On Measuring the Value of a Nonmarket Good Using Market Data

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Our purpose is to present in detail numerical methods of measuring the value of nonmarket goods using market data, under either weak neutrality, weak complementarity, or any other preference restriction meeting the requirements discussed in this paper. It has been claimed in a number of places in the literature that numerical methods cannot be used to measure the value of nonmarket goods unless the very restrictive Willig conditions are satisfied. We show that this claim is mistaken, and that numerical methods can be used whether or not the Willig conditions are satisfied. Our numerical methods are more flexible than the existing analytical method because ours can be used with any Marshallian demand system.

Using Line Integration to Measure with Market Data the Value of a Change in a Nonmarket Good

A Generalization of Earlier Measures: Total Value in Terms of a Line Integral

Following Neill (1986, 1988, 1991, 1995), LaFrance and Hanneman, and Larson (1991, 1992b), let $\mathbf{p} = (p_1, \ldots, p_{n-1})$, represent the prices of market goods $x_1, \ldots, x_{n-1}$. Let all other market goods be represented by a composite commodity, x_n, with unit price. Let z be a parameter describing (the quantity or quality of) some nonmarket good. The amount of z consumed is not chosen or bought by the consumer but rather is given to the consumer exogenously. Let a representative utility-maximizing consumer with income *y* have Marshallian demands $\mathbf{x}(\mathbf{p}, z, y) = (x_1(\mathbf{p}, z, y), \ldots, x_{n-1}(\mathbf{p}, z, y))$ for the non-composite goods, and a Marshallian demand $x_n(\mathbf{p}, z, y) \equiv y - \mathbf{px}(\mathbf{p}, z, y)$ for the composite good. With u representing utility, denote the consumer's expenditure function by $m(\mathbf{p}, z, u)$, and the corresponding Hicksian demands by $\mathbf{x}^c(\mathbf{p}, z, u) = (x_1^c(\mathbf{p}, z, u), \ldots, x_{n-1}^c(\mathbf{p}, z, u))$ and $x_n^c(\mathbf{p}, z, y) \equiv m(p, z, u) - \mathbf{px}^c(\mathbf{p}, z, u)$.

Let u^0 be the level of maximized utility at initial prices $\mathbf{p}^0 = (p_1^0, \ldots, p_{n-1}^0)$, income y^0, and the level z^0 of the nonmarket good. Then let the amount of the nonmarket good change from z^0 to z^1. Given that prices and income remain constant at $\mathbf{p}^0$ and y^0, the total value of this change (in terms of compensating variation) to the individual is

$$TV\left(z^0, z^1\right) = m\left(\mathbf{p}^0, z^0, u^0\right) - m\left(\mathbf{p}^0, z^1, u^0\right). \tag{1}$$

The challenge to the applied economist is to measure (1) using the information available from the estimate of the demand system $\mathbf{x}(\mathbf{p}, z, y)$.

Assume that $m(\mathbf{p}, z, u^0)$ has piecewise continuous first partial derivatives of $\mathbf{p}$ and z. Then the following can be established from line integral theory (Kaplan, p. 293) and Shephard's lemma:

$$TV\left(z^0, z^1\right) = \int_L \left[\sum_{i=1}^{n-1} \frac{\partial m\left(\mathbf{p}, z, u^0\right)}{\partial p_i} dp_i + \frac{\partial m\left(\mathbf{p}, z, u^0\right)}{\partial z} dz\right] = \int_L \left[\sum_{i=1}^{n-1} x_i^c\left(\mathbf{p}, z, u^0\right) dp_i + \frac{\partial m\left(\mathbf{p}, z, u^0\right)}{\partial z} dz\right], \tag{2}$$

where L is a path of integration in ($\mathbf{p}$, z)-space from point ($\mathbf{p}^0$, z^1) to point ($\mathbf{p}^0$, z^0). Furthermore, the line integrals in (2) are path independent (Kaplan (Theorem I, p. 292), so L may be any (piecewise smooth) path between endpoints ($\mathbf{p}^0$, z^1) and ($\mathbf{p}^0$, z^0). The path independence of the line integral on the far right-hand side of (2) makes it a general measure which explains and encompasses previous methods of measuring $TV(z^0, z^1)$.

While the line integral framework of (2) does not entirely remove the requirement of identifying the marginal willingness to pay function $\partial m(\mathbf{p}, z, u^0)/\partial z$, it does provide considerable flexibility to the researcher to meet that requirement. For (2) reveals that, given knowledge of the Hicksian demand functions, it is not necessary to identify $\partial m(\mathbf{p}, z, u^0)/\partial z$ along its entire domain, nor even along the "straight line" between ($\mathbf{p}^0$, z^1) and ($\mathbf{p}^0$, z^0). Rather it is at most necessary to identify $\partial m(\mathbf{p}, z, u^0)/\partial z$ along some arbitrary path of integration L which runs between points ($\mathbf{p}^0$, z^1) and ($\mathbf{p}^0$, z^0). Equation (2) gives the researcher the increased flexibility of choosing a "convenient" path of integration, which has a subpath along which the marginal willingness to pay function $\partial m(\mathbf{p}, z, u^0)/\partial z$ may be identified. Equation (3) illustrates. Because the path of integration L between $(\mathbf{p}^0, z^1)$ and $(\mathbf{p}^0, z^0)$ in (2) is arbitrary, we can break it into three arbitrary subpaths S_1, S_2, S_3 that join to form path L, and write,

$$TV\left(z^0,z^1\right)=\int_L\left[\sum_{i=1}^{n-1}x_i^c\left(\mathbf{p},z,u^0\right)dp_i+\frac{\partial m\left(\mathbf{p},z,u^0\right)}{\partial z}dz\right]$$

$$=\underbrace{\int_{S_1}\sum_{i=1}^{n-1}x_i^c\left(\mathbf{p},z,u^0\right)dp_i}_{D_1}+\underbrace{\int_{S_2}\sum_{i=1}^{n-1}x_i^c\left(\mathbf{p},z,u^0\right)dp_i}_{D_2}+\underbrace{\int_{S_3}\sum_{i=1}^{n-1}x_i^c\left(\mathbf{p},z,u^0\right)dp_i}_{D_3}$$

$$+\underbrace{\int_{S_1}\frac{\partial m\left(\mathbf{p},z,u^0\right)}{\partial z}dz}_{B_1}+\underbrace{\int_{S_2}\frac{\partial m\left(\mathbf{p},z,u^0\right)}{\partial z}dz}_{B_2}+\underbrace{\int_{S_3}\frac{\partial m\left(\mathbf{p},z,u^0\right)}{\partial z}dz}_{B_3}. \quad (3)$$

The chief difficulty in finding total value from (3) lies in dealing with the B_2 term.

Numerical Approach

Difficulties in the application of the concepts of weak neutrality and weak complementarity remain. Thus far in the literature, the concepts have been applied to models with very particular functional forms for demand—forms that enable analytical integration from a Marshallian demand function back to a quasi-expenditure function. Since these few functional forms do not always best fit studies' empirical estimation of demands, there is a need for the development of another, more flexible method of using the assumptions of weak complementarity, weak neutrality. Our discussion in the previous section lays the foundation for the remainder of the paper, in which we present a numerical method of measuring the value of nonmarket goods using market data, under weak neutrality, weak complementarity, and other conditions. The numerical method can be implemented with any system of Marshallian demand functions.

Earlier Suggestions about Using, and the Impossibility of Using, a Numerical Approach

Both Larson (1992b) and Flores suggested applying Vartia's numerical method to approximate the value of nonmarket goods using market data. Larson (1992b, pp. 108-109) developed an expression (his equation (6)) for the value of changes in z in terms of Marshallian demand parameters for a good that is Hicks neutral to the nonmarket good.[2] Larson briefly suggested that numerical techniques similar to those of Vartia could be used to approximate his equation (6).

Flores pursued Larson's idea by providing three iterative equations based on Vartia's three algorithms to approximate Larson's equation (6). But the numerical analysis was not the central focus of Flores's analysis, and his brief discussion is subject to some important limitations. The key obstacle in implementing a numerical algorithm is that the integral in it cannot be approximated by direct application of Vartia's algorithm.[3] Below we follow up on Flores's interesting line of inquiry, presenting an applicable method for calculating the value of changes in the nonmarket good under weak complementarity or weak neutrality.

Bockstael and McConnell (1993) discussed restrictive conditions on consumer preferences, which they called Willig conditions. Contrary to Larson's and Flores's suggestions, Bockstael and McConnell claimed (p. 1254) that when the level of a non-market good changes, in general it is not possible to use Vartia-type numerical techniques to obtain an exact measure of the resultant welfare change. They write (p. 1254),

> *Some Marshallian demand functions can be analytically integrated back to expenditures functions that embody weak complementarity, allowing solution for the compensated demand. Analytical integration is not always possible, however. In the standard price change case numerical integration as suggested by Vartia (1983) is commonly employed. In assessing quality changes, the Willig condition is instrumental in applying Vartia's techniques. … When the Marshallian demand shifts, the new Marshallian and the appropriate Hicksian cross at some unknown point. … To apply the numerical integration techniques of Vartia, one needs some means of identifying the new intersection point. The Willig condition provides such a means.*

Palmquist (2005, p. 104, footnote 1) follows Bockstael and McConnell to write,

> *Larson … analytically solves the differential equation implied the Marshallian demand. Numerical solutions as in Vartia … might seem an attractive alternative. However, as Bockstael and McConnell … show, determining the bounds for the numerical integration requires additional information such as that provided by the Willig condition discussed here.*

In a similar vein, Smith and Banzhaf (2004, p. 456) cite Bockstael and McConnell,

> *…the conventional strategies used to recover Hicksian welfare measures from Marshallian demands do not provide sufficient information. … To estimate Hicksian surplus for a change in* [a non-market good] *from the Marshallian demand for* [a market good]*, preferences must also satisfy the Willig 1978 condition.*

In the following, we will show that under weak complementarity or weak neutrality, Vartia-type numerical techniques indeed can be used to obtain exact measures of the welfare effects of a change in a non-market good from knowledge of the

Marshallian demand system of the market goods. Our numerical methods can be applied whether or not the Willig conditions are met. The implications of our findings are that in the general case inexact welfare measures, such as those discussed in Bockstael and McConnell, Palmquist, and Smith and Banzhaf (2004) need not be derived, since exact measures can be calculated from the same information.

Numerical Calculation of the Value of a Change in a Non-market Good

Our approaches extend Vartia's algorithm to the problem of measuring the value of nonmarket goods using Marshallian demand parameters. Our procedures measure the value of changes in the nonmarket good under the conventional restrictions on preferences, weak complementarity and weak neutrality along the choke-price subpath. Our approaches are based on the line integral framework of equation (2), and under weak complementarity can be used with any well-defined system of Marshallian demand functions, whether or not it can be analytically integrated back to obtain an explicit expenditure function, and whether or not the Willig conditions are satisfied.

Procedure 1

We have already shown that under weak complementarity along S_{choke}, nonuse value is zero, and therefore the use value of the change in z is equal to total value of the change in z: $UV(z^0, z^1) = TV(z^0, z^1) = D_1 + D_3$, as defined in (3). We illustrate for the case of n – 1 = 1 market good that is not the numeraire. Then (3) implies

$$TV\left(z^0,z^1\right)=\int_{p^0}^{\tilde{p}\left(z^1,u^0\right)}x^c\left(p,z^1,u^0\right)dp-\int_{p^0}^{\tilde{p}\left(z^0,u^0\right)}x^c\left(p,z^0,u^0\right)dp\,, \qquad (4)$$

The second integral on the right-hand side of (4) can be found using Vartia-type numerical methods. One begins the numerical procedure at quantity $x^c(p^0, z^0, u^0) = x(p^0, z^0, y^0)$, which can be observed if the functional form of the Marshallian demand is known.

By taking small changes in the output price p, one can numerically trace the Hicksian demand curve $x^c(p, z^0, u^0)$ by using a Vartia-type algorithm, adjusting income in each step by a trapezoid representing the change in consumer surplus when price p changes, Then one can approximate $TV(z^0, z^1)$ as nearly as desired by moving p up to the choke price $\tilde{p}(z^0, u^0)$, and summing the trapezoids. Bockstael and McConnell, and Palmquist claim, however, that in general numerical methods cannot be used to calculate the first integral on the right-hand side of (4). They point out that to calculate this integral, one would have to identify the quantity $x^c(p^0, z^1, u^0)$, which generally does not equal the observable quantity $x^c(p^0, z^1, u^0) = x(p^0, z^1, y^0)$. They explain that under the special restrictive case when Willig conditions hold, a result is that $x^c(p^0, z^1, u^0) = x(p^0, z^1, y^0)$, and so $x^c(p, z^1, u^0)$ can be identified, and so numerical methods can be used to calculate $TV(z^0, z^1)$.

We will show that even when Willig conditions do not hold, under weak complementarity along the choke price subpath, it is still possible to identity quantity $x^c(p^0, z^1, u^0)$ and therefore to use numerical methods to estimate (z^0, z^1). The key to our method is that even though $x^c(p, z^1, u^0)$ is not directly observable, we know that under weak complementarity, equation (5), which implicitly defines compensating variation for a change in the non-market good from z^0 to z^1, must hold:

$$x\left(p^0, z^1, y^0 - TV\left(z^0, z^1\right)\right) = x^c\left(p^0, z^1, u^0\right). \tag{5}$$

Because the algorithm to numerically calculate the first integral on the right-hand side of (4) must begin with the quantity $x^c\left(p^0, z^1, u^0\right)$ to hold, then the two equations (4) and (5) each contain two unknowns, $TV(z^0, z^1)$ and $x^c\left(p^0, z^1, u^0\right)$, and therefore can be solved numerically for the values of those unknowns. This solution algorithm does not depend on Willig conditions.

Next we provide an example of Procedure 1. We use the same model used by Palmquist (pp. 111-112). This model assumes a specific type of Stone-Geary utility function in its logarithmic form: $u(x_1, x_2, z) = z \ln(x_1+1) + \ln(x_2)$, where x_1 is the non-numeraire market good, x_2 is the numeraire (composite) market good, and z is the non-market good. Palmquist recognized that this utility function implies that the Marshallian demand for the non-numeraire market good takes the form $x(p, z, y) = (zy - p)/[(1+z)p]$, and that the expenditure function takes the form $m(p, z, u) = -p + (1+z)[(u-z\ln(z) + z\ln(p))/(1+z)]$. Initial income is assumed to be $y^0 = 10$, and the initial price level is $p^0 = 1$. From the expenditure function, Palmquist calculates directly the compensating variation for a change in quality from $z^0 = 5$ to $z^0 = 6$ to be 3.0015, and the equivalent variation to be 4.9528.

Palmquist states directly (p. 111) that the Willig conditions are not met by the preferences summarized in the model outlined in the previous paragraph. Further, he states (p. 114) that, *"it is only possible to confidently derive welfare measures for quality changes directly from estimates of the demand for the weakly complementary private good if the path independence* [i.e. Willig] *conditions hold.* (We have added the terms in brackets.) In fact, compensating and equivalent variation can be readily calculated numerically from the Marshallian demand function in this example. In Appendix 1, we present a short Gauss program that calculates the compensating variation as 3.0015, the same as Palmquist's analytical result. We also used this program to correctly calculate compensating variation for models in other papers, including Larson's (1992) bass fishing linear model, which also does not satisfy the Willig conditions. A very similar program can be written to numerically calculate equivalent variation.

Procedure 2

Our procedure can be implemented with a relatively simple computer program (an example, written for GAUSS is available from the authors.) The researcher chooses the restriction on assumptions, weak neutrality or weak complementarity along S_{choke} (line 3).[4] The integrals approximated are D_1, D_3, and B_2 in (4). These integrals allow us to approximate $TV(z^0, z^1)$, $UV(z^0, z^1)$, and $NUV(z^0, z^1)$ under either weak complementarity or weak neutrality along S_{choke}. Any desired level of accuracy can be reached by setting the size of the increments Δp and the number of increments for z (lines 4 and 5). The researcher also specifies the demand function, the initial levels of income (y^0), price (p^0), and the nonmarket good (z^0), and the final level of the nonmarket good (z^1) (lines 7-11).

The procedure involves two loops, one nested within the other. In the first ("outer") loop, the value of z is raised in small increments Δz from the initial value, z^0, to the final value, z^1 (the loop starts on line 21). For each z^i in the outer loop, the second ("inner") loop uses Vartia's method to trace the Hicksian demand curve from p^0 to the minimal Hicksian choke price, $\hat{p}(z^i, u^0)$ (lines 27-32).

Figure 1 illustrates this inner loop process for $z = z^0$. The inner loop measures the value of access to the nonmarket good, which for $z = z^0$ is the integral ($-D_3$) in equation (4). In the first price iteration, the price rises from p^0 to $p^0+\Delta p$ while demand falls from $x(p^0, z^0, y^0) = x^c(p^0, z^0, u^0)$ to $x(p^0 + \Delta p, z^0, y^0)$ along the Marshallian demand curve. The (approximate) change in consumer surplus associated with this price increase, represented by area ΔS_1, is used to adjust income upward, raising demand to $x(p^0+\Delta p, z^0, y^0 + \Delta S_1)$. If the price increment is small, this point is "close" to $x^c(p^0 + \Delta p, z^0, u^0)$. Note that the size of the price increments has been exaggerated in Figure 1 for illustrative purposes; in tests described below, we use $\Delta p = 0.1$, resulting in hundreds of increments between p^0 and $\hat{p}$. In the second price iteration, income is further adjusted by ΔS_2, so $y = y^0 + \Delta S_1 + \Delta S_2$. The process continues until a Hicksian choke price is reached (i.e., the price is high enough such that Hicksian demand is zero). The sum of the ΔS_i is approximately the area behind the Hicksian demand curve $x^c(p, z^0, u^0)$ above p^0, and the approximation can be made as accurate as desired by shrinking the size of the price increment Δp. This area is a measure of the value of access to the nonmarket good ($-D_3$ from equation (4)).

The iterations of the outer loop, which increase the value of z by an increment Δz, are illustrated in Figure 2. (In Figure 2, it is assumed that there are R such increments: $z^1 - z^0 = R\Delta z$.) In the second iteration of z, the inner loop process of tracing the Hicksian demand curve is repeated starting from $x(p^0, z^0 + \Delta z, y^0)$. The difference between the area behind the original Hicksian demand curve and the new one (area UV_1 in Figure 2) is a measure of the use value associated with the increase in z from z^0 to $z^0+\Delta z$ (line 35). Under weak neutrality, the updated nonuse value is approximated using a trapezoidal approximation over z^0 to $z^0 + \Delta z$ (lines 36-48). If weak complementarity is assumed, these calculations are skipped and nonuse value remains at zero.

The estimates of use value and nonuse value are used to adjust the income level for the third and subsequent iterations of z. As shown in Figure 2, the third Hicksian demand curve is traced upward using the inner loop process from the point $x(p^0, z^0 + 2\Delta z, y^0 - UV_1)$.[5] For small changes in z, this is near $x^c(p^0, z^0 + 2\Delta z, u^0)$. In general, at the end of the i^{th} iteration in z, income is adjusted using the interim estimates of use values (sum of areas UV(1) to UV(i - 2)) and nonuse values (NUV(1) to NUV(i - 2)) if the assumption is weak neutrality (line 50). This adjusted income is used in iteration i + 1 so that utility is maintained (approximately) constant as z varies. The final estimate of use value is the difference in area behind the Hicksian demand curve $x^c(p^0, z^1, u^0)$, representing D_1 , and the area behind Hicksian demand curve $x^c(p^0, z^0, u^0)$, representing -D_3. The intermediate steps are necessary to estimate the income adjustments needed to keep utility constant as z increases.

The GAUSS program in the appendix is written to follow Larson's (1992b) bass fishing example under weak neutrality along the choke-price subpath, with R = 5 iterations in z and price increments of 0.1 (lines 3-11). (The assumption of weak neutrality can be changed to weak complementarity simply by assigning assum = 2 in line 3. Using our numerical approach, this program calculates the use value at -\$7.689 and the nonuse value at \$10.109, which can be shown using the expenditure function in (11) to be correct to three digits past the decimal point. (Larson reports approximated results of -\$7.69 and \$10.10.) Additional iterations make our numerical approximation correct to more digits past the decimal point.

Conclusions

We have shown that in measuring the value of changes in a nonmarket good using market data, line integration techniques can provide several advantages over the more traditional use of definite integrals. We present a numerical method of measuring the value of a change in a nonmarket good. Unlike the analytical approach, which is limited to demand functions that can be integrated back to their quasi-expenditure functions, our numerical approach can be used with any well-defined system of Marshallian demand functions. We show that numerical methods may be useful even when the Hicksian choke price is infinite. We explore the possibility of extending the numerical approach to a multiple-good system. The flexibility, simplicity, and relative accuracy of this approach may make it a useful tool in applied research into the value of nonmarket goods.

Finally, a word needs to be said about the statistical implications of applying either of the weak complementarity or weak neutrality assumptions to actual data. If it is assumed that the arbitrary subpath S_2 in (4) is the choke-price subpath S_{choke}, then much of what needs to be known may be subject to large statistical error bounds. That is, data do not usually contain price and quantity observations that occur near choke prices, and so estimations of choke prices will often be far "out of sample." Thus, the standard errors of the estimates of choke prices will be large, and this may adversely affect the size of the statistical errors of the measurements of total value of a change in the nonmarket good. If $\partial x_i^c\left(\mathbf{p},z,u^0\right)/\partial z$ (for any i = 1, … , n – 1) or $\partial\mu\left(\mathbf{p},z,u^0\right)/\partial z$ is known along a subpath that runs closer to (p, z) values observed in the data, then statistical error bounds will be much less of a problem. The trick, of course, is knowing $\partial x_i^c\left(\mathbf{p},z,u^0\right)/\partial z$ (for some i = 1, . . . , n - 1) or $\partial\mu\left(\mathbf{p},z,u^0\right)/\partial z$. Reasonable intuitive arguments have been made for assuming $\partial\mu\left(\mathbf{p},z,u^0\right)/\partial z = 0$ everywhere along a choke-price subpath of integration. Good intuitive arguments about the functional forms of any $\partial x_i^c\left(\mathbf{p},z,u^0\right)/\partial z$ or $\partial\mu\left(\mathbf{p},z,u^0\right)/\partial z$ may be more difficult to develop. The Proposition in this paper has shown more general conditions under which the value of a change in a nonmarket good can be measured using market data. Better understanding of the more general conditions should lead to better applied research. Still, for many practical applications, these conditions, though more general, remain quite restrictive.

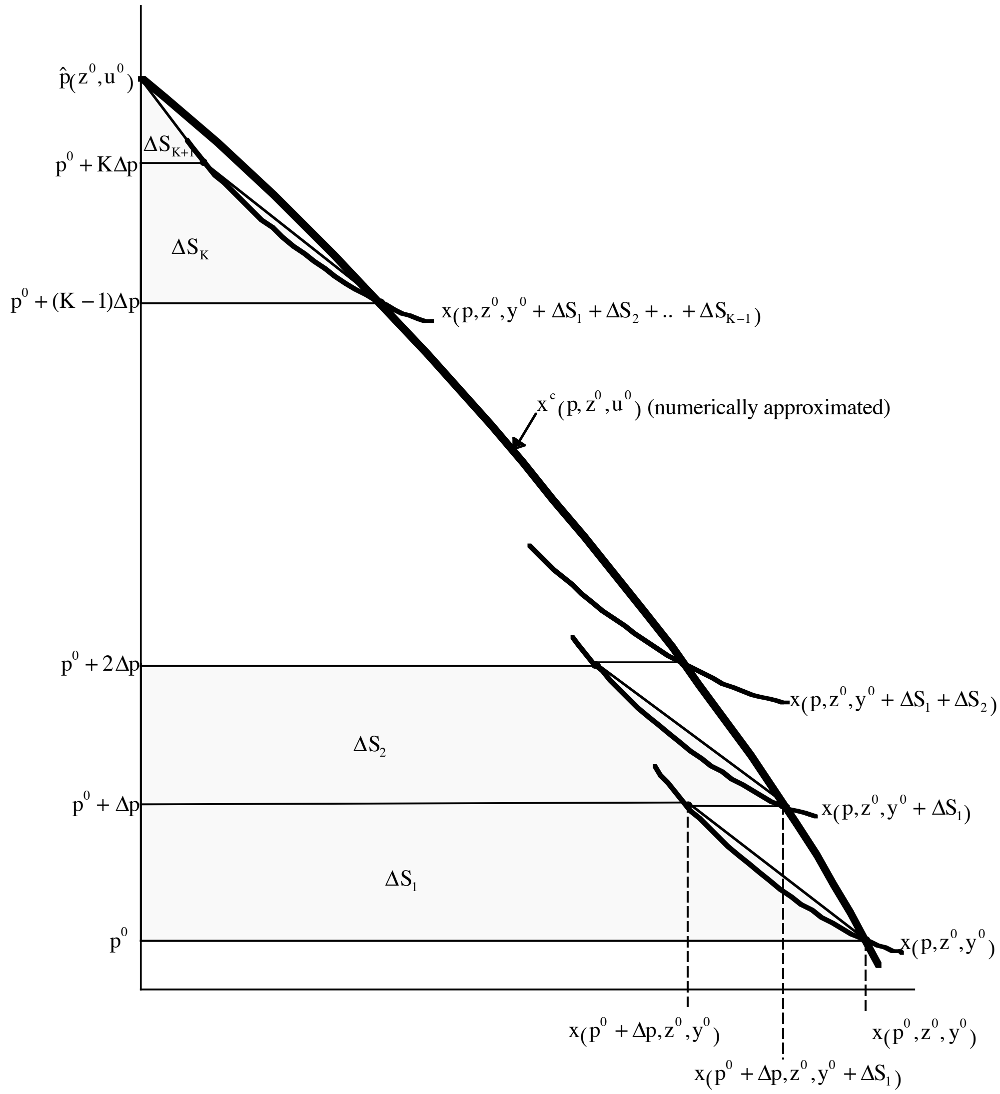

Figure 1. Approximating the area behind a Hicksian demand curve.

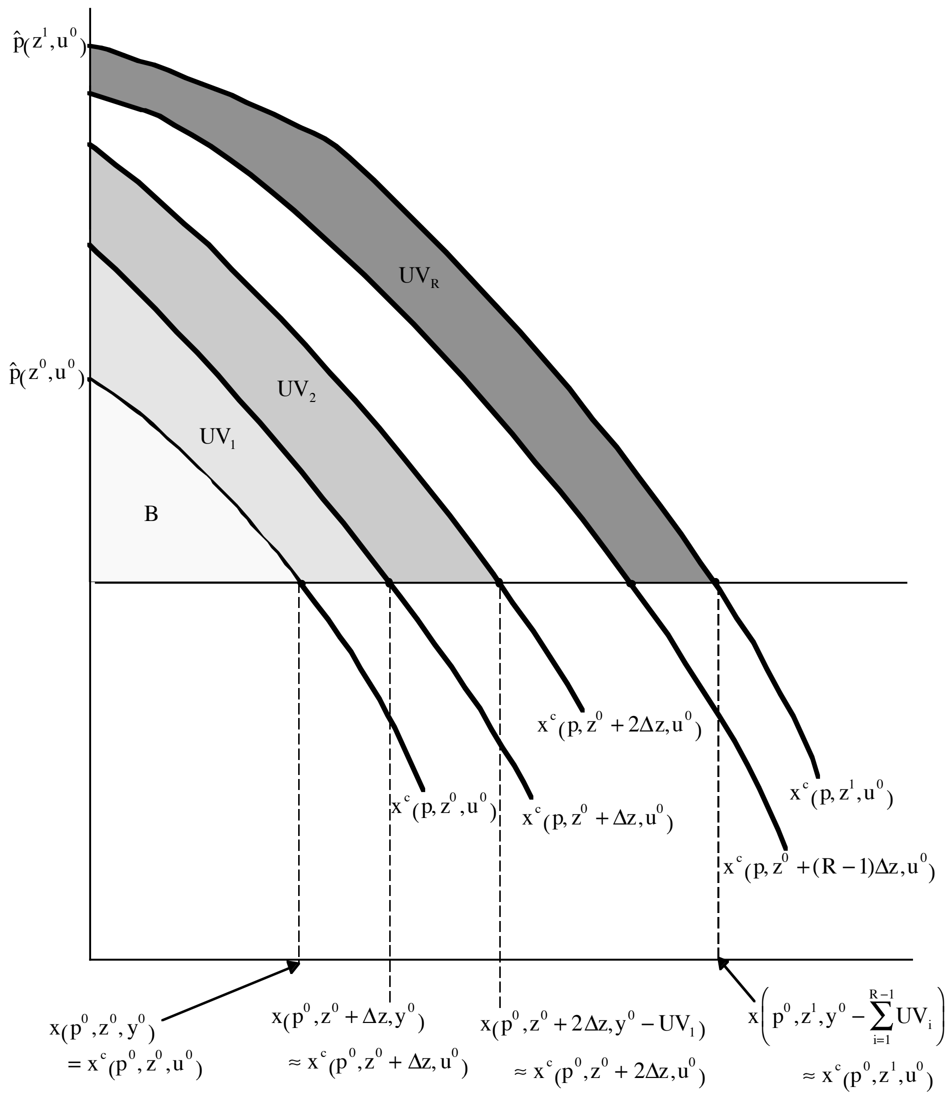

Figure 2. Approximating the change in area behind Hicksian demand curves.

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Notes

[2] Hicks neutrality for good i means that $\partial x_i^c(p, z, u^0)/\partial z = 0$ for all $z \in [z^0, z^1]$ and for all $\mathbf{p} \in R^n{}_+$. That is, good i must be weakly neutral *everywhere* in ($\mathbf{p}$, z)-space, not just along a subpath within ($\mathbf{p}$, z)-space.

[3] In particular, we lack a starting point $x_i^c(\mathbf{p}^0, z^1, u^0)$ from which to calculate the area behind $x_i^c(\mathbf{p}, z^1, u^0)$ over $\mathbf{p}^0$ to $\mathbf{p}^*$. (This area corresponds to ($-D_3$) in equations (4) and (7b).) We cannot obtain the starting point $x_i^c(\mathbf{p}^0, z^1, u^0)$ without knowing the amount of money which would compensate the consumer for the change in z, which is, of course, the objective of the whole exercise. This is the same point made by Bockstael and McConnell (p. 1254), and repeated by Palmquist (p. 104).

[4] The computer program could be adapted for other subpaths of integration.

[5] Here we assume weak complementarity, which implies nonuse value is zero. Under weak neutrality, we begin the inner loop process from the point $x(p^0, z^0 + 2\Delta z, y^0 - UV_1 - NUV_1)$.