Farm-Level Risk-Balancing Behavior and the Role of Latent Heterogeneity

Tamirat S. Aderajew, Xiaoxue Du, Joost M.E. Pennings, and Andres Trujillo-Barrera

The risk-balancing hypothesis (RBH) suggests that farms will take less business risk as their financial risk increases, but existing literature provides empirical evidence that the RBH might be invalid under certain circumstances. We present a unified model that explains the conditions under which the RBH holds or is invalidated by recognizing the role of latent heterogeneity among farms. We generalize the RBH idea and trace the source of credit risk back to latent heterogeneity among farms. We then apply recent literature to longitudinal data from a panel of Dutch farms and classify segments using a finite mixture regression fixed-effects model and find that the RBH may not apply to all groups in the same way.

Key words: farm business, finite mixture regression fixed-effects model, risk-balancing hypothesis

Introduction

How do farmers adjust their debt use in response to changes in the risk of their business? The risk-balancing theory, introduced by Gabriel and Baker (1980), suggests that a farm’s total risk can be decomposed into business risk and financial risk. Business risk is the risk a farm faces because of production, marketing, institutional, personal, or technological changes, while financial risk arises through the use of debt-financed assets. The risk-balancing hypothesis suggests that farms have an optimal level of total risk. Thus, when a farm’s business risk decreases, an increase in financial risk would be expected in order to maintain the optimal level of total risk.

The original risk-balancing hypothesis has inspired many extensions. Collins (1985), Featherstone et al. (1988), Escalante and Barry (2003), de Mey et al. (2014), and Bampasidou, Mishra, and Moss (2017) provide evidence of farm risk balancing. However, the role of (latent) heterogeneity in farms’ risk balancing has received less attention. As defined by Pennings and Garcia (2004), latent heterogeneity refers to two interrelated ideas: First, the impact of risk on behavior may not be equal across market participants (Haushalter, 2000; Du et al., 2015). Factors that play an important role in some farmers’ risk perception and risk management may be inconsequential for others. In this context, rather than all farmers responding to risk balancing in the same way, we expect segments of farmers to respond similarly. Second, latent heterogeneity refers to the fact that segments are not directly observable prior to analysis. Instead, these segments are determined by grouping together farmers who show similar associations between the determinants of risk balancing and their behavior.

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Most previous studies have analyzed farms by classifying them using a priori knowledge and observable characteristics, such as specific farm types (e.g., dairy, cash grains, or livestock), size, age, or region. The implicit assumption behind this seems to be that the same decision-making process applies to those farms, and farms within these groups are thus treated as homogeneous. However, assuming a priori that, for example, farmers from the same region or with similarly sized farms will behave similarly is not necessarily true. Farmer heterogeneity within these seemingly homogeneous farm types may originate from various sources, including farming motivations, external production conditions, or risk preferences, none of which are directly observable. It has been shown that classification of farmers based on observables may fail to identify critical relationships. For instance, Pennings and Garcia (2004) and Pennings and Garcia (2010) show that heterogeneity may mask important effects at the aggregate level when studying farmer hedging behavior. Hence, to gain more insight into risk-balancing behavior, we must use a model that simultaneously classifies farms into segments on the basis of the relationship between risk and the determinants of risk-balancing behavior and estimates the influence of these determinants on risk balancing in each segment identified. The main objective of this article is thus to account for the role of unobserved heterogeneity in farm risk-balancing behavior.

The latest developments in the panel data literature are also moving in that direction. Ando and Bai (2016); Sarafidis and Weber (2015); Bonhomme and Manresa (2015); Deb and Trivedi (2013) have shown that characterizing all observations in a single sample, while easier to estimate, may mask critical relationships contained within segments of that sample. Applications of these procedures reiterate the advantages of data-driven identification of latent group structures over “arbitrary” variables established a priori in order to classify a sample into groups (Su, Shi, and Phillips, 2016).

We build on the established frameworks and specifications of Gabriel and Baker (1980), Collins (1985), Wu, Guan, and Myers (2014), expanding the model with the effects of unobserved heterogeneity. We first build a simple theoretical model to show that, under certain conditions, the original risk-balancing hypothesis may or may not hold when there is unobserved heterogeneity. Second, fed by our conceptual model, we design a finite mixture regression fixed-effects model, which provides a unifying framework that classifies farmer observations into segments (classes), and we estimate a panel regression based on the explanatory variables of the model. Disentangling the whole sample into different segments, such that farms behave the same within one segment but dissimilar across segments (i.e., accounting for [latent] heterogeneity) allows us to better understand the relationship between financial risk and business risk.

We make two main contributions to the literature. First, we provide a unified model that explains conditions under which the risk-balancing hypothesis holds or is invalidated. In particular, we link these conditions to the role of latent heterogeneity among farms. The existing literature presents empirical evidence that the risk-balancing hypothesis could be invalid under certain circumstances (Uzea et al., 2014). With the exception of Wu, Guan, and Myers (2014)—who discuss how the credit risk that lenders face could cause a violation of the risk-balancing hypothesis—few papers provide a theoretical foundation for this phenomenon. In this paper, we generalize that idea and trace the source of credit risk back to latent heterogeneity among farms. Second, this paper is the first empirical paper to use latent structures in panel data (from a fixed-effects model) to deal with the latent heterogeneity of farms. Using longitudinal data from a panel of Dutch farms over a period of 15 years (2001–2015), we find that the risk-balancing hypothesis may not apply equally to different groups in the farming sector.

Related Literature

Volatile prices, changing policies, greater capital requirements, and the business operation process—from production to marketing—are common sources of risk in the agricultural sector (Hardaker et al., 2004). In order to build and sustain a competitive advantage, it is imperative for agricultural
producers to understand the sources of risk and mitigate the effects of exposure to these sources. Risk balancing constitutes an alternative tool to analyze and manage risk. The key notion of risk balancing is that total farm risk can be decomposed into business risk and financial risk (Gabriel and Baker, 1980).

Business risk is the inherent variability in the operating performance of a farm, independent of the way it chooses to finance its operations (i.e., its capital structure, Escalante and Barry, 2003). The most common business risks for farms are production (yield), price, institutional, technological, and personal risks (Hardaker et al., 2004). Uncontrollable weather conditions (e.g., heavy rainfall or lack of rain), insect infestations, and diseases induce yield (production) risks (Hardaker et al., 2015). Price risks are mainly attributable to the changes in output and input prices of agricultural products (Yonezawa and Richards, 2016; Harwood et al., 1999). Changes in government policies and regulations on agriculture (e.g., the abolition of milk quotas in Europe in 2015) are categorized under institutional risks (Barry and Ellinger, 2012). Technological risks are associated with the evolution of production techniques (Moschini and Hennessy, 2001), in that innovations can make previous investments obsolete or erode a competitive/comparative advantage. With respect to personal risks, farming is a sector in which family businesses and small partnerships are still common (Pennings and Garcia, 2004). As a consequence, factors such as illness, accidents, or family relationships may threaten the farm business.

Financial risk is caused by uncertainty regarding interest rates; changes in the cash flows of farm operations, which are used to repay farm loans; and changes in the market value of farm assets used as loan collateral (Langemeier, 2016). The use of borrowed funds means that a portion of the business profits must be reserved for making debt and interest payments (Harwood et al., 1999). Even farms that are entirely financed through equity still find their capital exposed to the probability of losing equity.

Risk Balancing

The risk-balancing hypothesis suggests that each farm has an optimal level of risk. If a farm’s total risk were to decrease through an exogenous reduction of its business risk, this drop in optimal risk level would then cause this farm to assume more financial risk. Conversely, exogenous shocks that increase the level of business risk, such as droughts or floods, may lead farms to opt for decreased borrowing, decreasing their financial risk and restoring the optimal total risk (e.g., Collins, 1985; Featherstone et al., 1988).

Many studies in the risk-balancing literature revolve around the leverage adjustments made by farms following changes in the level of business risk. Robison and Barry (1987) underline the importance of adjusting leverage as part of a risk-management strategy. Applying the expected utility model of optimal hedging, Turvey and Baker (1989) provide empirical evidence of risk-balancing behavior, where hedging is used as a risk-management instrument to lower business risk whenever financial risk increases due to a higher leverage ratio. Featherstone, Preckel, and Baker (1990) capture the sequential and stochastic nature of capital-structure decisions that farms make in response to expected net farm income. Similarly, Moss, Shonkwiler, and Ford (1990) show that aggregate debt is highly elastic (greater than 1) in relation to farm earnings (expected farm income).

In line with the leverage adjustment studies, Ramirez, Moss, and Boggess (1997) show that the ratio of debts to assets (measured as the volatility of expected returns on farm assets) is elastic for business risk. Langemeier (2016) shows that farms opt to adjust their exposure to business risk through, for instance, diversifying or purchasing crop insurance whenever their exposure to financial risk increases due to a rise in interest rates on farm loans. Recently, using U.S. data, Bampasidou, Mishra, and Moss (2017) model farm capital structure choices and show that earnings volatility is negatively related to the total amount of farm debt.

An interesting phenomenon in the risk-balancing literature is “the risk-balancing paradox,” whereby farm policies designed to reduce or buffer the level of business risk exposure might
actually expose farms to even higher total risk due to risk balancing. In this regard, Collins (1985) provides the first evidence of how policies aimed at increasing farm income might actually induce additional risk-taking behavior. Similarly, Featherstone et al. (1988) show that farms can go bankrupt from increasing leverage in response to income-raising policies. Ahrends, Collender, and Dixon (1994) find that policies intended to decrease farm business risk or augment farm profit induce farms to increase financial risk through the acquisition of more debt. They emphasized that the adjustment process is slow and time consuming. Iftt et al. (2013) also find evidence supporting the risk-balancing paradox, in which farm-financing decisions (debt use), participation in federal crop insurance (FCI), and financial risk were interrelated. They argue that, as business risk decreases, increased financial risk (i.e., a higher level of leverage) becomes acceptable for farms that have been operating at the optimal total risk level. However, Cheng and Gloy (2008) provide evidence that contradicts the phenomena of the risk-balancing paradox. Their analysis of risk adjustment by farms shows that policies aimed at reducing farm risks promote discipline in the use of debt to reduce the volatility of farm income, while simultaneously increasing the expected income from farm activities.

Empirical evidence of the risk-balancing behavior of European (EU-15) farmers is provided by de Mey et al. (2014), who examine the adjustments that farms made to financial risk levels in response to changes in business risk after exogenous shocks. A little over half of the farms investigated were found to be engaged in a weak form of risk balancing (i.e., showing some variation in total risk) as opposed to the strong form of risk balancing, where total risk is assumed to be constant over time. De Mey et al. (2016) extend the risk-balancing hypothesis from farm level to household level and provided empirical evidence of household risk-balancing behavior. Using data from Switzerland, they find that, in addition to strategic on-farm adjustments, farms also make off-farm adjustments at the household level in response to changes in farm business risk (e.g., by adjusting the levels of off-farm income and consumption).

Heterogeneity in Risk Balancing

The assumption of homogeneity among decision makers has been widely rejected in the analysis of economic behavior (Devadoss, Gibson, and Luckstead, 2016; Pennings and Garcia, 2004). While characterizing all observations using a single model is convenient, it may mask critical relationships. Factors that play an important role in the risk-balancing behavior of some farmers may be unimportant for others.

To account for the heterogeneity among risk balancers, previous studies have used observable variables like size, location, or farm type to segment the total population (de Mey et al., 2014, 2016; Escalante and Barry, 2003; Escalante and Rejesus, 2008; Uzea et al., 2014). This approach implicitly assumes that farms of the same type will respond similarly to changes in the determinants of risk-balancing behavior. A potential problem is that the assumptions of homogeneity among decision makers and the characterization of all observations by a single model may be misleading, particularly if the sample contains latent segments. Risk-balancing behavior is an outcome of an underlying decision-making process, reflected in the relationship between risk and risk balancing. If one seeks to understand the heterogeneity in risk-balancing behavior, one needs to understand the heterogeneity in that decision-making process: It is latent (not directly observable) as it is reflected in the impact of risk on risk balancing. Hence, to further increase insight into risk-balancing behavior, it is important to use an empirical model that simultaneously classifies farms into segments on the basis of the relationship between risk and the determinants of risk-balancing behavior and estimates the influence of these determinants on risk balancing for each segment identified.

Recent developments in panel-data literature further underpin the importance of recognizing and handling potential latent heterogeneity. Su, Shi, and Phillips (2016, p. 2216) argue that neglecting latent heterogeneity in panel data may lead to inconsistent estimations and misleading inference, and, until recently, “traditional panel data models deal with this challenge by avoidance.” Effort in the literature is devoted to determining the unknown panel structure without using external
variables or *a priori* knowledge, such as location or industry composition (Wang, Phillips, and Su, 2018). The panel-data literature approaches latent heterogeneity from within a unifying framework using statistical classification procedures such as clustering (Ando and Bai, 2016; Sarafidis and Weber, 2015; Bonhomme and Manresa, 2015), machine learning methods such as lasso (Su, Shi, and Phillips, 2016; Su and Ju, 2018), or finite mixture models (Kasahara and Shimotsu, 2009; Browning and Carro, 2010; Deb and Trivedi, 2013)). The application of these procedures reveals the advantages of data-driven identification of latent group structures (Su, Shi, and Phillips, 2016).

These developments in the panel-data literature stress the importance of identifying heterogeneity in the empirical analysis of how farms determine their capital structure, since risk-balancing behavior cannot be completely (and accurately) explained without accounting for latent groups. This is a fundamentally different approach from previous studies dealing with heterogeneity, in which the segments were determined *a priori* based on observable variables such as size, region, or farm type.

Most theoretical and empirical findings favor the risk-balancing hypothesis, but work on farm risk-balancing behavior to date has not explicitly addressed latent heterogeneity. Rather, a single set of parameters is typically estimated for any set of explanatory variables, under the assumption that all farms face similar constraints and behave in similar ways. This paper contributes to the existing risk-balancing literature by explicitly modeling, and thus accounting for, latent heterogeneity. The procedure emphasizes the role of theory in the empirical analysis, as the determinants of risk-balancing behavior are used both to explain risk-balancing behavior and to discriminate among groups of farmers. From a conceptual perspective, the procedure permits the determinants of risk-balancing behavior to have a different influence on actual risk-balancing behavior in each segment identified.

**Conceptual Framework**

Following the literature developed by Collins (1985) and Wu, Guan, and Myers (2014), this section uses a simple conceptual framework to discuss the role of latent heterogeneity in farms’ decisions on optimal capital structure. A farm makes decisions over debt-to-equity ratio (i.e., leverage ratio), denoted by $l$, to maximize expected utility of rate of return on its equity ($E$). Due to risks from business factors such as price or yield fluctuations, the rate of return of the total assets ($A$), denoted by $r_A$, is random. We assume that the mean of $r_A$ is $\bar{r}_A$ and that the variance of $r_A$ is $\sigma_A^2$.

In the conceptual framework, latent heterogeneity of farms could occur in two forms: (i) as observable to lenders but not to researchers and (ii) as unobservable to both lenders and researchers. To formalize the idea, note that the farm has to pay interest on the debt-financed assets ($D$); we use $i(\alpha)$ to denote the interest rate this farm faces, which is a function of certain characteristics, $\alpha$, of the farm. The continuous variable $\alpha$ can be interpreted as the farm’s credit worthiness, determined by the amount of collateral, among other factors. These characteristics are often observable by lenders but not by researchers. Meanwhile, since lenders do not have perfect information about the borrowing farm, the farm’s interest rate could also be random. One possible source of such randomness is the defaulting risk of a farm, as introduced by Wu, Guan, and Myers (2014). We assume that the mean of $i(\alpha)$ is $\bar{i}(\alpha)$ and the variance is $\sigma_i^2(\alpha)$. In sum, the farm faces random interest rate of

$$i(\alpha) = \bar{i}(\alpha) + \sigma_i^2(\alpha)\varepsilon,$$

where $\varepsilon$ is a random variable of mean 0 and variance 1. The rate of return on the farm’s equity is equal to the rate of return on assets plus the rate of return from financial leverage:

$$r_E = r_A + [r_A - i(\alpha)]l.$$ 

Since $r_E$ is a linear combination of other random variables, we can verify that the mean and variance of $r_E$ are
Figure 1. Total Risk Decomposition

Notes: The variance is well established as the unit of measurement for risk in the risk-balancing literature, but other finance literature commonly uses standard deviation as the measure. Since variance is nothing but a monotonic transformation of the standard deviation, there is a one-to-one correspondence between the two measures.

\[ E(r_E) = \bar{r}_A + [\bar{r}_A - \bar{i}(\alpha)]l \]

and

\[ \text{var}(r_E) = \sigma^2_A(1 + l)^2 + \sigma^2_i(\alpha)l^2 - 2\text{Cov}(A, i)(1 + l)l. \]

Equation (4) is the risk decomposition in the risk-balancing literature. Figure 1 provides an illustration of this decomposition. The total risk of the farm, defined as the variance of the return on equity, can be decomposed into two parts: business risk (BR) and financial risk (FR). Business risk is the risk a farm without leverage (i.e., \( l = 0 \)) faces and financial risk is the additional risk a leveraged farm faces. As Figure 1 shows, on the expected return–standard deviation utility map, the utility of a risk-averse farm increases in terms of the expected return on its equity but decreases in terms of its equity risk (or standard deviation). The solid lines are the capital market lines (CML), which represent the highest attainable combinations of expected return and risk. The light gray line is the CML for a farm without leverage. It can be shown that when \( l = 0 \), the total risk of the farm is the variance of the return on asset \( \sigma^2_A \) (BR). When a farm chooses to use leverage, the CML rotates to a steeper slope, as indicated by the dark gray line. While a leveraged farm may attain a higher utility by choosing higher expected returns, it bears more risk. The increment in total risk (\( \text{var}(r_E) \)) is therefore equal to the financial risk.

The risk-balancing hypothesis discussed in Collins (1985) suggests that a farm will assume more financial risk (i.e., increase its leverage ratio) as its business risk (\( \sigma^2_A \)) decreases. To examine whether this hypothesis still holds under latent heterogeneity, we conducted a comparative statics analysis. The farm’s expected utility maximization problem can be written as a mean-variance formulation:

\[ \max_l E(r_E) - \frac{\rho}{2} \text{var}(r_E), \]

where \( \rho \) is the farm owner’s risk-aversion coefficient.

\(^1\) The rate of return–standard deviation indifference curve can be traced back to Sharpe (1964).
The first-order condition yields
\begin{equation}
\bar{r}_A - \bar{i}(\alpha) - \frac{\rho}{2} \left[ 2\sigma^2_A(1 + l) + 2\sigma^2_i(\alpha)l - 2\text{Cov}(A, i)(1 + 2l) \right] = 0.
\end{equation}
Rearranging, we get the optimal leverage ratio \( l^* \):
\begin{equation}
l^* = \frac{\rho^{-1}[\bar{r}_A - \bar{i}(\alpha)] - [\sigma^2_A - \text{Cov}(A, i)]]}{\sigma^2_A + \sigma^2_i(\alpha) - 2\text{Cov}(A, i)}.
\end{equation}
Based on the optimal leverage ratio, we can conduct a comparative statics analysis. It should be noted that Collins (1985) relies on the critical assumption that farms face a constant interest rate \( i \).
In fact, our conceptual framework reduces to Collins’s (1985) model if we include this assumption.

**Remark 1.** When \( i(\alpha) \) is nonrandom, this model reduces to Collins (1985). That is \( \frac{dl^*}{\sigma^2_A} < 0 \).  

The intuition behind Remark 1 is that, as long as \( i(\alpha) \) is nonrandom, there is no latent heterogeneity among farms from a lender’s perspective and thus no distortion in the pricing of the debt-financed asset. Consequently, an exogenous shock that reduces the business risk would enable a farm to assume more financial risk while leaving the total amount of risk unchanged. Also note that, from a researcher’s perspective, neglecting the unobservable \( \alpha \) would not affect detecting risk-balancing behavior, but it would affect the estimate of the heterogeneity of farm responses to the same business risk shock. To see this, note that as long as \( \frac{dl^*(\alpha)}{d\alpha} \neq 0 \), we must have
\begin{equation}
\frac{\partial^2 l^*}{\partial \sigma^2_A \partial \alpha} = \rho^{-1}i'(\alpha)'(\sigma^2_A)^{-2} \neq 0.
\end{equation}
This means that, when interest rates are not identical for all farms, their leverage ratios will respond differently to business risk shocks.

When \( i(\alpha) \) is random, the risk-balancing hypothesis may or may not hold. The following proposition demonstrates this point:

**Proposition 1.** When \( i(\alpha) \) is random, the risk-balancing hypothesis holds (i.e., \( \frac{dl^*}{\sigma^2_A} < 0 \)) if and only if \( l^* > \frac{r^{-1}}{1-2r} \), where \( r \) is defined as \( \frac{d\text{Cov}(A, i)}{d\sigma^2_A} \).

The intuition behind Proposition 1 is that, when \( i(\alpha) \) is random, lenders do not have perfect information about the borrowing farms. The implications of this imperfect information are twofold: First, the pricing of the debt-financed asset could be distorted. This could keep farmers from using more leverage, not because they became less risk tolerant but rather because they would be paying a higher interest rate on the debt-financed asset. Second, the variance of the interest rate could be correlated with business risk; consider a scenario where \( r \) is positive. This could happen if the correlation coefficient between business risk \( \sigma^2_A \) and interest rate were positive. The implication of this positive correlation is that lenders would adjust the farm’s interest rate if it experienced an exogenous shock that would increase the farm’s business risk. Consequently, the increased interest rate may force the farm to lower its leverage ratio in rebalancing its business and financial risks.

The implication of Proposition 1 is that, in order to correctly identify a farm’s risk-balancing behavior, researchers must account for not only each individual farm’s latent heterogeneity, which affects \( r \), but also its leverage ratio, \( l^* \).

**Empirical Model**

*Farm-Level Risk-Balancing Behavior*

To examine whether farms exhibit risk-balancing behavior, we use a fixed-effects panel-regression model. The framework is based on insights from the conceptual model from the previous section.

\(^2\) See the Appendix for the proof.
\(^3\) See the Appendix for the proof.
and the specification of Wu, Guan, and Myers (2014), which in turn builds on the framework by Collins (1985). From equation (4), the determinants of financial risk (\( l^* \)) are business risk, \( \sigma_A^2 \); profitability \( r_A \); unobservable farms’ risk-aversion coefficient, \( \rho \), and expected interest rate, \( \tilde{i}(\alpha) \), and the covariance between \( i \) and \( r_A \), which are all latently heterogeneous. Since the fixed-effects model controls for unobservable but time-unvarying factors, it is able to control for \( \rho \) and \( \tilde{i}(\alpha) \).

We use a latent-class, fixed-effects mixture model to account for the covariance between \( i \) and \( r_A \). In addition, we use Proposition 1 to show that farms’ responses to a business risk shock also depend on the financial risk that these farms had been taking before the shock. Thus, we include lagged financial risk as an explanatory variable, regressing the financial risk on 1-year lagged business and financial risks as well as on farm size and age.

We use the debt-to-equity ratio as a proxy for financial risk (\( FR \)), and we approximate business risk (\( BR \)) using the standard deviation of farm profitability over the immediately preceding 3 years. Farm profitability (profit) is the ratio of net farm income to total assets. Farm profitability is expected to have a negative effect on financial risk as it helps to cover interest expenses and increases the level of debt coverage (de Mey et al., 2014; Escalante and Barry, 2003).

We include farm size as a farm-structural factor to explore the linkage with farm financial risk. Farm size is measured as the log of total farm output. Some literature argues that larger farms are likely to have more debt (Purdy, Langemeier, and Featherstone, 1997); however, farmers tend to prefer the use of internal over external sources of financing (Pecking theory). As such, it is not a priori clear whether larger farm size necessarily implies a higher leverage. We use year dummies to capture time-fixed effects. The estimated model is

\[
FR_{i,t} = \beta_1 FR_{i,t-1} + \beta_2 BR_{i,t-1} + \beta_3 Profit_{i,t-1} + \beta_4 FarmSize_{i,t} + \Gamma_t + \alpha_i + \epsilon_{i,t},
\]

where \( i \) and \( t \) are index farm and year, respectively; \( \alpha_i \) are individual fixed effects; \( \Gamma_t \) are year dummies; and \( \epsilon_{i,t} \) is the error term.

**Farm Risk-Balancing Behavior and Latent Heterogeneity**

The assumption of homogeneity among decision makers and the characterization of all observations by a single model may mask critical relationships and may be misleading if the sample consists of a number of unknown segments. Addressing this limitation requires disaggregation of the entire sample into segments.

To account for latent heterogeneity, we use a latent-class, fixed-effects mixture model (Kasahara and Shimotsu, 2009; Browning and Carro, 2010; Deb and Trivedi, 2013) in which observations are classified into multiple segments based on the relationship between financial risk and the explanatory variables. The classification is based on whether farms respond to the explanatory variables in similar ways. We assume that farms come from a population that consists of a mixture of unobserved (latent) segments \( S \), whereby each farm \( i \) in year \( t \) belongs to one, and only one, segment \( s \), which is not known in advance. While farms within each segment are assumed to be homogeneous with respect to the impact of the explanatory variables on behavior, the impact of these variables can vary across segments. The finite mixture is estimated via maximum likelihood estimation. The likelihood is computed by combining conditional likelihoods of each latent segment (class), weighted by the associated latent-class probability (StataCorp, 2017). More specifically, we closely follow the finite mixture model from Pennings and Leuthold (2000), Pennings and Garcia (2004), and StataCorp (2017).

The relative size (proportion) of \( S \) segments, \( \pi_s \), is restricted to

\[
0 < \pi_s < 1, \sum_{s=1}^{S} \pi_s = 1.
\]

---

4 Relying on the assumption that a farm’s risk attitude is time invariant.
We model the probability that farm $i$ in year $t$ belongs to segment $s$, $\pi_{its}$, depending on a vector of farm-specific variables, $Z_{it}$. The finite mixture models assume that the values of segment membership follow a multinomial distribution; thus, the probability, $\pi_{its}$, is

$$
\pi_{its} = \Pr(c_i = 1|x) = \frac{\exp(\eta_s'Z_{it})}{\sum_{j=1}^{g}\exp(\eta_j'Z_{ij})},
$$

where $Z_{it}$ is the fitted value for the $ith$ latent segment, $\eta$ is the vector of coefficients for that segment, and $s_i$ a segment indicator. Finite mixture models are computed via maximum likelihood estimation, where the likelihood is estimated by combining the likelihoods for each segment are weighted by the latent segment probabilities. Let $\theta$ be the vector of model parameters. For a given observation, let $y$ denote financial risk and $x$ be the vector of independent variables that explain financial risk. The model is assumed to follow a linear form, $y_{it} = x_{it}'\theta + \epsilon$, and an underlying probability distribution function, $f_i(y_{it}|x_{it}).$

As such, the combination of the previous elements of the likelihood would yield

$$
L_S(\theta) = \sum_{i=1}^{S} \pi_i f_i(y|x, s_i = 1, \theta),
$$

where $S$ is the categorical latent variable with $s$ latent segments, 1, ..., $s$. Initial values for the maximum likelihood estimation are obtained via an expectation-maximization algorithm.

Data and Descriptive Statistics

The empirical analysis uses an unbalanced rotating panel of a Dutch sample of the Farm Accountancy Data Network (FADN) from between 2001 and 2015. Data were provided by Wageningen Economic Research (previously LEI, Landbouw Economisch Instituut). The sample is representative of 80% of the farms and more than 90% of production in the Netherlands.

We have applied the following inclusion criteria for farms: First, continuous whole-farm data had to be available for at least 4 years between 2001 and 2015, since business risk is calculated based on a 3-year rolling window. To address outlier concerns, extreme values in the dataset were handled by dropping the top and bottom 0.5% observations for each variable from the analysis. These criteria reduced the total number of farms included in this study to 1,339 (89% of the original number of farms) and reduced the number of observations from 15,982 to 8,992. Table 1 report summary statistics for the variables. Since the data include four farm types (dairy, field-crop, horticulture, and livestock farms), summary statistics by farm type are also reported.

Results

Table 2 shows the results of the panel regression from 2004 to 2015 for the whole sample. The lag of financial risk (leverage) is positive and highly significant, suggesting that previous capital structure is the main determinant of current capital structure; as identified by Wu, Guan, and Myers (2014), it does justify the dynamic adjustment specification. Business risk, as proxied by the standard deviation of profitability, is not significantly different from 0 since such evidence of potential risk balancing is not supported when accounting for the whole sample. In line with our expectation, the estimated relationship between lagged asset profitability and financial risk is negative and significant. Higher farm profitability results in lower farm financial risk. Although small in magnitude, the signs of the coefficients for farm size are both negative and significant. This may imply a relatively low impact of farm size on the adjustment of capital structure. Overall, year-dummy coefficients show small
Table 1. Variables and Summary Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial Risk</td>
<td>0.766</td>
<td>0.905</td>
</tr>
<tr>
<td>Dairy farms</td>
<td>0.648</td>
<td>0.526</td>
</tr>
<tr>
<td>Field crops</td>
<td>0.446</td>
<td>0.595</td>
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<tr>
<td>Horticulture</td>
<td>0.925</td>
<td>1.217</td>
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<td>Livestock</td>
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<td>0.995</td>
</tr>
<tr>
<td>Profitability (return on assets (ROA))</td>
<td>0.025</td>
<td>0.047</td>
</tr>
<tr>
<td>Field Crops</td>
<td>0.026</td>
<td>0.034</td>
</tr>
<tr>
<td>Horticulture</td>
<td>0.040</td>
<td>0.066</td>
</tr>
<tr>
<td>Livestock</td>
<td>0.017</td>
<td>0.049</td>
</tr>
<tr>
<td>Business Risk</td>
<td>0.025</td>
<td>0.026</td>
</tr>
<tr>
<td>Dairy farms</td>
<td>0.011</td>
<td>0.010</td>
</tr>
<tr>
<td>Field crops</td>
<td>0.020</td>
<td>0.020</td>
</tr>
<tr>
<td>Horticulture</td>
<td>0.039</td>
<td>0.030</td>
</tr>
<tr>
<td>Livestock</td>
<td>0.030</td>
<td>0.029</td>
</tr>
<tr>
<td>Log of farm size</td>
<td>12.888</td>
<td>1.031</td>
</tr>
<tr>
<td>Dairy farms</td>
<td>12.568</td>
<td>0.689</td>
</tr>
<tr>
<td>Field crops</td>
<td>12.731</td>
<td>0.920</td>
</tr>
<tr>
<td>Horticulture</td>
<td>13.264</td>
<td>1.196</td>
</tr>
<tr>
<td>Livestock</td>
<td>13.020</td>
<td>1.132</td>
</tr>
</tbody>
</table>

Notes: Financial risk is the ratio of debt to equity. Profitability is the ratio of net farm income to total losses. Business risk is the standard deviation of profitability. Log of farm size is the natural log of total assets in € in nominal value.

effects, which are significant after the 2008 financial crisis and around 2015, when farms may have made adjustments in response to changes in the EU Common Agricultural Policy (CAP).5

One potential issue with the estimating equation is that \( FR_{t-1} \) is endogenous, as unobservables that affect a farm’s financial risk this year will be affected by the financial risk a farm faced last year. Since estimating equation (9) controls for individual and time fixed effects, we believe the impact of leftover time-varying confounding factors will be small. To explore the potential impacts of endogeneity, we use an instrumental variable (IV) approach as robustness check.

We use further lags of \( FR_{t-1} \) as IV for \( FR_{t-1} \). It should be noted that since we use a fixed effect model as our main estimating equation and a fixed effect model is equivalent to a first-difference model, \( FR_{t-2} \) is used to estimate equation (9). As a consequence, \( FR_{t-2} \) could also be potentially endogenous. We propose using \( FR_{t-3} \) and \( FR_{t-4} \) as IV for \( FR_{t-1} \). The identifying assumption here is that \( FR_{t-3} \) and \( FR_{t-4} \) are sufficiently lagged from \( FR_{t} \) so that the exclusion restriction is more likely to be satisfied.

Table 2 reports the IV results. From the first-stage estimation results, we find that both \( FR_{t-3} \) and \( FR_{t-4} \) are statistically significant and the first-stage F-statistics are far greater than 10 (\( F = 51.06 \)).

5 Common Agricultural Policy for 2014–2020 was finalized in 2013 (https://ec.europa.eu/agriculture/cap-post-2013/) and started its implementation in 2015. According to Nazzaro and Marotta (2016, p. 1), “The reform of Common Agricultural Policy for 2014–2020 aims at promoting greater competitiveness, efficient use of public goods, food security, preservation of the environment and specific action against climate change, social and territorial equilibrium, and a more inclusive rural development.” Direct payments and market measures are the main policy tool of CAP, and these changed for the 2014–2020 period. Not only did the amount of money provided by EU decreased, but changes in the criteria for the allocation of subsidies have also occurred (Nazzaro and Marotta, 2016). For instance, subsidies are given to producers that meet certain economic conditions. They should be active farmers, and their business should be feasible and economically sustainable, but since redistribution is one of the objectives of the reform, then the farm business cannot be very large. Also, the aim of the policy is to avoid long-term dependence on subsidies and to decrease market disruption.
Table 2. Panel-Regression Results, 2004–2015

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Lag financial risk</td>
<td>0.491</td>
<td>0.010***</td>
<td>0.620</td>
<td>0.132***</td>
</tr>
<tr>
<td>Lag business risk</td>
<td>−0.042</td>
<td>0.269</td>
<td>0.049</td>
<td>0.521</td>
</tr>
<tr>
<td>Lag profitability</td>
<td>−0.562</td>
<td>0.123***</td>
<td>−0.092</td>
<td>0.443</td>
</tr>
<tr>
<td>Farm size</td>
<td>−0.087</td>
<td>0.018***</td>
<td>−0.106</td>
<td>0.035***</td>
</tr>
<tr>
<td>Year</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2005</td>
<td>−0.052</td>
<td>0.022**</td>
<td>−0.048</td>
<td>0.024**</td>
</tr>
<tr>
<td>2006</td>
<td>−0.040</td>
<td>0.022*</td>
<td>−0.037</td>
<td>0.026</td>
</tr>
<tr>
<td>2007</td>
<td>−0.030</td>
<td>0.021</td>
<td>−0.014</td>
<td>0.024</td>
</tr>
<tr>
<td>2008</td>
<td>0.005</td>
<td>0.021</td>
<td>0.017</td>
<td>0.026</td>
</tr>
<tr>
<td>2009</td>
<td>−0.017</td>
<td>0.021</td>
<td>−0.018</td>
<td>0.028</td>
</tr>
<tr>
<td>2010</td>
<td>−0.055</td>
<td>0.021***</td>
<td>−0.034</td>
<td>0.027</td>
</tr>
<tr>
<td>2011</td>
<td>0.091</td>
<td>0.021***</td>
<td>0.103</td>
<td>0.029***</td>
</tr>
<tr>
<td>2012</td>
<td>−0.003</td>
<td>0.021</td>
<td>0.005</td>
<td>0.026</td>
</tr>
<tr>
<td>2013</td>
<td>0.011</td>
<td>0.021</td>
<td>0.012</td>
<td>0.025</td>
</tr>
<tr>
<td>2014</td>
<td>0.007</td>
<td>0.021</td>
<td>0.014</td>
<td>0.026</td>
</tr>
<tr>
<td>2015</td>
<td>−0.072</td>
<td>0.020***</td>
<td>−0.064</td>
<td>0.024***</td>
</tr>
<tr>
<td>Constant</td>
<td>0.016</td>
<td>0.016</td>
<td>0.010</td>
<td>0.019</td>
</tr>
</tbody>
</table>

First-stage instruments

| Lag business risk ($t - 3$) | 0.174 | 0.012*** |
| Lag business risk ($t - 4$) | −0.140 | 0.012*** |
| *F*-statistic for IV in first stage | 51.06 |

No. of obs. | 8,992 | 7,520 |

Notes: Single, double, and triple asterisks (*, **, ****) indicate significance at the 10%, 5%, and 1% level, respectively. Standard errors for instrumental variables (IV) estimation are calculated using bootstrap method with 500 iterations.

This evidence suggests that $FR_{t-3}$ and $FR_{t-4}$ are relevant in determining $FR_{t-1}$ and the performance of first-stage estimation is reasonably good.

There are three things worth noting regarding the results from the second stage of the IV estimation: First, our results are robust in terms of the signs of the variables of interest. $FR_{t-1}$ remains positive and significant and $BR_{t-1}$ remains insignificant in the IV estimation. Second, the magnitude of marginal impact of $FR_{t-1}$ increases by roughly 20% after the IV estimation. This suggests that the time-varying confounding factors tend to lead to a mild downward bias in the estimated $\beta_1$. Third, we see that the marginal impact of control variables also change. Notably, the marginal impact of lagged profitability was significant in the fixed effect model but becomes insignificant under IV estimation, which suggests that the confounding factors could also be correlated with the control variables.

Subsequently, we used a finite mixture model to explore the potential existence of latent segments. We ran a number of different models from one to six segments. Based on the information criteria (Akaike and Bayesian) reported in Table 3, we selected the model with two segments to explore the segmentation of the sample. Robustness checks using three and four segments suggest no further significant differences among groups beyond segmentation into two segments.

Table 4 shows the results of the panel regression for the two-segments finite mixture model. About 76.75% of the total observations are allocated to the first segment, while the remaining 23.25% fall into the second segment. Results show that previous leverage (financial risk) remains the main determinant of current capital structure in both segments. However, we observe differences between the segments and compared with a panel regression that does not account for latent
Table 3. Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) \((N = 8,992)\)

<table>
<thead>
<tr>
<th>Model</th>
<th>AIC</th>
<th>BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 segment</td>
<td>(-7,577.880)</td>
<td>(-7,705.753)</td>
</tr>
<tr>
<td>2 segments</td>
<td>(-6,694.001)</td>
<td>(-6,431.149)</td>
</tr>
<tr>
<td>3 segments</td>
<td>(-8,681.274)</td>
<td>(-8,283.444)</td>
</tr>
<tr>
<td>4 segments</td>
<td>(-9,126.825)</td>
<td>(-8,594.019)</td>
</tr>
<tr>
<td>5 segments</td>
<td>Did not converge</td>
<td></td>
</tr>
<tr>
<td>6 segments</td>
<td>(-9,330.381)</td>
<td>(-8,527.619)</td>
</tr>
</tbody>
</table>

Table 4. Panel-Regression Results, Two Segments Finite Mixture, 2004–2015

<table>
<thead>
<tr>
<th></th>
<th>Segment 1</th>
<th>Segment 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lag financial risk</td>
<td>0.895</td>
<td>0.005***</td>
</tr>
<tr>
<td>Lag business risk</td>
<td>(-0.173)</td>
<td>0.071**</td>
</tr>
<tr>
<td>Lag profitability</td>
<td>0.050</td>
<td>0.032</td>
</tr>
<tr>
<td>Farm size</td>
<td>(-0.025)</td>
<td>0.040***</td>
</tr>
<tr>
<td>2005</td>
<td>(-0.014)</td>
<td>0.050**</td>
</tr>
<tr>
<td>2006</td>
<td>(-0.004)</td>
<td>0.005</td>
</tr>
<tr>
<td>2007</td>
<td>(-0.024)</td>
<td>0.005***</td>
</tr>
<tr>
<td>2008</td>
<td>(-0.012)</td>
<td>0.005**</td>
</tr>
<tr>
<td>2009</td>
<td>(-0.032)</td>
<td>0.004***</td>
</tr>
<tr>
<td>2010</td>
<td>(-0.030)</td>
<td>0.004***</td>
</tr>
<tr>
<td>2011</td>
<td>0.011</td>
<td>0.004**</td>
</tr>
<tr>
<td>2012</td>
<td>(-0.021)</td>
<td>0.004***</td>
</tr>
<tr>
<td>2013</td>
<td>(-0.016)</td>
<td>0.004***</td>
</tr>
<tr>
<td>2014</td>
<td>(-0.004)</td>
<td>0.004</td>
</tr>
<tr>
<td>2015</td>
<td>(-0.022)</td>
<td>0.004***</td>
</tr>
</tbody>
</table>

Percentage of total observations 76.75% 23.25%

Notes: Single, double, and triple asterisks (*, **, ***)) indicate significance at the 10%, 5%, and 1% level, respectively.

heterogeneity. The results of Table 4 are discussed in combination with Table 5, which profiles the identified segments using a number of key variables.

Table 5 shows that farmers’ risk balancing behavior in the first segment is driven by financial risk, business risk, and farm size but not by profitability. In the second segment, risk-balancing behavior is driven by financial risk, profitability, and farm size but not by business risk. Hence, the impact of the determinants of risk-balancing behavior differs across segments. The value of accounting for latent heterogeneity becomes apparent when comparing Table 2 with Table 4.

Not accounting for latent heterogeneity would suggest that business risk has no impact on farmers’ risk-balancing behavior. Hence, not accounting for latent heterogeneity would have masked a critical relationship contained within segments of the sample. Table 5 shows that the second segment exhibits a much larger financial risk than the first segment (Table 5); hence, profitability seems crucial for the ability to adjust one’s financial structure (Table 4) (i.e., building equity or paying off debt). Previous business risk, on the other hand, does not seem to affect the feasibility of this adjustment process in this segment. The first segment shows a “healthy” leverage that is much less dependent on debt. Profitability is less influential in the second segment, whereas the impact of business risk becomes an important factor in risk-balancing decisions. Farm size is negative and significant in both groups but more influential in the second segment.

Looking at the farm types, we note a relatively larger proportion of observations from dairy farms and field crops in the first segment (86.79% and 87.23%, respectively). The first segment
Table 5. Descriptive Statistics for Segments

<table>
<thead>
<tr>
<th>Variable</th>
<th>Segment 1</th>
<th>Segment 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial risk</td>
<td>0.516 (0.478)</td>
<td>1.577 (1.281)</td>
</tr>
<tr>
<td>Profitability</td>
<td>0.026 (0.040)</td>
<td>0.024 (0.059)</td>
</tr>
<tr>
<td>Business risk</td>
<td>0.020 (0.020)</td>
<td>0.039 (0.034)</td>
</tr>
<tr>
<td>Log farm size</td>
<td>12.718 (0.971)</td>
<td>13.447 (1.026)</td>
</tr>
</tbody>
</table>

Farm type (observations/percentages per segment)

<table>
<thead>
<tr>
<th>Farm type</th>
<th>Segment 1</th>
<th>Segment 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dairy farms</td>
<td>2,457 / 86.79%</td>
<td>374 / 13.21%</td>
</tr>
<tr>
<td>Field crops</td>
<td>1,476 / 87.23%</td>
<td>216 / 12.77%</td>
</tr>
<tr>
<td>Horticulture</td>
<td>1,587 / 66.91%</td>
<td>785 / 33.09%</td>
</tr>
<tr>
<td>Livestock</td>
<td>1,381 / 65.85%</td>
<td>716 / 34.14%</td>
</tr>
<tr>
<td>Total</td>
<td>6,901 / 76.75%</td>
<td>2,091 / 23.25%</td>
</tr>
</tbody>
</table>

Notes: Values in parentheses are standard deviations.

still dominates for horticulture and livestock farms but relatively less so (66.91% and 65.85%, respectively). In general, the horticulture and livestock industries exhibit larger financial and business risks, consistent with the segmentation of the sample. For the model with two segments, we do not use IV approach for two reasons: First, it is uncertain in the literature whether IV or segmentation should be done first. Second, as shown in Table 3, our results are robust in terms of the signs of the variables of interest.

Conclusions

In this paper, we examine the role of latent heterogeneity in farm risk-balancing behavior. We first built a conceptual framework based on Wu, Guan, and Myers (2014) to show that—when there is latent heterogeneity among farmer—the risk-balancing hypothesis will hold if and only if the farmer’s optimal leverage ratio is above a certain threshold. This theoretical prediction has two main implications: First, when a farmer faces credit constraint, they may not be able to achieve the desired optimal leverage ratio, which in turn leads to a violation of the risk-balancing hypothesis. This could explain the rejection of the risk-balancing hypothesis in a number of empirical studies. Second, if latent heterogeneity does exist, it is crucial that researchers incorporate latent groups in their empirical analyses.

Building on models and specifications from the risk-balancing literature and recent developments in the panel data literature that incorporate latent groups, we examine whether segments of the sample should be disentangled to better understand the impact of financial risk and business risk in each segment on risk-balancing behavior. The empirical analysis uses longitudinal data from a panel of Dutch farms, collected over a period of 15 years between 2001 and 2015, and the model is estimated using finite mixture models. The model simultaneously allows statistical classifications of the segments based on the explanatory variables and estimates the effects of these variables on farm risk balancing for each identified segment.

Following the finite mixture procedure, we identify two segments in the sample. We find differences in results between the whole sample (Table 3) and the segments (Table 4). While we find no association between business risk and risk-balancing behavior in the whole sample, we find differences between the segments: One segment in which risk balancing is relevant and another in which it is not. This suggests that the risk-balancing hypothesis may not apply in the same way
to different groups in the farming sector; the assumption of homogeneity, as traditionally made in the literature, or the segmentation of the population according to observed variables may mask relationships regarding risk-balancing behavior.

Results suggest that the segments can exhibit very different risk-balancing profiles. While past financial risk (capital structure) is the main determinant of the current capital structure in both segments, we find that in the first segment (which has relatively lower financial risk), business risk is negatively associated with financial risk, while profitability does not play a significant role. However, in the second segment (which has more leverage), business risk does not seem influential, while profitability is.

The importance of previous leverage and farm profitability in farm risk-balancing behavior has at least two important implications for farms, financial institutions, and policy makers: First, farms rely heavily on the availability of and access to loans. Because farm businesses have little to no access to equity markets, policies aimed at helping farms manage their risk should prioritize access to credit facilities. Second, the relationship between farm profitability and risk balancing suggests that farms use their retained earnings as a buffer when exogenous shocks disturb the optimal total risk level. In general, the results suggest that more attention should be paid to both observed and unobserved factors in designing and implementing individual risk-management instruments and in assessing their impact on the farming sector.

This paper has limitations that motivate further research. One important limitation is that, although we used a fixed-effect model on the rich panel dataset, there could still be confounding unobservables, potentially of the time-varying type. Since any unobservables that are not time-varying and follow a time trend are absorbed by the fixed effects, we believe that the magnitude of leftover confounding unobservables will be small. In order to deal with time-varying confounding unobservables, this paper uses further lagged variables as IV. Future research may complement this study with behavioral data on, for example, farm risk aversion and risk perception. Another interesting extension would be to conduct a farm survey or experimental procedure on risk preferences. There are also opportunities to look for better instrumental variables. Future research also needs to look into improving IV procedure for latent heterogeneity models. In particular, the consequence of the order of segmentation and use of instrumental variables requires further investigation. It should also be noted that the standard errors of IV in latent heterogeneity models might be inaccurate. (See (Bonhomme and Manresa, 2015) for details on this point.) Bootstrapping the standard errors may partially solve the issue.

Future research may also test the synergistic relationships between risk balancing and alternative risk-management strategies and their impact on farm performance in terms of profitability, risk resilience, and viability. Another promising direction for future research would be to improve our understanding of the variations in farm risk-balancing behavior across countries and economies, including, for example, the economies of developed versus developing countries and countries with vastly different risk-management and farm-support systems, such as European countries and the United States. Finally, recent developments in the panel-data literature motivate the re-examination of the traditional models of farm risk balancing. Alongside the use of finite mixture models, other approaches that include a combination of machine-learning classification techniques with panel-data procedures (Su, Shi, and Phillips, 2016; Bonhomme and Manresa, 2015) are venues for further research.

[First submitted July 2018; accepted for publication October 2019.]

References


Appendix A

Proof for Remark 1

When $i(\alpha)$ is nonrandom, this model reduces to the model in Collins (1985). That is, $\frac{dl^*}{\sigma_A} < 0$.

Proof. When $i(\alpha)$ is nonrandom, we have $\sigma_i^2(\alpha) = \text{Cov}(A, i) = 0$; then, the optimal leverage ratio reduces to

$$(A1) \quad l^* = \frac{\rho^{-1}[\bar{r}_A - \bar{i}(\alpha)]}{\sigma_\alpha^2} - 1.$$

Comparative statics of $\frac{dl^*}{d\sigma_A^2}$:

$$(A2) \quad \frac{dl^*}{d\sigma_A^2} = -\rho^{-1}[\bar{r}_A - \bar{i}(\alpha)](\sigma_\alpha^2)^{-2}.$$

Since $\bar{r}_A - \bar{i}(\alpha)$ would indicate a negative expected return of equity (which is not possible), we must therefore have $\bar{r}_A - \bar{i}(\alpha) > 0$, which in turn implies that $\frac{dl^*}{d\sigma_A} < 0$. ■

Proof for Proposition 1

Proof. Let $\sigma^2$ denote the term $\sigma_A^2 + \sigma_i^2(\alpha) - 2\text{Cov}(A, i)$; the comparative statics of $\frac{dl^*}{d\sigma_A}$ is:

$$(A3) \quad \frac{dl^*}{d\sigma_A} = \frac{(r - 1)\sigma^2 - l^* \sigma^2(1 - 2r)}{(\sigma^2)^2}.$$

Thus, $\frac{dl^*}{d\sigma_A} < 0$ if and only if

$$(A4) \quad l^* > \frac{r - 1}{1 - 2r}.$$

■