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Splines and seasonal unit roots in weekly agricultural prices

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Abstract

In this paper, a methodological proposal is described to test for seasonal unit roots in weekly series of agricultural prices. When the deterministic seasonal component is not fixed over the sample, the tests for unit roots at seasonal frequencies tend to fail to reject the null hypothesis. This being the case, the original series are proposed to be filtered in order to remove the evolving deterministic seasonal component before applying standard procedures for testing for seasonal unit roots. In such a sense, the non-restricted evolving spline model (ESM) and the restricted evolving spline model (RESM) are shown to be useful parametric formulations to capture this type of deterministic seasonal pattern.

Keywords: agricultural prices, weekly series, unit roots, splines

JEL codes: C22, Q11

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1. Introduction

Weekly series of agricultural prices are increasingly available. In this type of series, seasonal effects do not usually remain the same over time. Therefore, dealing with these seasonal patterns is a complex task. In many papers in the field of agricultural economics, the seasonal effect at a season is assumed to be constant. Then, seasonal effects are removed by seasonal adjustment or modelled by means of seasonal dummies. However, wrong assumptions about the seasonal component may lead to draw wrong conclusions about the dynamic behaviour of the series and the transmission mechanisms between different price series. In particular, non-stationarity due to the presence of unit roots at seasonal frequencies needs to be tested.

To this aim, as Hylleberg (2011) pointed out, “the existence of seasonal unit roots in the data generating process implies a varying seasonal pattern where summer may become winter. In most cases, such a situation is not feasible and the findings of seasonal unit roots should be interpreted with care and taken as an indication of a varying seasonal pattern, where the unit root model is a parsimonious approximation and not the true DGP”. Furthermore, the finite sample distributions of HEGY type tests depend on the deterministic components included in the auxiliary regressions. In this paper, a non-fixed deterministic seasonal pattern is proposed to be modelled by means of evolving splines (Martín-Rodríguez and Cáceres-Hernández, 2012, 2013), able to capture gradual changes in the seasonal pattern as often observed in weekly series of agricultural prices. Once the deterministic seasonal component is removed, HEGY type tests for the null hypotheses of unit root at the seasonal frequencies (Cáceres-Hernández, 1996) are applied to three weekly series of Canary banana prices in Spanish markets.

The methodological proposal is described in the following section. Firstly, the restricted evolving spline model proposed in Martín-Rodríguez and Cáceres-Hernández (2012) is slightly modified in order to adjustments be not required in the estimates of the seasonal effects. Secondly, the specific regressors in the auxiliary regression to seasonal unit root tests are defined. Section 3 shows the results of applying these tests to both the original series of weekly prices of Canary banana from 2005 to 2013 and also the series obtained when the seasonal deterministic component modelled by means of the restricted evolving spline model is removed. Finally, concluding remarks are stated.

2. Evolving splines and test for seasonal unit roots

The following subsection deals with the proposal of a model to capture evolving seasonal patterns in weekly series. Next, in the second part of this methodological section, some procedures are explained for testing the null hypothesis of seasonal unit roots.

2.1. Restricted evolving spline model

In a weekly time series, $\{y_t\}_{t=1,...,T}$, such that

$$y_t = \mu_t + \gamma_t + \varepsilon_t ,\ t = 1,...,T , \tag{1}$$

where μ_t and γ_t are the trend or level component and the seasonal component, respectively, and ε_t is the irregular component, the seasonal pattern can be assumed to be completed in a period whose length does not remain the same over time[1]. Let s_c be the length of seasonal period c, $c = 1,...,m$. Let γ_t be defined as $\gamma_t = \gamma_{c,w}$ if the observation at time t and sub-period c corresponds to season j_c in such a way that $w = \frac{j_c}{s_c}$, $j_c = 1, ..., s_c$. According to Martín-Rodríguez and Cáceres-Hernández (2012), a restricted evolving spline model can be formulated to capture changes in the shape of the seasonal pattern over time.

The seasonal pattern in period c can be modelled by means of a periodic cubic spline. That is

$$\gamma_{c,w} = g_c(w) + \xi_{c,w} , \tag{2}$$

where $\xi_{c,w}$ is a residual term and $g_c(w)$ is a third-degree piecewise polynomial function,

$$g_c(w) = g_{c,i}(w) = g_{c,i,0} + g_{c,i,1}w + g_{c,i,2}w^2 + g_{c,i,3}w^3 ,\ w_{c,i-1} \le w \le w_{c,i} ,\ i = 1,...,k , \tag{3}$$

where $w_{c,0} = 0$ and $w_{c,k} = 1$. It is also assumed that $w_{c,i} = w_i$, $c = 1,...,m$, $i = 0,...,k$. The continuity of the spline function and of its first and second derivatives are enforced by the following conditions

$$g_{c,i}(w_i) = g_{c,i+1}(w_i) ,\ i = 1,...,k-1 ,\ g_{c,1}(w_0) = g_{c,k}(w_k) , \tag{3.a}$$

$$\nabla g_{c,i}(w_i) = \nabla g_{c,i+1}(w_i) ,\ i = 1,...,k-1 ,\ \nabla g_{c,1}(w_0) = \nabla g_{c,k}(w_k) , \tag{3.b}$$

$$\nabla^2 g_{c,i}(w_i) = \nabla^2 g_{c,i+1}(w_i) ,\ i = 1,...,k-1 ,\ \nabla^2 g_{c,1}(w_0) = \nabla^2 g_{c,k}(w_k) . \tag{3.c}$$

Furthermore, by assuming that

$$\sum_{i=1}^{k} \left(g_{c,i,0}\left(w_i - w_{i-1}\right) + g_{c,i,1}\frac{\left(w_i^2 - w_{i-1}^2\right)}{2} + g_{c,i,2}\frac{\left(w_i^3 - w_{i-1}^3\right)}{3} + g_{c,i,3}\frac{\left(w_i^4 - w_{i-1}^4\right)}{4} \right) = 0 , \tag{4}$$

[1] Note that the number of weeks in a year is not exact. Most years have 365 days, but a leap year has 366 days. Therefore, 53 weekly observations are occasionally registered in the same year depending on what is called a week. This being the case, an average of the observations corresponding to two contiguous weeks can be calculated to substitute the original observations in order to the length of the seasonal period remains to be 52. Furthermore, agricultural prices are usually observed for different weeks in different years. See, for example, Martín-Rodríguez and Cáceres-Hernández (2012, 2013).

the area under the spline over the whole seasonal period is restricted to be zero[2]. Then, the spline $g_c(w)$ can be expressed as a linear function

$$g_c(w) = g_{c,2,0} X^g_{c,1,w} + ... + g_{c,k,0} X^g_{c,k-1,w}, \tag{5}$$

where $X^g_{c,i,w}$, $i = 1, ..., k-1$, are appropriate functions of the proportion w and the break points $w_{c,i} = w_i$, $i = 0,...,k$, and $g_{c,2,0},...,g_{c,k,0}$ are free parameters to be estimated. Therefore, the seasonal pattern in the m subperiods in which the series is divided can be jointly modelled as $\gamma_t = g(t) + \xi_t$, where $g(t)$ is the evolving spline

$$g(t) = \sum_{c=1}^{m} \left[g_{c,2,0} X^g_{1,t} + ... + g_{c,k,0} X^g_{k-1,t} \right] D^{sp}_{c,t}, \tag{6}$$

where $D^{sp}_{c,t} = \begin{cases} 1, & t \in sub-period\ c \\ 0, & in\ other\ case \end{cases}$, $c = 1,...,m$.

To identify the changes in the shape of the seasonal pattern over time, the seasonal variation at any proportion w in period c can be also expressed as a function of the values of the seasonal effects at break points in such a period, γ_{c,w_i}, $i = 0,...,k$. In a similar sense to Koopman (1992) and Harvey *et al*. (1997), the spline $g_c(w)$ can be expressed as a linear function

$$g_c(w) = \gamma_{c,w_1} X^\gamma_{c,1,w} + ... + \gamma_{c,w_{k-1}} X^\gamma_{c,k-1,w}, \tag{7}$$

where $X^\gamma_{c,1,w},...,X^\gamma_{c,k-1,w}$ are appropriate functions of the proportion w and the break points $w_{c,i} = w_i$, $i = 0,...,k$, and $\gamma_{c,w_1},...,\gamma_{c,w_{k-1}}$ are free parameters to be estimated. Now, the evolving seasonal pattern over time can be modelled as $\gamma_t = g(t) + \xi_t$, where $g(t)$ is the evolving spline

$$g(t) = \sum_{c=1}^{m} \left[\gamma_{c,w_1} X^\gamma_{1,t} + ... + \gamma_{c,w_{k-1}} X^\gamma_{k-1,t} \right] D^{sp}_{c,t}, \tag{8}$$

where $X^\gamma_{i,t} = X^\gamma_{c,i,w}$, $i = 1,...,k-1$, if the observation at time t and sub-period c corresponds to season j_c in such a way that $w = \frac{j_c}{s_c}$.

[2] Such a condition is not taken into account by Martín-Rodríguez and Cáceres-Hernández (2012). This being the case, when the spline is introduced as a regressor into a time series model, the estimates of seasonal effects, and also the estimates of the level component, need to be corrected. Furthermore, in Martín-Rodríguez and Cáceres-Hernández (2012), the values of spline function, and first and second derivatives at the end of the seasonal period are not assumed to be equal to the corresponding values at the beginning of the seasonal period. In fact, the spline is assumed to be natural, in such a way that $\nabla^2 g_{c,1}(w_0) = 0$ and $\nabla^2 g_{c,k}(w_k) = 0$.

Therefore, the changes in the shape of the seasonal pattern over time can be explained by describing the evolution of the seasonal effects at break points. To this aim, a non-periodic spline is adjusted to the values of each one of these seasonal effects over time, $\{\gamma_{c,w_i}\}_{c=1,\dots,m}$, $i=0,\dots,k$. That is,

$$\gamma_{c,w_i} = g_i(w_c) + \xi_{i,c}, \tag{9}$$

where $\xi_{i,c}$ is a residual term and $g_i(w_c)$ is a third-degree piecewise polynomial function,

$$g_i(w_c) = g_{i,j}(w_c) = g_{i,j,0} + g_{i,j,1}w_c + g_{i,j,2}{w_c}^2 + g_{i,j,3}{w_c}^3,\ w_{c_{j-1}} \le w_c \le w_{c_j},\ j=1,\dots,r, \tag{10}$$

where $w_c = \dfrac{c}{m}$, $w_{c_0}=0$ and $w_{c_r}=1$. By imposing the following conditions

$$g_{i,j}(w_{c_j}) = g_{i,j+1}(w_{c_j}),\ j=1,\dots,r-1, \tag{11.a}$$

$$\nabla g_{i,j}(w_{c_j}) = \nabla g_{i,j+1}(w_{c_j}),\ j=1,\dots,r-1, \tag{11.b}$$

$$\nabla^2 g_{i,j}(w_{c_j}) = \nabla^2 g_{i,j+1}(w_{c_j}),\ j=1,\dots,r-1, \tag{11.c}$$

and also assuming that the spline is natural, that is to say,

$$\nabla^2 g_{i,1}(w_{c_0}) = 0, \tag{12.a}$$

$$\nabla^2 g_{i,r}(w_{c_r}) = 0, \tag{12.a}$$

the estimates $\{\gamma_{c,w_i}\}_{c=1,\dots,m}$, $i=0,\dots,k$, can be expressed as

$$g_i(w_c) = g_{i,1,0}Y^g_{i,1,0,w_c} + g_{i,1,1}Y^g_{i,1,1,w_c} + g_{i,2,0}Y^g_{i,2,0,w_c} + \dots + g_{i,r,0}Y^g_{i,r,w_c}, \tag{13}$$

where $Y^g_{i,1,0,w_c}, Y^g_{i,1,1,w_c}, Y^g_{i,2,0,w_c}, \dots, Y^g_{i,r,w_c}$ are appropriate functions of the proportion w_c and the break points w_{c_j}, $j=0,\dots,r$, and $g_{i,1,0}, g_{i,1,1}, g_{i,2,0}, \dots, g_{i,r,0}$ are free parameters to be estimated. Alternatively, the non-periodic cubic spline $g_i(w_c)$ can be also expressed as a function of the values of the seasonal effect γ_{c,w_i} at specific seasonal periods, γ_{c_j,w_i}, located at the break points w_{c_j}, $j=0,\dots,r$, as follows

$$g_i(w_c) = \gamma_{c_0,w_i}Y^\gamma_{i,c_0,w_c} + \dots + \gamma_{c_r,w_i}Y^\gamma_{i,c_r,w_c}. \tag{14}$$

Finally, from Equation (14), the parameters $\{\gamma_{c,w_i}\}_{c=1,\dots,m}$, $i=1,\dots,k-1$, could also be assumed to evolve over time according to the parametric model $\gamma_{c,w_i} = g_i(w_c) = \gamma_{c_0,w_i}Y^\gamma_{i,c_0,w_c} + \dots + \gamma_{c_r,w_i}Y^\gamma_{i,c_r,w_c}$. Then the evolving spline in Equation (7) can be written as a function of parameters $\{\gamma_{c_0,w_i}, \dots, \gamma_{c_r,w_i}\}_{i=1,\dots,k-1}$, as follows:

$$g(t) = \gamma_{c_0,w_1}U_{1,0,t} + \gamma_{c_1,w_1}U_{1,1,t} + ... + \gamma_{c_r,w_1}U_{1,r,t} + \\ \gamma_{c_0,w_2}U_{2,1,t} + \gamma_{c_1,w_2}U_{2,1,t} + ... + \gamma_{c_r,w_2}U_{2,r,t} + ... + \\ \gamma_{c_0,w_{k-1}}U_{k-1,0,t} + \gamma_{c_1,w_{k-1}}U_{k-1,1,t} + ... + \gamma_{c_r,w_{k-1}}U_{k-1,r,t} \tag{15}$$

where $U_{i,j,t} = \left[\sum_{c=1}^{m} Y_{i,c_j,w_c}^{\gamma} D_{c,t}^{sp} \right] X_{i,t}^{\gamma}$, $i = 1,..., k-1$, $j = 0,..., r$. Therefore,

$$g(t) = \sum_{i=1}^{k-1} \sum_{j=0}^{r} \gamma_{c_j,w_i} U_{i,j,t} \; . \tag{16}$$

Note that γ_{c_j,w_i} is the seasonal variation at proportion w_i of the seasonal period corresponding to the break point located at sub-period c_j. Therefore, the number of parameters to be estimated is equal to $(k-1)*(r+1)$.

This parametric model can be introduced into a time series model to estimate conjointly the seasonal component and the remainder of components in the original series. However, the formulation in Equation (16) is also useful to provide estimates of seasonal effects at any point in time over the sample, and, by subtracting these estimates, a filtered series can be obtained without changing deterministic seasonality. To filter the original series, a previous approximation to the seasonal variations, $\hat{\gamma}_t$, needs to be obtained. Then, from estimating the model

$$\hat{\gamma}_t = g(t) + \xi_t \, , \tag{17}$$

where $g(t)$ is the spline formulated in Equation (16), estimates of seasonal variations are obtained. Finally, values of a filtered series are obtained as follows

$$y_t^* = y_t - \hat{g}(t) \, , \tag{18}$$

where $\hat{g}(t)$ are the estimates of $g(t)$.

2.2. Test for seasonal unit roots

To apply conventional unit root tests at seasonal frequencies, the length of the seasonal period, s_c, is assumed to be 52.

Let x_t be the value of the series y_t once the filter described in section 2.1 is applied. That is to say,

$$x_t = y_t^* \, , \; t = 1,..., T \, , \tag{19}$$

where the data generating process for the series $\{x_t\}_{t=1,...,T}$ is such that

$$\varphi(B) x_t = \mu_t + \varepsilon_t \, , \; t = 1,..., T \, , \tag{20}$$

where $\varphi(B)$ is an autoregressive polynomial, and μ_t represents the deterministic component.

To test for roots corresponding to seasonal frequencies which have a unitary modulus (see Table 1), the procedure described in Cáceres-Hernández (1996), following Hylleberg *et al*. (1990), can be applied. The following auxilliary regression needs to be estimated,

$$\Delta_{52}\left(B\right)x_t = \mu_t + \pi_1 y_{1,t-1} + \pi_2 y_{2,t-1} + \sum_{k=3}^{27}\left[\pi_{k,1}y_{k,t-1} + \pi_{k,2}y_{k,t-2}\right] + \sum_{j=1}^{r}\Delta_{52}\left(B\right)x_{t-j} + \varepsilon_t, \qquad (21)$$

where $\Delta_{52}(B) = 1 - B^{52}$, and regressors $y_{1,t},..., y_{27,t}$ are defined as

$$y_{1,t} = \frac{\Delta_{52}(B)}{1-B}x_t = \left(1+B+B^2+...+B^{51}\right)x_t, \qquad (22.a)$$

$$y_{2,t} = -\frac{\Delta_{52}(B)}{1+B}x_t = -\left(1-B+B^2-...-B^{51}\right)x_t, \qquad (22.b)$$

$$y_{k,t} = -\frac{\Delta_{52}(B)}{1-2\cos(\theta_k B)+B^2}x_t,\ \theta_k = \frac{2(k-2)\pi}{52},\ k = 3,...,27. \qquad (22.c)$$

A number of lags of the dependent variable are included in order to ensure serial uncorrelation in the error term. Then, the hypothesis of unit root at zero frequency is rejected when the null hypothesis $\pi_1 = 0$ is rejected against $\pi_1 < 0$ by means of a t type test t_1. The hypothesis of unit root at Nyquist frequency is rejected when the null hypothesis $\pi_2 = 0$ is rejected against $\pi_2 < 0$ by means of another t type test t_2. As regards the remainder of seasonal frequencies, an F type test F_{k-2} about the significance of parameters $\pi_{k,1}, \pi_{k,2}$, can be applied to test for the presence of a pair of unit roots at seasonal frequency $\theta_k, k = 3,...,27$. Critical values to these tests have been obtained in Cáceres-Hernández (1996) for finite samples when a constant, seasonal dummy variables or a linear trend are included as deterministic components[3].

INSERT TABLE 1

3. Application to weekly agricultural price series

In this section, the tests for zero and seasonal frequencies described in Section 2 are applied to three weekly series of Canary banana prices in Spanish markets from 2005 to 2013: prices perceived by farmers, $\left\{y_t^F\right\}_{t=1,...,468}$, prices at wholesale markets, $\left\{y_t^W\right\}_{t=1,...,468}$, and consumer prices, $\left\{y_t^C\right\}_{t=1,...,468}$ (see Figure 1). HEGY type tests are also applied to the series seasonally adjusted by both, the non-restricted evolving spline model (ESM) in Equation (8) (see Figure 2), and also the restricted evolving spline model (RESM) in Equation (16) (see Figure 3). The dependent variable to estimate

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[3] Asymptotic critical values obtained by Meng and He (2012) to tests for seasonal unit roots in data at any frequency can also be applied to weekly data.

the seasonal component in equations (8) and (16) has been calculated as the difference between the original series and a 52-week moving average series.

INSERT FIGURE 1

INSERT FIGURE 2

INSERT FIGURE 3

The results of autocorrelation tests applied to the residuals of estimating Equation (20) lead to not include lags of the dependent variable into the auxiliary regression. To test for seasonal unit roots in the original series, a linear trend and seasonal dummies are included as deterministic components; whereas only a linear trend is included into the auxiliary regressions for the filtered series. As shown in Table 2.a, tests for seasonal unit roots in the original series fail to reject the null hypothesis of unit roots at several frequencies at the 5% or 10% significance level, whereas tests on the filtered series lead to reject the same null hypothesis at the same confidence level (see Table 2.b). These results have been obtained according to critical values in Tables 2.a and 2.b, although results are very similar when critical values obtained by Cáceres-Hernández (1996) or Meng and He (2012) are taken into account.

INSERT TABLE 2.a

INSERT TABLE 2.b

On the other hand, the HEGY type tests do not have good power properties against deterministic seasonal component alternatives (Ghysels *et al*., 1994). Therefore, when the null hypothesis of unit root is not rejected, stationarity tests at seasonal frequencies should also be applied. In such a sense, the KPSS test (Kwiatwokski *et al*., 1992) has been extended to the seasonal case (Taylor, 2003; Lyhagen, 2006; Khedhiri and Montasser, 2012; Montasser, 2014). According to the procedure described in Khedhiri and Montasser (2012), the statistics $\eta^{(0)}$, $\eta^{(\pi)}$ and $\eta^{(\theta_k)}$, $\theta_k = \frac{2(k-2)\pi}{52}$, $k = 3,...,27$, have been calculated to test for the null hypothesis of stationarity at zero and seasonal frequencies. To make the non-parametric correction of the estimate of the error variance to account for residual serial correlation, the maximum lag length, l, is set to be 3 or 8, following conventional criteria based on the sample size (Newey and West, 1987), and 52, taking the length of the seasonal period into account. A linear trend and seasonal dummies are included as deterministic components into the auxiliary regressions for the original series. In the case of the filtered series, the testing equations only include a linear trend. Although asymptotic distribution of statistical tests could be applied according to the results in Kwiatkowski *et al*. (1992), Khedhiri and Montasser (2012) and Montasser (2014), the decision has been made to obtain critical values by simulation exercises for the effective sample size in the weekly price series (8 years). The results shown in Table 3 correspond to the tests at the frequency π for the series of prices perceived by farmers, at the zero frequency for the series of wholesale prices, and at the frequencies 0, $\frac{5\pi}{26}$ and $\frac{10\pi}{26}$ for the series of consumer prices. Note that the increase on the maximum lag length eventually leads to the failure to reject the null hypothesis. However, when conventional criteria are applied, the null hypothesis of stationarity at the zero frequency is rejected for both the wholesale and the consumer price series. Some doubts remain about the stationarity at the frequency $\frac{5\pi}{26}$ for the retailing prices. On the other hand, whatever the maximum lag length, the null hypothesis of stationarity is clearly rejected at both the frequency π for the series of prices perceived by farmers and the frequency $\frac{10\pi}{26}$ for the series of consumer prices.

INSERT TABLE 3

4. Conclusions

In the standard HEGY type tests for seasonal unit roots, seasonal variations under the alternative hypothesis are assumed to be stationary around a fixed deterministic seasonal component modelled by seasonal dummies. However, weekly series of agricultural prices usually exhibit an evolving seasonal pattern. The results obtained in this paper show that this type of seasonal pattern tends to make HEGY type test wrongly fail to reject the null hypothesis of unit root at seasonal frequencies. Furthermore, parametric formulations such as ESM or RESM are capable of capturing the changes in the deterministic component of seasonal variations and, when the original series are filtered to delete such a deterministic seasonality, the HEGY tests can lead to reject a unit root hypothesis which is not rejected for the original series.

A note of caution concerning these results should be made. On the one hand, the sample size is small, although the critical values have been obtained by simulation exercises in which the effective sample size is the same as the one for estimating auxiliary regressions. On the other hand, the asymptotic distribution of HEGY type tests should be obtained when an evolving deterministic seasonal component is included in the auxiliary regressions. From estimating these auxiliary regressions, simulation exercises need to be done to obtain critical values for different sample sizes.

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Table 1. Unit roots in weekly data

Unit roots	Frequency	Period (weeks)	Cycles per year
$r_j=\cos(\theta_j)\pm i\mathrm{sen}(\theta_j)$	θ_j	$2\pi/\theta_j$	$52\theta_j/2\pi$
$r_1=1$	0	-	-
$r_{3,1}=\cos(\pi/26)+i\mathrm{sen}(\pi/26)$; $r_{3,2}=\cos(\pi/26)-i\mathrm{sen}(\pi/26)$	$\pi/26$	52	1
$r_{4,1}=\cos(\pi/13)+i\mathrm{sen}(\pi/13)$; $r_{4,2}=\cos(\pi/13)-i\mathrm{sen}(\pi/13)$	$2\pi/26$	26	2
$r_{5,1}=\cos(3\pi/26)+i\mathrm{sen}(3\pi/26)$; $r_{5,2}=\cos(3\pi/26)-i\mathrm{sen}(3\pi/26)$	$3\pi/26$	52/3	3
$r_{6,1}=\cos(2\pi/13)+i\mathrm{sen}(2\pi/13)$; $r_{6,2}=\cos(2\pi/13)-i\mathrm{sen}(2\pi/13)$	$4\pi/26$	13	4
$r_{7,1}=\cos(5\pi/26)+i\mathrm{sen}(5\pi/26)$; $r_{7,2}=\cos(5\pi/26)-i\mathrm{sen}(5\pi/26)$	$5\pi/26$	52/5	5
$r_{8,1}=\cos(3\pi/13)+i\mathrm{sen}(3\pi/13)$; $r_{8,2}=\cos(3\pi/13)-i\mathrm{sen}(3\pi/13)$	$6\pi/26$	26/3	6
$r_{9,1}=\cos(7\pi/26)+i\mathrm{sen}(7\pi/26)$; $r_{9,2}=\cos(7\pi/26)-i\mathrm{sen}(7\pi/26)$	$7\pi/26$	52/7	7
$r_{10,1}=\cos(4\pi/13)+i\mathrm{sen}(4\pi/13)$; $r_{10,2}=\cos(4\pi/13)-i\mathrm{sen}(4\pi/13)$	$8\pi/26$	26/4	8
$r_{11,1}=\cos(9\pi/26)+i\mathrm{sen}(9\pi/26)$; $r_{11,2}=\cos(9\pi/26)-i\mathrm{sen}(9\pi/26)$	$9\pi/26$	52/9	9
$r_{12,1}=\cos(5\pi/13)+i\mathrm{sen}(5\pi/13)$; $r_{12,2}=\cos(5\pi/13)-i\mathrm{sen}(5\pi/13)$	$10\pi/26$	26/5	10
$r_{13,1}=\cos(11\pi/26)+i\mathrm{sen}(11\pi/26)$; $r_{13,2}=\cos(11\pi/26)-i\mathrm{sen}(11\pi/26)$	$11\pi/26$	52/11	11
$r_{14,1}=\cos(6\pi/13)+i\mathrm{sen}(6\pi/13)$; $r_{14,2}=\cos(6\pi/13)-i\mathrm{sen}(6\pi/13)$	$12\pi/26$	26/6	12
$r_{15,1}=i$; $r_{15,2}=-i$	$13\pi/26$	4	13
$r_{16,1}=\cos(7\pi/13)+i\mathrm{sen}(7\pi/13)$; $r_{16,2}=\cos(7\pi/13)-i\mathrm{sen}(7\pi/13)$	$14\pi/26$	26/7	14
$r_{17,1}=\cos(15\pi/26)+i\mathrm{sen}(15\pi/26)$; $r_{17,2}=\cos(15\pi/26)-i\mathrm{sen}(15\pi/26)$	$15\pi/26$	52/15	15
$r_{18,1}=\cos(8\pi/13)+i\mathrm{sen}(8\pi/13)$; $r_{18,2}=\cos(8\pi/13)-i\mathrm{sen}(8\pi/13)$	$16\pi/26$	26/8	16
$r_{19,1}=\cos(17\pi/26)+i\mathrm{sen}(17\pi/26)$; $r_{19,2}=\cos(17\pi/26)-i\mathrm{sen}(17\pi/26)$	$17\pi/26$	52/17	17
$r_{20,1}=\cos(9\pi/13)+i\mathrm{sen}(9\pi/13)$; $r_{20,2}=\cos(9\pi/13)-i\mathrm{sen}(9\pi/13)$	$18\pi/26$	26/9	18
$r_{21,1}=\cos(19\pi/26)+i\mathrm{sen}(19\pi/26)$; $r_{21,2}=\cos(19\pi/26)-i\mathrm{sen}(19\pi/26)$	$19\pi/26$	52/19	19
$r_{22,1}=\cos(10\pi/13)+i\mathrm{sen}(10\pi/13)$; $r_{22,2}=\cos(10\pi/13)-i\mathrm{sen}(10\pi/13)$	$20\pi/26$	26/10	20
$r_{23,1}=\cos(21\pi/26)+i\mathrm{sen}(21\pi/26)$; $r_{23,2}=\cos(21\pi/26)-i\mathrm{sen}(21\pi/26)$	$21\pi/26$	52/21	21
$r_{24,1}=\cos(11\pi/13)+i\mathrm{sen}(11\pi/13)$; $r_{24,2}=\cos(11\pi/13)-i\mathrm{sen}(11\pi/13)$	$22\pi/26$	26/11	22
$r_{25,1}=\cos(23\pi/26)+i\mathrm{sen}(23\pi/26)$; $r_{25,2}=\cos(23\pi/26)-i\mathrm{sen}(23\pi/26)$	$23\pi/26$	52/23	23
$r_{26,1}=\cos(12\pi/13)+i\mathrm{sen}(12\pi/13)$; $r_{26,2}=\cos(12\pi/13)-i\mathrm{sen}(12\pi/13)$	$24\pi/26$	26/12	24
$r_{27,1}=\cos(25\pi/26)+i\mathrm{sen}(25\pi/26)$; $r_{27,2}=\cos(25\pi/26)-i\mathrm{sen}(25\pi/26)$	$25\pi/26$	52/25	25
$r_2=-1$	π	2	26

Table 2.a. Tests for unit roots at frequency θ in the original series

θ	y_t^F	y_t^W	y_t^C	Critical values[1]	
				5%	10%
0	-3.480	-3.089	-2.510	-3.057	-2.792
π	-2.473	-3.012	-2.759	-2.571	-2.291
				90%	95%
π/26	7.491	4.759	7.690	4.483	5.327
2π/26	7.983	4.106	4.859	4.481	5.382
3π/26	3.487	4.298	3.600	4.418	5.260
4π/26	6.493	12.741	6.863	4.489	5.282
5π/26	4.459	5.644	4.671	4.421	5.261
6π/26	5.643	5.247	8.905	4.509	5.349
7π/26	6.921	3.498	8.733	4.477	5.337
8π/26	5.177	2.887	11.653	4.448	5.263
9π/26	9.498	3.508	9.949	4.463	5.341
10π/26	3.247	6.521	3.272	4.444	5.277
11π/26	7.452	6.311	4.857	4.486	5.305
12π/26	8.184	6.084	9.765	4.445	5.311
13π/26	8.829	5.204	8.147	4.468	5.281
14π/26	4.392	5.937	7.352	4.469	5.345
15π/26	6.793	5.705	6.504	4.408	5.267
16π/26	8.903	5.899	6.561	4.452	5.296
17π/26	6.319	6.348	4.540	4.444	5.321
18π/26	7.565	7.002	5.258	4.448	5.301
19π/26	7.077	4.928	14.190	4.421	5.247
20π/26	4.984	8.217	2.785	4.438	5.303
21π/26	8.664	6.964	4.063	4.424	5.262
22π/26	8.778	5.941	7.809	4.410	5.244
23π/26	8.405	7.255	3.454	4.477	5.291
24π/26	8.268	5.667	5.730	4.441	5.284
25π/26	10.228	4.357	4.129	4.448	5.292

[1] Critical values have been obtained by Monte Carlo simulation experiments. The data generating process is a seasonal random walk where the disturbance term has unit variance. Twenty thousand replications were conducted. Testing equations include a constant, seasonal dummies and a trend. The effective sample size to estimate auxiliary regressions was 416 (8 years of weekly data).

Table 2.b. Tests for unit roots at frequency θ in the filtered series

θ	$y_t^{F,ESM}$	$y_t^{W,ESM}$	$y_t^{C,ESM}$	$y_t^{F,RESM}$	$y_t^{W,RESM}$	$y_t^{C,RESM}$	Critical values[1]	
							5%	10%
0	-3.316	-2.527	-3.104	-4.880	-3.979	-3.886	-3.202	-2.927
π	-1.700	-3.400	-2.678	-1.732	-3.525	-2.632	-1.803	-1.500
							90%	95%
π/26	15.199	10.425	11.128	20.507	15.904	18.247	2.245	2.917
2π/26	8.103	10.454	10.685	22.881	20.862	15.231	2.173	2.832
3π/26	5.312	4.750	4.701	9.836	10.632	6.596	2.198	2.809
4π/26	4.942	3.603	6.495	6.899	4.989	6.186	2.159	2.797
5π/26	5.800	4.727	2.133	6.316	6.119	2.490	2.131	2.749
6π/26	5.773	6.076	9.894	6.501	7.229	9.381	2.171	2.793
7π/26	7.196	5.951	7.579	7.307	6.125	7.712	2.166	2.814
8π/26	5.996	4.954	11.212	5.999	4.898	11.172	2.170	2.801
9π/26	8.443	5.078	8.809	8.451	5.117	8.389	2.181	2.799
10π/26	3.497	7.163	2.716	3.437	7.462	2.790	2.186	2.833
11π/26	8.870	4.671	6.555	9.984	5.069	6.097	2.154	2.789
12π/26	9.046	5.919	12.451	9.714	5.722	11.902	2.135	2.794
13π/26	10.600	8.673	11.452	10.754	8.756	10.739	2.162	2.771
14π/26	5.682	5.302	8.586	5.680	4.942	7.978	2.134	2.771
15π/26	9.718	6.654	6.584	10.024	6.327	6.578	2.158	2.842
16π/26	10.725	8.258	8.515	10.742	8.819	8.101	2.189	2.814
17π/26	6.856	6.795	4.500	7.351	7.298	3.901	2.119	2.704
18π/26	9.064	7.263	8.959	9.274	8.025	9.036	2.194	2.865
19π/26	11.945	7.439	16.572	12.126	7.524	16.535	2.198	2.838
20π/26	6.496	9.076	5.754	6.919	9.220	5.492	2.166	2.772
21π/26	5.482	12.043	4.615	5.635	11.843	4.423	2.163	2.766
22π/26	9.924	10.104	6.663	10.272	10.247	6.870	2.196	2.835
23π/26	11.769	11.608	5.072	11.764	10.984	4.891	2.135	2.775
24π/26	6.046	4.994	5.974	6.289	5.279	6.211	2.161	2.848
25π/26	8.929	5.299	4.534	9.178	5.442	5.271	2.186	2.840

(1) Critical values have been obtained by Monte Carlo simulation experiments. The data generating process is a seasonal random walk where the disturbance term has unit variance. Twenty thousand replications were conducted. Testing equations include a constant and a trend. The effective sample size to estimate auxiliary regressions was 416 (8 years of weekly data).

Table 3. Tests for stationarity at frequency θ in weekly price series

θ	y_t^F			Critical values (1)		$y_t^{F,ESM}$			$y_t^{F,RESM}$			Critical values (2)	
l	3	8	52	90%	95%	3	8	52	3	8	52	90%	95%
π	1.145	0.534	0.317	0.395	0.519	26.69	12.05	6.18	26.80	12.11	6.21	1.198	1.655
θ	y_t^W			Critical values (1)		$y_t^{W,ESM}$			$y_t^{W,RESM}$			Critical values (2)	
l	3	8	52	90%	95%	3	8	52	3	8	52	90%	95%
0	0.365	0.170	0.066	0.135	0.169	0.816	0.371	0.102	0.707	0.324	0.101	0.121	0.149
θ	y_t^C			Critical values (1)		$y_t^{C,ESM}$			$y_t^{C,RESM}$			Critical values (2)	
l	3	8	52	90%	95%	3	8	52	3	8	52	90%	95%
0	0.345	0.158	0.052	0.135	0.169	0.495	0.226	0.067	0.481	0.219	0.066	0.121	0.149
5π/26	1.257	1.414	0.481	0.337	0.418	1.405	1.166	0.329	1.284	1.134	0.331	1.029	1.289
10π/26	1.543	1.019	0.379	0.344	0.422	9.261	4.986	1.511	8.879	4.853	1.488	1.043	1.314

(1) Critical values have been obtained by Monte Carlo simulation experiments. The data generating process was a white noise with unit variance. Twenty thousand replications were conducted. Testing equations include a constant, seasonal dummies and a trend. The effective sample size to estimate auxiliary regressions was 416 (8 years of weekly data).

(2) Critical values have been obtained by Monte Carlo simulation experiments. The data generating process was a white noise with unit variance. Twenty thousand replications were conducted. Testing equations include a constant and a trend. The effective sample size to estimate auxiliary regressions was 416 (8 years of weekly data).

Figure 1. Weekly series of Canary banana prices in Spanish markets, y_t^F, y_t^W, y_t^C.

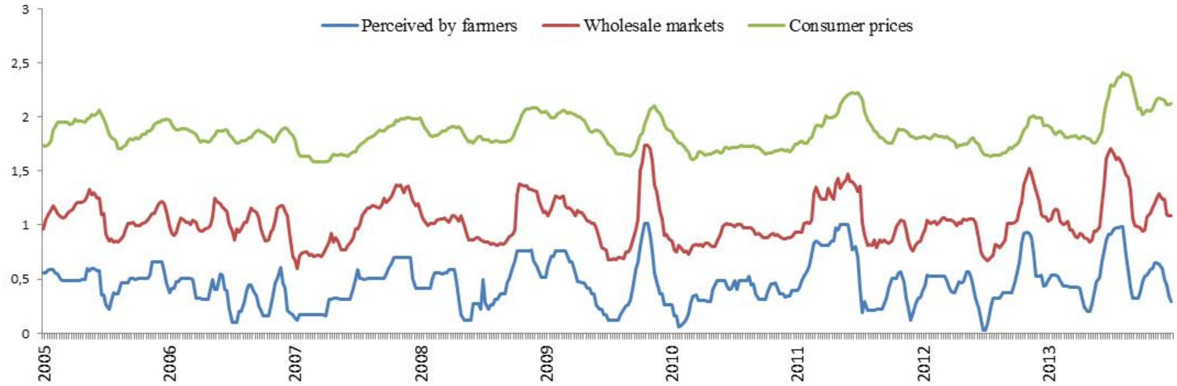

Figure 2. Weekly price series filtered by ESM (k=6), $y_t^{F,ESM}$, $y_t^{W,ESM}$, $y_t^{C,ESM}$.

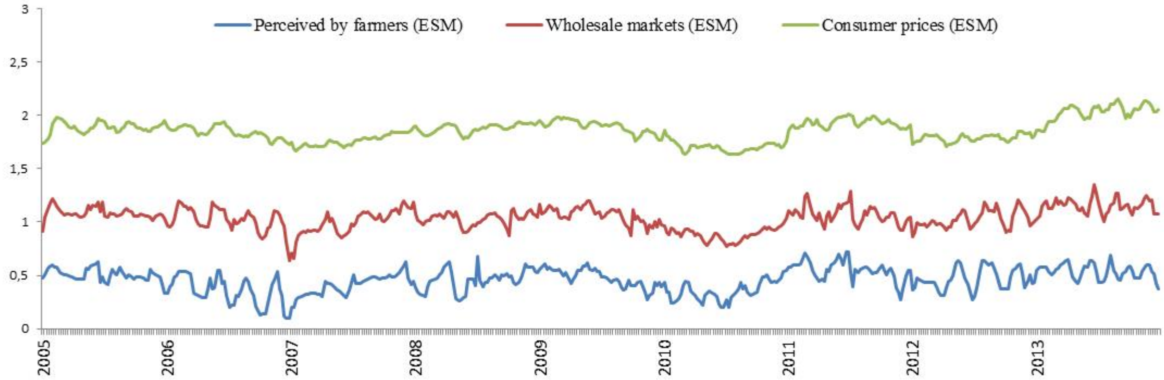

Figure 3. Weekly price series filtered by RESM (k=6, r=6), $y_t^{F,RESM}$, $y_t^{W,RESM}$, $y_t^{C,RESM}$.