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Towards a General Method of Estimating Productivity-Soil Depth Response Relationships

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Abstract

Four functional forms were investigated to determine their usefulness in estimating the relationship between soil erosion and crop yields. These functional forms were linear, polynomial, Cobb-Douglas, and Mitscherlich-Spillman. The Mitscherlich-Spillman is shown to better meet the theoretical requirements. A variable defined as the mechanical composition of the plant rooting zone is shown to be a superior predictor variable to topsoil depth. Soybean yield data for Georgia Cecil soils were fitted to a Mitscherlich-Spillman function.

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Introduction

Soil erosion on U.S. croplands has been publically recognized as a serious agricultural problem since the 1930's. It has been established that soil erosion can detrimentally affect soil productivity. Encouraged with this knowledge and by the government's willingness to devote public resources for soil conservation, the USDA has spent nearly $18 billion on soil conservation progarms in the last fifty years (U.S. GAO (7)). Emphasis has been placed on preventing soil displacement without regard to the degree of damage resulting from that displacement. In recent reports, the General Accounting office (7) and Crosson and Stout (2) have suggested that the USDA allocate its soil conservation resources using criteria based on the harmful effects of soil erosion and obtain the data to determine these effects.

Productivity loss is one of the major effects of soil erosion. Knowledge of yield-soil depth relationships is necessary in determining those effects. Such knowledge would help conservation planners direct efforts to those problem areas where productivity loss is the greatest or where the losses could be averted most efficiently.

Economic models of the effect of soil erosion all require explicit or implicit characterizations of the productivity relationships. Walker (9) formulated an erosion damage function which portrays the economic consequences through time of a farmer deferring the adoption of a conservation practice for one more year. This function incorporates a nonlinear yield-soil depth

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relationship in a dynamic economic model. McConnell (5) uses the yield-soil depth relationship as well as soil loss in a dynamic model of agricultural production to determine when the private path of erosion differs from the socially optimal path. Christensen and McElyea (1) applied a yield-soil depth function to a model which determines the value of topsoil lost through erosion. The Erosion Productivity Impact Calculator (EPIC) is an attempt to model erosion's effect on a national scale (12). Increased knowledge of the relationship between topsoil depth and yield is needed to validate and improve EPIC's predictions.

Attempts at estimating crop yield-soil depth functions have been made by a few researchers for a limited number of crop-soil combinations. Walker (9) estimated a wheat yield-soil depth function for Palouse soil using nonlinear regression analysis. Hoag and Young (3), using ordinary least squares, fit wheat yield-soil depth data to a Mitscherlich-Spillman nonlinear function for the Palouse soils of Washington state. Langdale et al. (4), using ordinary least squares, fit corn yield-soil depth data for Georgia Cecil soil to a polynomial response function. These efforts have given new insights into the nature of some yield-soil depth relationships and demonstrate techniques available to model these relationships for other crop and soil combinations.

This paper reviews theoretical and practical issues involving the choice of appropriate explanatory variables and functional forms for the different crop and soil combinations likely to be encountered in future response function estimations.

Organization

The paper is organized in four parts. First is a discussion of the basis for choosing the appropriate regressor in estimating a yield-topsoil depth response function. Second, the attributes of a proper yield-soil depth response function based on agronomic theory are presented. Characteristics of four commonly used functional forms, linear, polynomial, Cobb-Douglas, and Mitscherlich-Spillman, are compared with the theoretically proper attributes. Regression techniques for estimating these forms are also presented. Third, an estimation of a yield-topsoil depth response function for soybeans produced on Georgia Piedmont Cecil soils is described. Regression results of a number of different functional form-regressor combinations are shown. Fourth, general guidelines for the estimation of yield-topsoil depth functions for varied topsoil and crop combinations in the United States are proposed.

Choosing the Regressor in the Yield-Soil Depth Relationship

Erosion is the movement of soil by wind or water. The resulting reduction in topsoil depth changes the soil characteristics and alters the capacity of the soil to support plant growth (Crosson (2)). Pierce et al. (6) have identified soil rooting depth, available water capacity, plant nutrient storage, surface runoff, soil tilth and organic matter content as major soil characteristics which promote plant growth.

Spillman and Lang (7), reporting on the pioneering work of Mitscherlich and others, defined a limiting factor as that factor which is present in insufficient amounts to provide for maximum plant yield. In the case of soil erosion, soil characteristics which provide for maximum yield degrade and become as a group, the limiting factor. The yield-soil depth response function requires a regressor which is a single measurable quality. This regressor is a proxy for all of those soil rooting zone characteristics which collectively is the limiting factor. The soil rooting zone characteristics which limit or provide for growth do so by providing nutrients, moisture, and an environment for root penetration.

In some cases measured topsoil depth has been found to be a statistically acceptable proxy for those characteristics. This occurs on those soils where the mechanical composition (the sand, silt, and clay proportions) of the rooting zone, remains unchanged during the erosion process. This is a characteristic of soils with topsoil (Ap horizon) and subsoil (Bt horizon) with similar mechanical composition, for example the Palouse soils of the Pacific Northwest. In these soils, organic matter content is the major factor which decreases during the erosion process and varies directly and linearly with topsoil depth.

In cases where the textural composition of the plant rooting zone changes as erosion occurs, a variable which reflects the change in sand, silt, and clay composition may be superior to topsoil depth as a regressor. This occurs when the subsoil is of different mechanical composition than the topsoil, for example Georgia Piedmont Cecil soils, where the Ap horizon overlays a distinctly more clayey Bt horizon. During the erosion process, the Ap horizon decreases in depth and its clay content increases as plowing mixes clayey Bt material into the Ap. In these types of soils, the single variable topsoil depth does not reflect the changing rooting zone characteristics caused by the changing mechanical composition. A more appropriate regressor is the proportion of the factors in the plant rooting zone which decreases as erosion occurs. For example, in the case of the Cecil soil, the proportion of sand and silt decrease as erosion occurs and thus the sand and silt proportion of the rooting zone is likely to be the limiting factor in the yield-soil depth response regression.

When the soil in question has topsoil and subsoil horizons of similar mechanical composition, topsoil depth is probably a satisfactory regressor. If the soil has dissimilar topsoil and subsoil horizons and the mechanical composition of the topsoil changes as erosion occurs, then a variable which is the decreasing proportion of the eroding soil's mechanical components should be a better regressor.

Choosing the Functional Form for the Yield-Soil Depth Relationship

The functional form chosen for a yield soil depth response relationship should conform to the following four characteristics of agronomic response (Hoag and Young (3)).

1. The response of crop yield to additional topsoil should conform to the Law of Diminishing Returns.
2. When the topsoil depth is zero, positive yields should be possible.
3. The attainable yield should have a finite maximum.
4. Topsoil depth in excess of the maximum rooting zone should not decrease yields.

Corresponding to these characteristics are four requirements of the functional form for a yield-topsoil depth response function. This functional form should: (1) exhibit diminishing marginal returns to topsoil, (2) allow for a positive intercept, (3) have a finite maximum yield, and (4) display nonnegative marginal returns to the limiting factor throughout (Hoag and Young (3)).

Previous studies have used several functional forms to represent yield-topsoil depth relationships (3,4,9). Characteristics of the linear, polynomial, Cobb-Douglas, and Mitscherlich-Spillman forms are examined according to the four characteristics.

Linear Form

A linear form of the yield-soil depth function is:

$$Y = a + b \cdot f(X) \tag{1}$$

where Y = crop yield

a = the yield at zero topsoil depth
b = the slope of the function
$f(X)$ = the limiting factor which is a function of topsoil depth.
X = topsoil depth

The linear form is graphically represented in Figure 1. The linear form of the yield-topsoil depth function satisfies only two of the four theoretical criteria. A positive intercept is possible and nonnegative marginal returns are present. The constant slope violates the conditions of diminishing returns and maximum attainable yield. The linear functions can be easily estimated using ordinary least squares regressions. Thus the linear form is

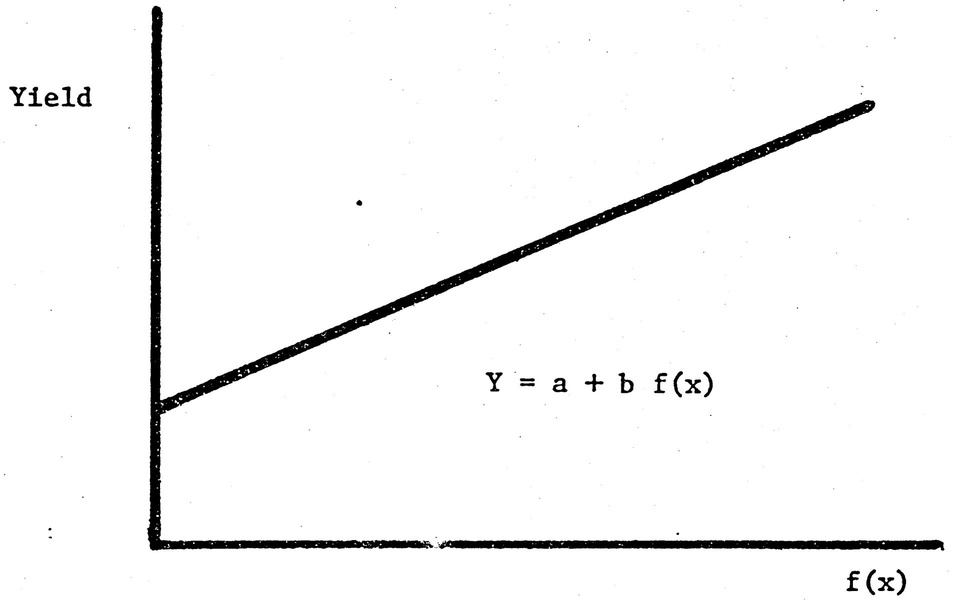

Figure 1. Graphical representation of the linear yield-soil depth function

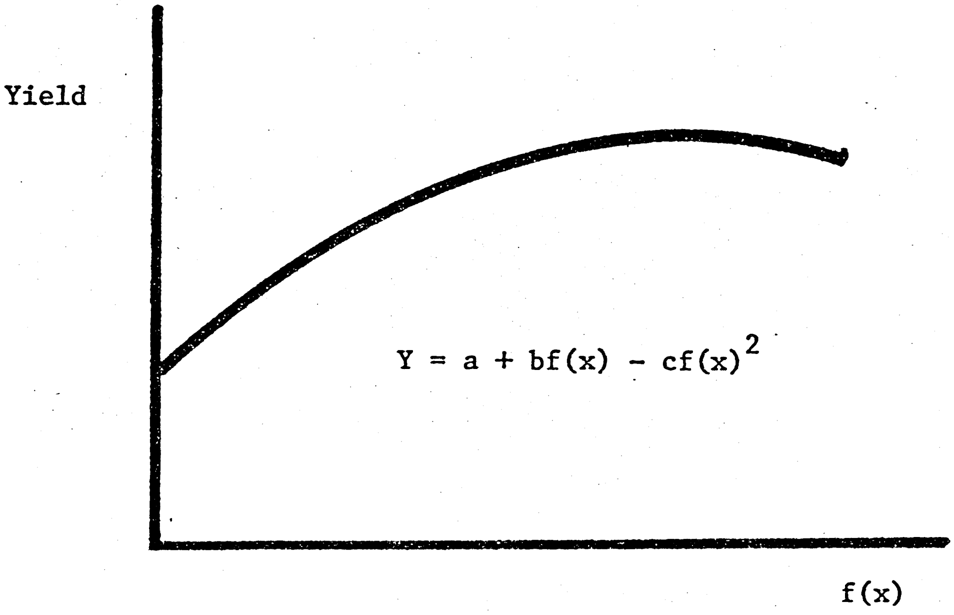

Figure 2. Graphical representation of the polynomial yield-soil depth function

x = soil depth
f(x) = function of soil depth

easily estimated but naive representation of the yield-soil depth relationships. It has limited use in economic analysis of soil erosion.

Polynomial Form

A polynonomial form of the yield-soil depth function is:

$$Y = a + bf(X) - c\ (f(X))^2 \qquad (2)$$

where

Y = crop yield
a = the yield at zero topsoil depth
b and c = positive slope coefficients
$f(X)$ = limiting factor which is a function of topsoil depth
X = topsoil depth

The polynomial form is graphically respresented in Figure 2. The polynomial form satisfies three of the four theoretical criteria. A positive intercept is possible. Diminishing marginal returns are present. Yield reaches a finite maximum value. The nonnegative marginal returns condition is violated. However, if an outside constraint specifies that yield is considered constant beyond the depth where yield is a maximum, this condition is satisfied. This form can be easily estimated by a linear transformation of the regressions using ordinary least squares. Thus, the polynomial form is a theoretically satisfactory, easily estimated representation of the yield soil depth relationship which could be useful in economic analysis of soil erosion.

Cobb-Douglas Form (Power Function)

A Cobb-Douglas form of the yield soil depth function is:

$$Y = af(X)^b \qquad (3)$$

where

Y = crop yield
a = the yield when topsoil depth is zero
b = the transformation rates for different values of the limiting factor
$f(X)$ = the limiting factor, which is a function of topsoil depth
X = topsoil depth

This function is estimated in the form:

$$\ln(Y) = \ln(a) + b\ln(f(X)) \qquad (4)$$

The Cobb-Douglas form is graphically represented in Figure 3. The Cobb-Douglas satisfies two of the four theoretical criteria: diminishing marginal returns and nonnegative marginal returns are present. The Cobb-Douglas form does not reach a finite maximum and yield is zero when the limiting factor is zero. The former can be remedied by assuming a constant maximum beyond a certain value of the limiting factor. Thus the Cobb-Douglas form is a partially theoretically satisfactory, easily estimated representation of the yield soil depth relationship which could be useful in econmic analysis of soil erosion.

Mitscherlich-Spillman Form

A Mitscherlich-Spillman or M-S form of the yield-soil depth function is:

$$Y = a + b\ (1 - R^{f(X)}) \tag{4}$$

where

Y = crop yield
a = the yield when the limiting factor is zero
b = the increment added to the intercept value which results in the maximum yield.
R = is a constant defining the ratio of successive increments to total yield
$f(X)$ = the limiting factor, a function of topsoil depth
X = topsoil depth

This function is estimated in the form:

$$Y = (a + b) - be^{-cf(X)} \tag{5}$$

where

$e^c = R$
c = a proportionately constant.

The M-S form is graphically shown in Figure 4. The M-S function satisfies all four theoretical criteria. It exhibits a positive intercept, decreasing marginal returns, a maximum yield value, and nonnegative marginal returns. Estimation of the M-S function is complicated by its algebraic form which cannot be linearly transformed for estimation of all three parameters. Two

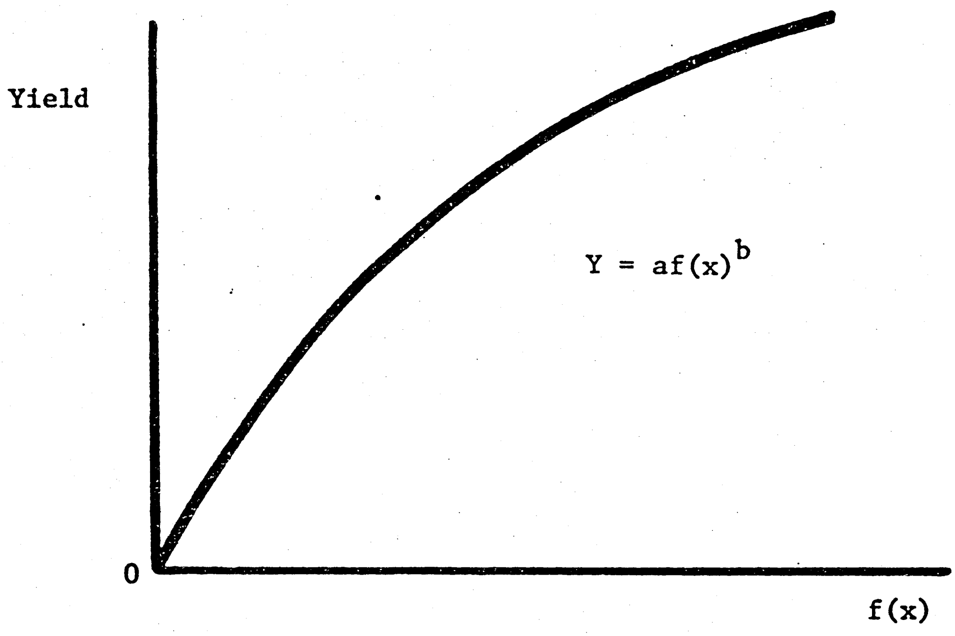

Figure 3. Graphical representation of the Cobb-Douglas yield-soil depth function

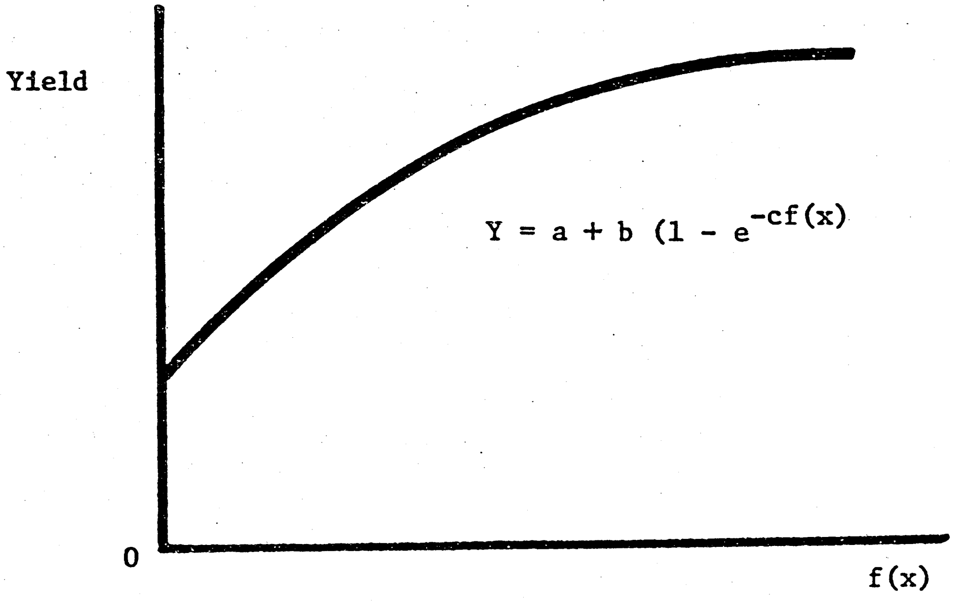

Figure 4. Graphical representation of the Mitscherlich-Spillman yield-soil depth function

estimation procedures can be used. Nonlinear least squares, an iterative procedure, is one choice. Walker (9) used this method. If more than one year's data is available, the estimates for c, the proportionately constant from the nonlinear estimation can be used in other function estimates of the same crop and soil type (3). Thus the Mitscherlich-Spillman is theoretically satisfactory without modification. Although somewhat more difficult to estimate than other functonal forms, it could be most useful in analyzing the economics of soil erosion.

An Application

In 1982, scientists from the Southern Piedmont Conservation Center at Watkinsville, Georgia collected soybean yield and topsoil depth data from 23 Piedmont Georgia fields with Cecil and Pacolet soils (10, 11). This data came from a cooperative research project between the Agricultural Research Service, USDA, the Soil Conservation Service, USDA, the Economic Research Service, USDA, the University of Georgia, and 25 independent farmer-cooperators. The Cecil series is predominant in the Southern Piedmont physiographic region of Georgia and is found in a belt South of the Southern Appalachians from Alabama to Virginia. The Cecil and Pacolet soil series are members of the clayey, kaolintic, thermic family of Typic Hapludults. These soils have sandy clay loam to clay subsoil horizons underlying a sandy loam surface horizon when only slightly eroded. The Bt is moderately dense and somewhat restrictive to root proliferation and drainage.

In each of the 23 fields studied, three areas classified as slightly, moderately, or severely eroded were located by inspecting Ap horizon samples for estimated clay content. For each erosion state in each location three topsoil depth (depth of the Ap horizon) measurements were taken and corresponding soybean yields were measured at harvest. Also, a single topsoil plus subsoil depth (depth to the bottom of the Bt horizon) and clay content were measured at each erosion state area. A total of 207 yields and 207 pairs of soil depth measurements (Ap and Bt) were taken. Further analysis of the data produced 68 mean yield and topsoil depth figures for the 23 locations with three erosion state areas. A soybean yield-soil depth function was estimated using these data.

Three different variables, Ap horizon depth: the sum of Ap and Bt horizon depth, and the sand and silt proportion of the rooting zone of the soybean plant were used. The Ap horizon depth, denoted X, is commonly referred to as topsoil depth. The sum of Ap and Bt horizon depths, denoted BX, was used as regressors by Langdale et al. (4). The sand and silt composition of the rooting zone was chosen as a regressor because it decreases with soil depth as erosion occurs. The environment for nutrient uptake in the rooting zone changes with such a change in mechanical composition. This sand and silt composition can be computed by:

$$\begin{aligned} SSC &= \frac{NCCA\ (X) + (Z - X)\ NCCB}{Z} \qquad (6) \\ &= (NCCA - NCCB)\ \frac{X}{Z} + NCCB \end{aligned}$$

where

SSC = The sand and silt proportion of the primary rooting zone of the soybean plant
NCCA = The nonclay (sand and silt) proportion of the Ap horizon
NCCB = The nonclay (sand and silt) proportion of the Bt horizon
X = Ap horizon depth
Z = Depth of the primary rooting zone

For soybeans on Cecil sandy loam soils, NCCB = .5 and Z = 12 inches.

$$SCC = (NCCA - .5)\ \frac{X}{12} + .5 \qquad (6)$$

When the topsoil depth = 0, SSC = NCCB = .5. Therefore, for regression purposes, the value of SSC is decreased by .5 to obtain correct intercept values.

The three theoretically acceptable functional forms which exhibit decreasing marginal returns, the polynomial, Cobb-Douglas, and the Mitscherlich-Spillman, were estimated using the three alternative limiting factors, X, BX, and SSC. For the M-S function using the SSC variable the c value (c = 7.28) was obtained by nonlinear least squares from 1983 soybean yield and soil depth data. For the X and BX regressor no results were obtained for the M-S function because the iterative regression process failed to converge. All regression parameters were estimated using OLS. Table 1 shows the regression results for the different regressor, functional form combinations.

Comparing the three regressors used as the limiting factor, the SSC variable regressed on a M-S function yields the best statistical results. All parameters are signficiant at the .01 levels and the R^2 is the highest for all models with significant parameters.

Conclusions

Topsoil depth is not necessarily the best regressor for yield topsoil depth response functions. In the case where a shallow topsoil overlays an unproductive subsoil of different mechanical composition, the composition of the crop rooting zone can be a better proxy for those soil characteristics which promote plant growth. A general method is thus proposed for the estimation of yield-soil depth response functions. First, the regressor chosen should reflect whether or not the mechanical composition of the eroding topsoil changes. This regressor should then be fitted to a function which best satisfies theoretical agronomic precepts and produces satisfactory statistical results. This method, which produced superior estimation of the yield soil depth response function for soybeans on Cecil soil, should provide a basis for improved estimation of crop productivity-soil depth relationships for the varied crop and soil combinations found in the crop producing regions of the United States. Additional research is needed to identify and measure the limiting factors for major erosive soils throughout the country, for purposes of explanation and problem solving.

Table 1. Regression Results for Alternative Regressors and Functional Forms

Regressor	Functional Form	Results	F-value	R^2
X	Polynomial	$Y = 6.19 + 3.8X + .057X^2$ (.556) (.998) (.187)	25.78**	.425
X	Cobb-Douglas	$Y = 5.87\ X^{.89}$ (5.78)** (4.85)**	23.60**	.252
X	Mitscherlich-Spillman	No results		
XB	Polynomial	$Y = 4.08 + 1.20XB - .0097(XB)^2$ (.40) (1.6) (-.73)	11.048**	.2307
XB	Cobb-Douglas	$Y = 3.55\ XB^{.609}$ (2.28)* (3.58)**	12.81**	.1499
XB	Mitscherlich-Spillman	No results		
SSC	Polynomial	$Y = 15.89 + 103.25(SSC - .5) - 34.7(SSC - .5)^2$ (6.85)**(2.917)** (-.338)	48.57**	.587
SSC	Cobb-Douglas	$Y = 34.87\ (SSC - .5)^{.11}$ (35.12)** (3.77)**	14.27**	.165
SSC	Mitscherlich-Spillman	$Y = 44.0 - 32.86e^{-7.28(SSC - .5)}$ (22.2)** (-8.84)**	78.25**	.5355

X = Ap horizon depth
XB = Ap + Bt horizon depth
SSC = sand and silt proportion of the rooting zone
Y = soybean yield
T-value in parentheses
** Significant at .01 level
* Significant at .05 level