Acceptable reforms of agri-environmental policies *

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Abstract

We consider a model of regulation for nonpoint source water pollution through non linear taxation/subsidization of agricultural production. Farmers are heterogenous along two dimensions, their ability to transform inputs into final production and the available area they possess. Asymmetric information and participation of farmers to the regulation scheme put constraints on the optimal policy that we characterize. We show that a positive relationship between size of land and ability may exacerbate adverse selection effects. We then introduce acceptability constraints and show that the intervention under acceptability amounts to reallocate production towards inefficient farmers who benefit from the reform at the expense of efficient producers. Last, we calibrate the model using datas on a french watershed (Don watershed). Simulations indicate that satisfying a high degree of acceptability does not entail high welfare losses compared to low degree of acceptability.

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1 Introduction

The use of economic incentive instruments such as green taxes in environmental policy has become increasingly popular in many countries and in particular in Europe. However, new taxes are often unpopular in polluting sectors that usually prefer traditional command and control regulation. When designing a green tax scheme, the regulator has thus to take into account the potential losses imposed to the polluters when comparing with the pre-reform (or status quo) situation. In other words, the design of green reforms is subject to a political acceptability constraint.

On the one hand, the tax scheme can be designed such that no producer looses through a reimbursement rule using lump sum transfer. This means that any polluting firm would enter voluntarily in the environmental regulation scheme given that there are no losses compared to the status quo situation. Such a highly acceptable policy comes usually at an inefficiency cost: in order to be acceptable by everybody, the environmental regulation should not impose too much effort. Otherwise, a high level of abatement would be too costly. On the other hand, the government might impose the term of regulation without fearing about losses imposed to polluters. In such a case, the strong commitment power of the government allows it to push forward a mandatory regulation. Of course, such a political power might be limited by laws protecting polluters’ revenues by limited liability up to a given amount. The resulting policy would be highly efficient from an environmental perspective but would gain very low political support in the polluting industry. (ex taxe CO2, rapport OCDE)

The concern about political support for reform is particularly deep in the agricultural sector in Europe. Rising concerns about environmental quality depressed by highly intensive agriculture have given birth to a series of reforms relating the level of subsidy to the environmental effort of farmers. Up to now, those policies have largely relied on voluntary adoptions by farmers and have gained little success. Some have argued that offered compensations were insufficient to cover the compliance cost and that uncertainty about the durability of envi-
ronmental voluntary programs have undermined their success. A large rate of participation is obviously needed to attain environmental protection goals, so that ensuring this rate in a voluntarily way might be excessively costly. This situation raises the issue of how to regulate agricultural pollution abatement while sustaining a given level of political support.

In this paper, we consider water pollution caused by agricultural activities and we analyze how to regulate farmers’ behavior while taking into account the profit potential losses induced by the reform. Of course, it is well known that regulating agricultural pollution is quite difficult because of the non point source feature: individual emissions cannot be measured and controlled easily (see Shortle and Horan (2001) for a survey). To overcome this difficulty, we consider the regulation of nonpoint source water pollution by an environmental agency through non linear taxation/subsidization of agricultural production and land. Those variables are more easily observable than individual emissions. The population of farmers we consider is heterogenous along two dimensions: their ability to transform inputs into final production and the available area they possess. Productive ability is assumed to be private information to the farmers while available area is an observable characteristic which may be part of the regulatory scheme. Hence, private information makes self-selection necessary. In addition, we assume that farmers’ profits are protected by limited liability such that the reform cannot yield negative profits.

We introduce political acceptability as part of the constraints that the regulator has to take into account. Following Lewis et al. (1989), the regulator has to satisfy a given proportion of farmers through intervention and a farmer is satisfied if he does not loose from regulation compared to the status quo situation.\(^1\) When this proportion is zero, the resulting policy is such that the political acceptability constraint is non binding and hence it can be labelled as a mandatory policy, which however does not allow the agency to impose negative profits to farmers because of limited liability constraints. On the contrary, when this proportion is

\(^1\)The properties induced by the use of this criterion have been rarely studied in the economic literature. A recent exception is Demange and Geoffard (2002) who analyzed the physician regulation.
one, the policy is voluntary by its very nature as any farmer is ready to accept the reform.

Bontems et al. (2004) analyze the mandatory case in details and show that asymmetric information yields to a larger range of less efficient farmers that are induced to quit the sector and a lower production level for any type of farmers compared to what prevails when individual ability is perfectly known to the agency. Overall, the total level of pollution is reduced because first the less efficient and most polluting farms are shut down and second because the remaining active farms are required to produce less intensively and therefore pollute less compared to the pre-reform situation.

This paper is devoted to the case where the proportion of farmers who have to be satisfied is strictly positive. The identity of winners is endogenous and determined by the agency as part of the optimal policy. Two regulatory regimes are identified. When the required level of acceptability is low, the regulatory pattern is similar to the mandatory one except that all farmers are now provided with a strictly positive level of profit which allows the agency to meet the acceptability constraint. When this level of acceptability is high, the regulatory pattern is quite different and entails pooling for a subset of farmers at the equilibrium, due to the presence of countervailing incentives. The regulatory regime depends on the observable farm area. We indicate the optimal rule of allocation of satisfied farmers according to their characteristics. Importantly, an acceptable reform amounts to reallocate production towards less efficient farmers who benefit from the reform at the expense of more efficient producers.

In the last part of the paper, we calibrate the model using data from a French watershed (Don watershed) and we simulate the optimal policy under different scenarii concerning the weight of acceptability constraints from farmers. For this, we elaborate a specific method to classify farmers along a one dimensional type that represent their productive ability. We also use the results of a hydrological model calibrated on the watershed in order to estimate nitrogen emissions. Damages are estimated by computing the retreatment cost necessary to meet water quality standards actually enforced in Europe (upper limit on nitrogen water concentration). We show that, on our particular application case, satisfying a high degree of
acceptability does not entail high welfare losses compared to a low degree of acceptability. However, the environmental damages varies highly with the level of political support required.

The paper is organized as follows. Section 2 is devoted to assumptions and notations and analysis of regulation under complete information. Regulation when self selection is necessary is analyzed in section 3 while section 4 is devoted to the empirical application to the Don watershed. Section 5 concludes. Most proofs are relegated into an appendix.

2 The model

2.1 The farmer’s behavior

We consider a farmer whose final output $y$ (say milk) is produced using a quantity $s$ of land devoted to feed crops and a polluting input such as fertilizers. Let us denote $c(y, \theta)$ the production cost per unit of land. We introduce heterogeneity among farmers through parameter $\theta$ that belongs to the set $\Theta = [\underline{\theta}, \bar{\theta}]$ and that represents the farmer’s ability to transform feed crops into the production of milk. Parameter $\theta$ can be understood as a function of several on-farm characteristics (management skills, soil quality, genetic value of the herd...).

We assume that the cost of production $c(., \theta)$ is increasing convex in $y$. We also normalize the set of types by assuming that cost $c$ is decreasing in $\theta$. In addition, the marginal cost of producing milk is also decreasing with ability ($c_y \theta < 0$). In other words, a more efficient farmer (higher type) is also associated with lower optimal rates of input use.\(^2\)

Total land available for the production we consider is denoted by $S$ which belongs to the set $S = [\underline{S}, \bar{S}]$. Land capacity $S$ is distributed according to land density function $h(S)$ with cumulative $H(S)$. We also normalize the total land devoted to agriculture to unity. For each land capacity $S$, there is a continuum of farmers characterized by their ability, with mass unity. Farmers are distributed along this line segment according to ability density function $f(\theta; S)$ with cumulative function $F(\theta; S)$. We assume that $f(\theta; S) > 0$ for every $\theta$ and $S$. Note

\(^2\)In addition, we make the following technical assumptions: $c_{\theta y y} < 0$, $c_{\theta y} > 0$. These assumptions will help us in obtaining optimal policies that effectively separate the different farmers.
that we also assume that the set Θ does not depend on S. However, this assumption does not presume that θ and S are independent. Moreover, the risk ratios \( \frac{1-F(\theta;S)}{\bar{F}(\theta;S)} \) and \( \frac{F(\theta;S)}{\bar{F}(\theta;S)} \) are assumed to be strictly increasing in θ.

Let us describe the private optimum of the profit-maximising farmer with ability \( \theta \) and land capacity \( S \), assuming there is no intervention by the environmental agency (status quo situation). It is given by solving the following program:

\[
\max_{s,y} \pi(s,y,\theta) \equiv (py - c(y,\theta))s \\
\text{s.t. } s \leq S
\]

where \( p \) is the (exogenous) product price. We assume that market and production conditions are such that all farms are active in the status quo situation. Hence the constraint \( s \leq S \) is binding everywhere. We denote \( y^\circ(\theta) \) as the optimal production level of type-\( \theta \) farmer. Given our assumptions on \( c(.,.) \), it is easy to check that both the optimal production level \( y^\circ(\theta) \) and the corresponding profit level \( \pi^\circ(\theta) \) are increasing in the farmer’s ability \( \theta \).

### 2.2 The Environmental Agency objective

Agricultural lands not only differ according to their productive efficiency but also in the environmental impacts of production. We assume that pollution is represented by a pollution production function per hectare, denoted \( g(y,\theta) \), that estimates the emissions using simulation models, where \( g(.,.) \) is increasing with \( y \) and depends on \( \theta \). We do not impose assumptions on the sign of \( g(.,.) \) that may vary: this is indeed the case on our empirical application. As a consequence, a more efficient farmer may or may not pollutes less at the margin.

\footnote{This assumption can be relaxed but at a much higher cost of complexity for the analysis. In that case, it is possible to diminish incentives costs because private information is partially verifiable by the regulator (see Green and Laffont 1986).}

\footnote{These regularity conditions are made in order to prevent the incidence of “pooling” in the optimal policy resulting from the probability function, that is policy in which the same allocation is selected for different values of \( \theta \).}

\footnote{Alternatively, we could have considered that the optimal cropped area \( s \) is interior, simply by adding a fixed cost \( k(s) \) for land in the profit function: \( \pi(s,y,\theta) \equiv (py - c(y,\theta))s - k(s) \). This extension is straightforward and does not modify drastically the features of the optimal policy derived in the paper.}
ceteris paribus. We denote $E$ the total pollution level for the watershed:

$$E = \int_S^\infty \int_\theta \int_{\mathbb{S}} sg(y, \theta) f(\theta; S) d\theta h(S) dS$$

The problem of the regulatory (utilitarian) agency is to maximize a social welfare function $W$, written as the sum of taxpayers surplus, the farmers total surplus and the environmental damage $D$. We assume that the social cost of pollution $D(E)$ depends on total pollution emitted and is increasing, convex. Note that we do not include any consideration to the consumer surplus in the welfare function. As the regulation is implemented on a small watershed in our empirical application, we expect that any variation in the agricultural production given the regulation will have negligible impacts on total production sold on the market.

As emphasized in the introduction, both the land effectively used and the level of production are assumed to be observable and verifiable by anybody. Suppose that the regulator requests the $\theta$-type farmer with land capacity $S$ to produce output $y(\theta)$ using land $s(\theta) \leq S$ and offers the monetary transfer $t(\theta)$.

The corresponding profit of the $\theta$-type farmer can be written as follows:

$$\pi(\theta) = (py(\theta) - c(y(\theta), \theta))s(\theta) + t(\theta)$$

Note that if the regulator requests $s(\theta)$ to be strictly inferior to $S$ then set-aside is part of the regulatory proposal for a $\theta$-type farmer with land capacity $S$. In particular, the regulator may find optimal to shut down some farmers by assigning them $s(\theta) = 0$.

The regulatory's objective can be expressed as

$$W \equiv \int_S^\infty \int_\theta \left[ \pi(\theta) - (1 + \lambda) t(\theta) \right] f(\theta; S) d\theta h(S) dS - D \left( \int_S^\infty \int_\theta s(\theta) g(y(\theta), \theta) f(\theta; S) d\theta h(S) dS \right)$$

where $\lambda$ is the (positive) marginal cost of public funds. Eliminating the transfer $t(.)$ and

\[ \text{For the purpose of clarity, we omit to condition the allocation } \{s(\theta), y(\theta), t(\theta)\} \text{ on the land capacity } S. \]
using (1), we obtain a reduced formulation of the regulator’s objective function:

\[ W = \int \int (1 + \lambda)(p(y(\theta)) - c(y(\theta), \theta))s(\theta) - \lambda \pi(\theta)]f(\theta; S)d\theta h(S)dS 
- D \left( \int \int s(\theta)g(y(\theta), \theta)f(\theta; S)d\theta h(S)dS \right) \]

Finding the optimal policy amounts to finding the land allocation \( s(\cdot) \), the production level \( y(\cdot) \) and the net profit \( \pi(\cdot) \) (or equivalently the transfer \( t(\cdot) \)) that are feasible in a sense to be precised below and that maximize the welfare function.

### 2.3 Incentive compatibility and acceptability constraints

Feasible allocations are first constrained by the information set of the regulator. Information asymmetry arises from the impossibility for the regulator to identify each farmer’s ability. Alternatively, one may assume that ability is observable but that institutional or political constraints prevent the regulator from perfectly discriminating farmers on that basis. Thus, a self-selecting policy remains the only option available to the regulatory agency. Following the Revelation Principle (Myerson, 1982) we can restrict our attention to the set of direct revelation mechanisms \( \{s(\theta), y(\theta), t(\theta)\}_{\theta \in \Theta} \).

By choosing a particular contract among all offered contracts, say a production level \( y(\hat{\theta}) \), a forage area \( s(\hat{\theta}) \) in exchange of a tax/subsidy \( t(\hat{\theta}) \), a farmer implicitly announces to be of type \( \hat{\theta} \). Hence, incentive compatibility requires to satisfy the following constraints:

\[ \pi(\theta) \geq \pi(\theta, \hat{\theta}), \quad \forall \theta, \forall \hat{\theta} \quad \text{(IC)} \]

where \( \pi(\theta, \hat{\theta}) = ps(\hat{\theta})y(\hat{\theta}) - s(\hat{\theta})c(y(\hat{\theta}), \theta) + t(\hat{\theta}) \) and \( \pi(\theta) \equiv \pi(\theta, \theta) \), which ensures that any farmer finds optimal to reveal his true type.

The regulator’s problem is to design policies which are not only incentive compatible but also politically acceptable. Consequently, the regulator has to operate under the additional constraint that the policy has to be acceptable compared to the status quo policy, and can therefore be implemented and enforced. This allows the regulator to take into account the
regulatory history in the agricultural sector before formulating the reform. Following Lewis, and al. (1989), suppose that only the farmers who benefit from the policy will support it, and that the probability that a farmer will support the policy is strictly positive if only if his profit after the reform is greater than before, i.e. \( \pi(\theta) \geq \pi^*(\theta) \). More precisely, we denote \( v(\pi(\theta) - \pi^*(\theta)) \) this probability and we assume for simplicity that \( v(.) = 1 \) whenever \( \pi(\theta) \geq \pi^*(\theta) \) and 0 either. The political implementation of the policy requires that the following acceptability constraint should be taken into account:

\[
\int_{S} \int_{\theta} v(\pi(\theta) - \pi^*(\theta))f(\theta; S)d\theta h(S)dS \geq \alpha \quad (AC)
\]

which indicates that the probability that the policy is accepted must be superior to a (exogenous) level \( \alpha \in (0, 1] \).\(^7\) In addition, we assume that any farmer is free to leave the agricultural sector following the reform. This implies that the profit obtained after the reform cannot be lower than the reservation profit. Assuming that the best outside option for any type of farmer yields a constant (not type dependant) profit, we normalize this reservation profit to 0 and we obtain the following individual rationality constraints:

\[\pi(\theta) \geq 0, \quad \forall \theta \quad (IR)\]

In this setting, the regulator is thus able to enforce its policy as long as it does not entail any net loss for farmers and provided that a proportion \( \alpha \) of farmers gets after the reform at least the income earned before.

Finally, we impose that the optimal production level after the reform cannot be greater than the level under the status quo situation (\( \forall \theta, S, \ y(\theta) \leq y^*(\theta) \)). Hence we forbid any re-allocation of production across farmers such that some of them may be given incentives to produce more than their private optimum absent any regulation. First, we believe that this constitutes a reasonable property of the optimal policy as it should facilitate political support for the regulatory intervention given that all farmers have to make some effort by reducing production. Second it helps us in deriving the optimal policy as will be clear below.

\(^7\)This way of modelling political constraints is also recently investigated by Demange and Geoffard (2002) in a model of physician regulation.
Given the various constraints faced by the regulator, the program to be solved is as follows:

$$\max_{s(\cdot), y(\cdot), \pi(\cdot)} W \quad \text{s.t.} \quad (\text{IC}), (\text{IR}), (\text{AC}), \ s(\theta) \leq S \ \text{and} \ y(\theta) \leq y^*(\theta) \ \forall \theta, S.$$

Obviously, it is clear that the addition of the acceptability constraint to the regulator’s maximisation program always induces a loss in welfare compared to the situation of a so-called mandatory policy which is obtained for the special case $$\alpha = 0$$. It remains important to ascertain the qualitative changes in the optimal regulatory policy when $$\alpha$$ varies. Moreover, when $$\alpha = 1$$, it is also interesting to note that our model belongs to the class of adverse selection models with type-dependant participation constraints (Jullien (2000)). However, in the general case where $$\alpha \in (0, 1)$$, the type-dependant participation constraint only applies to an endogenous subset of types (the “winners” of the reform) which must be identified as part of the optimal policy.

### 2.4 Regulation under complete information

Before analyzing the optimal self-selecting policy, we characterize in this section the optimal policy under complete information, ignoring incentive compatibility constraints (IC). The program to be solved is:

$$\max_{s(\cdot), y(\cdot), \pi(\cdot)} W \quad \text{s.t.} \quad (\text{IR}), (\text{AC}), \ s(\theta) \leq S \ \text{and} \ y(\theta) \leq y^*(\theta) \ \forall \theta, S.$$

Ignoring the constraint $$y(\theta) \leq y^*(\theta)$$ for the moment, we differentiate the objective w.r.t. $$y(\theta)$$ and we obtain the following necessary conditions:

$$(1 + \lambda)(p - c_y(y^{PI}(\theta), \theta)) = D' \left( E^{PI} \right) g_y((y^{PI}(\theta), \theta))$$

where $$PI$$ denotes perfect information and $$E^{PI} = \int_S \int_{S} s^{PI}(\theta)g(y^{PI}(\theta), \theta)f(\theta; S)d\theta h(S) dS$$. Equation (2) indicates that for an active farmer the optimal production level $$y^{PI}(\theta)$$ is determined by equalizing social marginal benefit and social marginal damage. Given the linearity

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8In this class of adverse selection models, countervailing incentives are generally part of the optimal policy because of the interaction between the incentive compatibility constraints and the type-dependant participation constraints. See also Lewis and Sappington (1989) and Maggi and Rodriguez-Cläre (1995) for earlier analysis on countervailing incentives.
of profit function with respect to the land input, the optimal production does not depend on the land capacity $S$. Moreover, it can be checked readily from equation (2) that the optimal regulation entails a decrease in production per hectare compared to the private optimum (i.e. $y^{PI}(\theta) < y^c(\theta)$ for any $\theta$).

Then, differentiating the objective w.r.t. $s(\theta)$ we get:

$$\frac{\partial W}{\partial s(\theta)} = \Omega(\theta)f(\theta; S).$$

where

$$\Omega(\theta) = (1 + \lambda)(py^{PI}(\theta) - c(y^{PI}(\theta), \theta)) - D'(E^{PI}) g(y^{PI}(\theta), \theta)$$

denotes the social net marginal surplus of land. We assume that $\Omega(\theta)$ is increasing and that $\Omega(\overline{\theta}) > 0$ otherwise all farmers should be shut down.\footnote{Differentiating $\Omega(\theta)$ w.r.t. $\theta$ and using (2) leads to $\Omega'(\theta) = -(1 + \lambda)c_{\theta}(y^{PI}(\theta), \theta) - D'(E^{PI}) g_{\theta}(y^{PI}(\theta), \theta)$. $\Omega(\theta)$ is increasing given that $c_{\theta} < 0$ and provided that $g_{\theta}$ is negative or not too positive, which is guaranteed in our empirical application.} It follows that when $\Omega(\overline{\theta}) < 0$ there exists a unique threshold type $\theta^{PI}(S)$ such that for any $S$, whenever $\theta \leq \theta^{PI}$, the farmer is not allowed to produce $(\forall S, s^{PI}(\theta) = 0, \forall \theta \leq \theta^{PI}(S))$. On the contrary, the more efficient farmers produce using their total land capacity $(\forall S, s^{PI}(\theta) = S, \forall \theta > \theta^{PI}(S))$. Finally, when $\Omega(\overline{\theta}) > 0$, then all farmers are allowed to produce at the optimum involving complete information.

To complete the description of the optimal policy under perfect information, it remains to characterize the redistribution policy (through the transfer) inside the population. As the objective is decreasing in the rent $\pi(\theta)$ left to any agent, it is clear that the regulator would like to bind all participation constraints at the optimum $(\pi(\theta) = 0, \forall \theta, \forall S)$. Of course, such a policy is not politically acceptable as long as $\alpha$ is strictly positive. In the limit case where all farmers must be compensated $(\alpha = 1)$, the regulator has to offer a compensation $t^{PI}(\theta) = \pi^c(\theta) - \pi^{PI}(\theta) > 0$ to any $\theta$-type farmer (with land size $S$) where $\pi^c(\theta)$ is the profit before the reform and $\pi^{PI}(\theta) = (py^{PI}(\theta) - c(y^{PI}(\theta), \theta)s^{PI}(\theta)$ is the profit gross of transfer under the reform.
In intermediate cases when $0 < \alpha < 1$, the identity of compensated farmers is determined by the shape of $\pi^c(\theta) - \tilde{\pi}^I(\theta)$. Unfortunately, this results in very various situations depending on the model specification. As we will see now, incomplete information puts more structure on the shape of the optimal profit and gives clearer predictions on the identity of compensated farmers.

3 Regulation under incomplete information

The regulator has now to solve the following program:

$$\max_{s(\cdot), y(\cdot), \pi(\cdot)} \mathcal{W} \text{ s.t. } (IC), (IR), (AC), s(\theta) \leq S \text{ and } y(\theta) \leq \tilde{y}(\theta) \quad \forall \theta, S.$$ 

Standards arguments (see Guesnerie and Laffont, 1984) indicate that incentive constraints can be replaced by the following set of constraints:

\begin{align*}
\pi'(\theta) &= -c_\theta(y(\theta), \theta)s(\theta) \quad \text{(IC1)} \\
-s'(\theta)c_\theta(y(\theta), \theta) - s(\theta)c_{\theta y}(y(\theta), \theta)y'(\theta) &\geq 0 \quad \text{(IC2)}
\end{align*}

Because the rate of growth of rents is positive ($\pi'(\theta) > 0$), then participation constraints reduce to $\pi(\theta) \geq 0$.

3.1 Preliminary analysis

Denote $y^a(\theta), s^a(\theta), \pi^a(\theta)$ as the optimal second best allocation offered to any type-$\theta$ farmer for a given land capacity $S$. For each land capacity $S$, we know that the set of farmers allowed to produce will be an interval $[\theta^a(S), \bar{\theta}] \subseteq \Theta$. Now, let us examine the constraint (AC). At least for some $S$, the continuous functions $\pi^a(\theta)$ and $\pi^c(\theta)$ must clearly intersect or touch at least once, otherwise (AC) would be violated or it would not bind, which is suboptimal.

We will proceed in two steps. First, we characterize the optimal policy for a given class of surface $S$ under the constraint that the proportion of farmers that gain from the reform is at least $\alpha(S)$ which is given, i.e.

$$\int_2^{\bar{\theta}} v(\pi(\theta) - \pi^c(\theta))f(\theta; S)d\theta \geq \alpha(S).$$
Second, we determine the optimal profile for the proportions $\alpha(S)$ under the constraint 
$$\int_S^\infty \alpha(S)h(S)dS = \alpha.$$ 

For that purpose, let us denote $W^S$ the welfare function (partially) restricted to the class of surface $S$, assuming that the policy for any $S' \neq S$ is fixed to the optimal level (i.e. $y^o(\theta)$, $s^o(\theta)$):
$$W^S = \int_\theta^\infty [(1 + \lambda)s(\theta)[py(\theta) - c(y(\theta), \theta) - \lambda\pi(\theta)] f(\theta; S)d\theta - D(E)$$
where the total level of pollution $E$ is rewritten as follows:
$$E = \int_S^\infty \int_\theta^\infty s^o(\theta)g(y^o(\theta), \theta)f(\theta; S')d\theta h(S')dS' + \int_\theta^\infty s(\theta)g(y(\theta), \theta)f(\theta; S)d\theta h(S) + \int_S^\infty \int_\theta^\infty s^o(\theta)g(y^o(\theta), \theta)f(\theta; S')d\theta h(S')dS'.$$

Actually, the following lemma indicates that, for a given $S$, the function $\pi^o(\theta)$ intersects once $\pi^o(\theta)$ from above at a point $\bar{\theta}(S)$ if $\alpha(S)$ is interior.

**Lemma 1** For a given $S$, if $\alpha(S) \in (0, 1)$, then

(i) $\pi^o(\theta)$ intersects $\pi^o(\theta)$ once and from above at a point $\bar{\theta}(S) \in \Theta$,

(ii) $\bar{\theta}(S)$ satisfies $\alpha(S) = F(\bar{\theta}(S); S)$.

**Proof.** see appendix A. ■

As shown by Lemma 1, it is important to note that incentive compatibility constraints and the constraint $\forall \theta, S$, $y(\theta) \leq y^o(\theta)$ together imply that the set of farmers who benefit from the reform is composed of the less efficient producers whether they are active or not. Furthermore, the relative position of the threshold type $\theta^a(S)$ and the intersection point $\bar{\theta}(S)$ is crucial for the pattern of the optimal policy. First the function $\pi^o(\theta)$ intersects $\pi^a(\theta)$ on the flat part of $\pi^a(\theta)$. This case occurs when $\alpha(S)$ is "small" and we have $\theta^a(S) > \bar{\theta}(S).$

Then necessarily $\pi^o(\bar{\theta}(S)) = \pi^a(\theta^a(S)).$ On the contrary, the function $\pi^o(\theta)$ intersects $\pi^a(\theta)$ on the increasing part of $\pi^a(\theta)$. This occurs when the proportion $\alpha(S)$ is "large" and we have $\theta^a(S) < \bar{\theta}(S).$ We will successively analyze both cases.
3.2 The case when the proportion $\alpha(S)$ is "small"

By a continuity argument, when $\alpha(S)$ is "small", we expect that the optimal policy looks like the optimal mandatory one (i.e. for $\alpha(S) = 0$). This is shown in the following proposition.

**Proposition 1** For any $S$ such that $\alpha(S)$ is "small", the set of farmers that are allowed to produce using their land capacity $S$ is $[\theta^a(S), \bar{\theta}]$ with $\theta^a(S) > \bar{\theta}(S)$ and the optimal level of production for any type $(\theta, S)$ farmer is given by:

$$p - c_y(y^a(\theta), \theta) = \frac{1}{1 + \lambda} D'(E^a) g_y(y^a(\theta), \theta) - \frac{\lambda}{1 + \lambda} c_{0y}(y^a(\theta), \theta) \frac{1 - F(\theta; S)}{f(\theta; S)}$$

where the total pollution level is

$$E^a = \int_{\theta^a(S)}^{\bar{\theta}} \int_{S}^{\bar{S}} S g(y^a(\theta), \theta) f(\theta; S) d\theta h(S) dS$$

Moreover, for any $\theta$ such that $\theta \leq \theta < \theta^a(S)$, production is not allowed ($s^a(\theta) = 0$) and $\pi^a(\theta) = \pi^a(\theta^a(S)) = \pi^o(\bar{\theta}(S))$.

**Proof.** See appendix B. ■

As indicated by proposition 1, when $\alpha(S)$ is "small", the optimal policy is similar to the mandatory one (i.e. when $\alpha(S)$ is nil), except that the production levels and the range of excluded farmers are modified because of the global pollution level $E^a$ which is a priori different. In particular, a positive distortion is added to the marginal damage when determining the optimal level of production for active farmers. Also, the minimum level of profit is now higher and equal to $\pi^o(\bar{\theta}(S)) > 0$ in order to fulfill the acceptability constraint.

Finally, it is obvious that if the optimal repartition of the weights $\alpha(S)$ across the classes of surface is such that $\alpha(S)$ is "small" everywhere, then the optimal policy is totally identical to the mandatory one, except for the basic level of profit given to all farmers. Formally, we would then have $E^o = E^* = (y^o(\theta), s^o(\theta), \theta^o(S)) = (y^*(\theta), s^*(\theta), \theta^*(S))$ for every $S$. However, the total welfare level is lower (and decreasing in $\alpha(S)$) compared to the mandatory one because of the social cost of giving a strictly positive minimum profit to every farmer. To sum up, in
that particular case, the introduction of the acceptability constraint only implies to translate upwards the minimum profit level for all farmers and does not change anything else to the optimal policy nor to the optimal level of pollution.

### 3.3 The case when the proportion $\alpha(S)$ is "large"

We now turn to the opposite case where $\theta^a(S) < \bar{\theta}(S)$ which occurs when $\alpha(S)$ is "large". The optimal policy in that case is described in the following proposition.

**Proposition 2** For any $S$, when $\alpha(S)$ is "large", the set of farmers that are allowed to produce using their land capacity $S$ is an interval $[\theta^a(S), \bar{\theta}]$ with $\theta^a(S) < \bar{\theta}(S)$ and the optimal level of production for any type $(\theta, S)$ farmer is continuous and defined by:

$$p - c_y(y^a(\theta), \theta) = \frac{1}{1 + \lambda} D'(E^a) g_y(y^a(\theta), \theta) + \frac{\lambda}{1 + \lambda} c_{y\theta}(y^a(\theta), \theta) F(\theta; S), \forall \theta \in [\theta^a(S), \theta_1(S)],$$

$$y^a(\theta) = \tilde{y}(S), \forall \theta \in [\theta_1(S), \theta_2(S)],$$

$$p - c_y(y^a(\theta), \theta) = \frac{1}{1 + \lambda} D'(E^a) g_y(y^a(\theta), \theta) - \frac{\lambda}{1 + \lambda} c_{y\theta}(y^a(\theta), \theta) 1 - F(\theta; S), \forall \theta \in (\theta_2(S), \bar{\theta}],$$

where the total pollution level is

$$E^a = \int_{S} \int_{\theta^a(S)} S g(y^a(\theta), \theta) f(\theta; S)d\theta h(S)dS$$

and where $y^a(\theta)$ denotes the quantities that maximizes $W^S$ and with $\tilde{y}(S)$ maximizing expected welfare over $[\theta_1(S), \theta_2(S)]$ with $\theta^a(S) \leq \theta_1(S) \leq \bar{\theta}(S) \leq \theta_2(S) \leq \bar{\theta}$. Moreover, for any $\theta$ such that $\bar{\theta} \leq \theta < \theta^a(S)$, production is not allowed and $\pi^a(\theta) = \pi^a(\theta^a(S)) > 0$.

**Proof.** See appendix C.

As indicated in the proposition, the output profile is generally not efficient. Indeed, holding the total level of pollution constant, we find that inefficiently high levels of production are induced for low ability types while inefficiently low levels of production are induced for highly productive farmers. The intuition is as follows. We knows that the acceptability constraint imposes that $\pi^a(\bar{\theta}(S)) = \pi^a(\bar{\theta}(S))$. For all types such that $\theta > \bar{\theta}(S)$, the farmer’s profit increases with its type at the rate of $-Sc_9 > 0$. The agency will thus tend to induce...
lower output on this range in order to limit the rate of growth of informational rents. On the contrary, for \( \theta < \tilde{\theta}(S) \), the farmer’s profit decreases with its type at the rate of \( Sc_{\theta} < 0 \) and consequently, the agency tends to limit rents by inducing high levels of output. The pooling interval on \([\theta_{1}(S), \theta_{2}(S)]\), where all types of farmers are required to produce at the same constant level \( \tilde{y}(S) \) appears because around \( \tilde{\theta}(S) \) the two tendencies exhibited above would require that output would decrease with ability which contradicts incentive compatibility (IC2). Thus, the agency has to induce the same level of production \( \tilde{y}(S) \) in the interval \([\theta_{1}(S), \theta_{2}(S)]\) around \( \tilde{\theta}(S) \). Also the farmers in this range receive the same transfer level.

To sum up, the optimal policy involves an intermediary range of types producing the same level of production which maximize expected welfare while the farmers with lower ability are required to overproduce and the farmers with higher ability are required to underproduce. Compared to the situation where acceptability is not required (mandatory policy), it is clear that political participation induces the agency to reorganize production in a way that benefit to the farmers with low ability. This is because essentially the revenue under status quo increases more rapidly than the revenue after intervention. Indeed for an active farmer, under the status quo we have \( \pi^{\omega}(\theta) = -Sc_{\theta}(y^{\omega}(\theta), \theta) \) and incentive compatibility requires that \( \pi'(\theta) = -Sc_{\theta}(y(\theta), \theta) \). Moreover, we have assumed that \( y(\theta) \leq y^{\omega}(\theta) \) and \( c_{\theta} \) is decreasing with \( y \). Hence the revenue is more equally distributed in the population, so that the low-types farmers benefit at the expense of high-types farmers. Subsidies given to the low-types farmers in order to induce them to leave the sector are financed through implicit taxation of high-types farmers. However, in our model, it is clear that the total level of pollution always decreases after intervention. Indeed, given that for any active farmer we have \( y(\theta) \leq y^{\omega}(\theta) \) and that the range of active farmers is smaller after regulation, total pollution is lower.

3.4 The optimal allocation of proportions \( \alpha(S) \)

In this subsection, we indicate the rule that determines the optimal allocation of proportions \( \alpha(S) \) across the surface classes. The regulator seeks to solve the following maximization
program:

\[
\max_{\alpha(S)} W = \int_S \mathcal{W}^S h(S) dS \\
\text{s.t.} \\
\mathcal{W}^S = \int_S [(1 + \lambda) s^a(\theta) (py^a(\theta) - c(y^a(\theta), \theta) - \lambda \pi^a(\theta) - D(E^a))] f(\theta; S) d\theta \\
\alpha = \int_S [\alpha(S) h(S)] dS
\]

We introduce a state variable \(\mu(S)\) defined by \(\mu(S) = \alpha(S) h(S)\) with the conditions \(\mu(S) = 0\) and \(\mu(S) = \alpha\). Let \(\eta(S)\) denote the associated co-state variable. The lagrangian can be written as follows:

\[
\mathcal{L} = \int_S \mathcal{W}^S h(S) dS + \int_S [\eta(S) [\alpha(S) h(S) - \mu(S)] dS \\
= \int_S \{\mathcal{W}^S + \eta(S) \alpha(S)\} h(S) dS + \int_S [\eta(S) \mu(S) dS - \eta(S) \mu(S) + \eta(S) \mu(S)]
\]

and pointwise maximization leads to the following necessary conditions (for an interior optimum):

\[
\frac{\partial \mathcal{L}}{\partial \mu} = \eta(S) = 0 \Rightarrow \eta(S) = \eta \quad \forall S \\
\frac{\partial \mathcal{L}}{\partial \alpha(S)} = \frac{\partial \mathcal{W}^S}{\partial \alpha(S)} + \eta = 0
\]

Using the envelop theorem, it is easy to prove that

\[
\frac{d \mathcal{W}^S}{d \alpha(S)} = -\lambda \left( \pi^a(\tilde{S}) + Sc\phi(\tilde{y}, \tilde{\theta}(S)) \right) \frac{\partial \tilde{\theta}(S)}{\partial \alpha(S)} \\
= \frac{\lambda S \left( c\phi(y^a(\tilde{S}), \tilde{\theta}(S)) - c\phi(\tilde{y}(S), \tilde{\theta}(S)) \right)}{f(\tilde{\theta}(S); S)} < 0.
\]

Thus the marginal impact of \(\alpha(S)\) on expected welfare which is negative should be equalized across the different classes of surface.

4 Empirical application to the Don watershed

For this application, we used the same set of data from dairy farms collected for year 2000 on the Don watershed (Loire Atlantique, France) by Bontems et al. (2004). We shall summarize
here the main features of this empirical work. First, the critical issue in the application is to define a unidimensional measure called “ability” $(\theta)$ which allows to reduce the dimensionality of the set of acute parameters. Second, we have to estimate the density functions $h(S)$ and $f(\theta; S)$ describing the population of farmers. Last, production cost, pollution and damage cost functions are estimated.

4.1 Determination of parameters

The parameter $\theta$ represents the ability for a given farmer to transform his forage area into dairy production. Its determination proceeds in two steps. Firstly, some experts in local extension services built a relative (one-dimensional) classification of the surveyed farms, using observed variables such as the amount of dry matter produced by grasslands, the amount of milk produced per cow, the balance in forage feed between proteins and energy and the amount of required concentrates. Secondly these variables have been combined to recover the classification proposed by the experts, and hence to determine a function which defines the one-dimensional parameter $\theta$. A farmer who manages to obtain high grass yields, who breeds high productive cows, who gives them good quality forages and who needs low rates of concentrates to produce milk is considered to possess a good ability to transform his forage crops into milk, and thus his $\theta$-parameter is high.

The distributions $h(S)$ and $f(\theta; S)$ have been estimated using a discrete approximation of Beta density functions. Hence, farms are classified into ten different classes of surface while ability falls into hundred different classes for each $S$.

Following Yiridoe et al. (1997), the amount $x(y, \theta)$ of N fertilizer used is expressed as a quadratic function of production $y$. The cost function $c(y, \theta)$ is then written as follows:

$$c(y, \theta) = w\vartheta(\theta)x(y, \theta) + \omega(\theta)y + \varphi$$

where $w$ is the inorganic N price, $\vartheta(\theta)$ is a proportionality factor of fertilizers in forage operation costs, and $\omega(\theta)y + \varphi$ represents the cost related to breeding operations.

\[\text{See Bontems et al. (2004) for more details.}\]
Moreover, individual emissions have been estimated using a local hydrologic model (Turpin et al., 2001) from data collected on the surveyed farms and other data concerning soils, hydrology and local climate on the Don watershed. Type-dependent emissions have been determined from these estimations, using a quadratic function and a constrained maximum likelihood method (Gouriéroux and Montfort, 1995). Finally, the cost of social damage is valued as being the cost of water treatment for transforming brut water into drinking water (Falala, 2002).

4.2 Analysis (to be completed)

Instead of exploiting the first order conditions exhibited in the theoretical part of this paper, the optimal policies for the mandatory and the political regulation cases have been more easily determined by solving the regulator maximisation program directly. For this purpose we used Gams software (Brooke et al., 1998) and Conopt optimisation procedure (Drud, 1994).

We will present successively the influence of the required proportion $\alpha$ of winners on production levels, after tax profits and welfare. Figure 1 depicts optimal production levels for different scenarios concerning $\alpha$. The depicted situations correspond to Proposition 2 where $\alpha$ is "large". As described in proposition 2, pattern of $y^\alpha$ includes two increasing parts and three pooling zones where all types are offered the same contract. The first pooling zone corresponds to the set of excluded farmers which are the less productive ones (low $\theta$). In our application, the range of excluded farmers do not vary highly with $\alpha$. Then, for all active farmers, the regulatory intervention implies a decrease in production whatever the type and land capacity, except for a limited range where the constraint $y^n(\theta) \leq y^\alpha(\theta)$ is binding. For the lower active types, the gap between the status quo and the optimal production levels is quite small.

The second pooling phenomenon appears because of the conflict between the acceptability (AC) and incentive (IC) constraints. Starting from this pooling zone towards the highest type,
the reform amounts to implement a larger decrease in production than before the pooling
zone. The last pooling zone appears for the higher types of farmers and is due to the binding
of second order conditions (IC2).

To sum up, we have imposed that every farmer must reduce his production level per
hectare \((y^a(\theta) \leq y^0(\theta))\) under the reform. We have shown that on our empirical application
an acceptable reform of agri-environmental policy implies to ask more effort in terms of
production reduction to high types compared to low ones, whatever the land capacity and
the required acceptability level.

Figure 2 depicts profit patterns for a low value of \(\alpha(S)\) corresponding to proposition 2
and for a high value of \(\alpha(S)\) corresponding to proposition 3. For low values of \(\alpha(S)\), the
simulated threshold value \(\tilde{\theta}(S)\) is lower than \(\theta^a(S)\), which means that some inactive farmers
loose money and this is true also for all active farmers. For high values of \(\alpha(S)\), \(\theta^a(S)\) becomes
lower than \(\tilde{\theta}(S)\). Consequently some active farmers now benefit from the reform. When \(\alpha(S)\)
is high, the loss of money imposed to the efficient farmers \((\pi^a - \pi^0)\) is lower than for smaller
values of \(\alpha(S)\).

Moreover the inefficient farmers receive a net subsidy while efficient farmers are taxed : the efficient farmers bear the cost of the reform.

Figure ? indicates that the optimal proportion \(\alpha(S)\) (weighted by the density \(h(S)\)) is
increasing in \(\alpha\) for any class \(S\). To compare different cases depending on the proportion
of farmers \(\alpha\) who benefit from the regulation we use indicators, constructed as a relative
variation between the status quo and the regulated situations. We denote \(P(\alpha)\) the relative
variation of total profit:

\[
P(\alpha) = \frac{\int_S \int_\theta \pi^a(\theta)f(\theta; S)d\theta h(S)dS - \int_S \int_\theta \pi^0(\theta)f(\theta; S)d\theta h(S)dS}{\int_S \int_\theta \pi^0(\theta)f(\theta; S)d\theta h(S)dS}
\]

And so are defined \(V(\alpha), M(\alpha), D(\alpha)\), the relative variation of welfare, milk production and
damage.

Figure 3 depicts the patterns of these indicators with respect to \(\alpha\). For the smallest values
Figure 1: Optimal production levels for laissez-faire situation ($y^0$) and different values of $\alpha$.

Figure 2: Farmers profit as a function of $\theta$-type for several cases: laissez-faire situation ($\pi^0$), mandatory regulation ($\pi^*$), low and high values of $\alpha$ ($\pi^\alpha$).
of \( \alpha \), we have the first regime for all \( S \) and proposition 2 applies, with \( y^a(\theta) = y^* (\theta) \) which is independent of \( \alpha \). Thus \( M(\alpha) \) and \( D(\alpha) \) are constant: introducing acceptability constraints does not change the optimal policy compared to the mandatory case, except for the basic level of profit given to all farmers. This explains the decreasing level of profit loss (\( \mathcal{P}(\alpha) \) is increasing with respect to \( \alpha \)). The farmers pay in expectation a tax to the regulator. As a consequence, the social welfare function is decreasing with respect to \( \alpha \).

For values of \( \alpha \) higher than \(?\), Proposition 3 applies for some \( S \) and \( \theta^a(S) \) is strictly positive. Note that for the application, the monetary value of damage is rather low\(^{11} \) when compared with farms profit and tax paid by the farmers (Figure 3 depicts variations in percentage of the status quo level). The evolution of the welfare is mostly drawn by the variation of profit and transfer.

When \( \alpha \) exceeds, then all classes of surfaces fall under Proposition 3. Introducing acceptability constraints when the proportion \( \alpha \) of farmers who benefit from the regulation is growing lead to a decrease of social welfare with \( \alpha \).

When \( \alpha \) is sufficiently high, the status quo situation is preferred by the regulator to intervention (\( \mathcal{V}(\alpha) \) becomes non positive). Except for the lower levels of \( \alpha \), welfare gain compared to the status quo does not vary much with the level of acceptability.

5 Conclusion

We have considered a model of regulation for nonpoint source water pollution through nonlinear taxation/subsidization of agricultural production. Farmers are heterogenous along two dimensions, their ability to transform inputs into final production and the available area they possess. Asymmetric information and participation of farmers to the regulation scheme put constraints on the optimal policy that we characterize. We have shown that a positive relationship between size of land and ability may exacerbate adverse selection effects. By introducing acceptability constraints we show that the intervention under acceptability

\(^{11}\text{Damage value in the status quo situation represents 0.62 \% of the farmers’ profit.}\)
Figure 3: Relative variation with respect to $\alpha$ of damage ($\Delta D$), farmers’ profit ($\Delta \pi$), milk production ($\Delta y$), transfert from farmers to the regulatory agency ($\Delta t$) and social welfare ($\Delta W$). The mandatory regulation is not depicted here.
amounts to reallocate production towards inefficient farmers who benefit from the reform at the expense of efficient producers. Last, we have calibrated the model using data on a French watershed (Don watershed). Simulations indicate that satisfying a high degree of acceptability does not entail high welfare losses compared to low degree of acceptability.

References


Appendix

A Proof of lemma 1

Proof of part (i): Using the envelop theorem, we get before any regulation \( \pi^o(\theta) = -Sc_\theta(y^o(\theta), \theta) \).

As \( c_\theta < 0 \) and \( y^o(\theta) \leq y^o(\theta) \), we have:

\[
-c_\theta(y^o(\theta), \theta) \leq -c_\theta(y^o(\theta), \theta)
\]

But we have also \( s^o(\theta) \leq S \), so using equation \((??)\) we obtain:

\[
0 \leq \pi^o(\theta) \leq \pi^o(\theta)
\]

If \( \alpha(S) > 0 \), then \( \pi^o(\theta) \) intersects \( \pi^o(\theta) \) once and from above, either on the flat part or on the increasing part of \( \pi^o(\theta) \).

Proof of part (ii): given (i), for the constraint \( R \) \((??)\) to be binding at the optimum, \( \tilde{\theta}(S) \) must satisfy \( \alpha(S) = \int_{\theta^o(S)}^{\tilde{\theta}(S)} f(\theta; S) d\theta = F(\tilde{\theta}(S); S) \).

B Proof of proposition 1

Assume that \( 0 < \tilde{\theta}(S) < \theta^o(S) \). Then, integrating (IC1), we get:

\[
\pi(\theta) = \begin{cases} 
\pi^o(\tilde{\theta}(S)) & \text{for } \theta \leq \theta^o(S) \\
\pi^o(\tilde{\theta}(S)) - \int_{\theta^o(S)}^{\tilde{\theta}(S)} c_\theta(y(u), u)s(u)du & \text{for } \theta > \theta^o(S)
\end{cases}
\]  

Using (3), we rewrite the objective \( W^S \) as follows:

\[
W^S = \int_{\theta^o(S)}^{\tilde{\theta}(S)} [(1 + \lambda)s(\theta)(py(\theta) - c(y(\theta), \theta)) f(\theta; S) d\theta - \lambda \int_{\theta^o(S)}^{\tilde{\theta}(S)} \left[ \pi^o(\tilde{\theta}(S)) \right] f(\theta; S) d\theta

- \lambda \int_{\theta^o(S)}^{\tilde{\theta}(S)} c_\theta(y(u), u)s(u)du \left[ \pi^o(\tilde{\theta}(S)) \right] f(\theta; S) d\theta - D(E)
\]

Integrating by parts, we obtain:

\[
W^S = \int_{\theta^o(S)}^{\tilde{\theta}(S)} \left[ (1 + \lambda)s(\theta)(py(\theta) - c(y(\theta), \theta)) + \lambda s(\theta)c_\theta(y(\theta), \theta) \frac{1 - F(\theta; S)}{f(\theta; S)} \right] f(\theta; S) d\theta

- \lambda \pi^o(\tilde{\theta}(S)) \int_{\theta^o(S)}^{\tilde{\theta}(S)} f(\theta; S) d\theta - D(E)
\]
It remains to maximize $W^S$ w.r.t. $s(.,.)$ and $y(.,.)$. It is then clear that the optimal allocation $(s(.,.),y(.,.))$ follows the same rule as the one under a mandatory policy.

We now check that equation IC2 is satisfied. For any type-($\theta,S$) farmer who is allowed to produce, equation IC2 can be written as:

$$-S c_{\theta y}(y(\theta), \theta)y'(\theta) \geq 0$$

Derivating the first-order condition for $y$ with respect to $\theta$, and dropping any argument, we get:

$$-(1 + \lambda)(c_{yy}y' + c_{\theta y}) - D'(E)g_{yy}y' - D'(E)g_{\theta y} + \lambda \left( \frac{1 - E}{f} \right)(c_{\theta yy}y' + c_{\theta \theta y}) + \lambda c_{\theta y} \frac{d}{d\theta} \left( \frac{1 - E}{f} \right) = 0$$

This equation can be written as:

$$y' = \frac{(1 + \lambda)c_{\theta y} + D'(E)g_{\theta y} - \lambda \left( \frac{1 - E}{f} \right)c_{\theta \theta y} - \lambda c_{\theta y} \frac{d}{d\theta} \left( \frac{1 - E}{f} \right)}{-(1 + \lambda)c_{yy} - D'(E)g_{yy} + \lambda \left( \frac{1 - E}{f} \right)c_{\theta yy}}$$

We have already made the following technical assumptions: $c_{yy} < 0$, $c_{\theta yy} < 0$, $c_{\theta \theta y} > 0$ and $\frac{d}{d\theta} \left( \frac{1 - E}{f} \right) < 0$. Thus, if in addition $g_{\theta y} < 0$ and $g_{yy} > 0$, then we have $y'(\theta) > 0$, $\forall \theta$ and the second order conditions are fulfilled.

### C Proof of proposition 2

Given Lemma 1, we can integrate (IC1), and we get:

$$\pi(\theta) = \begin{cases} 
\pi^c(\tilde{\theta}(S)) + \int_{\theta}^{\tilde{\theta}(S)} c_{\theta}(y(u), u)s(u)du & \text{for } \theta \leq \tilde{\theta}(S) \\
\pi^c(\tilde{\theta}(S)) - \int_{\theta}^{\tilde{\theta}(S)} c_{\theta}(y(u), u)s(u)du & \text{for } \theta > \tilde{\theta}(S) 
\end{cases} \quad (5)$$

Using the definition of profits given by (5), we get for $W^S$:

$$W^S = \int_{\theta}^{\tilde{\theta}(S)} [(1 + \lambda)s(\theta)(py(\theta) - c(y(\theta), \theta))] f(\theta; S)d\theta$$

$$- \lambda \int_{\theta}^{\tilde{\theta}(S)} \left[ \pi^c(\tilde{\theta}(S)) + \int_{\theta}^{\tilde{\theta}(S)} c_{\theta}(y(u), u)s(u)du \right] f(\theta; S)d\theta$$

$$- \lambda \int_{\tilde{\theta}(S)}^{\tilde{\theta}(S)} \left[ \pi^c(\tilde{\theta}(S)) - \int_{\theta}^{\tilde{\theta}(S)} c_{\theta}(y(u), u)s(u)du \right] f(\theta; S)d\theta - D(E)$$
Integrating $W^S$ by parts, we get:

$$W^S = \int_{\tilde{\theta}(S)}^{\tilde{\theta}} \left[ (1 + \lambda)s(\theta)(py(\theta) - c(y(\theta), \theta)) - \lambda \pi^S(\tilde{\theta}(S)) \right] f(\theta; S) d\theta$$  \hspace{1cm} (6)

$$- \lambda \int_{\tilde{\theta}(S)}^{\tilde{\theta}} \left( s(\theta)c_0(y(\theta), \theta) \frac{F(\theta; S)}{f(\theta; S)} \right) f(\theta; S) d\theta$$

$$+ \lambda \int_{\tilde{\theta}(S)}^{\tilde{\theta}} \left( s(\theta)c_0(y(\theta), \theta) \frac{1 - F(\theta; S)}{f(\theta; S)} \right) f(\theta; S) d\theta - D(E)$$

Before derivating (6), note that the expression of the derivatives will depend on whether $\theta$ is greater or lower than $\tilde{\theta}(S)$.

### C.1 Analysis for $\theta < \tilde{\theta}(S)$

For $\theta < \tilde{\theta}(S)$, derivating (6) with respect to $y(\theta)$, we obtain the following first-order condition for an (interior) optimum:

$$\frac{\partial W^S}{\partial y(\theta)} = 0 \Rightarrow (1 + \lambda) \left[ p - c_y(y^a(\theta), \theta) \right] - g_y(y^a(\theta), \theta)D'(E^a) - \lambda c_{y\theta}(y^a(\theta), \theta) \frac{F(\theta; S)}{f(\theta; S)} = 0$$  \hspace{1cm} (7)

Similarly, with respect to $s(\cdot)$, we obtain the following expression for the social marginal surplus of land:

$$\frac{\partial W^S}{\partial s(\theta)} = (1 + \lambda) \left[ py^a(\theta) - c(y^a(\theta), \theta) \right] f(\theta; S) - \lambda c_{\theta}(y^a(\theta), \theta)F(\theta; S) - g(y^a(\theta), \theta)D'(E^a) f(\theta; S)$$

Note that,  

$$\text{sign}(\frac{\partial W^S}{\partial s(\theta)}) = \text{sign}(B(\theta))$$

where

$$B(\theta) = (1 + \lambda) \left[ py^a(\theta) - c(y^a(\theta), \theta) \right] - \lambda c_{\theta}(y^a(\theta), \theta) \frac{F(\theta; S)}{f(\theta; S)} - D'(E^a)g(y^a(\theta), \theta)$$  \hspace{1cm} (8)

Using (7) and dropping any argument, we get from (8):

$$B' = -(1 + \lambda)c_0 - \lambda c_{\theta\theta} \frac{F}{f} - \lambda c_0 \frac{d}{d\theta} \left( \frac{F}{f} \right) - D'(E^a)g_{\theta}.$$ 

Note that when $c_{\theta\theta}$ is small with respect to the other terms, $B'(\theta) > 0$. Assuming $B(\tilde{\theta}) < 0$ and $B(\tilde{\theta}) > 0$, there exists a unique threshold type $\theta^a(S)$ such that for every $\theta < \theta^a(S)$,
\[ \partial W^S / \partial s(\theta) < 0 \text{ and consequently } s^a(\theta) = 0 \text{ so that all sufficiently inefficient farmers are shut down. Moreover, for any } \theta > \bar{\theta}(S), \partial W^S / \partial s(\theta) > 0 \text{ and consequently } s^a(\theta) = S. \] 

The threshold type separating active from inactive farmers is given by:

\[
(1 + \lambda)(py^a(\theta^a(S))) - c(y^a(\theta^a(S)), \theta^a(S)))f(\theta^a(S); S) - D'(E^a)g(y^a(\theta^a(S)), \theta^a(S))f(\theta^a(S); S) = \lambda c_0(y^a(\theta^a(S)), \theta^a(S))F(\theta^a(S); S)
\]

This equation means that the regulator equalizes the marginal surplus (net of damage) of keeping active the type-\(\theta^a(S)\) farmers with the corresponding incentive cost for all smaller type farmers (in proportion \(F(\theta^a(S); S)\)).

### C.2 Analysis for \(\theta > \bar{\theta}(S)\)

For \(\theta > \bar{\theta}(S)\), derivating (6) w.r.t \(y(\theta)\) yields to:

\[
(1 + \lambda)(p - c_y(y^a(\theta), \theta)) = g_y(y^a(\theta), \theta)D'(E^a) - \lambda c_y(y^a(\theta), \theta) \frac{1 - F(\theta; S)}{f(\theta; S)}
\]

Once again, the optimal production level follows the same rule as for the optimal mandatory policy and for the optimal policy when \(\alpha(S)\) is "small".

### C.3 Pooling around \(\tilde{\theta}(S)\)

From (7) and (9), it is clear that \(y^a(\theta)\) decreases discontinuously at \(\theta = \bar{\theta}(S)\) (that is, \(\lim_{\theta \to \bar{\theta}(S)^-} y^a(\theta) > \lim_{\theta \to \bar{\theta}(S)^+} y^a(\theta)\)). Thus, it is not possible to implement \(y^a(\theta)\) without violating incentive compatibility constraints (more precisely (??)). The optimal policy involves pooling with \(y^a(\theta) = \bar{y}(S)\) in a neighborhood \([\theta_1(S), \theta_2(S)]\) around \(\bar{\theta}(S)\) (Lewis and Sappington, 1989).

It remains to determine \(\theta_1(S), \theta_2(S)\) and \(\bar{y}(S)\). First, it is easy to show that \(y^a(\theta)\) is continuous (see Lewis and Sappington (1989)). Then, rewriting the profit function for any \(\theta\)-type farmer, we get:

\[
\pi^a(\theta) = \begin{cases} 
\pi^a(\bar{\theta}(S)) + \int_{\theta_1(S)}^{\bar{\theta}(S)} c_0(y^a(u), u)s^a(u)du + \int_{\theta_1(S)}^{\bar{\theta}(S)} c_0(\bar{y}(S), u)s^a(u)du \text{ pour } \theta \leq \bar{\theta}(S) \\
\pi^a(\bar{\theta}(S)) - \int_{\theta_2(S)}^{\bar{\theta}(S)} c_0(\bar{y}(S), u)s^a(u)du - \int_{\theta_2(S)}^{\bar{\theta}(S)} c_0(\bar{y}(S), u)s^a(u)du \text{ pour } \theta > \bar{\theta}(S) 
\end{cases}
\]
Denote \( AW(s(\theta), y(\theta), \theta) = (1 + \lambda)s(\theta)(py(\theta) - c(y(\theta), \theta) - \lambda \pi(\theta) - D(E) \). Then \( W^S = \int_\theta^{\tilde{\theta}} [AW(s^a(\theta), y^a(\theta), \theta)] f(\theta; S)d\theta \). Differentiating \( W^S \) with respect to \( \tilde{y}(S) \), we get:

\[
\frac{dW^S}{d\tilde{y}(S)} = \int_{\theta_1(S)}^{\theta_2(S)} \frac{\partial AW(s^a(\theta), \tilde{y}(S), \theta)}{\partial y} f(\theta; S)d\theta = 0.
\]