How Reliable Is It to Obtain Price Flexibilities from Inverting Price Elasticities?

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Introduction

Food price elasticities, defined as the percentage of changes in quantities demand for foods corresponding to given changes in food prices, and food price flexibilities defined reversely, are widely used in agricultural policy and program analyses. Most available food demand studies estimate price elasticites but not price flexibilities, probably because the Antonelli demand equation (a counterpart of the Slutsky demand equation) is not well known in demand modeling. Agricultural economists, however, often use flexibility measures for making agricultural pricing decisions to reflect that quantities and income are given in farm market demand relationships with price adjustments providing the market-clearing mechanism. Because of limited empirical flexibility estimates, most agricultural economists take the reciprocal of a directly estimated elasticity, or more rigorously the inversion of an elasticity matrix at the retail level, as flexibility measures. They then shift the model to the farm level by using price transmission or markup equations. In this paper, the major objective is to address the relationships between the empirical estimated price elasticities and flexibilities of food demands. In particular, I will provide a conceptual discussion and some empirical evidence to assess the reliability of this common practice of obtaining flexibility measures by inverting a matrix of directly estimated elasticities.

Conceptual problem

Although the matrices of elasticities and flexibilities are in deed reciprocal to each other in an economic model, the two matrices are in general not the inverse of one another in a statistical sense. For illustration, let's consider a set of single demand equations with only one independent variable: $q = a\, p + u$ and $p = b\, q + v$, where q and p are the relative changes of quantity and price of a commodity, parameters a and b are price elasticity and flexibility, respectively, and u and v are stochastic disturbance terms. For an economic model without the stochastic disturbance terms, there is no doubt that the price flexibility b is always a reciprocal to the price elasticity a. But for a statistical model with the stochastic disturbance terms, the estimates by applying the commonly used ordinary least squares give $a^* = (\boldsymbol{p}'\boldsymbol{q}) / (\boldsymbol{p}'\boldsymbol{p})$, and $b^* = (\boldsymbol{p}'\boldsymbol{q}) / (\boldsymbol{q}'\boldsymbol{q})$, where $\boldsymbol{p}$ and $\boldsymbol{q}$ are two-column vectors of price and quantity observations. According to the Cauchy-Schwarz inequality (Rao 1965, p.42), for any vectors $\boldsymbol{p}$, $\boldsymbol{q}$ of real elements, $(\boldsymbol{p}'\boldsymbol{q})^2 \leq (\boldsymbol{p}'\boldsymbol{p})\, (\boldsymbol{q}'\boldsymbol{q})$. Therefore, the following inequality should hold for the estimates: $a^* b^* \leq 1$. In other words, even though a simple form of demand equation, one cannot anticipate that the estimated price elasticity and its corresponding price flexibility should be reciprocals of each other.

For a general demand system with discernible cross-commodity effects, the estimated price flexibilities are certainly not the inverse of the estimated price elasticities. Why? First, for any well-known estimation procedure, the sum of residuals is minimized along the quantity axis in the estimation of an ordinary (quantity dependent) demand system, whereas the sum of residuals is minimized along the price axis in the estimation of an inverse (price dependent) demand system. Second, by inverting a demand matrix, one ignores the stochastic nature of the statistical estimates and treats the point estimates of the demand parameters as exact numbers. Third, the inverted results are quite sensitive to the numerical structure of a demand matrix being inverted, and that could cause unstable results. Fourth, there is a problem of interpreting the reciprocal of a directly estimated elasticity as flexibility measures. Direct-price flexibility in an inverse demand system reflects the price change of a commodity in response to a marginal change in its quantity by holding all other quantities and per capita expenditure constant. The reciprocal of direct-price elasticity in an ordinary demand system, however, should be interpreted differently as the price change of a commodity in response to a marginal change in its quantity by holding the same for all other prices and per capita expenditure.

Empirical evidence

Some empirical evidence is provided in this paper to compare the difference between the estimated price flexibilities and those inverted price flexibilities, which are generated from inverting an estimated price elasticity matrix. The previous estimates of two U.S. food demand systems in Huang (1991 and 1993), one an ordinary and the other one an inverse demand system for 39 food categories and 1 nonfood sector, will be used for the evaluation. The ordinary demand system is specified as a set of 40 linear differential-form equations. This demand system is estimated by applying the constrained maximum likelihood method with the parametric constraints of symmetry, homogeneity, and Engel aggregation. Similarly, using the distance function approach, the compensated inverse demand system consisting of the same 39 food categories and 1 nonfood sector is specified in the differential-form. Again, the parametric constraints of symmetry, homogeneity, and scale aggregation are incorporated into the estimation by applying the constrained maximum likelihood method. These estimates of compensated price flexibilities are then used to obtain the uncompensated price flexibilities through a linkage equation derived from the Antonelli demand equation.

At the beginning, the matrix of estimated price elasticities including 1,600 direct-and cross-price elasticities is inverted to obtain the inverted price flexibilities. These inverted price flexibilities are then used to compare with the estimated price flexibilities. The numerous results of price flexibilities make it difficult to perform a detailed comparison of the estimates for each food category, but the entries of direct price elasticities and flexibilities are shown in table 1 for a close comparison. In the table, the estimates of direct-price elasticities and flexibilities are listed in column A and C. The inverted direct-price flexibilities listed in column B are the basic demand information for use in evaluating the reliability of a common practice for obtaining flexibility measures from inverting a matrix of directly estimated elasticities. As anticipated, the inverted price flexibilities are substantially different from those of the estimated price flexibilities. Among the estimated direct price flexibilities, for example, the direct-price flexibilities for meats are beef (-1.156), pork (-1.142), and chicken (-1.239). On the other hand, the inverted direct-price flexibilities for meats are beef (-1.907), pork (-1.396), and chicken (-1.783). The ratios of inverted direct-price flexibilities to those estimated directly are listed in the last column of the table. For most food categories, the ratios in absolute value are larger than 1 implying that the inverted price flexibilities are substantially different from those estimated directly.

Conclusion

It is not proper to use the inverted elasticity as flexibility measures in agricultural policy and program analyses because of sizable measurement errors and inadequate interpretation of demand responses. The flexibilities from a directly estimated inverse demand system should be used to assess the price effects of quantity changes. To evaluate quantity effects of price changes, however, only elasticities from a directly estimated ordinary demand system should be used. Perhaps a statement from Frederick Waugh (1964) best addresses this point: " I prefer to use the price flexibilities themselves rather than their reciprocals. If, for any reason, the elasticity of demand is wanted, I would prefer to use the other regression equations, using quantities as the dependent variables." In other words, direct estimated price elasticities and flexibilities are preferable in either case.

References

Huang, K.S. *U.S. Demand for Food: A Complete System of Quantity Effects on Prices*. USDA ERS TB-1795, 1991.

Huang, K.S. *A Complete System of U.S. Demand for Food*. USDA ERS TB-1821, 1993.

Rao, C.R. *Linear Statistical Inference and Its Applications*. New York: John Wiley & Sons, 1965.

Waugh, F.V. *Demand and Price Analysis*. TB-1316. U.S. Dept. Agr., Econ. Res. Serv., 1964.

Table 1--Comparison of directly estimated and inverted price flexibilities

Food category	Estimated direct-price elasticity (A)	Inverted direct-price flexibility (B)	Estimated direct-price flexibility (C)	Ratio (B)/(C)
Beef	-0.621	-1.907	-1.156	1.65
Pork	-0.728	-1.396	-1.142	1.22
Other meats	-1.874	0.224	-0.198	-1.13
Chicken	-0.372	-1.783	-1.239	1.44
Turkey	-0.535	-1.526	-0.594	2.57
Fresh and frozen fish	0.121	0.910	-0.157	-5.80
Canned and cured fish	-0.372	-1.722	-0.036	48.38
Eggs	-0.110	-5.705	-3.689	1.55
Cheese	-0.247	-1.649	-0.366	4.51
Fluid milk	-0.043	-0.436	-0.294	1.48
Evaporated and dry milk	-0.276	-1.650	-0.095	17.35
Wheat flour	-0.078	-11.041	-0.313	35.32
Rice	0.066	6.573	-0.236	-27.87
Potatoes	-0.098	2.619	-0.711	-3.69
Butter	-0.243	0.467	-0.502	-0.93
Margarine	-0.009	4.477	-0.130	-34.49
Other fats and oils	-0.139	-1.344	-0.647	2.08
Apples	-0.190	3.285	-0.413	-7.95
Oranges	-0.849	-2.036	-0.756	2.69
Bananas	-0.499	-0.966	-0.335	2.89
Grapes	-1.180	0.316	-0.419	-0.75
Grapefruits	-0.455	-6.629	-0.637	10.41
Other fresh fruits	-0.416	-1.048	-0.083	12.66
Lettuce	-0.090	-1.140	-0.774	1.47
Tomatoes	-0.622	-2.477	-0.719	3.45
Celery	-0.078	-3.709	-0.687	5.40
Onions	-0.207	-5.705	-1.367	4.17
Carrots	-0.534	-3.262	-0.101	32.30
Other fresh vegetables	-0.215	0.820	-0.177	-4.64
Fruit juices	-0.558	-2.035	-0.781	2.61
Canned tomatoes	-0.169	0.602	-0.360	-1.67
Canned peas	-0.534	-0.463	-0.228	2.03
Canned fruit cocktail	-0.740	-0.678	-0.115	5.92
Peanuts and tree nuts	-0.169	5.788	-0.263	-22.02
Other processed F&V	-0.151	-1.666	-0.493	3.38
Sugar	-0.037	0.335	-2.480	-0.14
Sweeteners	-0.052	1.823	-0.155	-11.73
Coffee and tea	-0.176	-0.973	-2.515	0.39
Frozen dairy products	-0.078	3.278	-0.196	-16.71
Nonfood	-0.980	-0.890	-0.902	0.99

Note: Estimated price elasticities (column A) and flexibilities (column C) are compiled from Huang 1993 and 1991, respectively. The notation in the table is F&V (fruits and vegetables).