Contract Designs and Participation in the Conservation Reserve Program in the Era of Biofuel Production

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Abstract

This article has presented a farmer decision making model of participation in the Conservation Reserve Program (CRP) under the current rising bio-fuel production. The decision is specified as an optimal stopping problem and farming return is assumed following stochastic process with the uncertainty of growth rate. Nonlinear Kalman filter approach is used to continuously upgrade the new information and estimate the random growth rate with the minimum error. The problem is formulated as a linear complementarity problem that is solved numerically using a fully implicit finite difference method. It is found that participation in the CRP is sensitive to financial incentive, and shortening contract length is also an effective method to promote land enrollment in the CRP. These results have implications for the design and implementation of conservation programs.

Key Words: Conservation Reserve Program, Nonlinear Kalman Filter, Farmer Participation
The U.S. Conservation Reserve Program (CRP), enacted in 1985, is by far the most important conservation program in terms of scale and budget. It aims to reduce erosion, improve water quality, establish wildlife habitat, and provide other environmental benefits through retiring highly erodible and environmentally sensitive cropland from production. In its first twenty years of implementation, the program has prevented an estimated 450 million tons of soil from eroding and provided millions of acres of wetlands and buffers (USDA-FSA 2006). However, operation of the CRP is becoming more challenging due to ethanol production as the agricultural sector has been adjusting to a new role of bio-fuel production. Over the past few years, annual ethanol production in the U.S. has increased rapidly from less than 3 billion gallons in 2003 to over 9 billion gallons in 2008 (Gruenspecht 2009), and was projected to exceed 12 billion gallons by 2010 (USDA 2008a). The USDA forecasts that the acreage of corn, the major feedstock for ethanol production, will reach 93 million acres by 2010 from 78 million acres in 2006 (USDA 2008b). The ethanol driven structural shift has been crowding land out of alternative uses and pressuring environmental conservation programs. As of June 2008, CRP enrollment stood at 34.7 million acres, approximately 2 million acres lower than September 2007 (Cowan 2008). Notwithstanding the temporary ease of pressure due to the recession, many predict that economic recovery will soon push up the ethanol demand and will again increase the pressure. CRP acreage would be about 30.5 million acres on October 1, 2009—about 1.5 million acres below the cap if USDA holds no general signups and offers no further contract extensions (The Biomass Research and Development Board 2009). Therefore, it is more difficult to maintain the impressive achievement of the CRP. Given competing land use between conservation and intensive production, a better understanding of farmers’ land use choice is important for
identifying potential policy options to keep land enrollment in the program and preserve public interests in environmental benefits from conservation.

**Literature Review**

Studies about what drives land-use change include two broad categories: the econometric model approach and the real options approach. The econometric-model approach focuses on empirically identifying economic and policy factors with discrete choice models. Multinomial-choice model (Skaggs, Kirksey, and Harper 1994), logistic model (Janssen and Ghebremicael 1994; Isik and Yang 2004; Lubowski, Plantinga, and Stavins 2008), and ordered probit model (Cooper and Osborn 1998) can be found in the literature. Various factors affecting farmer participation in the conservation programs have been identified. Demographic characteristics such as farmer age and education, farm attributes such as farm tenure, location within the state, soil erosion rate, and economic factors such as farming return and anticipated levels of federal price/income support and bid cap have been found to influence enrollment in the programs. In general, under econometric framework land-use choices are modeled in a static way, without looking into the dynamic aspects of participation decision process and the mechanism underlying the process. Although the econometric approach can accommodate and examine different determinants of farmer participation in the CRP, the static and often ad hoc nature of econometric models call for alternative approaches that could generate more insights into the dynamic decision making process and the factors underlying it.

The real option approach in the literature is based on underlying theory on land use decision making process and emphasizes the effect of uncertainty and irreversibility when making conversion decisions. The option to retiring land is valuable because land return, the
main driver of land use change, is uncertain and land use change is irreversible under the current CRP design. When farmers delay conversion, more information about future returns is gained before making the irreversible enrollment. In the process, there exists a threshold expected return which will trigger participation. The optimal conversion thresholds from real option models reflect both the expected relative returns from alternative land uses and the conversion option value. Therefore, real option model provides a potential explanation of the process and mechanism underlying land-use changes. The real option approach has been applied in a growing body of literature on land use decisions. Capozza and Li (1994) use a real option framework to analyze land conversion decisions when land-capital ratio or capital intensity is variable and demonstrate that the option to varying intensity increases the hurdle rent and delays development. The conversion option has also been used to explain the failure to participate in land conversion. For example, the Conservation Reserve Program has been found ineffective in promoting cropland conversion to more permanent uses, e.g. forests (Schatzki 2003).

This paper follows real option approach to develop an optimal-stopping framework to model cropland enrollment in the CRP in the era of biofuel production. Traditionally, the uncertainty of agricultural return from cropland is summarized in a stochastic process, for example, Geometric Brownian Motion, with known parameters, such as mean and standard deviation or drift and volatility (Capozza and Li 1994, Schatzki 1998; Carey and Zilberman 2002). However, the implicit assumption, more specifically, known drift or return growth rate, is no longer true in the ear of biofuel production because of structure change in agriculture resulting from biofuel demand. One contribution of the paper is to capture the effect of biofuel production on agricultural returns. The movement of agricultural return is assumed to follow the process of Brownian motion with stochastic growth rate of certain distribution (Brennan 1998) to
capture the structural change. The response of participation triggers to distribution parameters reflects the magnitude of the effect of biofuel production on enrollment. The non-linear Kalman filtering approach (Kalman 1960) is introduced to derive the optimal estimators of the stochastic growth rate. The Kalman filter is a recursive algorithm which allows one to upgrade model estimates using new information. Compared to other estimation procedure, this approach generates estimated parameters with better statistical properties in terms of efficiency and forecasting. In this paper, we make a methodological contribution by deriving the numerical solution of the CRP participation decision problem under parameter uncertainty to a specified degree of accuracy using a technique that can handle a fairly general class of specifications for more uncertainty sources.

The paper departs from earlier land-use studies due to examining more different factors. First, we examine the effect of contract characteristics on participation in the CRP to identify effective contract designs that efficiently increase incentives for land enrollment. Little research has been done on the relationship between CRP enrollment and CRP contract characteristics, e.g. contract length. As we know, once enrolled in the CRP, lands will be locked in the contract for 10 or 15 years. However, the rigid contract design has undesirable consequences for landowners. For example, in the context of increasing ethanol production, landowners who participate in the CRP are losing out economically due to the forgone high agricultural returns. Another contract characteristic is financial incentive, which is implemented in other conservation programs. Farmers’ participation might be sensitive to financial incentives in addition to the land rental payments. In the paper, we account for the effects of these two characteristics on participation triggers in the model. Sensitivity tests are undertaken to determine the effects which changes in the movement of contract length or financial incentives of various parameters have on
enrollment triggers. The analysis can provide policy makers with guidance regarding how to increase enrollment in an effective manner through adjustments in contract design and financial incentives. The comparative-static analysis provides a number of important new results. We show that participation triggers is sensitive when contract length is shortened, while financial incentive for landowners can also be an effective instrument to help increase participation in the program. The comparative static analysis provides testable empirical implication for the model. Second, sensitivity tests were undertaken to determine the effects changes in the uncertainty of agricultural return on enrollment triggers. Farmers will wait till a lower agricultural return to warrant smaller loss of opportunity cost when standard deviation of the logarithm of agricultural returns increases.

**Parameter Uncertainty in Farming Return**

The participation problem is similar in nature to an American put option, with rental payment by government as the exercise price. The American option can be best described as an optimal stopping problem: landowners have the option (to participate) and wait for the best time at which they would stop waiting and exercise the option if they would participate at all. It is well known that valuing the American option is implemented under the assumption that asset return is following certain Ito processes. In the literature the often used stochastic diffusion process is Geometric Brownian Motion:

\[
dR_t = \theta R_t dt + \sigma R_t dW_t, \tag{1}
\]

where \( \theta \) is a drift term, \( \sigma \) is a constant volatility parameter, and \( W_t \) is a Brownian motion, defined in a complete probability space. However, this model always makes a strong assumption that parameters of this process are known in advance. That is, all possible sources of uncertainty that
affect farming return are singled out and summarized in the form of a log-normal distribution with known parameters such as growth rate and volatility. The assumptions made about farming return are too restrictive and inconsistent with the reality because the landowners do not really know the parameters of the truly probability distribution from which farming returns are drawn. Whereas farming return is clearly observable, it is hard to argue that parameters of stochastic processes representing the variable can be observed as well. In practice, these parameters must be estimated from the time-series of observable historical farming returns and are assumed to be constant. In the process of estimation, an additional ‘estimation error’ will be inevitably introduced (Gennotte 1986). In the ethanol surge, the error of growth rate (the drift term in eq. (1)) thus derived may be substantial, because the rising corn demand has caused structural changes in the sector and as a result the characteristics of farming return distribution differ substantially from previous ones. Therefore, it is more realistic to assume that farmers have only partial information about the growth rate in the bio-fuel era and form or update their expectations about farming return, conditional (rather than unconditional) on the new information they have at the time of decision.

When making participation decision, farmers have to take into account the fact that the market (or more specifically, the resulting agricultural return growth rate distribution) is no longer the same as the past because of the biofuel factor. To capture this uncertainty, we allow growth rate ($\theta$) of agricultural return from cropland to be stochastic and unobservable in this study. We use a two-stage stochastic Gordon growth process to model $\theta_t$. The model assumes it goes through a relatively short period of high growth rate ($\alpha$) first, and then changes to the average long-term industry growth rate ($\beta$) when the ethanol industry matures and reaches a long-term equilibrium. The maturity may result from the technological breakthrough in the cost-
effective production of cellulosic ethanol, from a wide variety of cellulosic biomass feed-stocks such as cereal straws, switchgrass, and other agricultural residuals. The model is specified as:

\( \theta_t \in \{\alpha, \beta\}, \quad \alpha > \beta \)  

where \( \alpha \) and \( \beta \) are two states. The time of staying in state \( \alpha \) denoted by \( T_\alpha \) follows an exponential distribution with parameter \( \lambda \):

\[
prob(T_\alpha \geq t) = \exp(-\lambda t).
\]

We use Kalman filter approach to estimate \( \theta_t \). The Kalman filter is a set of mathematical equations that implement a predict-corrected type estimator that is optimal in the sense that it minimizes the estimated error covariance (Kalman 1961). We allow farmers to update their information and thus learn about the true parameter distribution with each new farming return realization. Examination of past data at time zero provides some initial idea about growth rate and we can model this knowledge as a prior probability distribution of this random parameter. Based on new observations of farming return over time, prior distribution is upgraded in a continuous mode. Liptser and Shiryayev (1977) have derived the basic equation of optimal nonlinear filtering in the partially observable random process. The conditional probability of growth rate reverting to the average profit level on the observable farming return is defined as:

\[
p_t = prob(\theta_t = \beta | \Omega_t^R)
\]

Based on the information set \( \Omega_t^R \) available to farmers at time \( t \), farmers form expectations about the value of \( \theta_t \),

\[
m_t = E(\theta_t | \Omega_t^R) = \alpha (1 - p_t) + p_t \beta.
\]

Under the uncertainty of parameters, the Brownian motion \( W_t \) is also unobserved. The innovation process \( \tilde{W}_t \), derived from the observable process, is defined as the normalized deviation of the return from its conditional growth rate.
\( d\tilde{W}_t = \frac{1}{\sigma} \left( \frac{1}{R_t} dR_t - m_t \right) dt. \)

Substituting (1) into (6) yields

\( dW_t = d\tilde{W}_t - \frac{1}{\sigma} (\theta_t - m_t) dt. \)

Substituting \( dW_t \) into Eq.1, the stochastic process \( R_t \) becomes

\( dR_t = R_t m_t dt + R_t \sigma d\tilde{W}_t. \)

Farmers seek to extract information on future expected farming return from their observation of past returns, and then update the information when new return realization comes up. The conditional mean \( m_t \) evolves according to (Refer to Appendix A for the derivation).

\( dm_t = \lambda (\beta - m_t) dt + \frac{1}{\sigma} (\alpha - m_t)(m_t - \beta)d\tilde{W}_t. \)

**Farmer’s Decision under Uncertainty**

The CRP provides an annual per-acre rent payment (which varies prior to entry, but is fixed once enrolled in the CRP) to farmers to take highly erodible or environmentally sensitive cropland out of production for 10 or 15 years. The program also provides cost-share assistance to participants who establish approved cover on eligible cropland. The cost-share assistance can be an amount no more than 50 percent of the participants' costs in establishing approved practices. Other financial incentive payments are also offered. The farmer who decides to enroll in the program must enter into a \( T \) year contract (10 or 15 years).

Consider a risk neutral farmer facing a decision whether to convert a unit of land from corn production to conservation in the CRP program. The optimal decision in relation to participation will be determined in a continuous-time, infinite horizon framework\(^3\). If the farming return from production at time \( t \) is defined by eq. 1, then the expected farming return forgone over the \( \bar{T} \) year farming at year \( t \) is:
Since after participation the rental payment $Q$ will be locked in the length of the contract duration without inflation adjustment, the expected land rental payment received over the $\bar{T}$ year CRP contract at year $t$ is

\[
M(Q) = \int_{t}^{\bar{T}+t} Q e^{-\rho t} dt - (1 - k)C + \pi = \frac{Q}{\rho} - (1 - k)C + \pi
\]

where $C$ is the total restoration costs when participating in the CRP, and $k$ is the portion that is paid by government, who pays the part as the incentive for participation. $\pi$ is additional financial incentives, e.g. a one-time sign-up incentive payment. This payment is made soon after the contract has been signed and approved.

Denote $M(R_t, Q) = M(Q) - M(R_t)$ as the land conversion opportunity. The minimum requirement for participation is that $M(R_t, Q)$ be positive. The farmer’s land conversion decision can be specified as a question of at which year participating in the program can generate the maximum positive opportunity value. The optimal stopping problem can be represented as:

\[
J(R_t, Q) = \max_{t} E[e^{-\rho t} M(R_t, Q)]
\]

Thus we arrive at the following optimal stopping problem under the uncertainty of growth rate:

\[
J(R_t, m_t) = \max_{t} E[e^{-\rho t} M(R_t, Q)]
\]

\[
dR_t = R_t m_t dt + R_t \sigma d\bar{W}_t
\]

\[
dm_t = \lambda(\beta - m_t) dt + \frac{1}{\sigma}(\alpha - m_t)(m_t - \beta) d\bar{W}_t
\]

In this framework, landowners try to maximize the option value by choosing the optimal time $t$ to enter the CRP contract with duration $\bar{T}$ in the era of ethanol production. In the process, they continuously update the estimate of the growth rate of farming returns based on new return realizations. When the uncertain parameter is introduced into the optimal-stopping problem,
there is no analytical solution. The real option problem is formulated as a linear complementarity problem, and is solved numerically using an implicit finite difference approach (Wilmott, Dewynne, and Howison 1993).

**Linear Complementarity Problem**

Given certain regularity conditions, there will be a critical value of return $R^*$ such that participating in the CRP is optimal if $R_t < R^*$, while continue farming is optimal if $R_t \geq R^*$. The solution to the participation problem involves finding the free boundary $R^*$. Dynamic optimization techniques are used to derive the participation threshold (Dixit and Pindyck 1994). Using Ito’s lemma, we derive a partial differential equation in the continuous region (Refer to Appendix B for derivation):

\[
\rho J - \left[ R_t m_t J_R + \lambda (\beta - m_t) J_m + \frac{1}{2} R_t^2 \sigma^2 J_{RR} + \frac{1}{2\sigma^2} (\alpha - m_t)^2 (m_t - \beta)^2 J_{mm} + R_t (\alpha - m_t) (m_t - \beta) J_{Rm} \right] = 0,
\]

where $J_R$ denotes the derivative of $J$ with respect to $R_t$, $J$ denotes the put option value. Wilmott, Howison, and Dewynne (1993) formulate a full optimal stopping problem (eq.13) as a linear complementarity problem (LCP) to find a transformation that reduces the free boundary problem to a fixed boundary problem. We define an expression as $HJ$:

\[
HJ = \rho J - \left[ R_t m_t F_R + \lambda (\beta - m_t) F_m + \frac{1}{2} R_t^2 \sigma^2 F_{RR} + \frac{1}{2\sigma^2} (\alpha - m_t)^2 (m_t - \beta)^2 F_{mm} + R_t (\alpha - m_t) (m_t - \beta) F_{Rm} \right]
\]

where $\rho J$ represents the return required on participation opportunity for the rational farmer to holding the participation option. The expression within brackets represents the actual return on the time interval $dt$.

Then the LCP can be specified as
This LCP describes the strategy with regard to holding versus exercising the option to participation in the CRP. If $HJ \geq 0$, it states that the required return for holding the option must be at least as great as the actual return. If $HJ > 0$, it means that the required return from holding the option exceeds the actual return, and it is optimal to exercising the option immediately. If $HJ = 0$, it means that the required return equals the actual return, it is optimal to continue holding the option. If $J \geq M(R_t, Q)$, it means that the option value is no less than opportunity values from immediately participating in the CRP. $J$ would never drop below opportunity values from participating immediately because the rational investor would participate in the CRP before that could happen. When $J = M(R_t, Q)$, it is optimal to participate in the CRP immediately. When $J > M(R_t, Q)$, holding the participation option is a wise choice. The equality of $HJ \cdot [J - M(R_t, Q)] = 0$ states that at least one of $HJ = 0$ and $J - M(R_t, Q) = 0$ becomes a strict equality. If both $HJ$ and $J - M(R_t, Q)$ equal zero, then farmers are indifferent to participation in the CRP and non participation.

**Parabolic LCP**

In order to solve American free boundary problem, we can take an implicit finite-difference approximation of the linear complementarity problem (eq.16). In our model, LCP is time-independent, or elliptic type. Although it seems that elliptic LCPs are relatively easy to handle compared to parabolic LCPs that has an additional time dimension, it is not true in practice because there is no satisfactory method dealing with elliptic LCPs (Abasov 2005). In the process of discretizing parabolic partial differential inequalities, the matrices with tridiagonal structure
arise, which generates a highly efficient algorithm for solving inequalities. Meanwhile, good convergence of the implicit finite-difference approximation to the solution of the partial differential inequalities is also associated with diagonal dominance of those matrices. However, discretization of elliptic partial differential inequalities cannot produce the matrices with tridiagonal structure. The reason is that there is no time variable in elliptic LCPs. This suggests an idea to introduce an artificial time variable into partial differential inequality of elliptic LCP and solve the resulting parabolic LCP. We follow Abasov (2005) to implement the idea in the optimal stopping framework: (a) choose a sequence of \( t_i \), such that \( \lim t_i = \infty \); (b) for each \( i \) find an optimal value function \( J(R, m, t) \); (c) since it can be shown that the sequence of solutions \( J_i \) will converge to \( J \), we can choose sufficiently large \( t_i \) to reach the convergence value. After incorporating time component into the LCP, the parabolic LCP is as follows:

\[
\begin{align*}
(J_i)'_t + HJ_i & \geq 0 \\
J_i & \geq M(R_t, Q) \\
[(J_i)'_t + H] J_i & = [J_i - M(R_t, Q)] = 0
\end{align*}
\]

The subscript denotes derivative. Discretizing the LCP will yields matrices with dominating diagonals.

**Boundary Conditions**

For a numerical solution of LCP, we must specify the boundary conditions. Note that because the linear complementarity formulation does not depend explicitly on the free boundary we do not have to specify the value matching and smooth pasting conditions.

**Boundary condition 1.** It can be seen that from the eq.1 that once \( R_t \) reaches zero, it will stay there forever because of \( dR_t = 0 \). Therefore, it is optimal to exercise the option immediately for farmers.
Boundary condition 2. At a very high farming return \( R_{\text{max}} \), the put option is deeply out of the money and we can simply set the option value equal to zero or \( J(R_{\text{max}}, m, t) = 0 \) (in the implementation, \( R_{\text{max}} \) is set three times the average farming return).

Boundary condition 3. Since we are searching for solution on rectangular domain formed by \((R_t, m_t)\) in the process of implementation we also use numerical boundary conditions on two other boundaries, that is, \( m_{\text{min}} = \beta \) and \( m_{\text{max}} = \alpha \). In particular, we use BC2\(^4\) (Tavella and Randall 2000) boundary condition on \( m_{\text{min}} \) and \( m_{\text{max}} \).

Terminal condition 1. The terminal condition follows from the observation that at \( t = \infty \) there is no uncertainty about the true value of growth rate. As a result, \( m_t \) remains constant, hence \( J(R_t, m, \infty) \) is found as a solution of one-dimensional problem

\[
\begin{align*}
J(R_t, m, \infty) &= \max_t E[e^{-\rho t} M(R_t, Q)] \\
dR_t &= mR_t dt + \sigma R_t dW_t
\end{align*}
\]

which can be solved in closed form. After the transformation we can reach the put option value (refer to Appendix C for derivation), that is,

\[
J(R_t, m, \infty) = \begin{cases} M(R^*, Q) \left( \frac{R^*}{R_t} \right)^\gamma & R_t \geq R^* \\
M(R_t, Q) & R_t \leq R^*
\end{cases}
\]

where the threshold value \( R^* \) is defined as

\[
R_t^* = \frac{\gamma}{\gamma - 1} \cdot \frac{(\rho - m)M(Q)}{(1 - e^{-\rho (m - m)^2})}
\]

\[
\gamma = \frac{-(m - \frac{1}{2} \sigma^2) - \sqrt{(m - \frac{1}{2} \sigma^2)^2 + 2\rho \sigma^2}}{\sigma^2}
\]

When incorporating more uncertainty into the model, it is unlikely to result in closed-form solutions. We have to resort to a number of numerical techniques to solve complex optimal problems, and thus limit the method’s application to a more realistic setup. However, the
development of options theory in finance literature and the solution techniques in mathematics lend themselves to more sophisticated real options problems. The numerical algorithm for determining the value of the option involves the discretization of the linear complementarity problem using an implicit finite difference method. Details can be found in Appendix D.

**Data**

The model developed above is applied to Michigan farmers’ participation in the CRP. In Michigan, 276,151 acres are enrolled in the CRP in 2007, among which 191,660 acres are the general contract\(^5\). The total rental payment in 2007 for the general contract is 11,113 thousand dollars, and average payment is about 57.98 dollars/acre (USDA/FSA 2007). When participating in CRP, landowners are making a decision to retire the environmentally sensitive lands from production purposes. The opportunity cost of CRP participation is the forgone net agricultural returns. Since eligible land is “cropland (including field margins) that is planted or considered planted to an agricultural commodity 4 of the previous 6 crop years from 1996 to 2001, and which is physically and legally capable of being planted in a normal manner to an agricultural commodity”, cropland return in Michigan is used to be the main opportunity cost. Two major crops in Michigan, corn and soybean, are harvested 2.35 and 1.74 million acres respectively in 2007, the proportion of which to total harvested acres of field crops are 36.5% and 27% (Michigan Agricultural Statistics 2007-2008). Therefore, a weighted state-level average of the net returns per acre for the two crops are computed and used. State-level marketing-year-average prices and yields are from the National Agricultural Statistics Services (NASS). Data on cash costs as a share of revenue at the regional level are from the Economic Research Service (ERS). State acreage from NASS provided weights for averaging across individual crops. Government
payments are also important for farmers when making land use decisions; therefore, per acre government payments are estimated to be included in cropping returns. However, the estimation is difficult due to constant changes in the farm acts, e.g., in 1996 income support payments tied to commodity prices were eliminated in favor of production flexibility contract payments, while under 2002 farm act, deficiency payments were eliminated in favor of counter-cyclical payments. The proportion of state-level government payment to crop receipts is used to be a share of per acre government payment. In this paper, state-level federal program payments are from Michigan Agricultural Statistics and include receipts from deficiency payments, support price payments, indemnity programs, disaster payments, and production flexibility contract payment and so on. Conservation program payments are excluded. Annual cropland return from 1970 to 2008 is collected to form 39 samples.

Return data are aggregated, average returns, reflecting the average productivity of all feasible crop production land. In general, the land eligible for CRP participation is of low productivity. Land productivity is mainly determined by the physical characteristics of the soil. For example, grain crop yields decrease as slope increases and erosion becomes more severe. Therefore, it is necessary to make adjustments to crop yield to precisely estimate the return of land eligible for CRP. Previous studies have sought to measure the role of land quality in explaining differences in agricultural productivity. Using crop yield as a proxy measure for soil productivity, Den Biggelaar et al. (2004) uses the data from 179 plot-level studies from 37 countries to calculate absolute and relative yield losses of soil erosion for various crops, aggregated by continent and soil type and found that for most crops, soils, and regions, yields decline by 0.01-0.04 percent per ton of soil loss. In North America case, corn and soybean mean yield losses are equally 0.01 percent per ton of soil loss. Estimating annual erosion-induced
yield losses requires information on the rate at which soil is being lost to erosion. We rely on the Universal Soil Loss Equation (USLE), which estimates average annual soil loss from sheet and rill erosion as a function of rainfall, soil erodibility, slope (both steepness and length of slope), land cover and management, and conservation practices. We use USLE soil loss data (in tons per acre per year) estimated from 1997 National Resources Inventory to compute the weighted average soil loss for Michigan land enrolled in the CRP in 1997. This resulted in a 1997 average erosion rate of 6.0 tons per acre. Annual yield loss rates are estimated by multiplying the percentage yield loss per ton of soil loss by the estimated annual erosion rate. We estimated that corn and soybean yield loss from soil erosion in Michigan is on average 0.06 percent per year, which is consistent with other estimations. Therefore, cropland returns in the land eligible for the CRP can be adjusted according to the relationship between land quality and agricultural productivity.

Parameter Estimates

With Ito’s lemma it is known that if farming return follows Geometric Brownian Motion (GBM), the logarithm of return will follow simple Brownian motion with drift. Then $\ln(R_t)$ will be described as

$$d\ln(R_t) = (\theta - 0.5\sigma^2)dt + \sigma dW_t$$

(24)

To test whether $\ln(R_t)$ follows a process of Brownian motion with a drift, eq.24 must be approximated in discrete time. This can be done as

$$\ln(R_t) - \ln(R_{t-1}) = (\theta - 0.5\sigma^2)\Delta t + \sigma \varepsilon_t \sqrt{\Delta t}$$

(25)

where $\varepsilon_t$ is a normally distributed random variable with mean 0 and standard deviation 1. We performed an augmented Dickey–Fuller (ADF) test on the logarithm of return series to
investigate whether the data-generating process is a random walk (and hence nonstationary). Let 
\( c_0 = (\theta - 0.5\sigma^2)\Delta t \), and \( e_t = \sigma \epsilon_t \sqrt{\Delta t} \). The null hypothesis of a unit root can be tested using the 
\( t \) statistic from the regression:

\[
\Delta \log R_t = c_0 + c_1 \log R_{t-1} + \sum_{i=2}^{q} c_i \Delta \log R_{t+1-i} + e_t
\]

To appropriately select the lag length \( q \) in the above equation, we started with a relatively long 
lag length and test down the model. Once a tentative lag length has been determined, the Ljung- 
Box Q-tests are conducted to ensure that no significant autocorrelations are in the residuals. 
After investigations, \( q = 5 \) was selected for the given return series. The statistics of the 
augmented Dickey-Fuller tests, and the associated hypothesis and their critical values are 
summarized in Table 1. From table 1, \( t \)-value is larger than the critical values at the standard 
significant levels, implying the null hypothesis of unit root (\( H_0: c_1 = 0 \)) is not rejected.

<Table 1>

Therefore, the analysis will be undertaken assuming that cropland return follows GBM. 
In making this assumption we are ignoring the lagged dependent variable term which was found 
to be insignificant in the Dickey Fuller test. If we assume that return follows a process of GBM, 
the maximum-likelihood estimates of the drift \( \theta \) and the variance \( \sigma^2 \) for cropland return \( R_t \) will 
be \( \theta = \phi + 0.5\varphi^2 \) and \( \sigma = \phi \), where \( \phi \) is the mean and \( \varphi \) is the standard deviation of the series, 
\( \ln(R_t) - \ln(R_{t-1}) \). After the estimation, we can get \( \theta = 0.0225 + 0.5 \times 0.2115^2 = 0.0672 \) and 
\( \sigma = 0.2115 \).

Based on eq.3, the expected time of staying in the first state is \( 1/\lambda \). After the growing 
stage, the ethanol industry will be in equilibrium. Tokgoz et al. (2008) consider that the long-run 
equilibrium may be achieved in 2016-17 where investors are indifferent between building and 
not building a new ethanol plant. Investment in new ethanol plants will take place the market
price of corn allows a prospective plant to cover all the costs of owning and operating an ethanol plant. This indifference occurs because the ethanol industry stops growing in response to earlier negative earnings. Economic production of cellulosic ethanol resulting from the technological breakthrough may necessitate the equilibrium condition to be achieved. In the paper, we assume that $\lambda = 0.125$. Higher growth rate of cropland return in the future is referred to the report of USDA Agricultural Projections to 2018. The report shows that although increases in corn-based ethanol production in the United States are projected to slow, ethanol demand remains high and affects production, use, and prices of farm commodities throughout the sector. As a result, although net farm income initially declines from the highs of 2007 and 2008, it remains historically strong and rebounds in the projections. With the projected corn price, yield data we reach a 15.35% annual corn return growth rate and a 13.91% annual soybean return growth rate covering the periods between 2008 and 2016-17. In views of the proportion between planted corn and soybean, we estimate the growth rate $\alpha$ in the first state is 14.81%. We use the original rate 6.72% as one in the second rate. Among other parameters, a discount rate $\rho$ of 20% is used. And the cost share proportion ($k$) establishing land cover is 50%. The total restoration cost ($C$) when participating in the CRP depends on the land cover cost. In general, the cost is $60. In some specific program, participants will also receive a sign-up incentive payment ($\pi$) equal to $100 or $150 per acre upon enrollment into the program.

**Empirical Results**

Option value, $J(R_t, m_t)$, is determined using the numerical method described in Appendix D. Figure 1 plots the value of the option of participating in the CRP for the baseline case where
\[ \alpha = 0.1481, \beta = 0.0672, \lambda = 0.125, k = 0.5, C = 60, \sigma = 0.2115, \rho = 0.2, \bar{T} = 10, Q = 80, \]
and \( \pi = 150. \)

<Figure 1>

In this figure, the vertical axis measures the option value \( J(R_t, m_t) \), which joint smoothly onto the horizontal plane of the colored grids in the 3-dimensional space. On the vertical lines of this plane the growth rate is constant and the farming return varies. On the slanted (diagonal) lines the return is constant while the growth rate varies. The option value increases (decreases) as the farming return and/or its growth rate decreases (increases). Table 2 show all the option values in the grid and free boundaries are highlighted in the cells, on which option value equals the opportunity value of immediately participating in the CRP and the landowner is therefore indifferent to the two activities. On the lower side of the boundary (including the revenue-growth plane), opportunity value of participation is lower than the option value; the landowner would choose to wait and not to participate. The boundary represents the critical returns and growth rates that would trigger participation in the CRP. On the other side of the boundary, with lower agricultural returns and growth rates, the option has a positive, larger value and it’s optimal to exercise the option and participate immediately.

<Table 2>

The main findings are as follows: 1) option value decreases with farming returns \( R_t \) and return growth rate \( m_t \), which is illustrated in figure 1; 2) the option value increases with \( \lambda \) (recall that \( 1/\lambda \) represents the average time of staying in the high growth state), which means that the longer the high growth state persists, the lower the value of the option, suggesting that in a time of extended high growth, the conservation option is of less value to landowners; 3) the boundary declines with the landowners’ updated expectation of the growth rate; 4) the boundary also
declines with the growth rate at the high growth stage $\alpha$. All results presented above are intuitive: higher farming return, higher expectations of the growth rate, and longer average time of staying in the state of higher growth will result in an lower option value, which means the farmer will wait longer before participating in the CRP or would not participate in the CRP. This result is consistent with the observation in the last few years when farming returns grew rapidly due to the demand from biofuel production.

**Sensitivity Analysis**

The parameters in the baseline impact on participation triggers in the model. Therefore, sensitivity tests were undertaken to determine the effects changes in various parameters had on the participation boundaries. First, CRP contract characteristics have played a role in participation in the CRP. However, not each characteristic is good to facilitate enrollment. An understanding of these motivating factors from contract characteristics could be helpful to policy makers in improving contract design and implementation of such crucial conservation programs and their cost-effectiveness. Second, an analysis is undertaken to determine the effects changes in volatility of farming return had on boundary values.

**Contract length**

As we know, once enrolled in the CRP, lands will be locked in the contract for 10 or 15 years. The rigid contract design may well deter landowners from participating in the CRP because of the risk of being locked-in. Farmers’ sensitivity to contract length is investigated by selecting various contract length values. Figure 2 plots the boundary of participation and non-participation in the Return-Growth space, under different contract lengths, 5, 10, 15, 20 and 30 years, while
keeping other parameters unchanged ($\alpha = 0.1481, \beta = 0.0672, \lambda = 0.125, k = 0.5, C = 60, \sigma = 0.2115, \rho = 0.2, Q = 80,$ and $\pi = 150$).

*Figure 2*

In this figure, landowners would participate when the agricultural return and its growth rate are low, in the area below the boundaries, and they would not participate otherwise. The downward sloping boundaries indicate that at higher growth rates, CRP participation requires lower return threshold. The figure shows that there are no clear-cut conclusions whether landowners will more readily participate when offered shorter contracts of 5 years compared to the longer ones of 20 or 30 years. But we find that participation triggers rise from 50-60$/acre to about 70-80$/acre when contract length is shortened from 10 years to 5 years. The jump is substantially greater than those when contract length is extended.

*Table 3*

Table 3 gives the elasticity of participation triggers to changes of contract length from the baseline case (10-year contract). It suggests that when contract length is extended from 10 to 15 years till 30 years, the sensitivity of participation trigger to contract length is gradually decreasing. It means that if only longer-term contracts are offered, threshold return triggering participation in the CRP have no substantial difference among these contracts. When contract length is shortened from 10 to 5 years, the degree of sensitivity of return trigger is prominently increased. Among the majority of different growth expectations, its elasticities are two times more than those in the case from 10 to 15 years. It indicates that shortening contract length is an effective method to encourage enrollment.

The results have implications for the design of conservation programs promoting shifts in behavior and cost-effectively preserve environmental benefits. First, when the CRP land was
squeezed out by ethanol production, shortening contact lengths is a good scheme to attract more land in the CRP. Second, for high environmentally sensitive cropland a longer term contract should be offered to reach cost-effectiveness because participation threshold is more insensitive to contract length. Third, the results are in favor of contracts with different lengths. Currently, only 10- or 15-year contract is offered to farmers. Its distribution of contract length is too concentrated to offer more options to farmers or to supply policy maker more effective instruments to reach optimal cost-benefits of such programs.

*Financial Incentive*

Except annual rental payments and cost-share assistance, Farm Service Agency(FSA) also provides CRP participants additional financial incentives, for example, one-time signing bonus. Farmers’ sensitivity to financial incentive is investigated by selecting various incentive values. Figure 3 plots the boundary of participation and non-participation in the Return-Growth space, under different incentives, 50, 100, 150, 200, 250, and 300$/acre, while keeping other parameters unchanged (\(\alpha = 0.1481, \beta = 0.0672, \lambda = 0.125, k = 0.5, C = 60, \sigma = 0.2115, \rho = 0.2, \bar{T} = 10, \text{and } Q = 80\)).

<Figure 3>

Compared to the contract length scheme, this figure gives a completely different scene. The boundary is more sensitive to the financial incentive. For example, in the baseline, if the farmer expects a 6.7% growth rate, he will accept the 10-year contract when the farming return reached $65 per acre, but if given an incentive of $300, he will be willing to participate even if the farming return reached $85. Therefore, providing sign-up incentives in addition to the land rental payments would substantially increase participation in these programs. This justifies the
practice adopted in the supplementary programs such as the continuous CRP and the Conservation Reserve Enhancement Program (CREP), which aim to improve water quality and wildlife by offering additional financial incentives to landowners for conservation. Table 4 provides the elasticity analysis. In sum, the average elasticity rates are greater than those in different contract lengths scenario expect one case—contract length is shortened from 10 to 5 years. Meanwhile, financial incentive is basically equally effective in all situations, either high or low growth rate. It suggests that financial incentive is always an effective tool to motivate participation in the CRP.

<Table 4>

The Effect of Volatility
Real option allows researchers to build uncertainty, irreversibility, and the ability to wait to make a decision into one framework and determines the impact three factors have on the profitability of the decision. The opportunity value of participation in the CRP incorporates these factors. In the sector, we study the effects of uncertainty and irreversibility on participation triggers. We keep other parameters unchanged in the baseline while varying the standard deviation of the logarithm of cropping returns. The impact of changes in volatility within the model can be seen in figure 4.

<Figure 4>

Net present value method (NPV) ignores risk or uncertainty inherent to cropland return. Standard NPV will overestimate the participation triggers by not including the value of waiting for new information to reduce the uncertainty of the cash flows (Pindyck 1988). Figure 4 shows that uncertainty in the cropland return contributes approximate $14/acre the increase in
participation triggers relative to the baseline. Figure 4 also demonstrates the effect of different $\sigma$ on the trigger value. As expected, while the variance of the model increased, the triggers decreased. It means that the producer will wait a lower cropland return to participating in the CRP. Table 5 gives the elasticity analysis of participation in the CRP on different uncertainty parameters. Reducing the variance can increase participation boundaries. The implication is that government programs aimed to reduce income risk can play a role in encouraging farmer to enter in the CRP. However, since these programs also support income increase, the aggregate effect on participation is not clear.

<Table 5>

**Concluding Remarks**

This paper proposes a land use decision model under uncertainty in the era of bio-fuel production. The decision is modeled as an optimal stopping problem in the real option framework, and farming return is assumed to follow stochastic process with parameter uncertainty. The model captures the structures changes in the agricultural sector caused by biofuel production and further accommodates the potential shocks in relation to the possible technological breakthroughs in cellulosic ethanol production. The decision model provides a useful tool both for landowners to determine optimal land use and the optimal timing of land use conversion, and for policy makers to make informed environmental policies and to make more efficient use of taxpayers’ money. Methodologically, we use nonlinear Kalman filter approach to continuously upgrade the information and estimate the random growth rate with the minimum error. The problem is formulated as a linear complementarity problem, which is addressed with a fully implicit finite difference approach.
Sensitivity analysis was conducted for key parameters. As with ordinary financial put options, an increase in farming returns decreases the value of the real option to participate in conservation. Importantly, we find the threshold returns (critical value) is elastic to contract lengths from 10 to 5 years, which means that shortening contract length may be effective in motivating participation in the CRP. Meanwhile, financial incentive is found to be an effective means to stimulate participation, justifying the observed supplementary program practices. When public policies are increasingly relying on the use of land retirement and conversion programs to achieve environmental policy goals, the results of this study have implications for the design of conservation contracts promoting enrollment in the CRP.
The Food, Conservation, and Energy Act of 2008 imposes a 32-million-acre maximum for CRP starting October 2009, which is 7.2 million acres below the former cap established in the 2002 farm bill.

In the farmers’ information set, agricultural return $R_t$ is an observable component, while $\theta_t$ is an unobservable component.

Although it is only the limiting case of the discrete time problem and nonstandard, the continuous framework has an analytic convenience.

Boundary conditions proposed by Tavella and Randall are to apply the pricing equation itself as a boundary condition rather than other financial argument to which to appeal for one condition. BC1 is to postulate a linear dependence of option value on the price, while BC2 is to discretize the drift terms and volatility terms with second order one-sided different operators. BC2 will have smaller error when we compute option value using finite difference method.

General contract is contrasted with continuous contract. General contract means land sign-up enrollment occurs only during designated sign-up periods, while in continuous contract land can be enrolled at any time under CRP continuous signup, which is generally appropriate for environmentally desirable land devoted to certain conservation practices, e.g. buffer.

This report provides projections for the agricultural sector through 2018. Projections cover agricultural commodities, agricultural trade, and aggregate indicators of the sector, such as farm income and food prices. Prospects for the agricultural sector in the near term reflect adjustments to the global economic slowdown and the U.S. recession.


U.S. Department of Agriculture. 2008b. “Agriculture Secretary, Deputy and FAS Discuss Conservation Reserve Program (CRP) Decision.”

http://www.usda.gov/wps/portal!/ut/p/_s.7_0_A/7_0_1OB?contentidonly=true&contentid=2008/07/0197.xml

http://www.msue.msu.edu/objects/content_revision/download.cfm/revision_id.441679/works
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Table 1. Results from Augmented Dick-Fuller Unit Root Tests

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Note: Observation number: 39. The null hypothesis: $c_1 = 0$. Dickey Fuller critical value: 1%, -3.58; 5%, -2.93; 10%, -2.60.
Table 2. Option Value of Participation in the CRP

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Table 3. Elasticity Analysis to Critical Value of Participation in the CRP on Changes in Contract Length (Baseline Case: $\bar{T} = 10$)

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Table 4. Elasticity Analysis to Critical Value of Participation in the CRP on Changes in Financial Incentive (Baseline Case: $\pi = 150$/acre)

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<td>0.35</td>
<td>0.38</td>
<td>0.38</td>
<td>0.25</td>
<td>0.41</td>
<td>0.27</td>
<td>0.27</td>
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<tr>
<td>$300$</td>
<td>0.31</td>
<td>0.31</td>
<td>0.42</td>
<td>0.33</td>
<td>0.33</td>
<td>0.25</td>
<td>0.36</td>
<td>0.36</td>
<td>0.27</td>
<td>0.40</td>
<td>0.30</td>
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Table 5. Elasticity Analysis to Critical Value of Participation in the CRP on Different Volatilities (Baseline Case: $\sigma = 0.2115$)

<table>
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<tr>
<th>$\sigma \backslash m$</th>
<th>0.067</th>
<th>0.075</th>
<th>0.083</th>
<th>0.092</th>
<th>0.100</th>
<th>0.108</th>
<th>0.116</th>
<th>0.124</th>
<th>0.132</th>
<th>0.140</th>
<th>0.148</th>
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<tr>
<td>0.10</td>
<td>-0.29</td>
<td>-0.47</td>
<td>-0.32</td>
<td>-0.32</td>
<td>-0.32</td>
<td>-0.16</td>
<td>-0.34</td>
<td>-0.17</td>
<td>-0.17</td>
<td>-0.19</td>
<td>-0.19</td>
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<tr>
<td>0.40</td>
<td>-0.26</td>
<td>-0.28</td>
<td>-0.28</td>
<td>-0.28</td>
<td>-0.28</td>
<td>-0.31</td>
<td>-0.31</td>
<td>-0.31</td>
<td>-0.22</td>
<td>-0.22</td>
<td></td>
</tr>
<tr>
<td>0.80</td>
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<td>-0.21</td>
<td>-0.21</td>
<td>-0.21</td>
<td>-0.21</td>
<td>-0.24</td>
<td>-0.23</td>
<td>-0.23</td>
<td>-0.22</td>
<td>-0.22</td>
<td></td>
</tr>
</tbody>
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Figure 1. Option Value of Land Conversion Opportunity (Baseline Case)
Figure 2. Critical Farming Return versus Growth Rate under Different Contract Lengths
Figure 3. Critical Value versus Growth Rate under Different Financial Incentives
Figure 4. Critical Value versus Growth Rate under Different Volatility Parameters
Appendix A

Assume the probability of random growth rate is:

(A.1) \[ p_t = \text{prob}(\theta_t = \beta|\Omega_t^R) = \text{prob}(\theta = \beta|R_t) \]

According to Bayes formula,

(A.2) \[ p_t = \frac{f(R_t|\theta_t=\beta)\text{prob}(\theta_t=\beta)}{f(R_t|\theta_t=\beta)\text{prob}(\theta_t=\beta) + f(R_t|\theta_t=\alpha)\text{prob}(\theta_t=\alpha)} \]

where \( f(\cdot) \) is the density function of \( R_t \).

(A.3) \[ \theta_t = \alpha \Rightarrow R_t = R_0\exp\left[\alpha - \frac{\sigma^2}{2} t + \sigma W_t\right] \]

(A.4) \[ \theta_t = \beta \Rightarrow R_t = R_0\exp\left[\beta - \frac{\sigma^2}{2} t + \sigma W_t\right] \]

\( R_t \) has a conditional lognormal distribution, therefore

(A.5) \[ f(R_t|\theta_t = \alpha) = \frac{1}{\sqrt{2\pi \sigma R_t}} \exp\left(-\frac{[\ln(R_t) - \ln(R_0) - (\alpha - \frac{\sigma^2}{2})t]^2}{2\sigma^2 t}\right) \]

(A.6) \[ f(R_t|\theta_t = \beta) = \frac{1}{\sqrt{2\pi \sigma R_t}} \exp\left(-\frac{[\ln(R_t) - \ln(R_0) - (\beta - \frac{\sigma^2}{2})t]^2}{2\sigma^2 t}\right) \]

\( T_\alpha \), the time of staying in state \( \alpha \), has an exponential distribution with parameter \( \lambda \).

(A.7) \[ \text{prob}(T_\alpha < t) = 1 - \exp(-\lambda t). \]

Substituting these above into \( p_t \) implies

(A.8) \[ p_t = \frac{1}{1 + B\exp\left((\alpha-\beta)[\ln(S_t) - \ln(S_0)] + \frac{\beta^2 - \alpha^2 + (\alpha - \beta)\sigma^2}{2\sigma^2 t}\right)} \]

where \( B = \exp(\lambda t) - 1. \)

According to the Geometric Brownian Motion \( dR_t = R_t m_t dt + R_t \sigma d\tilde{W}_t \),

(A.9) \[ R_t = R_0\exp\left[\left(m_t - \frac{\sigma^2}{2}\right)t + \sigma \tilde{W}_t\right], \]

Then
\( p_t = \frac{1}{1 + B \exp \left( \frac{(a-\beta)(2m_t-a-\beta)}{2\sigma^2} \right)} \)

Using Ito’s Lemma to \( p_t \),

\[ dp_t = -\left( \frac{a-\beta}{\sigma} \right) p_t (1 - p_t) d\tilde{W}_t + \lambda (1 - p_t) dt \]

For \( m_t = \alpha (1 - p_t) + p_t \beta \), we can derive

\[ dm_t = \lambda (\beta - m_t) dt + \frac{1}{\sigma} (\alpha - m_t) (m_t - \beta) d\tilde{W}_t \]
Appendix B

The derivation is through Ito’s Lemma. We know that \( dR_t = R_t m_t \, dt + R_t \sigma d\bar{W}_t \) and \( dm_t = \lambda(\beta - m_t)dt + \frac{1}{\sigma}(\alpha - m_t)(m_t - \beta)d\bar{W}_t \). Take the square of the first equation, we get

(B.1) \( (dR_t)^2 = (R_t m_t \, dt)^2 + 2R_t^2 m_t \sigma dt d\bar{W}_t + R_t^2 \sigma^2 (d\bar{W}_t)^2 \).

We already know that \((dt)^2\) and cross product \( dt \, d\bar{W}_t \) are equal to zero in the mean square sense, and \((d\bar{W}_t)^2 = dt\), therefore

(B.2) \( E(dR_t)^2 = R_t^2 \sigma^2 dt \).

Similarly, we can get

(B.3) \( E(dm_t)^2 = \frac{1}{\sigma^2}(\alpha - m_t)^2(m_t - \beta)^2 dt \)

(B.4) \( E(dR_t \, dm_t) = R_t(\alpha - m_t)(m_t - \beta)dt \)

Applying Ito’s Lemma to \( J(R_t, m_t) \), we can derive a partial differential equation

(B.5) \( dJ(R_t, m_t) = J_R dR_t + J_m dm_t + \frac{1}{2} [J_{RR} (dR_t)^2 + 2J_{Rm} dR_t dm_t + J_{mm} (dm_t)^2] \)

Substituting B.2, B.3, and B.4 into B.5, we can get

(B.6)

\[
E[dJ(R_t, m_t)] = \left[ R_t m_t J_R + \lambda(\beta - m_t)J_m + \frac{1}{2} R_t^2 \sigma^2 J_{RR} + \frac{1}{2} \sigma^2(\alpha - m_t)^2(m_t - \beta)^2 J_{mm} + R_t(\alpha - m_t)(m_t - \beta)J_{Rm} \right] dt
\]

The Bellman equation for eq.13 is expressed as:

(B.7) \( \rho J(R_t, m_t) dt = E[dJ(R_t, m_t)] \)

Therefore,

(B.8) \( \rho J - \left[ R_t m_t J_R + \lambda(\beta - m_t)J_m + \frac{1}{2} R_t^2 \sigma^2 J_{RR} + \frac{1}{2} \sigma^2(\alpha - m_t)^2(m_t - \beta)^2 J_{mm} + R_t(\alpha - m_t)(m_t - \beta)J_{Rm} \right] = 0 \)
Appendix C

The simple optimal stopping problem without parameter uncertainty is

\[ J(R_t, m, \infty) = \max_t E[e^{-\rho t} M(R_t, Q)] \]
\[ dR_t = mR_t \, dt + \sigma R_t \, d\bar{W}_t \]

Using Ito’s lemma, the fundamental differential equation of this optimal stopping problem is an ordinary differential equation.

\[ \frac{1}{2} R_t^2 \sigma^2 J_{RR} + R_t m J_R - \rho J = 0 \]

where \( J_R \) and \( J_{RR} \) are the first and second derivatives of \( J \) with respect to \( R_t \). Let \( R^* \) represent the threshold value, which triggers participation in the CRP. This partial differential equation is solved subject to the boundary conditions.

The continuity condition is

\[ J(R^*) = M(R^*, Q) = M(Q) - \frac{R^*(1-e^{-(\rho-m)\gamma})}{\rho - m} \]

The smooth pasting condition is

\[ J_R(R^*) = M_R(R^*, Q) = -\frac{(1-e^{-(\rho-m)\gamma})}{\rho - m} \]

In addition we have

\[ J(\infty) = 0 \]

which says that when farming return approaches infinity, the land conversion option is worthless.

The general form of the solution to equations is

\[ J(R_t, m, \infty) = \begin{cases} M(R^*, Q) \left( \frac{R_t}{R^*} \right)^\gamma & \text{if } R_t \geq R^* \\ M(R_t, Q) & \text{if } R_t \leq R^* \end{cases} \]

where \( \gamma = -\frac{(m-\frac{1}{2} \sigma^2) - \sqrt{(m-\frac{1}{2} \sigma^2)^2 + 2 \rho \sigma^2}}{\sigma^2} \), which is the negative root of the fundamental quadratic equation \( \frac{1}{2} \sigma^2 \gamma (\gamma - 1) + m \gamma - \rho = 0 \), and the threshold \( R_t^* = \frac{\gamma}{\gamma - 1} \cdot \frac{(\rho - m) M(Q)}{(1-e^{-(\rho-m)\gamma})} \).
Appendix D

This Appendix describes the numerical approach used to solve the linear complementarity problem for valuing the option of participating in the CRP. For a general discussion of numerical methods of option valuation, refer to Wilmott et al. (1993). A finite difference scheme is used in this paper, which involves reducing a continuous partial differential equation to a discrete set of difference equations. In choosing a discretization approach, attention must be given to the properties of stability and convergence. Stability is a problem if the discretized model is sensitive to small errors that arise from the finite precision of computer algorithms. Convergence refers to whether the solutions of the discretized model converge to the solutions of the partial differential equations when the discretization is increasingly refined. Two basic finite difference methods are the implicit method and the explicit method. When applied to the diffusion equation backward and forward difference approximation for the derivative of option value with respect to time lead to explicit and implicit finite-difference schemes, respectively (Wilmott et al. 1993). Implicit finite-difference method is robust because it can overcome the stability and convergence limitations imposed on the explicit finite-difference method. Implicit finite difference method allows us to use a large number of mesh points without having to take ridiculously small time-steps. In the paper we use an implicit difference scheme—the Crank-Nicolson scheme, an average of the fully implicit and explicit methods.

We handle the three-dimensional \((R, m, t)\) discretization for eq. 17. Consider a three-dimensional grid with the horizontal plane formed by farming returns \(R\) and the growth rate \(m\) and the vertical axis \(t\). We divide the \(R\)-axis into equally spaced nodes a distance \(\Delta R\) apart, \(m\) – axis into equally spaced nodes a distance \(\Delta m\) apart, and \(t\)-axis into equally spaced nodes a
distance $\Delta t$. This divides the $(R, m, t)$ plane up into a mesh, where the mesh points have the form $(s\Delta R, n\Delta m, i\Delta t)$. We then compute the option value, $J$, at these points.

(D.1) $R = \{R_0, R_1, R_2, \ldots, R_S\}$,

$m = \{m_0, m_1, m_2, \ldots, m_N\}$,

$t = \{t_0, t_1, t_2, \ldots, t_I\}$.

The finite difference method involves replacing partial derivatives by approximations based on Taylor series expansions near the point or points of interest. At any point on the grids $(R, m, t) = (R_s, m_n, t_i)$ and the value of the option is $J_{s,n,i}$. A formula of approximating the partial derivatives using the implicit difference method can be found in Wilmott et al. (1993). Replacing these derivatives in eq.17 yields difference scheme:

(D.2) $HJ_i = [a_{s-1,n-1}J_{s-1,n-1} - a_{s-1,n}J_{s-1,n} + a_{s-1,n+1}J_{s-1,n+1} + a_{s,n-1}J_{s,n-1} + a_{s,n}J_{s,n} + a_{s,n+1}J_{s,n+1} + a_{s+1,n}J_{s+1,n} - a_{s+1,n-1}J_{s+1,n-1} + a_{s+1,n+1}J_{s+1,n+1} + a_{s+2,n}J_{s+2,n} + a_{s+2,n-1}J_{s+2,n-1} + a_{s+2,n+1}J_{s+2,n+1} + a_{s+2,n+2}J_{s+2,n+2} + a_{s+2,n+3}J_{s+2,n+3}]/2$,

where $a_{s-1,n-1} = -\frac{R_{s-1}(a-m_{n-1})(m_{n-1}-\beta)}{4\Delta R\Delta m}$,

$a_{s-1,n} = \frac{R_{s-1}m_n}{2\Delta R} - \frac{R^2_{s-1}\sigma^2}{2\Delta R^2}$,

$a_{s-1,n+1} = \frac{R_{s-1}(a-m_{n+1})(m_{n+1}-\beta)}{4\Delta R\Delta m}$,

$a_{s,n-1} = -\frac{\lambda(m_{n-1}-\beta)}{2\Delta m} - \frac{(a-m_{n-1})^2(m_{n-1}-\beta)^2}{2\sigma^2\Delta m^2}$,

$a_{s,n} = \frac{R^2\sigma^2}{\Delta R^2} + \rho + \frac{(a-m_n)^2(m_n-\beta)^2}{\sigma^2\Delta m^2}$,

$a_{s,n+1} = -\frac{(a-m_{n+1})^2(m_{n+1}-\beta)^2}{2\sigma^2\Delta m^2} + \frac{\lambda(m_{n+1}-\beta)}{2\Delta m}$,

$a_{s+1,n-1} = \frac{R_{s+1}(a-m_{n-1})(m_{n-1}-\beta)}{4\Delta R\Delta m}$,

$a_{s+1,n} = -\frac{R^2_{s+1}\sigma^2}{2\Delta R^2} - \frac{R_{s+1}m_n}{2\Delta R}$.
\[ a_{s+1,n+1} = -\frac{R_{s+1}(\alpha-m_{n+1})(m_{n+1}-\beta)}{4\Delta R \Delta m} \]

The superscript \( i \) on the right-hand side means that all variables within the braces are evaluated at \( t_i \).

(D.3) \( (J_i)_t' = \frac{J_{s,n,i} - J_{s,n,i-1}}{\Delta t} \)

The discretized partial differential equation must also be specified when the growth rate is at its maximum and minimum points. We use BC2 boundary condition on \( m_{\text{min}} = \beta \) and \( m_{\text{max}} = \alpha \). One-sided difference for \( J_{mm} \) (the second derivative of \( J \) with respect to \( m \)) and forward-backward difference for \( J_{Rm} \) (the cross derivative of \( J \) with respect to \( R \) and \( m \)) are used at \( m_{\text{max}} = \alpha \).

(D.4) \[ J_{mm} = \left[ \frac{J_{s,n-2} - J_{s,n-1} + J_{s,n-2}}{\Delta m^2} \right]^i \]
\[ J_{mm} = \left[ \frac{J_{s,n+1} - J_{s,n} + J_{s,n-1}}{\Delta m \Delta R} \right]^i \]

Then in eq. 17,

(D.5) \[ HJ_i = [a_{s-1,n-1}J_{s-1,n-1} + a_{s-1,n-1}J_{s-1,n} + a_{s,n-2}J_{s,n-2} + a_{s,n-1}J_{s,n-1} + a_{s,n}J_{s,n} + \]
\[ as_{s+1}J_{s,n+1} + as_{s+1}J_{s,n+1}]I_i, \]

where \( a_{s-1,n-1} = -\frac{R_{s-1}(\alpha-m_{n-1})(m_{n-1}-\beta)}{\Delta R \Delta m} \),

\[ a_{s-1,n} = \frac{R_{s-1}(\alpha-m_{n})(m_{n}-\beta)}{\Delta R \Delta m} + \frac{R_{s-1}m_{n}}{2\Delta R} - \frac{R_{s-1}^2\sigma^2}{2\Delta R^2}; \]
\[ a_{s,n-2} = -\frac{(\alpha-m_{n-2})^2(m_{n-2}-\beta)^2}{2\sigma^2 \Delta m^2} - \frac{\lambda(m_{n-2}-\beta)}{2\Delta m} \]
\[ a_{s,n-1} = \frac{(\alpha-m_{n-1})^2(m_{n-1}-\beta)^2}{\sigma^2 \Delta m^2} + \frac{R_{s}(\alpha-m_{n-1})(m_{n-1}-\beta)}{\Delta R \Delta m}; \]
\[ a_{s,n} = \frac{R_{s}^2 \sigma^2}{\Delta R^2} + \rho - \frac{(\alpha-m_{n})^2(m_{n}-\beta)^2}{2\sigma^2 \Delta m^2} - \frac{R_{s}(\alpha-m_{n})(m_{n}-\beta)}{\Delta R \Delta m} + \frac{\lambda(m_{n}-\beta)}{2\Delta m}; \]
\[ a_{s+1,n} = -\frac{R_{s+1}^2 \sigma^2}{2\Delta R^2} - \frac{R_{s+1}m_{n}}{2\Delta R}. \]
Similarly, one sided difference for $J_{mm}$ and backward-backward difference for $J_{Rm}$ are used at $m_{\text{min}} = \beta$:

\begin{align}
(D.6) & \quad J_{mm} &= \left[\frac{J_{s+2,n} - 2J_{s,n+1} + J_{s,n}}{\Delta m^2}\right]^i \\
J_{mm} &= \left[\frac{J_{s+1,n} - J_{s-n} - J_{s-1,n+1} + J_{s-1,n}}{\Delta m \Delta R}\right]^i
\end{align}

Then in eq. 17,

\begin{align}
(D.7) & \quad HJ_i &= \left[ a_{s-1,n}J_{s-1,n} + a_{s-1,n+1}J_{s-1,n+1} + a_{s,n}J_{s,n} + a_{s,n+1}J_{s,n+1} + a_{s,n+2}J_{s,n+2} + \right. \\
& \quad \left. a_{s+1,n}/s+1,ni, \right]
\end{align}

where $a_{s-1,n} = \frac{R_{s-1}m_n}{2\Delta R} - \frac{R_{s-1}^2\sigma^2}{2\Delta R^2} - \frac{R_{s-1}(\alpha - m_n)(m_n - \beta)}{\Delta R \Delta m};$

\begin{align*}
a_{s-1,n+1} &= \frac{R_{s-1}(\alpha - m_{n+1})(m_{n+1} - \beta)}{\Delta R \Delta m}; \\
a_{s,n} &= \frac{R_s^2\sigma^2}{\Delta R^2} + \rho + \frac{\lambda (m_n - \beta)}{2\Delta m} - \frac{(\alpha - m_n)^2(m_n - \beta)^2}{2\sigma^2 \Delta m^2} + \frac{R_s(\alpha - m_n)(m_n - \beta)}{\Delta R \Delta m}; \\
a_{s,n+1} &= \frac{R_s(\alpha - m_{n+1})(m_{n+1} - \beta)}{\Delta R \Delta m}; \\
a_{s,n+2} &= -\frac{(\alpha - m_{n+2})^2(m_{n+2} - \beta)^2}{2\sigma^2 \Delta m^2}; \\
a_{s+1,n} &= \frac{R_{s+1}^2\sigma^2}{2\Delta R^2} - \frac{R_{s+1}m_n}{2\Delta R}. 
\end{align*}

Equations D.2, D.3, D.5 and D.7 can form a system equation, which can be written in matrix form and solved by iteration (Insley 2002).