The Value of Reducing Temporal Input Nonuniformities

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The producer value of reducing temporal uncertainty concerning the level of soil nitrate is estimated for corn production in Iowa. The reduction in uncertainty is obtained through use of a late-spring nitrate test. Parametric representations of conditional densities of soil nitrates are used along with an estimated production function to estimate optimal nitrogen fertilizer applications under both uncertainty and certainty for a representative risk-neutral Iowa corn farm. Results indicate that decreasing uncertainty could reduce average fertilizer applications by up to 38% and that producer returns could be increased by up to $22.08/acre.

Key words: fertilizer rates, input nonuniformities, soil nitrates, soil test.

Introduction

Nonuniformities in the distribution of agricultural inputs on fields can have large effects on crop yields and input decisions. Past studies have focused primarily on spatial nonuniformities from uneven irrigation applications or uneven irrigation availability because of variations in soil attributes (e.g., Nielson, Biggar, and Erh; Warrick and Gardner; Letey, Vaux, and Feinerman; and Feinerman, Letey, and Vaux). An exception is that of Chiao and Gillingham who considered how uneven applications of fertilizer affect optimal application rates. They also estimated the producer willingness to pay for reductions in application uncertainty.

Another type of nonuniformity is temporal nonuniformity, which is defined as year-to-year variations in input availability given a constant application rate. Temporal uncertainty concerning the level of nutrient present in the soil, for example, may influence decisions as much as spatial uncertainty when nutrients are subject to random losses. For example, year-to-year fluctuations in soil nitrate levels during critical growing periods may be large because of losses from leaching and denitrification and gains from fixation of atmospheric and organic nitrogen sources. These loss and gain rates are random, depending on weather events and crop yields (Hanley). Consideration of temporal nonuniformities has been limited to studies of stochastic nutrient carryover rates of nitrogen in semiarid regions (Stauber, Burt, and Linse) and phosphorus in tropical agriculture (Lanzer and Paris). The focus of this study is on temporal nonuniformities caused by stochastic loss and gain rates of soil nitrates before and after nitrogen fertilizer is applied in the spring.[1]

Because of random loss and gain rates, producers of rain-fed crops typically apply most of their nitrogen fertilizer without knowledge of relevant soil nitrate levels at the time of application and, more importantly, at the time of rapid plant uptake. In the Corn Belt,

nitrogen fertilizer typically is applied just before or at planting in early spring or with fall fieldwork. Traditional soil tests in the fall or early spring could give an indication of soil nitrate levels at the time of application, but they are not widely used. Potentially large, and random, nitrate gains and losses through leaching and denitrification between the time of testing and plant use make such tests unreliable predictors of nitrate levels when plants start rapid uptake in late spring and early summer.[2]

Recent studies (Magdoff, Ross, and Amadon; Blackmer et al.; and Fox et al.) have shown good correlations between nitrate concentrations obtained from a late-spring soil nitrate test when corn plants are 6 to 12 inches tall and subsequent corn yields. The soil test is conducted late enough to reflect the effects of fall and most spring weather and early enough to allow producers to apply additional fertilizer to correct nitrate deficiencies. The test has been suggested as a tool to increase producers' expected profits by decreasing temporal uncertainty about soil nitrate levels. In addition, the test is suggested by some as an example of how research can result in production technologies that decrease the negative externalities of modern agriculture practices. Of course, the extent to which this second benefit is achieved depends on the response of producers to reductions in temporal uncertainty. Will producers increase or decrease average applications of nitrogen fertilizer?

The overall objective of this article is to estimate the potential value to a representative risk-neutral producer of a reduction in temporal uncertainty from adoption of the late-spring soil nitrate test. As part of the value estimation, the changes in average nitrogen applications also are estimated. The reported estimates in this article of changes in expected profits and average fertilizer applications place an upper bound on actual changes that will result from adoption by producers because it is assumed that (*a*) all temporal uncertainty is eliminated by testing and (*b*) there is no spatial uncertainty. No effort is made to measure the value of changes in externalities (nitrate contamination of water supplies) associated with a change in average nitrogen fertilizer applications. The results of this research, however, are relevant to the ongoing debate about how best to control nitrate contamination. Is it possible that voluntary adoption of risk-reducing technologies lowers nitrogen applications sufficiently to decrease the demand for direct regulation?

The article proceeds as follows. The second section presents a conceptual framework that shows how uncertainty about soil nitrate levels affects profit-maximizing nitrogen applications. The third section presents parametric representations of density functions of soil nitrate levels. These estimated densities are used in the fourth section to estimate expected profit-maximizing nitrogen fertilizer applications with and without use of the soil test and to calculate the producer value of the test. Conclusions are presented in the final section.

Nitrogen Applications under Nonuniform Availability

The response of profit-maximizing producers to uncertainty about the level of nitrates present in the soil is developed in this section.[3] Let N_a be the level of applied nitrogen fertilizer and let N_s be the level of nitrogen already present in the soil. Let both be measured in equivalent units so that yield, y, is a function of total nitrogen: $y = F(N_a + N_s)$. Assume for now that F is continuous with $F_N > 0$ and $F_{NN} < 0$, where the subscripts on F denote partial derivatives. Let $g(N_s)$ be the relevant density function of soil nitrogen at the time N_a is applied. Throughout this article, $g(N_s)$ will be interpreted as representing a producer's beliefs, based on year-to-year variations in soil nitrogen loss and gain rates, about the single nitrogen level in a field. It is implicitly assumed that there is no spatial variation in soil nitrate levels in the field. The risk-neutral producer's problem is to choose N_a to maximize the expected value of profits, π, with the expectation, E, being taken with respect to N_s:

$$(1) \qquad\qquad E(\pi) = PE[F(N_a + N_s)] - P_N N_a,$$

where P is the price of output and P_N is the price of nitrogen fertilizer. The necessary

condition for maximum expected profits is to equate the expected marginal product of applied nitrogen, N_a, to the ratio P_N/P.

The effect of increasing uncertainty about N_s can be obtained by replacing N_s with $\delta\tilde{N} + \bar{N}_s$, where $\bar{N}_s$ is the mean of N_s and $\tilde{N} = (N_s - \bar{N}_s)$, and by differentiating the first-order condition with respect to N_a and δ. An increase in δ represents a mean-preserving spread in N_s. Thus,

$$(2) \qquad \frac{\partial N_a}{\partial \delta} = \frac{E(F_{NN}\tilde{N})}{-F_{NN}}.$$

The sign of (2) equals the sign of $\text{cov}(F_{NN}, N)$ because the denominator is positive and $\tilde{N}$ has a mean of zero. The sign of $\text{cov}(F_{NN}, N)$ equals the sign of F_{NNN}. Increasing uncertainty about the availability of nitrogen present in the soil at planting will increase nitrogen applications if the nitrogen marginal product function is convex. If the function is concave, then increasing uncertainty decreases applications. This is a specific example of the general result first developed by Rothschild and Stiglitz.

Many popular functional forms, such as the Cobb–Douglas and Mitscherlich functions, exhibit a convex marginal product function. Some polynomial functions can exhibit a concave marginal product function, with eventually negative marginal products, which, by (2), implies that increasing uncertainty about the amounts of nitrogen in the soil will result in decreased optimal nitrogen rates. Increasing uncertainty about soil nitrate levels with a quadratic production function has no effect on optimum nitrogen rates because a quadratic function implies a linear marginal product function.

To see why a convex marginal product function leads to greater optimal fertilizer applications under uncertainty than under certainty, suppose a producer who faces input uncertainty is considering applying the fertilizer rate that is needed when N_s is at its average level. Another unit of nitrogen above this rate can lead to lower profits when soil nitrogen is abundant. The loss in profits is, at most, equal to the price of nitrogen fertilizer. However, another unit of nitrogen will lead to higher profits when soil nitrogen is scarce. The gain is the value of marginal product less the price of nitrogen. A convex marginal product function implies that the average gain from the additional unit of nitrogen will be greater than the average loss, so the additional unit of nitrogen increases expected profits. Thus, the optimal rate under uncertainty is greater than the optimal rate under certainty. Optimal fertilizer rates under input uncertainty and a convex marginal product function are consistent with a decision rule that says "apply extra fertilizer just in case it is needed."

A popular functional form for which the third derivative is always zero is the linear response and plateau (LRP) function. The LRP functional form is appropriate if plant-level production functions operate on a limiting nutrient concept. Let $y = \min\{\alpha + \beta(N_a + N_s); y_p\}$ denote the LRP model, where y_p is the nonrandom plateau yield. The producer's objective is to maximize expected profits, where expected yield is

$$(3) \qquad E(y) = \int_0^{(y_p - \alpha - \beta N_a)/\beta} [\alpha + \beta(N_a + N_s)]g(N_s)dN_s + \int_{(y_p - \alpha - \beta N_a)/\beta}^{N_{\max}} y_p g(N_s)dN_s,$$

where $N_{\max}$ is the maximum level of soil nitrogen possible and zero is the assumed minimum. Berck and Helfand's results imply that (3) is an increasing concave function in $N_a + N_s$. Thus, the necessary condition for expected profit maximization is to set the derivative of (3) equal to the price ratio P_N/P. To demonstrate how uncertainty affects nitrogen use with the LRP function, assume that N_s follows a rectangular distribution: $g(N_s) = 1/(b - a)$, $a \le N_s \le b$. Then

$$(4) \qquad \frac{\partial E(y)}{\partial N_a} = \frac{y_p - \alpha - \beta(N_a + a)}{b - a}.$$

Thus, the optimal nitrogen application under uncertainty is

(5)
$$N_a^* = \frac{\frac{P_N}{P}(b - a) - (y_p - \alpha - \beta a)}{-\beta}.$$

How does this compare with average nitrogen applications under certainty?

Assume that uncertainty concerning N_s can be eliminated by allowing the level of N_s to be observed before nitrogen applications. Under certainty, the optimal applied nitrogen level is either zero or is the difference between the observed level of soil nitrate and the agronomic critical value of nitrogen [the "kink" in the production function defined by $(y_p - \alpha)/\beta$]. It is zero if the slope of the LRP function is less than the price of nitrogen relative to output price, or if the observed soil nitrate level is greater than the critical value. Assuming that optimal nitrogen application levels are always positive, the average level of nitrogen use under certainty is $N_k = (y_p - \alpha - \beta \bar{N}_s)/\beta$. The optimal nitrogen rate under uncertainty given by (5) is equal to N_k if the slope of the LRP function when nitrogen is binding is equal to twice the relative price of nitrogen. When the slope is greater than twice the price, then the expected profit-maximizing nitrogen rate is greater under uncertainty than under certainty. The intuition of this result is that with symmetric distributions there is a probability of .5 that $N_s > \bar{N}_s$ and a probability of .5 that $N_s < \bar{N}_s$. Thus the expected benefit of applying an additional unit of nitrogen fertilizer above N_k is half the slope of the LRP function. The real cost of the fertilizer is the price ratio. Thus, when the price ratio is less than half the slope of the LRP function, the benefits are greater than the costs, and the profit maximizer increases fertilizer use.[4]

The extent to which optimal nitrogen decisions change because of uncertainty depends in part on the amount of uncertainty that producers face. Parametric estimates of $g(N_s)$ are provided in the next section. The estimates are derived from experimental data on Iowa corn-producing plots from 1985–90.

Estimating the Distribution of Soil Nitrate

Many corn producers in the Corn Belt apply their nitrogen fertilizer before planting. The relevant soil nitrate concentrations (for yields) occur later in the growing season. Blackmer et al. and Binford, Blackmer, and Cerrato used data generated from a series of experiments conducted in Iowa from 1985 to 1990 to determine correlations between yields and late-spring nitrate levels. These experiments involved applying various rates of nitrogen (ranging from 0 to 300 lbs./ac.) shortly before planting in late April or early May, testing for soil nitrate concentrations in early June, and measuring yields. An individual plot received the same rate of nitrogen fertilizer each year. These data can be used to estimate the density functions of soil nitrate concentrations that are relevant for producers who apply fertilizer before planting. It is likely that the data underestimate the amount of uncertainty concerning nitrate levels for those producers who apply their fertilizer in the fall or who apply manure in the spring. Many of the experimental sites involved both continuous corn and corn after soybeans rotations, allowing the effects of crop rotation on the distribution of soil nitrates to be estimated. For a complete description of the data, see Blackmer et al. and Binford, Blackmer, and Cerrato.

A nonparametric approach to estimating the probability density functions of nitrate concentrations (e.g., Silverman) is appropriate if an accurate portrayal of the densities is the primary objective. If the density functions are to be used for optimization, then a parametric approach is more appropriate, but a functional form must be specified. For the present problem, nitrate concentrations are nonnegative, and kernel estimates of the densities (Silverman) indicate that at all nitrogen rates the distributions are skewed to the
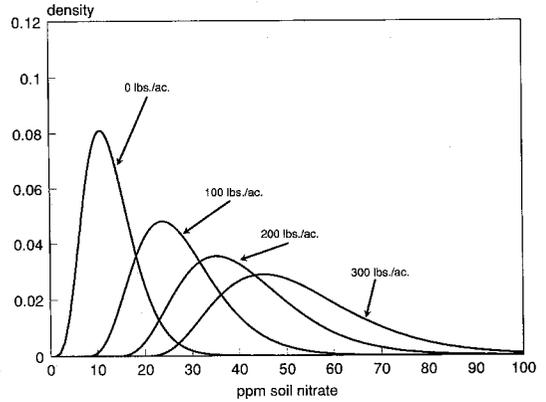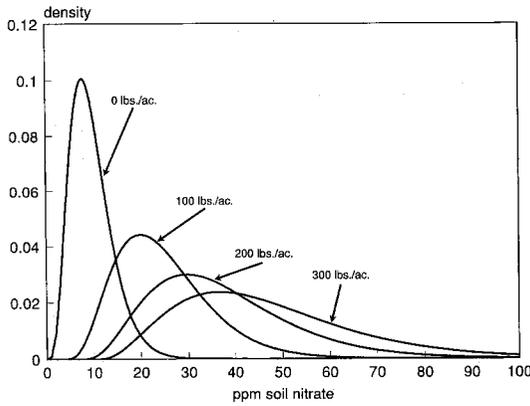
Figure 1(a).   **Soil nitrate density functions at various N-rates of fertilizer for continuous corn**

Figure 1(b).   **Soil nitrate density functions at various N-rates of fertilizer for corn after soybeans**

right. Thus the log-normal or gamma distributions seem appropriate. The three-parameter gamma distribution was chosen for this study:

$$(6) \qquad g(N_s) = \frac{(N_s - \gamma)^{\theta-1}\exp[-(N_s - \gamma)/\lambda]}{\lambda^\theta \Gamma(\theta)} \qquad (\theta > 0, \lambda > 0; N_s > \gamma).$$

It is reasonable to expect that nitrate concentrations in the late spring depend on nitrogen applications before planting. This dependence can be captured by making the parameters in (6) linear functions of applied nitrogen:

$$(7) \qquad \theta = \theta_0 + \theta_1 N_a,$$

$$\lambda = \lambda_0 + \lambda_1 N_a, \quad \text{and}$$

$$\gamma = \gamma_0 + \gamma_1 N_a.$$

The resulting composite function was estimated using the maximum-likelihood procedure ML in TSP. Separate functions for the two rotations were estimated. The null hypothesis that the parameters of (6) are not functions of applied nitrogen was rejected for both rotations ($\chi^2 = 189.52$ for continuous corn, $\chi^2 = 131.35$ for corn after soybeans). The γ_0 estimates for the two equations were small and not significantly different from zero, so the restrictions that they equal zero were imposed. These restrictions imply that the lower bound of nitrate concentration is zero when no nitrogen fertilizer is applied.

The maximum-likelihood parameter estimates and estimated standard errors imposing the two intercept restrictions are presented in table 1. It is assumed that soil nitrate levels on each site, conditional on a nitrogen fertilizer level, are independently and identically distributed. The number of observations used for estimation was 1,251 for the corn after corn equation and 750 for the corn after soybeans equation. The predicted effects of increasing nitrogen applications on nitrate concentrations are shown in figures 1(a) and 1(b) for the two rotations.[5] The estimated means and standard deviations of the gamma distributions are given in table 2 for both rotations. As applied nitrogen increases, the mean, the variance, and the minimum levels all increase for both rotations. The estimated means increase approximately linearly with applied nitrogen. The variance of nitrate levels for the corn after corn rotation increases at a faster rate than the variance of the soybean rotation, suggesting that continuous corn involves greater uncertainty about nitrate levels than corn after soybeans.

The estimated densities for the continuous corn rotation represent the unconditional, equilibrium densities of soil nitrate concentrations on a field assuming that a constant

Table 1.   Estimated Parameters and Standard Errors of the Distribution of Soil Nitrate Concentrations

	Rotation	
Parameter[a]	Corn after Corn	Corn after Soybeans
θ_0	4.920	5.940
	(.30)[b]	(.59)
θ_1	−.00478	−.00468
	(.0010)	(.0021)
λ_0	1.963	2.178
	(.139)	(.188)
λ_1	.0279	.0167
	(.0016)	(.0016)
γ_1	.0366	.0657
	(.0064)	(.0083)

[a] See equations (6) and (7) in the text for the interpretation of the parameters. The restriction that $\gamma_0 = 0$ was imposed for both rotations.
[b] Estimated standard errors are given in parentheses.

level of nitrogen fertilizer is applied before planting every year. The densities are unconditional because they do not take into account the effects of postapplication weather on observed nitrate levels in June. The densities represent equilibrium densities because the data reflect average carryover of nitrogen from the previous year given that the same amount of nitrogen fertilizer is applied each spring, as was done in the experiments. That is, the estimated density corresponding to 100 lbs./ac. of applied nitrogen is relevant when 100 lbs./ac. are applied on a particular plot or field every year. A different density would result if 100 lbs./ac. were applied this year but 200 lbs./ac. were applied the previous year.[6]

The Value of Testing

Estimation of the value of soil testing requires the specification and estimation of a production function. As demonstrated in the second section, the form of the production function plays an important role in determining the effects of reductions in uncertainty concerning soil nitrate levels. But the ongoing debate about the most appropriate functional form will not be continued here. Previous analysis of these data supports the existence of a yield plateau and an approximately linear response to soil nitrates prior to the plateau. (See figs. 2 and 3 in Binford, Blackmer, and Cerrato.) Thus, the LRP function will be used in this analysis.

Table 2.   Estimated Moments of the Gamma Distribution for Various Levels of Applied Nitrogen

	Soil Nitrate Density Characteristics[a]					
	Minimum (ppm)		Mean (ppm)		Standard Deviation (ppm)	
Applied Nitrogen (lbs./ac.)	Corn after Corn	Corn after Soybeans	Corn after Corn	Corn after Soybeans	Corn after Corn	Corn after Soybeans
0	0.0	0.0	9.66	12.94	4.35	5.31
50	1.83	3.29	17.55	20.48	7.27	7.20
100	3.66	6.57	24.77	27.63	10.02	9.00
150	5.49	9.86	31.33	34.38	12.60	10.71
200	7.32	13.14	37.22	40.75	15.02	12.34
250	9.15	16.43	42.44	46.73	17.25	13.88

[a] Given the definitions of θ, λ, and γ in equation (7) in the text and the estimates given in table 2, the minimum level of soil nitrates is equal to γ, the mean equals $\theta\lambda + \gamma$, and the standard deviation equals $\sqrt{\theta}\lambda$.

Table 3. Estimated LRP Production Function Parameters

	Rotation	
	Corn after Corn	Corn after Soybeans
Intercept	51.64	78.25
	(7.06)[a]	(10.26)
Slope	3.95	3.66
	(.377)	(.496)
Critical Level[b]	24.44	24.44
	(.82)	(.82)
Plateau Yield	148.21	166.79
	(1.69)	(2.08)

[a] Estimated standard errors are in parentheses.
[b] The functions were estimated subject to the restriction that the critical levels for the two rotations are identical.

The independent variable in the LRP function is the concentration of soil nitrate observed when the corn plant is 6 to 12 inches tall, which occurs in early June. Therefore, estimation of the LRP function can be accomplished directly from observations on yields and soil nitrates without consideration of either the parameters of the nitrate densities or the level of applied nitrogen fertilizer. This approach is different than the approach of Berck and Helfand who only had data on input applications, not on input availability. Because of this data limitation, they were forced to estimate the parameters of the LRP function simultaneously with the parameters of the input distribution.

The data used to estimate the LRP production function were generated from the same experiments that generated the data on nitrate levels used to estimate the parameters of the gamma distribution. The estimated LRP function includes only one input—soil nitrate concentration. For this specification to be a valid representation of production, all other inputs need to be set at nonbinding levels. Thus, only the 1987 data are used to estimate the LRP because that year involved fairly good and uniform growing conditions across the experimental sites. Other potential limiting inputs on corn are phosphorus, potassium, and sulphur levels. These three inputs were raised to nonbinding levels at all experimental sites. Yield levels for the corn after soybeans rotation may be higher than for continuous corn because of pest-control benefits. This leads to the following representation of the LRP production function:

$$(8) \qquad y = \min[(\alpha + \alpha' D_s) + (\beta + \beta' D_s)N_s; \; y_p + \eta D_s],$$

where $D_s = 1$ if corn follows soybeans, and α, α', β, β', y_p, and η are parameters to be estimated. The estimated parameters and standard errors are given in table 3.[7] The standard errors were estimated according to the bootstrap method suggested by Paris and Knapp.

From table 3, the plateau yield for the corn after soybeans rotation is approximately 12.5% greater than the continuous corn plateau yield. Also, the slope of the LRP function for the soybean rotation is approximately 7.3% less than for continuous corn, although this effect is not significantly different from zero. The estimated agronomic critical level of 24.44 ppm soil nitrate was restricted to be the same for both rotations.[8] Estimated expected yields as functions of applied nitrogen are shown in figure 2 for the two rotations. These yields are given by equation (3), with $g(N_s)$ conditional on N_a given by equations (6) and (7). Uncertainty about nitrate levels smooths out the kink in the LRP function.

The estimated plateau yields are greater than the average corn yield in Iowa during a normal growing season, so, as indicated above, the subsequent empirical estimates of optimal nitrogen use and the value of the soil test may be most appropriate for an Iowa corn farm that does not have other limiting inputs. The slopes of the LRP function, 3.95 and 3.66, are much greater than the nitrogen–corn price ratio. For example, this ratio is .06 with a nitrogen price of $.15/lb. and a corn price of $2.50/bu. Given the results of
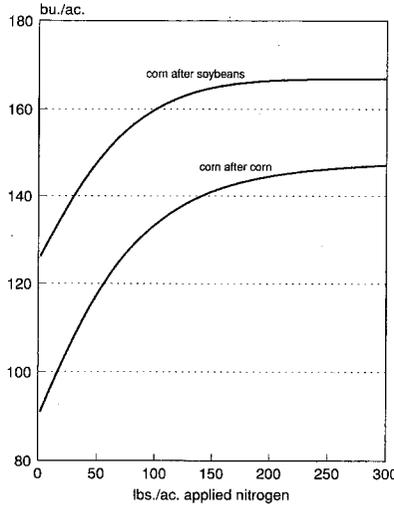
Figure 2.   Expected yield as a function of applied nitrogen fertilizer

the second section and the amount of uncertainty about soil nitrate levels indicated by the estimated moments in table 2, one should expect the profit-maximizing levels of nitrogen applications to be substantially greater under uncertainty than under certainty.

The value of risk reductions from the soil test for a risk-neutral producer is the change in expected profits from adoption of the test. Expected profits with and without the test are evaluated with respect to the same distribution of soil nitrate concentrations. As stated above, it will be assumed that there is no uncertainty about the level of soil nitrates once the soil test results are obtained.[9] Thus, the subsequent calculations place an upper bound on the benefits from testing, conditional on the assumption that the functional forms for the production function and the density function are appropriate. Without the test, fertilizer application is assumed to take place just before planting. The level of nitrogen fertilizer applied, N_a, is determined by maximizing

$$(9) \quad E(\pi) = P \int_{\gamma_1 N_a}^{N^*} (\alpha + \beta N_s) g(N_s \mid N_a) dN_s + P \int_{N^*}^{\infty} y_p g(N_s \mid N_a) dN_s - P_N N_a - AC,$$

where N^* is the agronomic critical nitrate level (24.44 ppm) and AC is the per-acre fertilizer application cost. The dependence of the parameters of the density of N_s on N_a is given by (7). The solution to (9) is assumed to be constant from year to year so that the interpretation of $g(N_s \mid N_a)$ as an equilibrium density is appropriate.

The fertilizer application scenario that incorporates the soil test is as follows. The producer applies a given amount of nitrogen fertilizer (which may be zero) at planting. Then soil nitrate levels are revealed with the late-spring soil test and, based on results of the test, the producer chooses the amount of fertilizer that will be sidedressed. If the soil test indicates that N_s is above N^*, no additional nitrogen is applied. If N_s is less than N^*, the producer will sidedress nitrogen if the benefits of increased nitrate concentrations are greater than the costs of achieving the increase. The benefits equal the value of the yield increase. The costs are the application cost and the materials cost. Hence, there is a level of nitrate concentration below N^* at which sidedressing will commence. This economic critical level is found by equating total benefits of additional nitrogen (TB) and total costs (TC). Given a soil test reading of the random variable N_s, TB and TC are as follows:

$$TB = P[y_p - (\alpha + \beta N_s)],$$
$$TC = kP_N(N^* - N_s) + AC,$$

where y_p is the plateau yield and k is a constant that transforms lbs./ac. applied nitrogen to ppm nitrate in the upper 12-inch layer of soil. The economic critical level, N^e, is given by

$$N^e = \frac{P(y_p - \alpha) - kP_NN^* - AC}{P\beta - kP_N}.$$

For nitrate concentrations below this level, additional nitrogen will be applied. For nitrate concentrations above N^e, the costs exceed the benefits, so no late nitrogen is applied.[10]

Feinerman, Choi, and Johnson show that the optimal fraction of total nitrogen to be applied at planting time is a function of the probability that adverse field conditions will prevent late applications. Let ρ be this probability. Let q be the probability that nitrate concentrations fall between N^* and N^e. Then expected revenue is

$$(10) \qquad PE(y) = P\rho\left[\int_{\gamma_1 N_a}^{N^*} f(N_s)g(N_s)dN_s + \int_{N^*}^{\infty} y_pg(N_s)dN_s\right]$$

$$+ P(1 - \rho)\left[\int_{N^e}^{N^*} f(N_s)g(N_s)dN_s + y_p(1 - q)\right];$$

$$q = \int_{N^e}^{N^*} g(N_s)dN_s,$$

where $f(N_s) = \alpha + \beta N_s$. The dependence of $g(N_s)$ on N_a has been suppressed in (10). Let r equal the probability that nitrate concentrations fall below the economic critical level. Then expected costs are

$$(11) \quad E(C) = P_NN_a + r(1 - \rho)kP_N[N^* - E(N_s \mid N_s < N^e)] + AC + r(1 - \rho)AC;$$

$$r = \int_{\gamma_1 N_a}^{N^e} g(N_s)dN_s.$$

Expected costs equal the sum of the material and application costs of nitrogen fertilizer applied in the early spring, N_a, and the expected material and application costs of side-dressed nitrogen fertilizer in the late spring. The probability of bearing this second cost is $r(1 - \rho)$. The producer can save the first application cost by not applying any nitrogen at planting ($N_a = 0$).

The producer who uses the soil nitrate test maximizes the difference between (10) and (11). Before this can be done, a value of k needs to be determined. A large value of k implies a large cost of raising nitrate concentrations in the late spring by sidedressing nitrogen, making early-season nitrogen applications relatively inexpensive. A low value of k makes early-season applications relatively expensive. The value that will be used in the subsequent calculations is the number of pounds of nitrogen fertilizer, applied just before planting, that are needed to raise expected late-spring nitrate concentrations by one ppm. This number is found by regressing observed nitrate concentrations, NO_3^+, on applied nitrogen. The resulting regression equation is

$$NO_3^+ = 10.759 + 3.039D_s + .131N_a,$$

where D_s is as defined in equation (8). There was not a significant difference in the response of NO_3^+ to N_a for the two rotations. This equation implies that it takes 7.63 lbs./ac. to raise nitrate concentrations by one ppm. This value of k will be used to maximize the expected profits of the producer who uses the soil test.[11]

The profit-maximizing levels of nitrogen use under uncertainty about soil nitrate concentrations for the two rotations are given in table 4. The profit maximizer applies 183.2 lbs./ac. of nitrogen fertilizer for the continuous corn rotation and 144 lbs./ac. for the corn after soybeans rotation, given a corn price of \$2.50/bu. and a fertilizer price of \$.15/lb.

Table 4.   Optimal Nitrogen Plans with No Information about Soil Nitrate Levels

	Rotation	
	Corn after Corn	Corn after Soybeans
Spring-Applied Nitrogen (lbs./ac.)	183.2	144.0
Probability N_s < 24.44[a]	.24	.19
Expected Yield (bu./ac.)[b]	143.6	164.4
Expected Profit ($/ac.)[c]	330.09	388.04

Note: The producer is assumed to know only the probability density function of soil nitrate levels.

[a] The plateau yield is reached at N_s = 24.44 ppm soil nitrate.

[b] The estimated production functions are given in table 3.

[c] Expected profit is defined as expected revenue less nitrogen material and application costs. The prices of corn and nitrogen are set to $2.50/bu. and $.15/lb., respectively. Application costs are $1.42/ac.

Planting corn after soybeans reduces nitrogen use by 39.2 lbs./ac., and results in an expected yield increase of almost 21 bu./ac. and increased returns over nitrogen fertilizer costs of $57.95/ac. The effect of the soybean rotation on the distribution of soil nitrates is such that even though the optimal fertilizer rate is less, the probability that soil nitrates fall below the agronomic critical value is lower.

The profit-maximizing levels of nitrogen applications under certainty about soil nitrate levels for various levels of uncertainty about the accessibility of fields in late spring are reported in table 5(a) for continuous corn, and in table 5(b) for the corn after soybeans rotation. When it is always possible to sidedress nitrogen fertilizer after the results of the soil test are known, then it is optimal to sidedress all the fertilizer in the late spring. This result is consistent with the simulations of Feinerman, Choi, and Johnson. The average amounts of nitrogen to apply in this instance are 112.9 lbs./ac. and 88.6 lbs./ac. for the two rotations. The elimination of uncertainty in combination with allowing producers to sidedress nitrogen in the late spring decreases optimal fertilizer applications for risk-neutral producers by a maximum of 38.3% for continuous corn and by 38.5% for the corn after soybeans rotation.[12]

When uncertainty about the ability to apply late nitrogen increases, the proportion of total nitrogen applied early increases. The lumpy nature of application costs at low probabilities of having inaccessible fields causes total nitrogen fertilizer use initially to decline for the continuous corn rotation. As the probability of not being able to enter fields increases, however, total nitrogen use eventually increases. With a probability of .2, the elimination of uncertainty about soil nitrates decreases expected nitrogen use by a maximum of 28.8% for the corn rotation and by 24.9% for the soybean rotation.

The value of testing soil nitrates is reported in the last row of tables 5(a) and 5(b). This value is the increase in expected per-acre profits from using the test, without consideration of the cost of the test. The estimated maximum expected value of the test is $22.08/ac. for the continuous corn rotation and $14.27 for corn after soybeans. These values decrease to $11.89 and $6.37, respectively, as the probability of not being able to apply late nitrogen increases to .2. The substantial increases in expected profits are due to the cost savings arising from decreased nitrogen use and by the increase in expected yield, which results from the ability to compensate for nitrogen deficiencies before they affect yields.[13] Of course, the net producer value of the test depends on the per-acre cost of the test. Estimates of this cost range from $.50 to $3, depending on the size of the field and the number of tests per field conducted. Although the optimal number of tests per field has yet to be estimated, the number depends on the cost per test, the homogeneity of the field, and the amount of measurement error inherent in the test.

Table 5(a). Optimal Nitrogen Plans with Perfect Information Concerning Soil Nitrate Levels for Corn after Corn

	Probability that Fields Are Inaccessible for Sidedressing Fertilizer				
	0	.05	.10	.15	.20
Nitrogen Applied at Planting (lbs./ac.)	0	69.2	87.0	100.4	111.1
Expected Sidedressed Nitrogen (lbs./ac.)[a]	112.9	42.7	31.0	24.1	19.3
Expected Total Nitrogen (lbs./ac.)	112.9	111.9	118.0	124.5	130.4
Probability Nitrogen Is Sidedressed	.99	.68	.55	.45	.38
Expected Yield (bu./ac.)	148.2	147.0	146.4	146.0	145.7
Expected Profit ($/ac.)[b]	352.17	348.10	345.61	343.64	341.98
Value of Information ($/ac.)[c]	22.08	18.01	15.52	13.55	11.89

Note: Information about soil nitrate levels is obtained after nitrogen fertilizer is applied in the spring.
[a] Nitrogen is sidedressed if late-spring soil nitrate levels are below 24.30 for continuous corn and 24.00 for corn after soybeans. This is the economic critical level as defined in the text.
[b] Gross revenue less cost of nitrogen fertilizer.
[c] The value of information equals the difference between expected profit under uncertainty from table 4 and expected profit using the test.

Table 5(b). Optimal Nitrogen Plans with Perfect Information Concerning Soil Nitrate Levels for Corn after Soybeans

	Probability that Fields Are Inaccessible for Sidedressing Fertilizer				
	0	.05	.10	.15	.20
Nitrogen Applied at Planting (lbs./ac.)	0	56.8	73.6	85.3	94.4
Expected Sidedressed Nitrogen (lbs./ac.)[a]	88.6	34.7	23.8	17.7	13.7
Expected Total Nitrogen (lbs./ac.)	88.6	91.5	97.4	103.0	108.1
Probability Nitrogen Is Sidedressed	.96	.64	.50	.40	.33
Expected Yield (bu./ac.)	166.8	165.9	165.6	165.3	165.2
Expected Profit ($/ac.)[b]	402.31	398.51	396.77	395.46	394.41
Value of Information ($/ac.)[c]	14.27	10.47	8.73	7.42	6.37

Note: Refer to table 5(a) notes.

Concluding Comments

This analysis has shown that the ability to reduce temporal uncertainty about soil nitrate levels through soil testing can significantly increase the expected profits of producers by increasing expected yields while reducing costs. The empirical results indicate reductions in uncertainty can cause the profit-maximizing nitrogen rates to be reduced by a maximum of almost 40% while expected profits are simultaneously increased by up to $22.08/ac. The estimated changes from the additional information indicate that widespread adoption of the test is potentially a viable, voluntary alternative to regulatory actions (taxes or quotas) aimed at decreasing nitrogen fertilizer applications.

The relatively large changes in expected profits and total amounts of applied nitrogen from adoption of the soil test are due to the large amount of uncertainty that exists concerning nitrate levels in the late spring. The empirical evidence suggests that for all relevant nitrogen fertilizer application levels, there is a relatively large probability that crop yields will respond to increased nitrate levels. Producers who do not use the test find it profitable to reduce the probability (and the associated opportunity costs) that nitrogen

is a limiting input, particularly when nitrogen is inexpensive relative to the expected gain from increased crop yields.

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Notes

[1] Feinerman, Choi, and Johnson examined the effects of nitrate losses on fertilizer decisions, but they assumed that the loss rate is a known proportion of applied nitrogen.

[2] Blackmer points out that under ideal conditions, it can take only a few days for soil micro-organisms to denitrify amounts of nitrogen equivalent to annual applications of fertilizer. Denitrification amounts are difficult to predict in climates with highly variable precipitation and temperature, such as the Corn Belt, because the conditions favoring denitrification depend on soil characteristics interacting with weather events. For arid or tropical climates, temporal uncertainty about nitrogen loss rates is reduced. See Blackmer for a detailed discussion of losses and transport of nitrogen from soils.

[3] See Babcock for a more general analysis of the effects of weather and input uncertainty on optimal input applications.

[4] The relation between optimal input application and uncertainty using the LRP function was developed initially by Letey, Vaux, and Feinerman who estimated that with very low water prices, uncertain water availability could lead to optimal irrigation rates of 50% to 100% greater than optimal rates under uniform availability.

[5] One would expect that the estimated density functions reported in table 1 overstate the amount of uncertainty that a particular experimental site might experience because no site effects are included in equation (7). Site-specific density functions were estimated to determine if individual sites have less uncertainty than that implied by the aggregate functions reported in table 1. Ten sites had adequate data (five years or more) to estimate site-specific density functions: five for continuous corn and five for corn after soybeans. Of the 10 estimated site-specific density functions, four are similar to the estimated densities shown in figures 1(a) and 1(b), three exhibit more uncertainty, and three exhibit less uncertainty. Thus, the two aggregate densities reported in table 1 can be viewed as representative of the amount of uncertainty that can be expected on Iowa corn fields.

[6] The results obtained by Binford and Blackmer indicate that nitrogen fertilizer carryover in these Iowa experiments is small. Their experiments tracked the uptake of ^{15}N-labeled ammonium sulfate the year after application. On average, 2% of nitrogen used by corn was recovered from the previous year's application.

[7] The estimated production function parameters reported in table 3 are representative of parameters that would have resulted if site-specific production functions had been estimated from 1987 data. Thus, the production functions presented in table 3 should be interpreted as representative production functions of Iowa corn production. The site-specific parameters are reported in Binford, Blackmer, and Cerrato.

[8] The point estimates of the critical levels for the two rotations when they were not restricted were 23.98 ppm for continuous corn and 24.60 ppm for corn after soybeans. These estimates are within one standard error of each other.

[9] The measurement of the amount of nitrate uncertainty remaining after the test results are observed and the effect of this residual uncertainty on optimal decisions are the subjects of ongoing research. Consideration of measurement error likely will affect three aspects of this analysis. First, the amounts of nitrate uncertainty reported in tables 1 and 2 and figures 1(a) and 1(b) overstate the actual amounts of uncertainty if nitrates are measured with error. Thus, the magnitude of the increase in optimal fertilizer rates would be less than that reported in table 4 if nitrates were measured without error. Second, the optimal amount of fertilizer applied after the soil test is conducted will be different if there remains input uncertainty after the soil test results are obtained. Because the cost of nitrogen deficiencies is greater than the cost of fertilizer, it is likely that the optimal average application of late fertilizer will be greater than the levels reported in tables 4 and 5. The third effect of measurement error is that it introduces an errors-in-variable problem when the response of yields to soil nitrates is estimated. In this case, additional information about the moments of the error term should be incorporated into the estimation procedure (Judge et al., pp. 714–17).

[10] It is assumed that the slope of the production function when nitrogen is binding is greater than the ratio of nitrogen price to output price. Thus, it will always be optimal for a producer to reach the agronomic critical level if nitrate levels are below the economic critical level.

[11] Increasing nitrate concentrations by sidedressing fertilizer in the late spring is probably more efficient than increasing nitrate concentrations by incorporating nitrogen before planting. There are two reasons for equating the two efficiencies. First, there are not enough data to estimate the efficiency of sidedressed nitrogen fertilizer in increasing nitrate concentrations. Second, current Iowa State University guidelines for use with the soil test suggest that farmers use a value of 10 for k. A value of 10 would make the efficiency of sidedressing fertilizer much lower, on average, than applying fertilizer just before planting. This value would imply that farmers could lower costs by applying a greater amount of nitrogen before planting rather than waiting until the results of the soil test are known. A practical reason given for understating the efficiency of sidedressed fertilizer is that the developers of the nitrate test want to be conservative in their recommendations in the early phase of adoption.

[12] Sensitivity analyses of the results indicate that the changes in average nitrogen applications when measured

in pounds are sensitive to the parameters of the LRP function but the changes are relatively robust when measured as percentages. In addition, the percentage estimates correspond well with the results of initial farmer trials of the soil test. On average, farmers who utilized the test reduced their per-acre nitrogen fertilizer applications by 35% (Leopold Center for Sustainable Agriculture).

[13] The value of the soil test estimated here does not account for the possible effects of changes in equilibrium market prices from adoption of the soil test.

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