but in the case of a unilateral storage control policy for U.S. wheat, a passive policy which does not explicitly try to control price variation is a close approximation to a more comprehensive strategy. Optimization using a pure expected net benefit function for the U.S. simultaneously reduces price variation about as much as would be desirable under a more complete criterion which deals directly with the price variability issue.

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