A New Approach to Investigate Market Integration: a Markov-Switching Autoregressive Model with Time-Varying Transition Probabilities

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Selected Paper prepared for presentation at the Agricultural & Applied Economics Association's 2012 AAEA Annual Meeting, Seattle, Washington, August 12-14, 2012

Abstract

In this study, we develop a new approach to investigate spatial market integration. In particular, it is a Markov-Switching autoregressive (MSAR) model with time-varying state transition probabilities. Studying market integration is an effective way to test whether the law of one price holds across geographically separated markets, in other words, to test whether these markets perform efficiently or not. In this model, we assume that the parameters depend on a state variable which describes two unobservable states of markets – non-arbitrage and arbitrage – and is governed by a time-varying transition probability matrix. The main advantage of this model is that it allows transition probabilities to be time-varying. The probability of being in one state at time t depends on the previous state and the previous levels of market prices. An EM (Expectation-Maximization) algorithm is applied in the estimation of this model. For the empirical application, we examine market integration among four regional corn (Statesville, Candor, Cofield, Roaring River) and three regional soybean markets (Fayetteville, Cofield, and Creswell) in North Carolina. The prices of these markets are quoted daily from 3/1/2005 to 6/30/2010. Six pairwise spatial price relationships for the corn markets, and three pairwise spatial price relationships for the soybean markets are examined. Our results demonstrate that significant regime switching relationships characterize these markets. This has important implications for more conventional models of spatial price relationships and market integration. Our results are consistent with efficient arbitrage subject to transactions costs.

1. Introduction

Market integration has been widely discussed and evaluated by studying the mechanism of price transmissions among interrelated markets. Studies that investigate market integration focus either on spatially separated markets or on vertically related markets. Markets with related goods are said to be integrated if prices from these markets move proportionally or follow similar patterns in the long run. Examining the integration of markets has profound impacts for market participants and researchers. A typical example concerns the spatial speculators. They make their market decisions by comparing the prices of the same good among different markets. In particular, spatial speculators can make profits if the price difference between two markets is higher than the transactions costs of delivering the good from the market with a lower price to the market with a higher price. In this case, these markets are considered as not being integrated, and non-integration is the main reason for spatial speculation or arbitrage. On the other hand, studying market integration is an effective way to test whether the law of one price (LOP) holds across geographically separated markets, in other words, to test whether these markets perform efficiently or not.

Early research on market integration mainly focused on the static correlation between prices from spatially separated markets. Spatial arbitrage exists only when the price difference is large enough to cover the transactions costs. The profits from arbitrage, however, will gradually fall to zero since more and more traders are getting involved. When arbitrage becomes unprofitable, price linkages between the two markets will gradually switch to a different pattern, and co-movements of prices will not be as easy to observe. By this argument, static correlation is not a valid way to investigate market integration. In many studies, static correlation was proved to be insufficient and was extended in multiple directions. A group of regime switching models became widely accepted in recent studies due to plenty of advantages. For example, it can separate the analyses into different situations (e.g., arbitrage and non-arbitrage) and it also takes into account the unobservable transactions costs.

To improve the performance of the regime switching models for testing for market integration, we propose a Markov-Switching error correction model with time-varying transition probabilities. The basic idea of this model is that, the model contains two regimes – arbitrage and non-arbitrage, and the switching between regimes is governed by a Markov chain. For example, the probability that the next period is in one regime depends on the current regime and current market prices. The main advantage of this model is its flexibility in the transition probabilities which are changing over time. Specifically, the transition probabilities at time t depend on the price levels at time t-1. Since the state (regime) variable is unobservable, an EM (Expectation-Maximization) algorithm is applied for the model estimation.

2. Previous Research

This section briefly presents innovations in the development of testing for market integration in agricultural economics and introduces some of the typical models in this area over the past half century.

In the context of spatial market integration, the first general approach to investigate spatial competitive equilibrium is developed by Takayama and Judge (1964). They reformulated the problem as a quadratic programming problem based on the previous work of Enke (1951) and Samulson (1952). In the following two decades, this approach was extended to a variety of dimensions of empirical work, for example, to compute optimal spatial locations, to examine the spatial boundary between markets, and to test for spatial market efficiency.

As the subsequent development of the Takayama and Judge’s point-location model, testing for market integration, or market efficiency, began to attract researchers’ attention since the 1980s. Early studies examined market integration by studying the correlation between prices from spatially separated markets. Ravallion (1986) proposed a stronger test procedure which he argued avoids inferential dangers from methods using static price correlation. Before that, static price correlations remained the most common measure of spatial market integration. This new method provides a dynamic relationship for market prices from different regions, considering both long-run integration and short-run integration. The subsequent research mainly focused on cointegration, error correction, and Granger causality frameworks (Fackler and Goodwin 2001).

The first application of the regime switching model for testing market integration was introduced by Sexton, Kling, and Carmen (1991). They extended Spiller and Huang's (1986) method of testing for market integration, and tested for three different regimes they defined – efficient arbitrage, relative shortage (the price difference is less than transactions costs), and relative glut (the price difference is greater than transactions costs). Regime switching models then became popular in this area since it allows for different price transmission behaviors under different market conditions such as arbitrage and non-arbitrage.

In the development of methodology for testing for market integration, very little work has considered transactions costs (or delivery costs). Although transactions costs play an important role in the analysis, these costs are difficult to observe or to correctly estimate using correlated variables. Therefore, most of the existing research only applied market price data to deal with market integration. The ignorance of transactions costs or treating them as constant, however, would cause some estimation bias such as to reject market integration even when no spatial arbitrage exists (Baulch, 1997). To improve the reliability of the test, Baulch (1997) proposed a parity bounds model (PBM) which takes transactions costs and trade flows into account. The PBM including transactions costs or variables highly correlated with transactions costs was extended to a variety of directions, but it was still criticized by some researchers about its limitations. Moreover, the difficulty in collecting data of transactions costs or correctly predicting them remained a problem in this area and led to the ignorance of transactions costs in the subsequent research. Most researchers continued to test for market integration only with price data.

The limitation of ignoring transactions costs and the nonstationary nature of price data led to the application of new empirical models with nonlinear techniques. A generally accepted method is the threshold error correction model which is first applied by Goodwin and Piggott (2001) to test for spatial market integration for corn and soybean markets in North Carolina. This model allows price transmission to be regime switching, and it is considered as a more appropriate way to deal with unobservable transactions costs which could be nonstationary.

More recent research refers to the Markov-switching vector error correction (MSVEC) model proposed by Brummer, von Cramon-Taubadel, and Zorya (2008), which studies the vertical price transmission between wheat and wheat flour in Ukraine. The application of the MSVEC model is motivated by the unstable policy environments in Ukraine. The subsequent work refers to the application of the Markov-switching vector autoregressive model (Ihle, von Cramon-Taubadel, and

Zorya, 2009) in agricultural economics. Both of these models are estimated under a constant transition probabilities framework.

3. Methodology

3.1. The Model

This section provides a new method, a Markov-Switching autoregressive (MSAR) model with time-varying transition probabilities, to investigate spatial market integration. An MSAR model is a subclass of the threshold (vector) autoregressive models. A specification of the threshold autoregressive model commonly used by the most recent literature is given by

$$\Delta y_t = \alpha(s_t) + \beta(s_t) y_{t-1} + \sum_{j=1}^{p-1} \phi_j(s_t) \Delta y_{t-j} + \varepsilon_t \,, \quad (3.1)$$

$$\varepsilon_t | \Psi_{t-1} \sim NID\big(0, \sigma(s_t)\big) \,,$$

where $y_t = \ln\left(p_t^a / p_t^b\right)$, p_t^a and p_t^b are the cash prices for a homogenous commodity at location a and b at time t, Ψ_{t-1} is the information set at $t-1$, $\alpha(s_t)$, $\beta(s_t)$, $\phi_1(s_t)$, … , $\phi_{p-1}(s_t)$, and $\sigma(s_t)$ are parameters which depend on the state variable s_t. $\beta(s_t)$ also represents the degree of "error-correction" that characterizes the departure from price parity. Assume that the per-unit revenue for spatial speculators transporting from location a to b is $(1-\kappa_{ab})p^b$, where κ_{ab} is the rate of transactions costs from location a to location b, and $0 < \kappa_{ab} < 1$. Therefore, the non-arbitrage conditions for location a and b are

$$(1-\kappa_{ab})p^b \le p^a \text{ and } (1-\kappa_{ba})p^a \le p^b,$$

or they can be rewritten as

$$(1-\kappa_{ab}) \le p^a/p^b \le 1/(1-\kappa_{ba}) \,.$$

After taking natural logarithms, the non-arbitrage condition is given by

$$\ln(1-\kappa_{ab}) \le y \le -\ln(1-\kappa_{ba}) \,. \quad (3.2)$$

From a number of studies, y behaves quite differently from the non-arbitrage case to the arbitrage case, and this property can be captured by threshold (vector) autoregressive or threshold (vector) error correction models (e.g., Goodwin and Piggott, 2001). It is generally believed that y follows something close to a unit root under the non-arbitrage condition, and this ensures a threshold error correction model to be appropriate to investigate market integration. In our study, we assume that the state variable s_t contains two unobservable states – non-arbitrage $(s_t = 1)$ and arbitrage ($s_t = 2$), and it is governed by the time-varying transition probability matrix Π_t:

$$\Pi_t = \begin{bmatrix} \pi_{11}(y_{t-1}) & \pi_{21}(y_{t-1}) \\ \pi_{12}(y_{t-1}) & \pi_{22}(y_{t-1}) \end{bmatrix}$$

where π_{ij} is the probability of switching from state i at time $t-1$ to state j at time t giving the level of y_{t-1}. Or, $\pi_{ij} = P(S_t = j|S_t = j, y_{t-1})$, and $\sum_{j=1}^{2} \pi_{ij} = 1$, for $i = 1,2$.

In this study we apply two types of probability functions for $\pi_{ii}(y_{t-1})$. The first one is a logistic function symmetric around the mean of y_t (or $\bar{y}$):

$$\pi_{ii}(y_{t-1}|\gamma_{ii}, c_{ii}) = \frac{1}{1 + \exp\{\gamma_{ii}(|y_{t-1} - \bar{y}|) - c_{ii}\}}, \quad i = 1,2 \quad (3.3).$$

We assume that $\gamma_{11} > 0$, and $\gamma_{22} < 0$. The maximum or minimum transition probability depends on both γ_{ii} and c_{ii}.

The second type of the probability function is a second order logistic function:

$$\pi_{ii}\left(y_{t-1}|c_{0,ii}, c_{1,ii}, c_{2,ii}\right) = \frac{1}{1 + \exp\left\{c_{0,ii}y_{t-1}^2 + c_{1,ii}y_{t-1} + c_{2,ii}\right\}}, i = 1,2 \quad (3.4).$$

Similarly, we assume that $c_{0,11} > 0$, and $c_{0,22} < 0$. For both π_{ii}'s, $\pi_{ij} = 1 - \pi_{ii}$.

Figure 1 shows the plots of equation (3.3) and (3.4) with different values of parameters. These plots imply that, when the previous state is non-arbitrage, the probability of shifting to an arbitrage state at the current period would be relatively low at extreme values of y_{t-1}. In other words, the probability of staying in the same state, non-arbitrage, is relatively high when the value of y_{t-1} is close to the mean of y_t in the case of equation (3.3). Similarly, when the previous state is arbitrage and the previous level of price deviation (y_{t-1}) is extremely high, then the probability of being in the same state at time t is relatively high, compared to the y_{t-1}'s that are close to $\bar{y}$ in the case of equation (3.3).

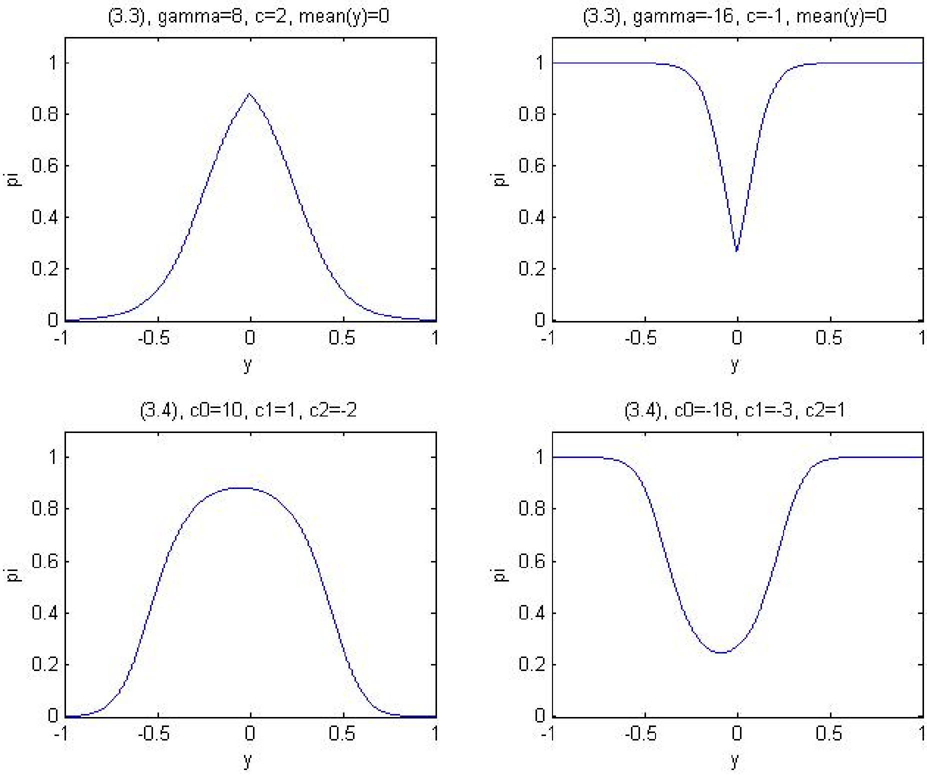

Figure 1. Examples of Equation (3.3) and Equation (3.4).

3.2. Model Estimation: The EM Algorithm

The estimation of the MSECM with time-varying transition probabilities can be done by applying the Expectation-Maximization (EM) algorithm. This algorithm was first developed by Hamilton (1990) to solve for Markov-Switching models with constant transition probabilities. Diebold, Lee, and Weinbach (1994) extended it to a time-varying transition probabilities framework. The main challenge for the estimation of this models is that, first, the state process $\{s_t\}$ is unobservable and depends on model parameters ($\alpha(s_t)$, $\beta(s_t)$, $\phi_1(s_t)$, … , $\phi_{p-1}(s_t)$, $\sigma(s_t)$); second, the model parameters also depend on the state process $\{s_t\}$. The EM algorithm has been considered as an effective way to deal with this two-way dependence problem so far.

In the first step of the EM algorithm, the expectation step, we initiate some starting values for the model parameters. Then, probabilities of being in each regime conditional on data up to t-1 are filtered by a particular filter (e.g., Hamilton filter), in order to obtain the filtered probabilities conditional on the data up to t. After filtering, the smoothed probabilities are obtained based on the filtered probabilities. The second step of the EM algorithm, the maximization step, computes the maximum likelihood estimates of the parameters using the smoothed probabilities. These two steps are iterated until the convergence criterion is achieved. Section 3.2.1 and 3.2.2 provide the details of this algorithm for the estimation of this model.

3.2.1. The complete-data log-likelihood function

In equation (3.1), we assume that $\varepsilon_t|\Psi_{t-1}\sim NID\big(0,\sigma(s_t)\big)$, which can be rewritten as

$$\Delta y_t-\alpha(s_t)-\beta(s_t)y_{t-1}-\sum_{j=1}^{p-1}\phi_j(s_t)\Delta y_{t-j}\;|\Psi_{t-1}\sim NID\big(0,\sigma(s_t)\big)$$

for $t\geq p+1$. Therefore, the conditional density function for y_t $(t\geq p+1)$ is

$$\left(y_t\middle|s_t=i,\underline{y_{t-1}};\alpha_i,\beta_i,\phi_{1,i},\phi_{2,i},\dots,\phi_{p-1,i},\sigma_i\right)$$
$$=\frac{1}{\sqrt{2\pi\sigma_i^2}}\exp\left\{-\frac{\left[y_t-\alpha_i-(1+\beta_i)y_{t-1}-\sum_{j=1}^{p-1}\phi_{j,i}\Delta y_{t-j}\right]^2}{2\sigma_i^2}\right\}$$

where $i=1,2$ indicates State 1, non-arbitrage, and State 2, arbitrage.

Let $\alpha=\left[\alpha_1,\beta_1,\phi_{1,1},\dots,\phi_{p-1,1},\sigma_1,\alpha_2,\beta_2,\phi_{1,2},\dots,\phi_{p-1,2},\sigma_2\right]'$, the parameters in equation (3.1), $\beta=[\gamma_{11},c_{11},\gamma_{22},c_{22}]'$ (or $\beta=\left[c_{0,11},c_{1,11},c_{2,11},c_{0,22},c_{1,22},c_{2,22}\right]'$ in the case of second order logistic transition probability function), the parameters in equation (3.3). We also need a probability for the beginning state (s_{p+1}), so we define $\rho=P\big(S_{p+1}=1\big)$. Therefore, the vector of all parameters in our model is

$$\theta=[\alpha',\beta',\rho]',$$

a $(2p+9)$-dimention vector in the case of equation (3.3).

The complete-data likelihood function (from $t=p+1$ to T) is given by

$$f\left(\underline{y_T}, \underline{s_T}\middle|\underline{x_p};\theta\right) = f\left(y_{p+1}, s_{p+1}\middle|\underline{x_p};\theta\right)\prod_{t=p+2}^{T} f\left(y_t, s_t\middle|\underline{y_{t-1}}, \underline{s_{t-1}}, \underline{x_p};\theta\right)$$

$$= f\left(y_{p+1}\middle|s_{p+1}, \underline{x_p};\theta\right) P(s_{p+1})\prod_{t=p+2}^{T}\left\{f\left(y_t\middle|s_t, \underline{y_{t-1}}, \underline{s_{t-1}}, \underline{x_p};\theta\right)\times\right.$$

$$\left.P\left(S_{p+1} = s_{p+1}\middle|\underline{y_{t-1}}, \underline{s_{t-1}}, \underline{x_p};\theta\right)\right\},$$

where $\underline{y_T} = [y_T, y_{T-1}, \dots, y_{p+1}]$, $\underline{s_T} = [s_T, s_{T-1}, \dots, s_{p+1}]$, and

$\underline{x_p} = [y_p, y_{p-1}, \dots, y_1]$. So, the log-likelihood function for the complete-data is

$$\log f\left(\underline{y_T}, \underline{s_T}\middle|\underline{x_p};\theta\right) = \log f\left(y_{p+1}\middle|s_{p+1}, \underline{x_p};\theta\right) + \log P(s_{p+1})$$

$$+\sum_{t=p+2}^{T}\left\{\log f\left(y_t\middle|s_t, \underline{y_{t-1}}, \underline{s_{t-1}}, \underline{x_p};\theta\right) + \log P\left(S_{p+1} = s_{p+1}\middle|\underline{y_{t-1}}, \underline{s_{t-1}}, \underline{x_p};\theta\right)\right\}.$$

For convenience, we will use the complete-data log-likelihood function with indicator functions in the estimation, which is given by

$$\log f\left(\underline{y_T}, \underline{s_T}\middle|\underline{x_p};\theta\right) = I(S_{p+1}=1)\left[\log f\left(y_{p+1}\middle|S_{p+1}=1, \underline{x_p};\theta\right) + \log\rho\right]$$

$$+I(S_{p+1}=2)\left[\log f\left(y_{p+1}\middle|S_{p+1}=2, \underline{x_p};\theta\right) + \log(1-\rho)\right]$$

$$+\sum_{t=p+2}^{T}\left\{I(S_t=1)\log f\left(y_t\middle|S_t=1, \underline{y_{t-1}}, \underline{x_p};\theta\right)\right.$$

$$+I(S_t=2)\log f\left(y_t\middle|S_t=2, \underline{y_{t-1}}, \underline{x_p};\theta\right)$$

$$+I(S_t=1, S_{t-1}=1)\log(\pi_{t,11}) + I(S_t=2, S_{t-1}=1)\log(1-\pi_{t,11})$$

$$\left.+I(S_t=1, S_{t-1}=2)\log(1-\pi_{t,22}) + I(S_t=2, S_{t-1}=2)\log(\pi_{t,22})\right\}$$

(3.7),

where $\pi_{t,11}$, and $\pi_{t,22}$ are transition probabilities calculated from equation (3.3) or (3.4).

3.2.2. The EM algorithm

The complete-data log-likelihood function cannot be used for estimation because the state variable s_t is unobservable. Therefore, following Diebold, Lee, and Weinbach (1994), we propose an EM algorithm to maximize the incomplete-data log likelihood. The procedure of the EM algorithm is show in Figure 2, and it consists of four steps:

(1) Pick a vector of starting values, $\theta^{(0)}$.

(2) Construct the expected log-likelihood function $E\left[\log f\left(\underline{y_T}, \underline{s_T} \middle| \underline{x_p}; \theta^{(0)}\right)\right]$ by replacing the I 's in equation (3.7) with the following smoothed probabilities:

$$P\left(S_t = 1 \middle| \underline{y_T}; \theta^{(0)}\right)$$

$$P\left(S_t = 2 \middle| \underline{y_T}; \theta^{(0)}\right)$$

$$P\left(S_t = 1, S_{t-1} = 1 \middle| \underline{y_T}; \theta^{(0)}\right)$$

$$P\left(S_t = 2, S_{t-1} = 1 \middle| \underline{y_T}; \theta^{(0)}\right)$$

$$P\left(S_t = 1, S_{t-1} = 2 \middle| \underline{y_T}; \theta^{(0)}\right)$$

$$P\left(S_t = 2, S_{t-1} = 2 \middle| \underline{y_T}; \theta^{(0)}\right)$$

(3) Set $\theta^{(1)} = \arg\max_\theta E\left[\log f\left(\underline{y_T}, \underline{s_T} \middle| \underline{x_p}; \theta^{(0)}\right)\right]$, (3.8).

(4) Iterate to convergence.

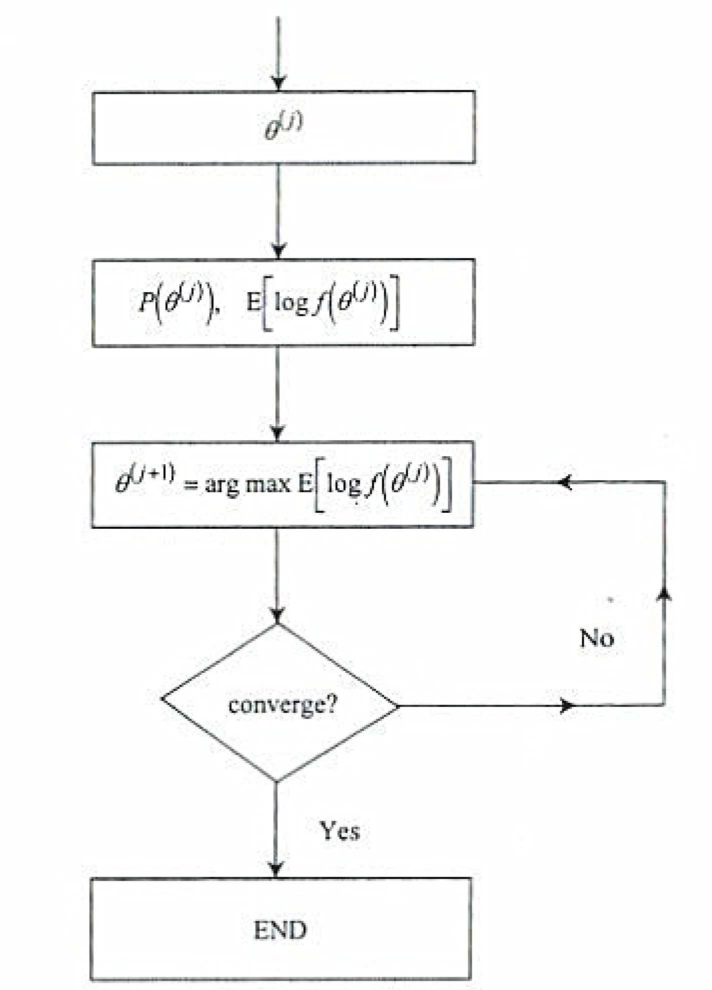

Figure 2. The EM Algorithm

Source: Diebold, Lee, and Weinbach (1994)

3.2.2.1 The Expectation Step

As in Diebold, Lee, and Weinbach (1994), the expected log-likelihood function with smoothed probabilities is given by

$$E\left[\log f\left(\underline{y_T}, \underline{s_T}\middle|\underline{x_p}; \theta^{(j)}\right)\right] = \rho^{(j)}\left[\log f\left(y_{p+1}\middle|S_{p+1} = 1, \underline{x_p}; \theta^{(j)}\right) + \log \rho^{(j)}\right]$$

$$+\left(1-\rho^{(j)}\right)\left[\log f\left(y_{p+1}\middle|S_{p+1} = 2, \underline{x_p}; \theta^{(j)}\right) + \log\left(1-\rho^{(j)}\right)\right]$$

$$+\sum_{t=p+2}^{T}\left\{P\left(S_t = 1\middle|\underline{y_T}; \theta^{(j)}\right)\log f\left(y_t\middle|S_t = 1, \underline{y_{t-1}}, \underline{x_p}; \theta\right)\right.$$

$$+P\left(S_t = 2\middle|\underline{y_T}; \theta^{(j)}\right)\log f\left(y_t\middle|S_t = 2, \underline{y_{t-1}}, \underline{x_p}; \theta\right)$$

$$+P\left(S_t = 1, S_{t-1} = 1\middle|\underline{y_T}; \theta^{(j)}\right)\log\left(\pi_{t,11}\right)$$

$$+P\left(S_t = 2, S_{t-1} = 1 \middle| \underline{y_T}; \theta^{(j)}\right) \log\left(1 - \pi_{t,11}\right)$$

$$+P\left(S_t = 1, S_{t-1} = 2 \middle| \underline{y_T}; \theta^{(j)}\right) \log\left(1 - \pi_{t,22}\right)$$

$$+P\left(S_t = 2, S_{t-1} = 2 \middle| \underline{y_T}; \theta^{(j)}\right) \log\left(\pi_{t,22}\right)\Big\} . \quad (3.9)$$

The smoothed probabilities for the j^{th} iteration is calculated from the following four steps:

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Step 1. Calculate the (conditional) densities (a $T \times 2$ matrix) from equation (B.1.1), and the transition probabilities (a $(T-1)\times 4$ matrix) from equation (3.3.3) (or (3.3.4), here we only discuss the case of equation (3.3.3)):

$$\begin{bmatrix} f\left(y_1 \middle| \Delta_t = 1; \theta^{(j)}\right) & f\left(y_1 \middle| \Delta_t = 2; \theta^{(j)}\right) \\ f\left(y_2 \middle| y_1; \Delta_t = 1; \theta^{(j)}\right) & f\left(y_2 \middle| y_1; \Delta_t = 2; \theta^{(j)}\right) \\ \vdots & \vdots \\ f\left(y_T \middle| y_{T-1}; \Delta_t = 1; \theta^{(j)}\right) & f\left(y_T \middle| y_{T-1}; \Delta_t = 2; \theta^{(j)}\right) \end{bmatrix}$$

$$\begin{bmatrix} \pi_2^{11} = \left[1+\exp\left\{-\gamma_{11}^{(j)}\left(|y_1| - c_{11}^{(j)}\right)\right\}\right]^{-1} & 1-\pi_2^{11} & 1-\pi_2^{22} & \pi_2^{22} = \left[1+\exp\left\{-\gamma_{22}^{(j)}\left(|y_1| - c_{22}^{(j)}\right)\right\}\right]^{-1} \\ \pi_3^{11} = \left[1+\exp\left\{-\gamma_{11}^{(j)}\left(|y_2| - c_{11}^{(j)}\right)\right\}\right]^{-1} & 1-\pi_3^{11} & 1-\pi_3^{22} & \pi_3^{22} = \left[1+\exp\left\{-\gamma_{22}^{(j)}\left(|y_2| - c_{22}^{(j)}\right)\right\}\right]^{-1} \\ \vdots & \vdots & \vdots & \vdots \\ \pi_T^{11} = \left[1+\exp\left\{-\gamma_{11}^{(j)}\left(|y_{T-1}| - c_{11}^{(j)}\right)\right\}\right]^{-1} & 1-\pi_T^{11} & 1-\pi_T^{22} & \pi_T^{22} = \left[1+\exp\left\{-\gamma_{22}^{(j)}\left(|y_{T-1}| - c_{22}^{(j)}\right)\right\}\right]^{-1} \end{bmatrix}$$

Step 2. Calculate the filtered joint conditional probabilities of (a $(T-1)\times 4$ matrix) by iterating the following steps:

Step 2.1 Calculate the joint conditional probabilities of $(y_t, \Delta_t, \Delta_{t-1})$ given $\underline{y_{t-1}}$ (4 numbers):

For $t = 2$, the joint conditional distribution is

$$f\left(y_2, \Delta_2, \Delta_1 \middle| y_1; \theta^{(j)}\right) = f\left(y_2 \middle| y_1, \Delta_2; \theta^{(j)}\right) P\left(\Delta_2 \middle| \Delta_1, y_1; \theta^{(j)}\right) P\left(\Delta_1\right),$$

For $t \geq 3$,

$$f\left(y_t,\Delta_t,\Delta_{t-1}\left|\underline{y_{t-1}};\theta^{(j)}\right.\right)=\sum_{\Delta_{t-2}=1}^{2} f\left(y_t\left|\underline{y_{t-1}},\Delta_t;\theta^{(j)}\right.\right)P\left(\Delta_t\left|\Delta_{t-1},\underline{y_{t-1}};\theta^{(j)}\right.\right)P\left(\Delta_{t-1},\Delta_{t-2}\left|\underline{y_{t-1}};\theta^{(j)}\right.\right).$$

The conditional density $f\left(y_t\left|\underline{y_{t-1}},\Delta_{t-1};\theta^{(j)}\right.\right)$ and the transition probabilities $P\left(\Delta_t\left|\Delta_{t-1},\underline{y_{t-1}};\theta^{(j)}\right.\right)$ are given by step 1. The filtered probabilities $P\left(\Delta_{t-1},\Delta_{t-2}\left|\underline{y_{t-1}};\theta^{(j)}\right.\right)$ are obtained from execution of step 2 for the previous t (See Step 2.2 to 2.3).

Step 2.2 Calculate the conditional likelihood of y_t (one number):

$$f\left(y_t\left|\underline{y_{t-1}};\theta^{(j)}\right.\right)=\sum_{\Delta_t=1}^{2}\sum_{\Delta_{t-1}=1}^{2} f\left(y_t,\Delta_t,\Delta_{t-1}\left|\underline{y_{t-1}};\theta^{(j)}\right.\right)$$

Step 2.3 Calculate the filtered probabilities for time t (four numbers):

$$P\left(\Delta_t,\Delta_{t-1}\left|\underline{y_t};\theta^{(j)}\right.\right)=\frac{f\left(y_t,\Delta_t,\Delta_{t-1}\left|\underline{y_t};\theta^{(j)}\right.\right)}{f\left(y_t\left|\underline{y_t};\theta^{(j)}\right.\right)},$$

where the numerator is from step 2.1 and the denominator is from step 2.2. Repeat step 2.2 to 2.3 for $t\geq 3$, and finally we can obtain the $(T-1)\times 4$ matrix of filtered joint probabilities.

Step 3. Calculate the smoothed probabilities (a $(T-1)\times 6$ matrix) by the following steps:

Step 3.1 For $t=2$, calculate the joint probability of $\left(\Delta_\tau,\Delta_{\tau-1},\Delta_t,\Delta_{t-1}\right)$ given $\underline{y_\tau}$, for $\tau=t+2,t+3,...,T$:

$$P\left(\Delta_\tau,\Delta_{\tau-1},\Delta_t,\Delta_{t-1}\left|\underline{y_\tau};\theta^{(j)}\right.\right)$$
$$=\frac{\sum_{\Delta_{t-2}=1}^{2} f\left(y_\tau\left|\Delta_\tau;\theta^{(j)}\right.\right)P\left(\Delta_\tau\left|\Delta_{\tau-1},\underline{y_{\tau-1}};\theta^{(j)}\right.\right)P\left(\Delta_{\tau-1},\Delta_{\tau-2},\Delta_t,\Delta_{t-1}\left|\underline{y_{\tau-1}};\theta^{(j)}\right.\right)}{f\left(y_\tau\left|\underline{y_{\tau-1}};\theta^{(j)}\right.\right)}$$

where $f\left(y_\tau \middle| \Delta_\tau;\theta^{(j)}\right)$ and $P\left(\Delta_\tau \middle| \Delta_{\tau-1}, \underline{y_{\tau-1}};\theta^{(j)}\right)$ are given by step 1, $f\left(y_\tau \middle| \underline{y_{\tau-1}};\theta^{(j)}\right)$ is given by step 2.2, and $P\left(\Delta_{\tau-1}, \Delta_{\tau-2}, \Delta_t, \Delta_{t-1} \middle| \underline{y_{\tau-1}};\theta^{(j)}\right)$ is obtained by the previous τ in step 3.1. When $\tau = t+2$, the third term in the numerator is given by the following expression:

$$P\left(\Delta_{t+1}, \Delta_t, \Delta_{t-1} \middle| \underline{y_{t+1}};\theta^{(j)}\right) = \frac{f\left(y_{t+1} \middle| \Delta_{t+1};\theta^{(j)}\right) P\left(\Delta_{t+1} \middle| \Delta_t, \underline{y_t};\theta^{(j)}\right) P\left(\Delta_t, \Delta_{t-1} \middle| \underline{y_t};\theta^{(j)}\right)}{f\left(y_{t+1} \middle| \underline{y_t};\theta^{(j)}\right)}.$$

For each τ, we obtain a (1×4) vector of probabilities corresponding to the four possible combinations of $(s_\tau, s_{\tau-1})$. Thus, upon reaching $\tau = T$, we have computed a $((T-3)\times 4)$ matrix. The last row of this matrix is used to calculate the smoothed joint probability for time $t = 2$, which is given by

$$P\left(\Delta_t, \Delta_{t-1} \middle| \underline{y_T};\theta^{(j)}\right) = \sum_{\Delta_T=1}^{2} \sum_{\Delta_{T-1}=1}^{2} P\left(\Delta_T, \Delta_{T-1}, \Delta_t, \Delta_{t-1} \middle| \underline{y_T};\theta^{(j)}\right)$$

Step 3.2 Repeat step 3.1 for $t = 3, 4, ..., T$, and obtain a $((T-1)\times 4)$ matrix of smoothed joint probabilities.

Step 4 Calculate the smoothed marginal probabilities by summing over the smoothed joint probabilities. For example,

$$P\left(\Delta_t = 1 \middle| \underline{y_T};\theta^{(j)}\right) = P\left(\Delta_t = 1, \Delta_{t-1} = 1 \middle| \underline{y_T};\theta^{(j)}\right) + P\left(\Delta_t = 1, \Delta_{t-1} = 2 \middle| \underline{y_T};\theta^{(j)}\right).$$

Finally, a $(T-1)\times 6$ matrix of smoothed probabilities is obtained.

3.2.2.2 The Maximization Step

Substitute the smoothed probabilities for iteration j from the expectation step into equation (B.2.1), and estimate the parameters that maximize equation (B.2.2) as in equation (B.2.1). Iterate the expectation step and the maximization step until convergence.

4. Results

4.1 Data

For the empirical application, we examine market integration among four regional corn (Statesville, Candor, Cofield, Roaring River) and three regional soybean markets (Fayetteville, Cofield, and Creswell) in North Carolina. The prices of these markets are quoted daily from 3/1/2005 to 6/30/2010. Six pairwise spatial price relationships for the corn markets, and three pairwise spatial price relationships for the soybean markets are examined. We discuss market integration among these nine pairs of markets by analyzing the estimates of parameters and the smoothed probabilities of the arbitrage and non-arbitrage regimes. Table 1 reports descriptive statistics for these nine y_t's. Figure 2 through Figure 4 shows the time series plots of these y_t's.

Table 1. Summary Statistics for y_t

Soybean Markets			
	Fayetteville - Cofield	Fayetteville - Creswell	Cofield - Creswell
Notation	b_{12}	b_{13}	b_{23}
Observations	1567	1567	1567
Mean	0.02105	0.06440	0.04335
Standard Deviation	0.02863	0.05250	0.05753
Minimum	-0.08863	-0.07648	-0.13470
Maximum	0.16097	0.40809	0.40809
Skewness	0.18858	3.27351	3.19179
Kurtosis	0.48361	14.21455	14.02739
ADF Tau (single mean)	-5.39***	-4.27***	3.56***

Corn Markets (1)			
	Statesville - Candor	Statesville - Cofield	Statesville - Roaring River
Notation	c_{12}	c_{13}	c_{14}
Observations	1567	1567	1567
Mean	-0.06383	-0.00905	-0.06697
Standard Deviation	0.05911	0.06190	0.05947
Minimum	-0.27831	-0.21474	-0.28682
Maximum	0.11421	0.65471	0.12382
Skewness	-0.37480	0.52567	-0.48185
Kurtosis	-0.01448	8.39005	0.17414
ADF Tau (single mean)	-4.51***	-5.71***	-4.86***

Corn Markets (2)			
	Candor - Cofield	Candor - Roaring River	Cofield - Roaring River
Notation	c_{23}	c_{24}	c_{34}
Observations	1567	1567	1567
Mean	0.05478	-0.00314	-0.05792
Standard Deviation	0.04316	0.02669	0.04658
Minimum	-0.20150	-0.15575	-0.54689
Maximum	0.56755	0.18540	0.19363
Skewness	0.93826	-0.52271	-0.54480
Kurtosis	15.55812	7.36462	9.14688
ADF Tau (single mean)	-7.16***	-11.26***	-7.43***

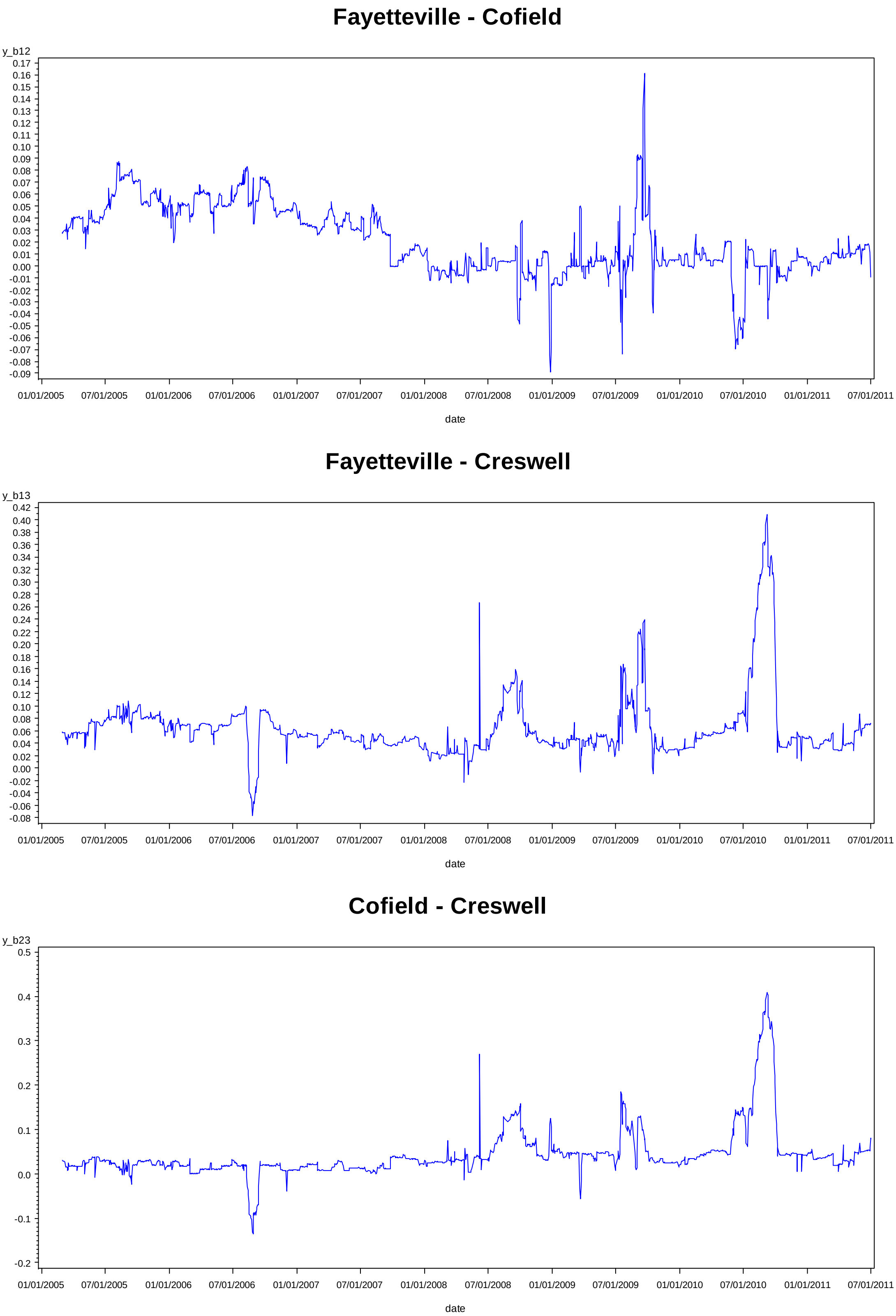

Figure 2. Time Series Plots of y_t's for the Soybean Markets

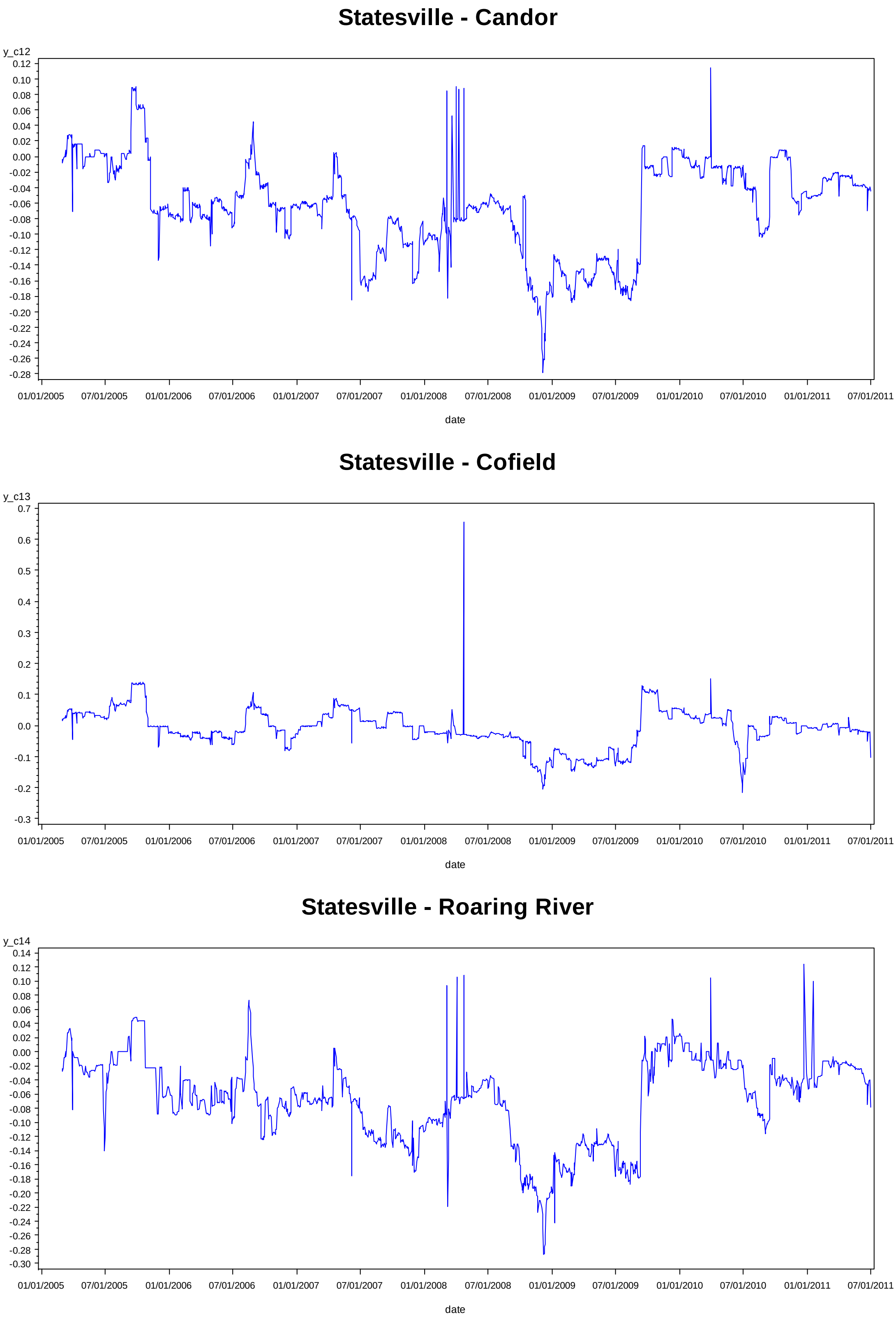

Figure 3. Time Series Plots of y_t's for the Corn Markets (1)

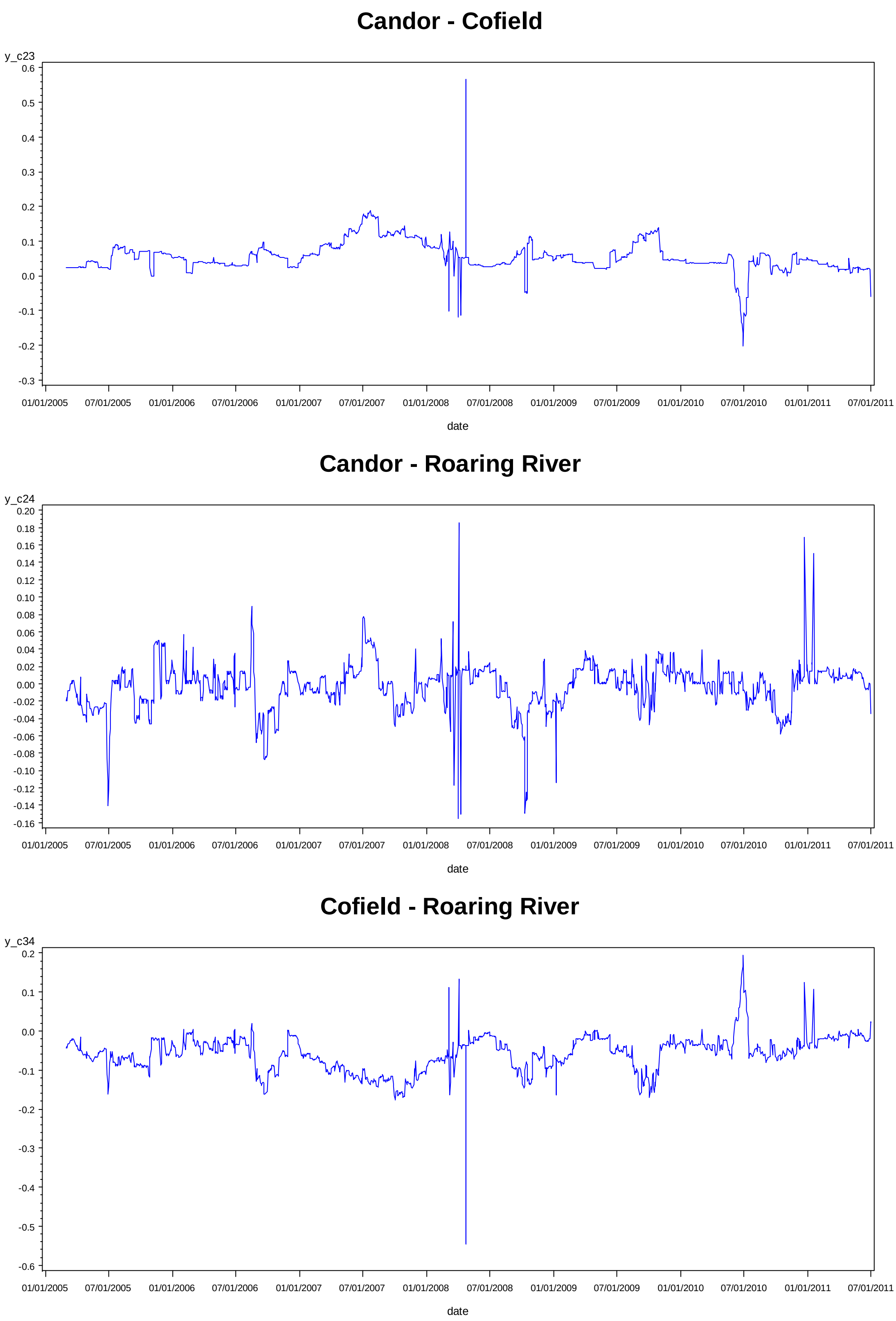

Figure 4. Time Series Plots of y_t's for the Corn Markets (2)

4.2 Results of the MSAR models

We first estimate the ordinary autoregressive (AR) model, and decide the lag length p in equation (3.1) by the BIC criterion. Then, we estimate the MSAR models by applying the EM algorithm. Table 2 through Table 4 shows the results of the MSAR models.

Table 2. MSAR Model Results for Soybean Markets

	b13	b14	b34
log likelihood	6549.2727	6309.0625	6359.2839
alpha1	0.0000	-0.0002	0.0000
	(0.0000)	(0.0001)***	(0.0001)
beta1	0.0023	0.0032	-0.0004
	(0.0011)**	(0.0009)***	(0.0013)
phi11	-0.0116	-0.0004	-0.0053
	(0.0049)**	(0.0027)	(0.0041)
phi21	-0.0103		
	(0.0043)**		
phi31	0.0020		
	(0.0041)		
phi41	-0.0004		
	(0.0042)		
phi51	-0.0032		
	(0.0037)		
sigma1	0.0009	0.0008	0.0013
	(0.0000)***	(0.0000)***	(0.0000)***
alpha2	0.0009	0.0033	0.0022
	(0.0008)	(0.0015)**	(0.0016)
beta2	-0.0620	-0.0386	-0.0258
	(0.0229)***	(0.0134)***	(0.0129)**
phi12	-0.1396	-0.3172	-0.3554
	(0.0594)**	(0.0507)***	(0.0623)***
phi22	-0.1744		
	(0.0631)***		
phi32	-0.0996		
	(0.0670)		
phi42	-0.2059		
	(0.0645)***		
phi52	-0.2343		
	(0.0731)***		
sigma2	0.0150	0.0235	0.0246
	(0.0005)***	(0.0007)***	(0.0009)***
gamma11	15.9205	16.0951	21.9931
	(6.4324)**	(4.4084)***	(4.9409)***
c11	1.7344	1.9154	2.6727
	(0.1709)***	(0.1320)***	(0.1629)***
gamma22	19.9978	7.3759	11.1878
	(5.3466)***	(2.6117)***	(2.6249)***
c22	-0.6431	0.2310	-0.0865
	(0.1745)***	(0.1321)*	0.1563

Table 3. MSAR Model Results for Corn Markets (1)

	c12	c13	c14
log likelihood	5850.4465	5934.0314	5649.9339
alpha1	0.0000	0.0000	0.0000
	(0.0001)	(0.0001)	(0.0001)
beta1	0.0004	-0.0007	-0.0018
	(0.0013)	(0.0012)	(0.0008)**
phi11	-0.0190	-0.0158	0.0008
	(0.0058)***	(0.0043)***	(0.0032)
phi21	-0.0084	-0.0088	0.0001
	(0.0046)*	(0.0035)**	(0.0027)
phi31		-0.0041	0.0028
		(0.0028)	(0.0025)
sigma1	0.0026	0.0024	0.0014
	(0.0001)***	(0.0001)***	(0.0000)***
alpha2	-0.0062	-0.0005	-0.0051
	(0.0039)	(0.0041)	(0.0022)**
beta2	-0.1001	-0.0934	-0.0610
	(0.0363)***	(0.0526)*	(0.0207)***
phi12	-0.6169	-0.8008	-0.5150
	(0.0822)***	(0.0906)***	(0.0568)***
phi22	-0.5886	-0.3097	-0.3459
	(0.1176)***	(0.2382)	(0.0714)***
phi32		0.1188	-0.1455
		(0.2916)	(0.0675)**
sigma2	0.0393	0.0576	0.0306
	(0.0018)***	(0.0028)***	(0.0010)***
gamma11	7.0592	6.7574	0.8367
	(4.0652)*	(2.6003)***	(2.1915)
c11	2.5254	2.5496	1.0625
	(0.2246)***	(0.1601)***	(0.1165)***
gamma22	12.8380	11.4877	16.3218
	(3.5345)***	(3.2591)***	(2.7996)***
c22	-1.3511	-1.2464	-1.5293
	(0.2725)***	(0.2592)***	(0.1992)***

Table 4. MSAR Model Results for Corn Markets (2)

	c23	c24	c34
log likelihood	6911.0321	5394.5747	5069.2568
alpha1	0.0001	-0.0001	-0.0002
	(0.0001)**	(0.0001)	(0.0001)
beta1	-0.0030	-0.0042	-0.0026
	(0.0009)***	(0.0030)	(0.0021)
phi11	0.0001	-0.0104	-0.0087
	(0.0022)	(0.0057)*	(0.0052)*
phi21	0.0020	-0.0054	-0.0019
	(0.0018)	(0.0049)	(0.0044)
phi31	-0.0009	0.0028	-0.0014
	(0.0015)	(0.0048)	(0.0037)
phi41		0.0004	
		(0.0044)	
phi51		0.0015	
		(0.0040)	
sigma1	0.0011	0.0021	0.0028
	(0.0000)***	(0.0001)***	(0.0001)***
alpha2	0.0042	-0.0009	-0.0070
	(0.0042)	(0.0014)	(0.0030)**
beta2	-0.0709	-0.2703	-0.1164
	(0.0495)	(0.0556)***	(0.0394)***
phi12	-0.7548	-0.2857	-0.6382
	(0.0878)***	(0.0721)***	(0.0683)***
phi22	-0.4200	-0.3272	-0.2205
	(0.1786)**	(0.0821)***	(0.1065)**
phi32	-0.0549	0.0408	-0.0672
	(0.1789)	(0.0802)	(0.0963)
phi42		-0.2073	
		(0.0832)**	
phi52		-0.2422	
		(0.0847)***	
sigma2	0.0478	0.0290	0.0410
	(0.0022)***	(0.0010)***	(0.0015)***
gamma11	20.3459	8.6714	3.1753
	(3.8980)***	(5.4311)	(3.0255)
c11	2.7705	1.3399	1.4233
	(0.1622)***	(0.1197)***	(0.1231)***
gamma22	11.3865	25.2587	20.1142
	(3.0523)***	(4.9962)***	(3.3827)***
c22	-0.8739	-1.2073	-1.4567
	(0.1980)***	(0.1565)***	(0.1826)***

5. Conclusion

In this study, we develop a new approach to investigate spatial market integration, which is a Markov-Switching autoregressive (MSAR) model with time-varying state transition probabilities. Our results demonstrate that significant regime switching relationships characterize these markets. This has important implications for more conventional models of spatial price relationships and market integration. Our results are consistent with efficient arbitrage subject to transactions costs.

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