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A MEASURE OF THE VALUE OF INFORMATION FOR THE COMPETITIVE FIRM UNDER PRICE UNCERTAINTY

by

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This paper addresses the problem of measuring the value of information to an agent in an environment where the agent is risk averse and choices are based on the utility of income and personal beliefs about the likelihood of uncertain outcomes. If the agent's beliefs about the likelihood of uncertain outcomes are less consistent with realized outcomes than would be the case of "more informed" beliefs, then it can be shown that the agent would be willing to forego an amount of income to become more informed and would experience an increase in utility. Empirical estimates of the value of information are important for obtaining insights into issues such as the informational efficiency of alternative market structures, the effects of the quality of agents conditional forecasts of market prices on the efficiency of resource use and the social profitability of information supplied by private enterprise and public agencies.

The conceptual framework of the competitive firm under price uncertainty has been developed by a number of authors including Sandmo (1971), Holthausen (1979), and Pope (1978, 1980). In these models the producer is assumed to formulate his subjective distribution based on information available at the time output decisions are made. The value of information in this context has been formulated using a Bayesian approach which provides the agent with normative decision rules to determine whether additional information would lead to an increase in expected utility. (Lindley, 1971; Anderson, Dillon, and Hardacker, 1977; Winkler, 1972).

Otherwise, the literature has given little emphasis to the question of measuring the value of information to the competitive firm under price uncertainty. Furthermore, the empirical application of Bayes theorem to explain the observed behavior of risk averse agents is fraught with major difficulties. While estimates of the agents' prior distribution of uncertain events may be obtained from observed choices, or in some cases solicited directly, the content of additional information, the process by which additional information becomes available and whether the agent behaves as though the prior is updated according to Bayes theorem raises major difficulties in applying the theorem to estimating the value of information from observed behavior of agents.

Three approaches to estimating the value of information are presented in this paper which avoids these difficulties, although, for two of the measures, knowledge of the agents utility function is required. These approaches are easier to use in empirical application even though they are similar to the Bayesian procedure.

The conceptual framework is stated in the next section. Then, procedures for measuring the value of information to a risk averse agent are derived, and the usefulness of these measures in applied research is discussed. A sample problem is presented in the appendix to demonstrate the procedure.

I. THE CONCEPTUAL FRAMEWORK

The competitive firm under price uncertainty will be described in a Bernoullian framework where the agent's utility function is a concave, continuous, and differentiable function of profits. Consider the primal-dual Lagrangean function

$$L^* = EU(\pi^*) - EU(\pi) \tag{1}$$

where $EU(\pi^*)$ is the indirect expected utility function and $EU(\pi)$ is the direct expected utility function. Profit is

$$\pi = Pq - C(q) \quad (2)$$

where P is stochastic output price, q is quantity, and $C(q)$ is the cost function. The indirect function is determined by solving for the optimal output quantity as a function of only the parameters and substituting back into the direct function. Hence, the indirect function is the maximum value of expected utility expressed solely as a function of the parameters. More specifically equation (1) is

$$L^* = EU[Pq^* - C(q^*)] - EU[Pq - C(q)]. \quad (3)$$

where q^* is the value that maximizes the direct utility of profit function, $EU(\pi)$. The first order condition for a minimum is:

$$\frac{\partial L^*}{\partial q} = -E[U'(\pi)(P - C'(q))] = 0 \quad (4)$$

where $U'(\pi) = \frac{dU}{d\pi}$ and $C'(q)$ is positive and continuous.

Equation (4) is the general first order condition with no reference made to the agent's distribution of output price. To describe the different output choices that occur when the agent's distribution of output price is based on different sets of information and to facilitate the derivation of various measures of the value of information, four states of information will be defined: the subjective, more informed, realized, and perfect states.

The Subjective State

The agent's subjective distribution, denoted by $f^0(p)$, represents his beliefs based on the information available at the time the

output decision was made. The optimal quantity of output can be determined by using equation (4) and taking the expectation with respect to $f^0(p)$. The first order condition can be represented by

$$E^0[U'(\pi)(P - C'(q))] = 0. \quad (5)$$

The agent's optimal output choice found by solving equation (5) is a non-stochastic variable which will be represented by q^0 . However, prior to the realization of output price, profit remains a stochastic variable:

$$\pi^0 = Pq^0 - C(q^0). \quad (6)$$

The utility that the agent expects to obtain from producing q^0 is $E^0U(\pi^0)$. Note that this expectation is based on his subjective distribution of price.

Realized State

The quantity produced, q^0 , is sold at a realized price denoted by p^R . However, because P is stochastic, the agent's price expectation may not equal p^R . Profit is no longer a stochastic variable and will be denoted by

$$\pi^R = p^R q^0 - C(q^0). \quad (7)$$

That is, given the output decision made in the subjective state q^0 and the realized price p^R , realized profits are given by π^R .

More Informed State

If the agent's beliefs were based on more information than in the subjective state, the distribution based on this information would be a more accurate description of the random price variable than $f^0(p)$.

Denote this more informed distribution by $f^m(p)$. The optimal output choice, denoted by q^m , in the more informed state can be determined by equation (4) with expectations taken with respect to the more informed distribution:

$$E^m[U'(\pi)(P - C'(q))] = 0. \quad (8)$$

Prior to realization of the output price, profit is a stochastic variable represented by

$$\pi^m = Pq^m - C(q^m). \quad (9)$$

The expected utility in the more informed state is $E^m U(\pi^m)$.

Perfect State

The perfect state of information represents what the agent's optimal output decision would have been if he had known the actual output price p^r at the time he made the output decision.^{1/} With p^r known, the agent need only maximize profits. In this instance, the primal-dual Lagrangean function is

$$J^* = p^r q^* - C(q^*) - P^r q + C(q),$$

and the first order condition is

$$\frac{\partial J^*}{\partial q} = -p^r + C'(q) = 0. \quad (10)$$

Let the optimal output choice given by equation (10) be denoted by q^t where profit is given by

$$\pi^t = p^r q^t - C(q^t). \quad (11)$$

II. The Value of Information

Three different measures of the value of information will be discussed. The first is an ex-post measure of the value of information. This measure is referred to as the value of perfect information. It is determined by comparing the profit realized π^r from the choice q^o in the subjective state with the profit π^t in the perfect state. The other two measures that will be discussed are ex-ante measures. In these cases, the actual output price is not assumed to be known, but rather decisions made in the subject state with information embodied in $f^o(P)$ are compared with those made in the more informed state with information embodied in $f^m(P)$.

An Ex-Post Measure

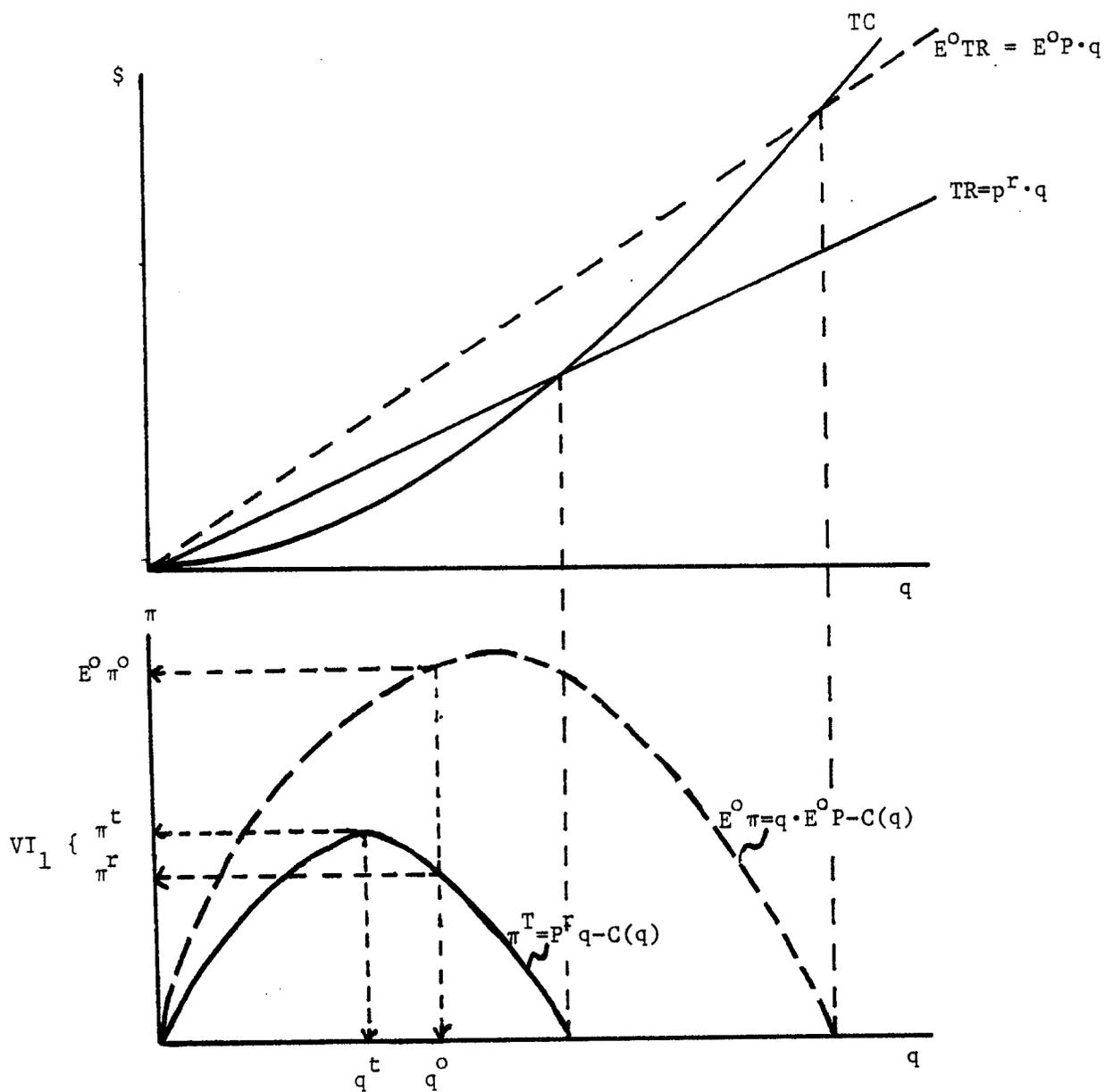
An ex-post measure of the value of information is defined to be the difference between profits earned in the perfect state, equation (11), when the agent has perfect information and profits earned in the realized state, equation (7). This value of information, denoted by VI_1 , is given by

$$VI_1 = \pi^t - \pi^r = (p^r q^t - C(q^t)) - [p^r q^o - C(q^o)]. \quad (12)$$

From the definition of the primal-dual problem, it follows that VI_1 is always non-negative.

The ex-post measure of the value of information is illustrated graphically in Figure 1 for the case where the agent expects a higher output price than is actually realized. The top graph in this figure separates the total cost and total revenue components of the profit function. Although total cost is known to the agent in the subjective state, total revenue is not. Hence, the solid straight line, TR, represents total revenue in the perfect state when p^r is

Figure 1



known, while the dashed line, $E^0 TR$, is the total revenue as perceived by the agent in the subjective state. From revenue, TR , and cost, TC , the profit function with perfect information, π^T , is shown in the lower graph where its maximum is given by π^t at q^t . Expected subjective profits (denoted by the dashed curve in the lower graph) are based on TC and the expected total revenue, $E^0 TR$. Because expected utility of profits is maximized in the subjective state and the agent is assumed to be risk averse, the optimal output in this state, q^0 , lies somewhere to the left of the maximum point of the expected subjective profits curve. Although $E^0 \pi^0$ is the profit expected by the agent in the subjective state, the realized profit is determined by the true profit function. Hence, if q^0 were produced, realized profits, π^r , is attained instead of $E^0 \pi^0$. Since π^t lies at the maximum point of the true profit curve, the realized profit, π^r , for any other output choice but q^t will result in a lower value. Consistent with (12), the value of information, VI_1 , is given by the difference between π^t and π^r .

The usefulness of this approach now becomes apparent. Even though $U(\pi)$ is generally not known, q^0 and π^r are observable; and π^t can be estimated. In this case, if the establishment of a forward market is being contemplated or if consideration is being given to a policy of announcing the price of output at the time production decisions are made, our procedure gives insights into output response and changes in profits in a rather straightforward manner.^{2/}

Ex-Ante Measures

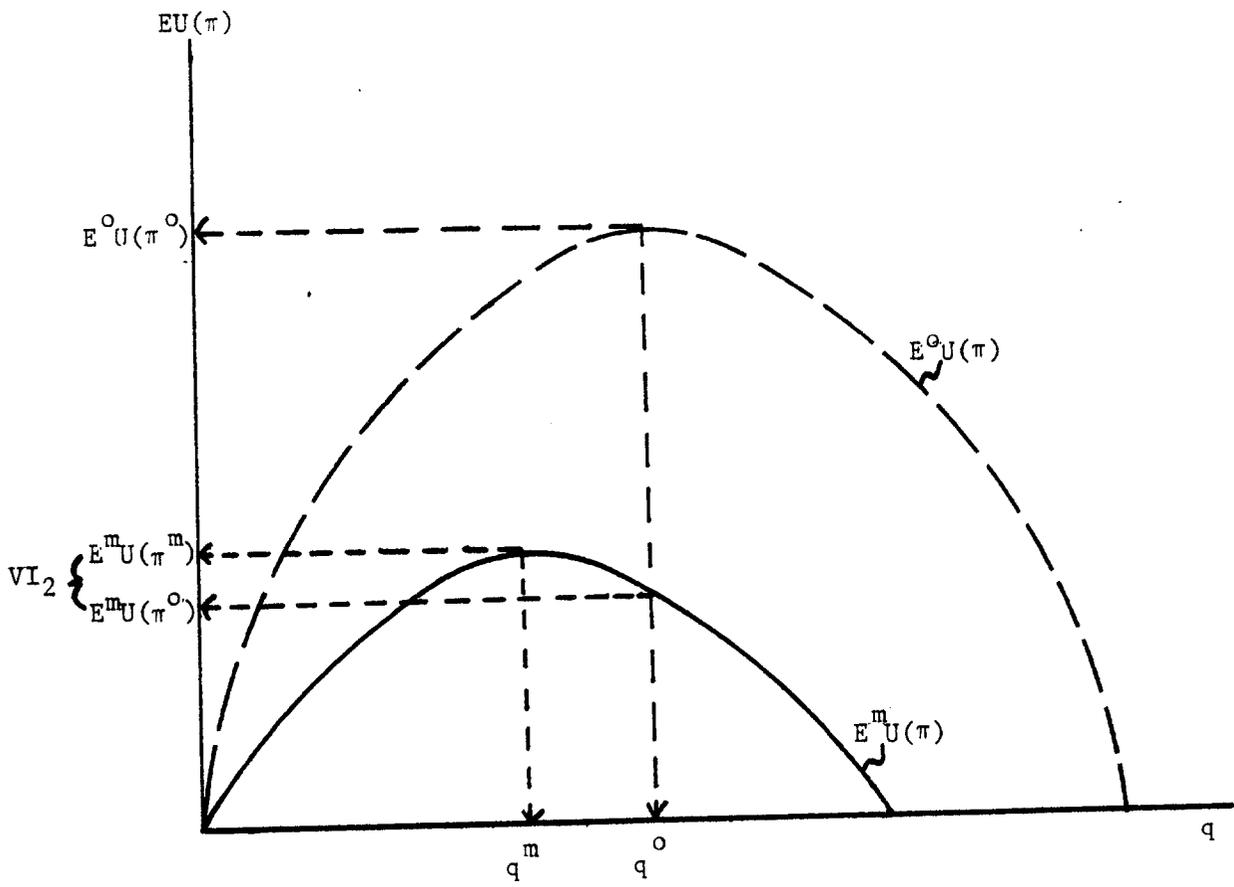
In the case of VI_1 , information is considered to be perfect because decisions made in the subjective state are compared to those made when actual output price is known at the time production commitments are made.

When P is stochastic, it is impossible to measure the value of "perfect" information ex-ante because output price cannot yet be observed. Recall that the agent's subjective distribution is formulated by using the information available to him at the time the output decision was made. For our purposes here the more informed distribution is defined as more descriptive of the stochastic variable P than the subjective distribution. Hence, the value of information can be computed by determining what it would be worth to the agent to know the more informed distribution rather than his subjective distribution. Two conceptualizations of this problem will be discussed in this section.

The first ex-ante measure discussed is based on expected utility in the more informed and subjective states. This measure, denoted by VI_2 , is illustrated in Figure 2. Curve $E^m U(\pi)$ represents the agent's expected utility of profits in the more informed state. The optimal quantity to be produced in the more informed state, q^m , lies at the maximum point on the $E^m U(\pi)$ curve. Expected utility of profits at q^m is denoted by $E^m U(\pi^m)$ on the vertical axis. For the case depicted in figure 2, the agent's expected utility of profits in the subjective state, based on $f^o(p)$ and $E^o U(\pi)$, is depicted by the broken curve. The maximization of $E^o U(\pi)$ yields the optimal quantity, q^o , produced in the subjective state with corresponding subjective utility of $E^o U(\pi^o)$. However, the expected utility of q^o in the more informed state is $E^m U(\pi^o)$. Hence, the value of information can be defined to be the difference in the more informed state between the expected utility of producing q^m and the expected utility of producing q^o :

$$VI_2 = E^m U(\pi^m) - E^m U(\pi^o). \quad (13)$$

Figure 2



It can be shown that VI_2 will always be non-negative by considering primal-dual equation (1). By derivation of quantity q^m , it is clear that $q^m = q^*$ in equation (1) when expectations are taken with respect to $f^m(p)$. By definition of the primal-dual problem, $EU(\pi^*)$ is the maximum value of expected utility that can be attained over all possible values of profit. Thus,

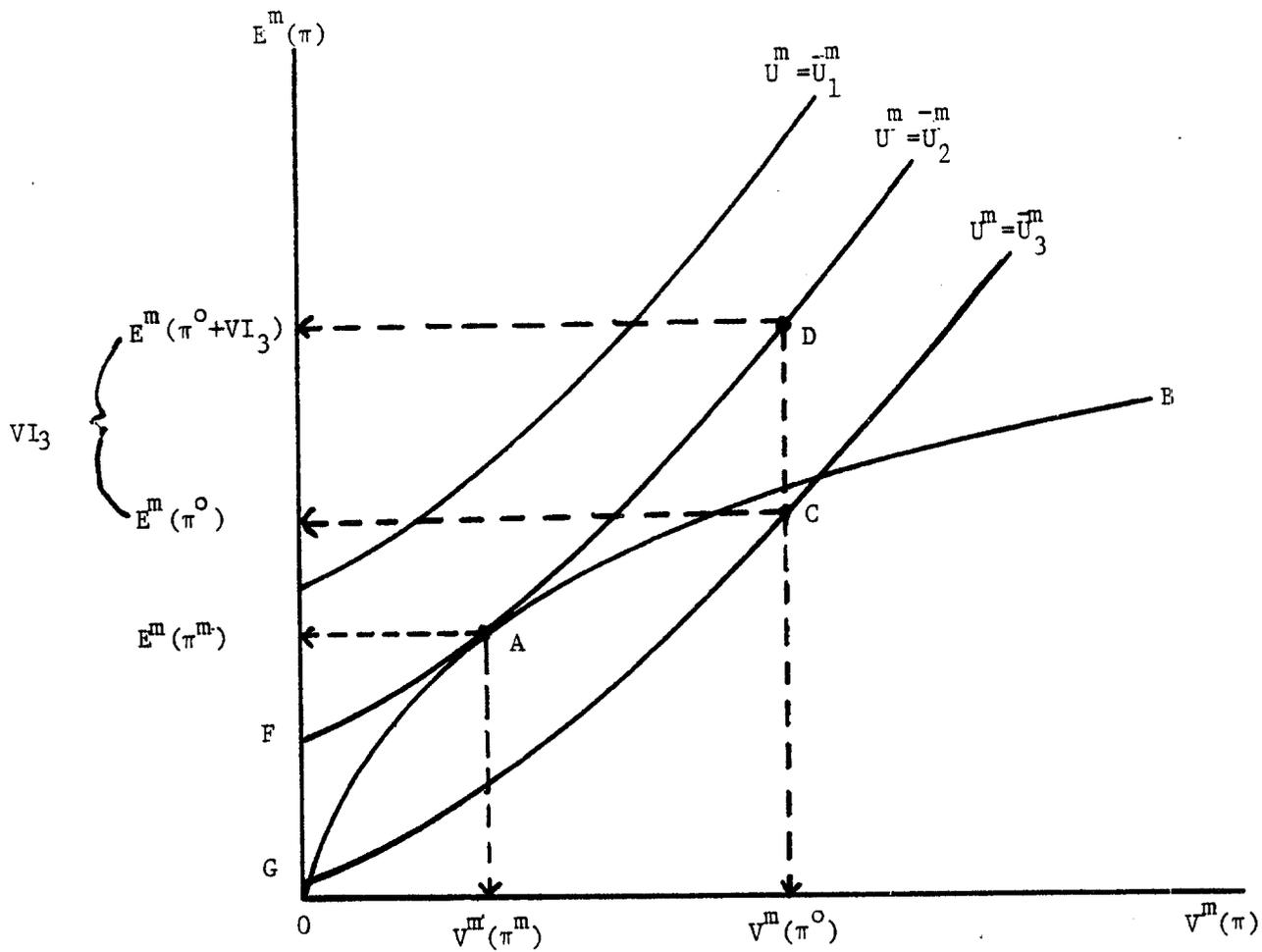
$$\begin{aligned} L^* &= EU(\pi^*) - EU(\pi) \geq 0 \\ &= E^m U(\pi^m) - E^m U(\pi) \geq 0 \end{aligned}$$

and hence VI_2 is non-negative.

This measure of the value of information, however, is not very useful because utility has only ordinal properties. To avoid the problems created by ordinal measures, a measure similar to equivalent variation in the certainty case is derived.

For illustration purposes consider the simple case when $f^m(p)$ has only two parameters a mean and variance. In Figure 3, the mean-variances (E, V) space has been given for the more informed state where OAB is the mean-variance frontier of response possibilities, and the U^m curves represent isoutility where $\bar{U}_1^m > \bar{U}_2^m > \bar{U}_3^m$. Point A corresponds to the optimal output level in the more informed state, q^m , and the random variable profits, π^m . Let, decisions made in the subjective state lead to production q^0 . Then the point corresponding to production level q^0 must lie on or below the mean-variance frontier OAB because this curve represents the set of all efficient output levels in the more informed state. Suppose that output level q^0 can be represented by point C which by necessity, lies on a lower isoutility curve \bar{U}_2^m . Let VI_3 be the amount of monetary payment that must be given to

Figure 3



an agent who produces q^0 so that his expected utility in the more informed state would have been the same as if he had produced q^m . VI_3 is a value (as opposed to utility) measure of the value of information. It is illustrated by the distance on the vertical axis between points C and D.

Stated in general terms, define a nonstochastic variable VI_3 such that

$$E^m U(\pi^m) = E^m U(\pi^0 + VI_3). \quad (14)$$

To show that VI_3 is non-negative, recall that $U'(\pi) > 0$ implies

$$U(\pi_1) \geq U(\pi_2) \quad \text{if } \pi_1 > \pi_2. \quad (15)$$

Since it has already been shown from the primal-dual problem that

$$E^m U(\pi^m) \geq E^m U(\pi^0), \text{ then by equation (14),}$$

$$E^m U(\pi^0 + VI_3) \geq E^m U(\pi^0). \quad (16)$$

By definition of expectations,

$$\int U(\pi^0 + VI_3) f^m(p) dp \geq \int U(\pi^0) f^m(p) dp. \quad (17)$$

But by the properties of integrals, expression (17) implies

$$U(\pi^0 + VI_3) \geq U(\pi^0).$$

Then by equation (15),

$$\pi^0 + VI_3 \geq \pi^0.$$

And hence, VI_3 is non-negative.

While the practical application of this approach is more complex than in the previous case, its advantage relative to other approaches lies in the ease of obtaining a monetary measure of the value of having the additional information embodied in $f^m(p)$. In this case, knowledge of the agent's utility function and $f^m(p)$ are required to compute the value of information. However, knowledge of his initial beliefs $f^0(p)$ are not required. Estimates of $f^m(p)$ may come about through public or private research to obtain insights into factors determining the distribution of P. Given knowledge of the agent's

utility function, our measure of the value of information becomes a key input into determining the social or private profitability of efforts to supply agents with the knowledge embodied in $f^m(p)$. As in the previous cases, this approach is also useful because value is measured relative to realized outcome q^0 . Hence, in addition to estimates of the value of information, estimates of changes in q and are also obtained.

III. Concluding Remarks

The theory of the competitive firm under price uncertainty has been addressed a number of times in the literature where the optimal output decision for the agent is generally derived given his subjective distribution of output price. Within this problem setting, less attention has been given to deriving practical monetary measures of the value of information resulting from a more accurate distribution of output price. In this paper, four states of information were defined to address this problem. Measures of the value of information were obtained using expected utilities and optimal output decisions in the different states.

The usefulness of this approach is that, in the case of VI_1 , the value of information can be measured in monetary units from observed data without knowledge of the agent's utility function. In the case where a more accurate distribution of output price can be made available to the agent, VI_3 , a monetary measure of the value of information is derived which requires knowledge of the agent's utility function. However, unlike other approaches, knowledge is not required of his beliefs about the posterior nor whether the agent behaves according to Bayes theorem. In each case, our approach measures the adjustment of output to new information.

FOOTNOTES

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1/ This situation may arise from forward pricing (Holthausen, 1979) or from a government guaranteed price.

2/ Since the form of the utility function is assumed to be unknown in the case of (12), realized profits, $(\pi^{\bar{A}})$ are not compensated for the risk the agent incurred in producing q^0 . See Appendix for discussion.

3/ Lindley (1971) describes a similar measure for the value of information, Z , given by $E^B U(\pi^B - Z) = E^B U(\pi^0)$, where expectations are taken with respect to the Bayesian posterior distribution $f^B(p)$. Although both Lindley's Z and our VI_3 are measures of the amount the agent is willing to pay to obtain more information, they may not be equal; and there is a subtle difference in interpretation. In the Bayesian approach Z is the amount of money which must be given up by the agent when he produces q^B so that he has the same amount of utility in the more informed state as producing q^0 . In our case, VI_3 is the amount of money that must be given to the agent when he produces q^0 so that his expected utility in the more informed state is the same as if he had produced q^m . Whether other measures, such as the distance A-E or the distance F-G provide equivalent measures to VI_3 depends on the form of $E^m U[\pi^m]$.

REFERENCES

- Holthausen, Duncan M. "Hedging and the Competitive Firm under Price Uncertainty." The American Economic Review 69 (December 1979): 989-995.
- Lindley, Dennis. Making Decisions. London: Wiley-Interscience, 1971.
- Pope, Rulon D. "The Expected Utility Hypothesis and Demand--Supply Restrictions." American Journal of Agricultural Economics 62 (November 1978): 619-627.
- _____. "The Generalized Envelope Theorem and Price Uncertainty." International Economic Review 21 (February 1980): 75-86.
- Roe, Terry and Nygaard, David. Wheat, Allocative Error and Risk: Northern Tunisia. University of Minnesota Economic Development Center Bulletin V, March 1980.
- Sandmo, Agnar. "On the Theory of the Competitive Firm Under Price Uncertainty." The American Economic Review 61 (March 1971): 65-73.
- Silberberg, Eugene. The Structure of Economics: A Mathematical Analysis. New York: McGraw Hill Book Company, 1978.
- Winkler, Robert L. Introduction to Bayesian Inference and Decision. New York: Holt, Rinehart and Winston, Inc. 1972.

APPENDIX: SAMPLE PROBLEM

Consider an expected utility function of the form:

$$(A.1) \quad EU(\pi) = E\pi - \alpha\sigma_{\pi},$$

where the risk parameter is α and σ_{π} is the standard error of profits. This form has been justified on the basis of a second order Taylor series expansion or as a decision function of Freund's negative exponential utility function under normality.

For clarity, let $\pi = P\sqrt{x} - p_1x$ where output price, P , is stochastic, x is input and p_1 is input price. It follows from the maximization of (A.1) that the indirect utility function is

$$EU(\pi^*) = p_1^{-1}(\bar{p} - \alpha\sigma)^2 \quad .25$$

where the agent's forecast of output price is described by $P \sim N(\bar{p}, \sigma)$.

For the case of (A.1), it has been shown by Pope (1978) that the optimal quantity supplied is given by

$$(A.2) \quad \partial EU(\pi^*) / \partial \bar{p} = q^*$$

while the optimum level of input use is given by $\partial EU(\pi^*) / \partial p_1 = -x^*$.

The first problem is to derive the value of perfect information given knowledge of q^0 and cost $C(q)$, although, values $(\bar{p}, \sigma, \alpha, p_1)$ are assigned in order that the results may be confirmed using (A.1) if desired. The ex-ante value VI_3 is derived next where knowledge of (A.1) is assumed. While various procedures have been used to estimate (A.1) the supposition that only (A.2) has been fit to data is employed. This supposition is also sufficient to derive $C(q)$.

To obtain the solution for the subjective state, the following initial values are assumed: $(\bar{p}, \sigma, \alpha, p_1) = (33, 3.6, .8, 1.5)$. It can be verified that these values yield the solution $(E^0U(\pi^0), q^0, C(q^0), E^0(\pi^0)) = (151.2, 10.04, 151.2, 180.12)$, where "o" denotes the agent's initial distribution of output price $f^0(p)$.

The ex-post measure of the value of perfect information, VI_1 , is given by (12). Let the price realized at the time output is marketed be denoted by $p^r = 24$. In this case,

$$\begin{aligned} (A.3) \quad VI_1 &= p^r q^t - C(q^t) - [p^r q^o - C(q^o)] \\ &= [24(8) - 96] - [24(10.04) - 151.2] = 6.24 \end{aligned}$$

The agent would be willing to pay or forfeit 6.24 to either know or be guaranteed, at the time of making production commitments, the price p^r .

The ex-ante value-measure, VI_3 , is shown for the case where the more informed distribution of output price $f^m(p)$ is described by $P \sim N(24, 3.0)$. Given the parameter α from (A.2) it can be verified that these values yield the solution $(E^m U(\pi^m), q^m, C(q^m), E^m(\pi^m)) = (77.76, 7.2, 77.76, 95.04)$. The right hand side of (14) is

$$E^m U(\pi^o + VI_3) = p^m q^o - C(q^o) - \alpha q^o \sigma^m + VI_3,$$

in which case

$$\begin{aligned} VI_3 &= p^m q^m - C(q^m) - \alpha q^m \sigma^m - [p^m q^o - C(q^o) - \alpha q^o \sigma^m] \\ &= 77.76 - 65.66 = 12.01. \end{aligned}$$

The value (12.01) is the maximum amount the agent would be willing to pay to adopt the more informed distribution $f^m(p)$.

Continuing with the supposition that only (A.2) has been fit to data, it can be shown that the ex-post measure given in (A.3) can be considered a lower bound to the value of perfect information for the risk averse agent if the compensation the agent demanded for the risk involved in producing q^o is viewed as a cost and hence removed from realized profits π^r . Recall from (7) that realized profits are $\pi^r = p^r q^o - C(q^o)$. In this case the

estimate of the ex-post value of perfect information, given $f^0(p)$, increases.

In the context of this problem, total cost (c) becomes

$$c = q^2 p_1 + \alpha q^0 \sigma = 151.2 + 28.92 = 180.12,$$

and

$$\overline{VI}_1 = p^r q^t - C(q^t) - [p^r q^0 - C(q^0, \alpha \sigma)] = 96 - 60.84 = 35.16.$$