Dynamics and Price Volatility in Farm-Retail Livestock Price Relationships

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This study uses an error correction model (ECM) to investigate dynamics in farm-retail price relationships. The ECM is a more general method of incorporating dynamics and the long-run, steady-state relationships between farm and retail prices than has been used to date. Monthly data for beef and pork are used to test the time-series properties for the ECM specification. The model is extended to study price volatility through the generalized autoregressive conditional heteroskedasticity (GARCH) process. Accommodation of the GARCH process provides a useful way of analyzing both mean and variance effects of policy or market structure changes.

Key words: cointegration, error correction models, GARCH process, long-run, unit root.

Introduction

Knowledge of farm-retail price relationships is important for many contemporary policy and commodity market analyses. Traditionally, this relationship has been specified by using a markup model (see Ward 1982; Heien; Lyon and Thompson; and Wohlgenant and Mullen, among others) or a reduced-form framework (e.g., Wohlgenant; Brorsen, Chavas, and Grant). Among these, Heien may have been first to formulate a dynamic model for agricultural pricing relationships. More recent studies have extended the empirical linkage between farm and retail prices to include dynamics and lag adjustments in the price determination process (e.g., Bailey and Brorsen; Schroeder and Goodwin; Babula and Bessler; Brorsen, Chavas, and Grant). These latter studies are based generally on a vector-autoregressive or a time-series framework and have established the importance of dynamics and lag adjustments in farm-retail price relationships, especially with shorter time period data.

Whereas past studies have highlighted the importance of short-run dynamics, the long-run structure of farm-retail price relationships has been neglected or studied in a limited context. Typically, the long-run behavior in agricultural pricing relationships has been inferred from time-series models identifying short-run dynamic behavior. Specifically, data are differenced to achieved stationarity, and the long-run effects are calculated as ratios of short-run parameters. This approach is restrictive in the sense that the long-run information is lost through differencing and does not account for the short-run nonstationary aspects of time-series data that are commonly observed. Furthermore, the standard errors of the long-run estimates, defined as the ratios of regression estimates, are difficult

to compute. Because moments of ratios of regression coefficients may not exist, approximation methods often are necessary.

Given that dynamics also are used to identify the long-run structure, one need not restrict econometric methods and techniques to merely identifying short-run dynamic adjustments. Instead, long- and short-run economic relationships can be estimated directly in an internally consistent systematic manner by adopting the error correction method. The error correction model (ECM) approach allows for direct estimation of the long-run, steady-state equilibrium condition implied by theory along with the short-run dynamic adjustments based on nonstationary properties of data. Thus, the model provides an opportunity to study retail-to-farm linkages in a framework accommodating both equilibrium hypotheses (e.g., Wohlgenant) and short-run dynamics (e.g., Brorsen, Chavas, and Grant) simultaneously.

Another feature of agricultural pricing relationships is the presence of price volatility in certain periods. Aradhyula and Holt reported that retail meat prices have become relatively volatile in recent periods. Such a phenomenon violates the basic assumption of homoskedastic variance resulting in a loss of efficiency among the estimated parameters. Modeling heteroskedasticity within the time-series framework not only overcomes this problem, but also provides information useful for evaluating the effects of external factors or shocks on both conditional means and the variances of farm and retail prices.

This study investigates farm-retail price relationships for beef and pork by using an estimation strategy that incorporates short-run dynamics, the steady-state relationship, and price volatility within a unified framework. The first two aspects are accommodated by extending the idea of cointegration[1] to link it to the error correction mechanism (Engle and Granger). The result is a dynamic econometric model that includes both short- and long-run effects. Price volatility, in the form of conditional heteroskedasticity, is incorporated by using the GARCH process developed by Engle and Bollerslev. The empirical analysis is carried out for beef and for pork, both of which have been studied extensively in terms of the factors affecting farm-retail price spreads (e.g., Wohlgenant; Holloway), but neither of which has been studied within a general dynamic framework.

Cointegration, Error Correction Models, and Price Dynamics

Time-series models often are used to analyze dynamic properties of price systems (Bessler; Bessler and Brandt) and interactions among farm, wholesale, and retail prices (e.g., Brorsen, Chavas, and Grant; Babula and Bessler). Vector autoregression (restricted and Bayesian) and transfer functions are used frequently in price analysis. These techniques were originally developed under standard statistical theory as it applies to stationary time-series data sets. If nonstationarity is observed, individual time series must be transformed to stationarity by using deterministic trends and seasonals, and/or differencing.

Differencing filters motivated by such stationarity requirements do not contain information on long-run, steady-state structure (Granger 1986; Harvey) that is of economic interest.[2] Furthermore, it is not necessary for all the variables in the regression equation to be stationary. All that is required is that the conditional distribution of the regression error be stationary (Hendry and Mizon). To overcome these problems, new methods have been suggested that accommodate nonstationarity properties of time-series data by means of cointegration systems.

Before proceeding further into model development, it is useful to set out certain terminologies and definitions. A variable, x_t, is said to be integrated of order d if it achieves stationarity after differencing d times, and is denoted as $x_t \sim I(d)$. Formally, a series x_{it} is integrated of order d, $x_{it} \sim I(d)$, if and only if $(I - B)^d x_{it}$ has a stationary, invertible, nondeterminsitic ARMA representation (see Engle and Granger), where B indicates the back lag operator. Thus, by definition for $x_{it} \sim I(0)$, x_{it} is stationary, and for $x_{it} \sim I(1)$, the first difference of the series is stationary.

The concept of cointegration for integrated series is provided formally in Granger (1981,

1986) and Engle and Granger. The cointegration concept states that an individual time series can wander extensively, yet, when paired with another series (or a set of series), the pairs will tend to move together consistently. Formally, N series in the vector x_t are cointegrated of order d [i.e., $x_t \sim CI(d - b)$] if all the N series are integrated of order d and there exists a linear combination of the N series $y_t = \Gamma'x_t$ such that $y_t = \Gamma'x_t \sim I(d - b)$ with $b > 0$, where Γ (known as the cointegrating vector) denotes the equilibrium condition between the elements of x. In this study, we deal with series that are only $I(1)$ and $I(0)$. For our purpose, it is simply stated that a set of variables integrated of order one, $I(1)$, are cointegrated if there exists a linear combination of these variables that are integrated of order zero, $I(0)$.

Under these conditions, Engle and Granger have shown that the time-series vector x_t can be modeled equivalently in the form of:

(*a*) a multivariate Wold representation, $(I - B)x_t = C(B)e_t$;

(*b*) a vector autoregressive (VAR) representation, $A(B)x_t = e_t$; and

(*c*) an error correction representation, $A^*(B)(I - B)x_t = -\Theta\Gamma'x_{t-1} + e_t = -\tau y_{t-1} + e_t$,

where

$$A(B) = I + A_1B + A_2B^2 + \ldots + A_pB^p,$$

$$A^*(B) = I + A_1^*B + A_2^*B^2 + \ldots + A_p^*B^p,$$

$$A_i^* = -\sum_{j=i+1}^{p} A_j,$$

$$C(B) = I + A_1B + A_2B^2 + \ldots + A_nB^n,$$

$$y_t = \Gamma'x_t; \quad \tau = \Theta\Gamma, \quad \text{and}$$

$$e_t \text{ is the error term.}$$

This result is due to the Granger representation theorem (Engle and Granger). An interesting result to note from the above equivalent representations is that cointegration implies the presence of levels of the variables in the error correction formulation.[3] Thus, the VAR models in differences may be misspecified if the variables are actually cointegrated.

A particularly appealing feature of the ECM is that the short-run responses of prices with respect to exogenous variables [$A^*(B)$] and their long-run relationships (Γ) are determined in a unified estimation framework. However, a linear steady-state structure is implied by the ECM. Given that agricultural pricing relationships usually are studied within a linear framework, the ECM attempts to reconcile time-series models and economic theory by merging short- and long-run effects.

A Dynamic Model for Price Relationships

The static relationship between farm and retail prices for livestock commodities can be specified as

(1) $$y_t = a_0 + bx1_t + cx2_t,$$

where y_t is the logarithm of farm price; $x1_t$ is the logarithm of retail price; $x2_t$ is the logarithm of the marketing cost index; a_0, b, and c are parameters; and t refers to time period. Equation (1) is a markup type of model,[4] augmented with a marketing cost variable, which has been used widely in empirical studies (e.g., Kinnucan and Forker; Wohlgenant and Mullen). Imbedded in this relationship is the assumption that retail prices determine farm prices.

Equation (1) is static and does not account for short-run dynamic adjustments in farm-retail price relationships. Because lag adjustments in price transmission and determination

are important (Heien; Bessler; Babula and Bessler; Bailey and Brorsen), the dynamic aspects are represented by a general distributed lag specification,

$$(2) \qquad y_t = a_0 + \sum_{i=1}^{m} a_i y_{t-i} + \sum_{j=0}^{n} b_j x1_{t-j} + \sum_{k=0}^{p} c_k x2_{t-k}.$$

This autoregressive, distributed lag model forms the basis for many analyses of dynamic schemes and long-run responses (Hendry, Pagan, and Sargan).

The error correction formulation is derived by transforming the general distributed lag model in equation (2) to incorporate explicitly the long-run, steady-state relationship between y and the exogenous variables (xs), along with the short-run dynamics. By repeated substitution, the steady-state relationships between y and other exogenous variables ($x1$, $x2$) can be deduced from (2) as (Harvey):

$$(3) \qquad y = \frac{a_0}{\left(1 - \sum_{i=1}^{m} a_i\right)} + \sum_{j=0}^{n} \frac{b_j}{\left(1 - \sum_{i=1}^{m} a_i\right)} x1 + \sum_{k=0}^{p} \frac{c_k}{\left(1 - \sum_{i=1}^{m} a_i\right)} x2$$

$$= \phi_0 + \phi_1 x1 + \phi_2 x2.$$

A number of (linear) transformations on the autoregressive-distributed lag model (2) are possible so as to directly identify the long-run, steady-state structure defined in (3) (Banerjee, Galbraith, and Dolado). Based on the original autoregressive distributed lag model in (2), the particular restrictions used for identifying the long-run structure in (3) are given by

$$(4) \qquad 1 - \theta = 1 - \sum_{i=1}^{m} a_i, \qquad\qquad \phi_0 = a_0/(1 - \theta),$$

$$\phi_1 = \sum_{j=0}^{n} b_j/(1 - \theta), \quad \text{and} \quad \phi_2 = \sum_{k=0}^{p} c_k/(1 - \theta).$$

Subtracting y_{t-1} from both sides of equation (2), and manipulating algebraically[5] to derive θ, ϕ_0, ϕ_1, and ϕ_2, as defined in (4), we get

$$(5) \qquad \Delta y_t = - \sum_{i=2}^{m} a_i (y_{t-1} - y_{t-i}) + b_0 \Delta x1_t - \sum_{j=2}^{n} b_j (x1_{t-1} - x1_{t-j})$$

$$+ c_0 \Delta x2_t - \sum_{k=2}^{p} c_k (x2_{t-1} - x2_{t-k})$$

$$+ (\theta - 1)[y_{t-1} - \phi_0 - \phi_1 x1_{t-1} - \phi_2 x2_{t-1}] + v_t,$$

where Δ is the difference operator such that $\Delta x_t = x_t - x_{t-1}$, $\theta = \sum_{i=1}^{m} a_i$, and v_t is the disturbance term. The unknown steady state parameters, ϕ_0, ϕ_1, and ϕ_2, can be estimated directly from equation (5) and inferences drawn on the long-run properties of the model.

Intuitively, the model in (5) states that the change in farm price is a function of both levels of and differences of dependent (farm price) and independent (retail price and marketing cost) variables. The salient feature of the error correction formulation can be found in the term within the square brackets in equation (5). This term reflects the past period's deviation from the steady-state solution in (3). Under stable conditions, this disequilibrium is corrected to the steady-state solution; hence, the term within the square brackets represents the mechanism for error correction (Harvey; Hendry, Pagan, and Sargan).

Although the error correction mechanism usually is captured with variables at lag $t - 1$, it could be introduced at another lag following a different normalization. In this respect,

the transformation adopted to derive the ECM is not unique. However, some researchers have argued that alternative transformations would produce numerically equivalent representations of the long-run multipliers (Banerjee, Galbraith, and Dolado). In the context of livestock price analysis with monthly data, an error correction mechanism at lag $t - 1$ seems reasonable. Also, the ECM provides a consistent analytical framework that combines short-run dynamics and the long-run, steady-state relationships among farm price, retail price, and marketing costs.

Time Series Properties of Livestock Prices

Dynamics of farm-retail price linkages are investigated for beef and pork commodities based on (5) using monthly data from January 1965 to December 1989. Both farm[6] and retail prices were collected from *Livestock and Meat Statistics* [U.S. Department of Agriculture (USDA) 1983, 1988] and from *Livestock and Poultry Situation* (USDA, various issues). Following Wohlgenant and Mullen, a marketing cost index was computed as the average of two indexes: the index for wage rates in the meat-processing industry and the producer price index for fuel related products and power. The data for wage rates were collected from *Employment and Earnings* [Bureau of Labor Statistics (BLS)], and data for the fuels products and power index were gathered from the *Survey of Current Business* (BLS).

Before estimating ECM, time-series properties of the data are examined to ensure the appropriate conditions for specifying ECM. As described earlier, the sufficient conditions for ECM specifications are that variables are integrated of the order of one and that they are cointegrated. The presumption that the variables are integrated of order one can be examined by using the unit root testing procedures.

Unit Root Tests

The presence of a unit root in economic time series commonly is tested by means of Dickey–Fuller (DF) or augmented Dickey–Fuller (ADF) tests (Dickey and Fuller; Fuller; Perron). The DF and ADF test statistics are derived for autoregressive (AR) models with lags of first differences of the series included as regressors. These test statistics are derived under the assumption that the sequences of innovations are identical and independent (normally) distributed with common variance. It has been shown that if a moving average representation of the series is important (instead of AR), a large number of lags of first differences of the variable are needed as regressors in the autoregressive correction of the ADF test (Schwert). This approach, therefore, involves the estimation of additional nuisance parameters reducing the effective number of observations.

Recently, Phillips, and Phillips and Perron derived testing procedures for the (null) unit root hypothesis under more general (weaker) conditions. In the current study, Phillips–Perron tests, referred to as Z-tests, are applied to the logarithm of each price series. Accordingly, the null hypothesis of a unit root is tested by using the following ordinary least square (OLS) regressions:

$$(6) \qquad\qquad Y_t = \hat{\alpha} Y_{t-1} + \hat{e}_t,$$

$$(7) \qquad\qquad Y_t = \mu^* + \alpha^* Y_{t-1} + e_t^*, \quad \text{and}$$

$$(8) \qquad\qquad Y_t = \tilde{\mu} + \tilde{\beta}(t - T/2) + \tilde{\alpha} Y_{t-1} + \tilde{e}_t,$$

where Y_t denotes the economic time series and T denotes the sample size.

Equation (6) contains neither a constant nor a trend, whereas equation (7) has a constant term. The test statistic for the hypothesis that $\hat{\alpha} = 1$ in equation (6) is represented by $Z(t_{\hat{\alpha}})$. In equation (7), two test statistics are calculated, $z(t_{\alpha^*})$ and $Z(\Phi_1)$, respectively, for the null hypotheses $\alpha^* = 1$ and $\mu^* = 0$, $\alpha^* = 1$. Equation (8) contains constant, trend,

Table 1. Unit Root Tests on Farm and Retail Prices for Beef and Pork

Variable	Z-Test Statistics					
	$Z(t_{\tilde{\alpha}})$	$Z(t_{\alpha^*})$	$Z(\Phi_1)$	$Z(t_{\tilde{\alpha}})$	$Z(\Phi_2)$	$Z(\Phi_3)$
Null hypothesis:	$\hat{\alpha} = 1$	$\alpha^* = 1$	$\alpha^* = 1$; $\mu^* = 0$	$\tilde{\alpha} = 1$	$\tilde{\alpha} = 1$; $\tilde{\beta} = 0$	$\tilde{\alpha} = 1$; $\tilde{\beta} = \tilde{\mu} = 0$
Beef farm price	1.18	−.89	4.08	−2.93	3.45	3.49
Beef retail price	2.27	−.48	5.79[a]	−2.10	3.61	4.02
Pork farm price	−.20	−2.18	2.47	−3.43	3.96	4.26
Pork retail price	1.21	−.67	2.84	−3.29	4.31	5.48
Marketing cost	1.54	−.97	3.43	−.89	2.08	2.21
Critical value[b]	−1.95	−2.88	4.68	−3.43	4.73	6.32

[a] Indicates statistical significance at the 5% level.
[b] Critical values are given for 95% probability level.

and lag terms; correspondingly, three hypothesis tests are performed. The test statistics $Z(t_{\tilde{\alpha}})$, $Z(\Phi_3)$, and $Z(\Phi_2)$ are computed for the null hypotheses that $\tilde{\alpha} = 1$; $\tilde{\beta} = 0$, $\tilde{\alpha} = 1$; and $\tilde{\mu} = 0$, $\tilde{\beta} = 0$, $\tilde{\alpha} = 1$, respectively. See Perron for the precise form of the algebraic expressions for these test statistics.

Unlike ADF tests, these test procedures do not require the estimation of additional nuisance parameters (differenced lagged terms) saving valuable degrees of freedom. The approach to calculating these tests also takes into consideration the correlation structure of the residuals (e_ts) in a nonparametric way.

The Z-test results for presence of a unit root in farm and in retail prices of beef and pork, and the marketing cost variable are presented in table 1. The results are computed by using a maximum lag of 16 for the autocovariances of the residuals, according to a weighting pattern suggested by Newey and West.[7] If the value of the calculated test statistic is smaller than the critical value, the null hypothesis of a unit root is not rejected. Only one out of the total of 30 statistics reported is significant at the 5% level. Thus, the null hypothesis of a unit root is rejected in only one case, the $Z(\Phi_1)$ statistic for retail price of beef. Therefore, the results support the presence of a unit root in the farm and the retail prices of beef and pork, as well as the marketing cost variable.

Cointegration Tests

A number of tests for cointegration of series are available (Phillips and Ouliaris). Among these, the residual based tests are popular because of their ease and convenience in empirical applications. Residual based tests involve estimating the cointegration regression

$$(9) \qquad y_t = \lambda_0 + \lambda_1 x1_t + \lambda_2 x2_t + \epsilon_t,$$

where ϵ_t indicates the residual (all other variables are as defined previously), and performing unit root tests on the estimated residuals, $\hat{\epsilon}_t$. The null hypothesis of no cointegration is tested through an equivalent form of the null hypothesis of a unit root in the residuals.

In this study, since more than one exogenous variable is involved, the cointegration system is of higher order. Engle and Granger recommend the augmented Dickey–Fuller (ADF) test for higher order systems, whereas Phillips developed two other residual based tests, known as Z_α and Z_t tests, that might have power properties superior to the ADF. However, the ADF and Z_t tests are asymptotically equivalent and are based on t-ratio procedures, whereas the Z_α test is a direct coefficient test. All three residual based tests, namely ADF, Z_α, and Z_t statistics, are employed in this study to test for cointegration between the dependent variable (farm price) and the independent variables (retail prices and marketing cost).

Table 2.   Tests of Cointegration Between Farm and Retail Prices of Both Beef and Pork

Test Statistic	Beef	Pork
ADF(1)	−5.76*	−4.36*
ADF(2)	−5.68*	−3.76
ADF(3)	−5.59*	−3.45
Z_α	−52.15*	−36.98*
Z_t	−5.36*	−4.39*

Notes: Asterisk indicates statistical significance at the 5% level. The cointegration regression contains an intercept and two exogenous variables. The critical values for ADF statistics at the 5% level of significance are −3.78 (Engle and Yoo, table 3) and −3.77 (Phillips and Ouliaris, table IIb), respectively, for 200 and 500 degrees of freedom. The approximate critical values at the 5% level for Z_α and Z_t statistics are, respectively, −26.09 and −3.77 (Phillips and Ouliaris, tables Ib and IIb).

The ADF test statistics are derived from the residuals based on (9) by means of the regression

$$(10) \qquad \qquad \Delta\hat{\epsilon}_t = -\rho\hat{\epsilon}_{t-1} + \sum_{l=1}^{p} \delta\Delta\hat{\epsilon}_{t-1} + \xi_t.$$

The ADF test statistics are computed by dividing the estimated ρ by its standard error. For details on the calculation of Z_α and Z_t test statistics, see Phillips and Ouliaris. It should be pointed out that the residual based tests are numerically dependent on the precise formulation of the cointegration regression, that is, whether it is mean corrected or detrended. In other words, these tests are not invariant to the normalization of the cointegration regression. However, to be consistent with the long-run, steady-state relationship specified in (3), only a constant term is included in the cointegration regression (9).

The null hypothesis of no cointegration between variables is rejected when the calculated values are smaller than their respective critical values. The results for cointegration of farm price with retail price and marketing cost are presented in table 2. The ADF statistics are reported for different lag lengths, up to a maximum of three. The ADF statistics for beef indicate that the null hypothesis of no cointegration is rejected at the 5% level of significance. For pork, one of the ADF statistics is significant. The ADF(2) statistic is approximately equal to its critical value. On the other hand, the Z_α and Z_t tests support the alternate hypothesis of cointegration between farm price and retail price and marketing cost at the 95% probability level. Overall, test results support the hypothesis of cointegration for the specified farm-retail price relationships, implying that the price relationships for both pork and beef commodities can be represented adequately by an ECM.

ECM and Garch Error Process

Having established the conditions for ECM specification between farm and retail prices, the next step is to identify the appropriate lag lengths for each commodity in equation (2). In choosing the appropriate lags for each commodity, one must keep parsimony in mind. A maximum of 24 lag lengths is considered, with the lag coefficients not contributing significantly to statistical performance of the model omitted. Once a model that adequately reflects the data generating process is identified, the ECM specification corresponding to equation (5) is derived. The ECM's parameters are estimated by maximum likelihood, and the results for both beef and pork are presented in tables 3 and 4, respectively. The subscripts to the parameters *a, b,* and *c* reported in tables 3 and 4 indicate the order of

Table 3. Maximum Likelihood Estimates of ECM and ECM/GARCH Models for Beef

Param-eters	ECM		ECM/GARCH	
	Estimated Coefficient	Asymptotic t-Values	Estimated Coefficient	Asymptotic t-Values
a_2	−.009	−.913	−.053	−.913
a_{11}	.020*	6.186	.235*	6.186
a_{13}	−.150*	−4.491	−.173*	−4.491
b_0	1.286*	10.187	1.141*	10.187
b_2	.173*	3.396	.176*	3.396
b_{11}	−.337*	−5.265	−.359*	−5.265
b_{13}	.109	1.653	.117	1.653
c_0	.104	.334	.273	.334
c_2	2.645	1.516	1.751	1.516
c_4	−2.167	−1.449	−1.584	−1.449
c_5	1.706	1.647	1.308	1.647
$\theta - 1$	−.100*	−5.269	−.104*	−5.269
Φ_0	−.073	−.071	−.031	−.071
Φ_1	.740*	2.141	.668*	2.141
Φ_2	.238	1.111	.308	1.111
α_0	.001*	13.460	.001*	2.109
α_1			.086*	2.449
β_1			.848*	18.056
Log (L)[a]	848.867		857.537	
Q-Statistics:[b]				
$Q(12)$	11.42		10.96	
$Q(24)$	27.09		29.18	
$Q^2(12)$	46.35*		23.92	
$Q^2(24)$	53.43*		28.94	

Note: Asterisk indicates significance at the 5% probability level.
[a] Log (L) denotes the log-likelihood values, up to a constant.
[b] The Q-statistics denote Box–Pierce–Ljung portmanteau tests for autocorrelation, which are distributed as chi-squares with degrees of freedom equal to the lag provided within the parentheses. The critical values at the 5% level of significance are 21.03 and 36.42, respectively, for 12 and 24 degrees of freedom.

lag length included in the model. For instance, lags 2, 4, and 5 are used for the marketing cost variable in the beef ECM models.

The results indicate that retail prices have a positive and significant effect on the long-run farm prices of both beef and pork. The long-run elasticities of farm price with respect to retail price are .74 and 1.28, respectively, for beef and for pork. In the Wohlgenant study, elasticities of farm price with respect to the retail demand shifter were 1.320 and 1.963, respectively, for beef and for pork.

Further information about the validity of the estimated ECM models is obtained by examining Box–Pierce portmanteau Q-statistics associated with fitted residuals ($\hat{V}_t$). Tables 3 and 4 report Q-statistics for the residuals associated with beef and pork ECMs, respectively. In both instances, the Q-statistics are smaller than the critical value 21.03 (36.42) at 12 (24) degrees of freedom. Thus, the null hypothesis that the residuals from the estimated ECMs are white noise cannot be rejected.

A different picture arises, however, when squared residual series, $\hat{V}_t^2$, are examined. As McLeod and Li report, the portmanteau test statistic $Q^2(m)$ associated with the first m-squared innovations will be distributed asymptotically as a $\chi^2(m)$ under the null hypothesis of no heteroskedasticity. In both beef and pork ECMs, $Q^2(12)$ is significant, while $Q^2(24)$ also is significant for pork at the 5% level. This indicates the presence of heteroskedasticity of some form in the farm-retail pricing relationships. As Bollerslev suggests, the

absence of serial correlation in the conditional first moments, coupled with the presence of serial correlation in the conditional second moments, is one indication of the presence of a GARCH error process.

Modeling Price Volatility and the GARCH Process

The GARCH process provides a convenient form of representing heteroskedasticity of unknown form in time-series data. Under GARCH, the conditional variance of a time series follows an autoregressive moving average (ARMA) representation of the squared residuals of the random process (Engle; Engle and Bollerslev). Let Ω_{t-1} be the set of all relevant and available information at time $t - 1$. The GARCH process for a normally distributed innovation series, v_t, is given by

$$(11) \qquad\qquad\qquad v_t \mid \Omega_{t-1} \sim N(0, h_t),$$

$$(12) \qquad\qquad\qquad h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i v_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j},$$

where

$$p \geq 0, \quad q \geq 0,$$

$$\alpha_0 > 0, \alpha_i \geq 0, \quad i = 1, \ldots, q,$$

$$\beta_j \geq 0, \quad j = 1, \ldots, p, \text{ and}$$

$$\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1.$$

The conditional variance equation in (12) describes a GARCH(p,q) process whereby the time-dependent, conditional variance is specified as a function of lagged squared innovations, v_{t-i}^2, and the past behavior of the variability, h_{t-j}. The nonnegativity restrictions on the α and β parameters and the requirement that the sum of all α_is and β_js is less than one in equation (12) are necessary to guarantee that the conditional and unconditional variances associated with the GARCH model are positive-valued. It should be noted that for $p = 0$, the process reduces to an autoregressive conditional heteroskedasticity (ARCH) process. Also, the β coefficients in equation (12) indicate the persistence effect in the variance of a GARCH process. Thus, the GARCH process not only provides a way to model heteroskedasticity of an unknown form, but also captures persistence in the conditional variances of time-series data sets.

Although time-series methods can be used to chose p and q, as Bollerslev suggested, a GARCH(1,1) process is probably appropriate in most empirical situations. Accordingly, this study adopts a GARCH(1,1) process for the innovations associated with the farm-retail price relationships.

The results of combining the ECM with GARCH(1,1) error process (referred to as ECM/GARCH) are obtained by using maximum likelihood methods to estimate the parameters of equations (5), (11), and (12) simultaneously. The log likelihood function used to estimate an ECM/GARCH model for a sample of T observations is given by

$$(13) \qquad\qquad \log(L) = (-T/2)\log(2\pi) - 0.5 \sum_{t=1}^{T} [\log(h_t) + (v_t^2/h_t)].$$

Estimation is carried out using the Davidson–Fletcher–Powell (DFP) algorithm with numerical derivatives after conditions for the nonnegativity of GARCH parameters are imposed. Following Cecchetti, Comby, and Figlewski, the nonnegativity constraint of the GARCH process was maintained by squaring the parameters in (12).

The results of the ECM/GARCH process for farm prices of beef and of pork are presented in the right-hand columns of tables 3 and 4, respectively. Note that the ECM is nested

Table 4. Maximum Likelihood Estimates of ECM and ECM/GARCH Models for Pork

Param- eters	ECM		ECM/GARCH	
	Estimated Coefficient	Asymptotic *t*-Values	Estimated Coefficient	Asymptotic *t*-Values
a_4	−.016	−.380	−.001	−.012
a_5	.087*	2.208	.056	1.355
a_{11}	.151*	2.596	.186*	3.076
a_{12}	.036	.497	.042	.578
a_{13}	−.153*	−2.699	−.120*	−2.005
b_0	2.209*	16.630	2.284*	16.584
b_4	−.206	−1.054	−.321	−1.689
b_{12}	−.272	−1.065	−.248	−1.013
b_{13}	.422*	2.886	.391*	2.847
c_0	−2.345	−1.657	−2.706	−1.920
c_2	1.464	.734	1.319	.677
c_4	−1.581	−.871	−1.286	−.660
c_5	.527	.426	.406	.296
$\theta - 1$	−.208*	−3.772	−.211*	−3.862
Φ_0	−.335	−1.072	−.016	−.461
Φ_1	1.284*	6.771	1.206*	6.281
Φ_2	−.403*	−2.669	−.353*	−2.368
α_0	.003*	13.714	.001	1.893
α_1			.106	1.620
β_1			.718*	5.649
Log (L)[a]	698.623		703.750	
Q-Statistics:[b]				
Q(12)	19.02		14.45	
Q(24)	28.26		25.31	
Q^2(12)	21.36*		4.73	
Q^2(24)	27.87		9.74	

Note: Refer to notes to table 3.

with ECM/GARCH(1,1) when $\alpha_1 = \beta_1 = 0$ in equation (12). Statistics for the likelihood ratio tests of the null hypothesis of constant conditional variances are computed by using the estimated ECM and the ECM/GARCH. This test statistic is asymptotically distributed as a chi-square with two degrees of freedom. The calculated likelihood ratio test statistics, 17.34 for beef and 10.25 for pork, are greater than the critical chi-square value of 5.99 (at the .05 level), indicating that the null hypothesis of constant conditional variances is rejected in favor of the ECM/GARCH model for beef and pork farm prices. The Box–Pierce Q-test statistics also are reported for the standardized residuals ($v_t/\sqrt{h_t}$) along with the square of the standardized residuals (v_t^2/h_t) from the estimated ECM/GARCH models. In each case, the calculated values for Q and for Q^2 are smaller than the critical values of the chi-square distribution at the 5% level; thus, no further first- or second-order serial dependence is observed in the estimated ECM/GARCH models.[8] However, the estimated long-run parameters differ only marginally from those of the ECM model without the GARCH process.

Discussion and Implications

The results of this study are useful for drawing inferences about many features of livestock price relationships. Time-series analysis of farm and retail prices of beef and pork and of the marketing cost variable suggested that individual economic series are nonstationary, and that farm prices are cointegrated with retail prices and marketing costs. This brings out two related issues in modeling farm-retail price linkages, namely dynamics and long-run structure.
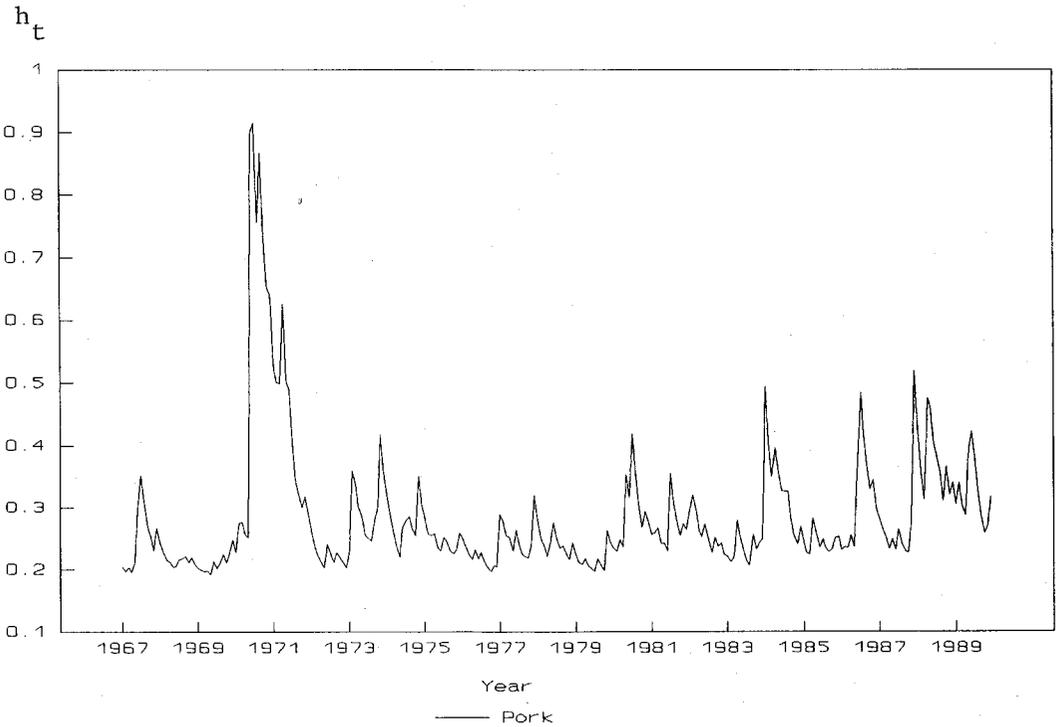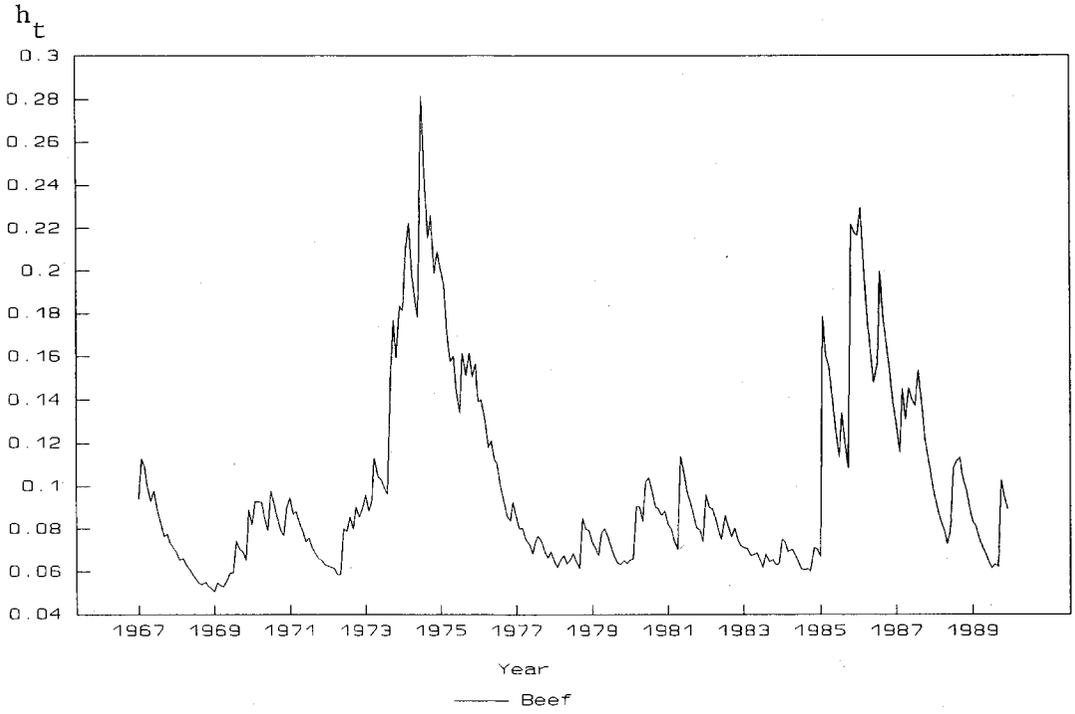
Figure 1. Estimated conditional variances in farm prices using ECM/GARCH models

Note: The h_t values are multiplied by 100 for convenience in presentation.

Unlike previous efforts where short-run dynamics and long-run relationships were treated separately, this study employed an alternative method of modeling time-series dynamics. The ECM framework used in this study provides a convenient form of deriving the long-run structure, while accounting for nonstationarity in the individual time series.

Albeit a sufficient condition, the cointegration results conform to the existence of a long-run, steady-state relationship between the farm prices of beef and pork and the retail prices and marketing costs of these goods. The empirical results indicate that own retail prices have a significant and positive effect on the respective farm prices of beef and pork in the long run. This finding suggests that agricultural policies such as price stabilization or food stamp programs directed at consumer demand may have a significant effect on long-run farm prices through movements in retail prices. The long-run effects of a 1% change in retail price were estimated at .67% and 1.2%, respectively, for farm prices of beef and pork. However, the long-run effect of the marketing cost variable on farm price was significant only for pork. For a 1% change in the marketing cost index, the farm price of pork changed by only .35%.

The lack of long-run estimates based on other dynamic schemes for beef and pork commodities makes it difficult to compare the estimates based on our ECM/GARCH models. Simulation studies have indicated that long-run estimates are sensitive to the magnitude of the lag coefficients and the assumptions about stationarity of independent variables (Bewley and Fiebig). Given that prices and marketing cost series are nonstationary, the long-run estimates derived based on the ECM/GARCH model are appealing. Further, since the dynamic model adopted here provides direct estimates of the long-run parameters, it is useful for hypothesis testing on the long-run performance of farm price determination.

Another aspect of the dynamic framework is the speed of adjustment toward the long-run equilibrium. This is reflected by the relative magnitude of the θ parameter. Estimated values for θ are .9 (table 3) and .8 (table 4), respectively, for beef and pork. Thus, for shocks within the system, farm prices of beef adjust more rapidly toward the long-run equilibrium than pork farm prices. For both commodities, the θ parameters are significant, indicating the importance of the error correction mechanism. Studies also have demonstrated that the speed of adjustment may be affected by the degree of competition and the market structure (Weaver, Chattin, and Banerjee). Inasmuch as different adjustment measures are available, the dynamic model presented here provides a direct method of estimating adjustment parameters to perform such analysis.

The empirical results also reveal that the innovations for farm-retail price relationships follow a GARCH(1,1) process. This shows that the conditional variances for farm prices of beef and pork are not constant. Figure 1 shows the temporal nature of the conditional variances in farm prices of beef and pork. The conditional variances of the beef farm price showed two distinct periods of high volatility—the mid-1970s and 1980s. Pork farm prices were highly volatile during the early 1970s. Although several jumps in pork price volatility were observed during the 1980s, the levels were not as high as for the early 1970s. In contrast, the price volatility in beef during the mid-1980s was very close to the level observed during the mid-1970s.

The GARCH process for prices also implies persistence in the price volatility. This may be important in some policy analyses. For instance, there is considerable debate over the effect of changing market structure on price volatility (Carlton; Ward 1988). Furthermore, there is continuing concern over the issues of structural change in the meat industry due to higher concentration in the processing sector, changes in eating habits and demographs, and increased health awareness and nutrition education programs.

Concluding Remarks

This study investigates dynamics in farm-retail price relationships within a general framework based on an Error Correction Model (ECM). A specification based on the ECM was

applied to monthly data for farm prices of both beef and pork and was found valid and appropriate for the study of farm-retail price linkages. The estimated ECM model was extended to model the time-varying, conditional variance of prices by using a GARCH error process.

In a changing environment such as the livestock market, a model is needed that combines the desirable aspects of dynamics, static equilibrium, and price volatility. The ECM/GARCH model is a step toward such a unified approach.

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Notes

[1] The concept of cointegration is described in the next section.

[2] In a differenced specification, the long-run elasticities tend to be infinite, which is of limited value for the purpose of long-term agricultural planning and policy evaluations.

[3] However, presence of cointegration is only a *sufficient* condition for ECM formulation.

[4] Wohlgenant's model includes both supply and demand shifters in farm and in retail price determination. Our specification is similar to the reduced-form equation in Brorsen, Chavas, and Grant, with no supply shifters. Since the purpose of this study is to investigate the dynamics of farm-retail price linkages, the commonly used markup model augmented with a marketing cost index is used. The analysis can be extended easily to other more refined models dictated by theory.

[5] As one reviewer correctly pointed out, the manipulation involved in the transformation imposes the following normalization:

$$a_0 = (1 - \theta)\phi_0; \qquad a_1 = \theta - \sum_{i=2}^{m} a_i;$$

$$b_1 = (1 - \theta)\phi_1 - b_0 - \sum_{j=2}^{n} b_j;$$

$$c_1 = (1 - \theta)\phi_2 - c_0 - \sum_{k=2}^{p} c_j$$

to identify the long-run structure with variables at lag $t - 1$.

[6] The data for farm prices are measured by the gross farm value in cents-per-pound equivalent to one pound of retail weight.

[7] The weighting procedure basically ensures a nonnegative estimate for variance. Perron also recommended a check on the sensitivity of the results to various lag lengths. Accordingly, the test statistics are calculated for lag lengths of 8, 12, and 20, and the results regarding unit roots are insensitive to these lags.

[8] For a GARCH(1,1) process, the fourth-order moment exists if $3\alpha_1^2 + 2\alpha_1\beta_1 + \beta_1^2 < 0$. Checks of the estimated GARCH parameters indicate that the fourth-order moment of v_t exists for each model. The estimated GARCH parameters also satisfy the stationarity conditions, $\alpha_1 + \beta_1 < 1$. Hence, the asymptotic properties of the maximum likelihood estimates are established.

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