Annual and Monthly SNAP Participation Rates

Mark Prell, Constance Newman, and Erik Scherpf
Annual and Monthly SNAP Participation Rates

Mark Prell, Constance Newman, and Erik Scherpf

Abstract

A key aspect of the U.S. Department of Agriculture, Food and Nutrition Service’s (FNS) Supplemental Nutrition Assistance Program (SNAP) is the extent to which it reaches its target population—the rate of participation among people who are eligible for SNAP benefits (eligibles). FNS publishes national and State-level estimates of participation among eligibles for an average month of the year. This report demonstrates the feasibility and usefulness of estimating an annual SNAP participation rate, which counts the number of individuals who participated at some time during the year as a share of individuals who were eligible at some time during the year. An annual SNAP participation rate matches the annual timeframe that several Federal surveys adopt when collecting data on SNAP participation. It also captures how well the program reaches all people who are eligible for it at any time during the year, independent of the number of months they were eligible. The monthly rate tends to capture those who have relatively more months of eligibility during the year. Results show that the national annual SNAP participation rate among eligibles for 2012 (the most recent year with suitable data) is about 70.7 percent—more than 10 percentage points below the monthly participation rate of 83.1 percent.

Keywords: Supplemental Nutrition Assistance Program, SNAP, participation rate, program performance, length-biased sampling, targeting, SNAP administrative data, underreporting, poverty

Acknowledgments

We thank James Sullivan of the University of Notre Dame; Laura Wheaton of the Urban Institute; Michael Wiseman of the George Washington University; Bob Dalrymple and Jenny Genser of USDA, Food and Nutrition Service (FNS); Kelly Kinnison formerly of USDA, FNS; and John Pender of USDA, Economic Research Service (ERS) for their peer reviews. We also thank ERS editor Susmita Pendurthi and ERS designer Curtia Taylor.
## Contents

Summary ................................................................................................................... iii

Introduction ............................................................................................................. 1

Complementary measures of SNAP participation ............................................. 2

The relationship between annual and monthly SNAP participation rates .......... 6

Estimated annual and monthly distributions of months of eligibility ............... 9

National annual and monthly SNAP participation rates ..................................... 16

Behavioral explanations for intrayear dynamics of eligibility and participation .... 20

Contrasting annual versus monthly eligibles by selected demographic subgroups ... 22

New York State case study ..................................................................................... 26

Conclusion ............................................................................................................. 29

References ............................................................................................................. 31

Appendix 1—A new version of length-biased sampling ...................................... 34

Appendix 2—Intrayear dynamics of eligibility and participation determine
SNAP participation rates ......................................................................................... 38
Annual and Monthly SNAP Participation Rates

Mark Prell, Constance Newman, and Erik Scherpf

What Is the Issue?

A key aspect of the U.S. Department of Agriculture, Food and Nutrition Service’s (FNS) Supplemental Nutrition Assistance Program (SNAP) is how well the program reaches its target population, measured by the participation rate among those eligible for SNAP benefits. Knowing who receives SNAP benefits, and who does not among those eligible, is important for assessing and improving program performance.

SNAP uses monthly circumstances to determine program eligibility and benefit amounts, and FNS publishes national and State-level estimates of participation among those eligible for an average month of the year (monthly SNAP participation rates). To complement the monthly participation rate, this report develops a new measure of SNAP’s reach—an annual SNAP participation rate, defined as annual participants (who receive SNAP for 1 or more months of the year) as a proportion of annual eligibles (who are eligible for 1 or more months). The average monthly SNAP participation rate gives more weight to individuals who have more months of eligibility during the year, while an annual participation rate gives equal weight to all eligible individuals, regardless of how long they were eligible during the year.

What Did the Study Find?

Estimates of program participants and eligibles differ markedly depending on the timeframe used and the group examined. For 2012:

- About 45 percent more individuals were estimated to be eligible for SNAP at some time during the year than were eligible in an average month of that year. There were only 20 percent more elderly eligible at some time during the year than in an average month, and only 6 percent more disabled people.
- About 57 percent of individuals who were ever eligible in the year were eligible for only part of the year (1 to 11 months), while the rest (43 percent) were eligible for the full year. Of those ever eligible in the year, about 66 percent of elderly individuals (age 66 years or older) were estimated to be eligible all 12 months, as were about 77 percent of disabled people.
- In contrast, about 84 percent more working poor were eligible for SNAP at some time during the year than in an average month, reflecting the transitory nature of eligibility for the working poor. Only about 25 percent of the working poor who were ever eligible during the year were eligible all 12 months.
An estimated 76.6 million people were eligible at some time during the year, while only about 54.1 million people ever participated. The resulting annual SNAP participation rate of 70.7 percent is 12.4 percentage points below the FNS estimate of the average monthly SNAP participation rate of 83.1 percent for fiscal 2012 (October-September).

While estimates of annual participants exceed monthly participants (54.1 versus 42.1 million), annual eligibles exceed monthly eligibles by an even larger proportion (76.6 versus 50.7 million), making the annual SNAP participation rate lower than the monthly rate.

New York State was examined as a case study for this report based on access to this State’s confidential micro-level SNAP records. With an estimated 5 million people eligible at some time during the year, the 2012 annual SNAP participation rate for New York State was 74.5 percent, which is below the State’s monthly participation rate of 79.7 percent.

**How Was the Study Conducted?**

The data for estimating the annual U.S. SNAP participation rate for 2012 come from the 2008 panel of the U.S. Census Bureau’s Survey of Income and Program Participation (SIPP), a nationally representative survey that provides longitudinal data by month. The data available in SIPP for estimating eligibility for SNAP are less complete than the data used by local SNAP offices to make official determinations of SNAP eligibility. Analysis for the New York State estimates was conducted at the U.S. Census Bureau using State-level SNAP administrative data and data from the American Community Survey.
Annual and Monthly SNAP Participation Rates

Mark Prell, Constance Newman, and Erik Scherpf

Introduction

A key measure of the performance of the U.S. Department of Agriculture, Food and Nutrition Service’s (FNS) Supplemental Nutrition Assistance Program (SNAP) is the extent to which the program reaches its target population, which is captured by the rate of SNAP participation among people who are eligible for SNAP benefits (eligibles). SNAP requires using monthly circumstances to determine program eligibility and the benefit allotment. A standard approach to measuring SNAP’s reach is to estimate a monthly SNAP participation rate using participation and eligibility estimates measured for an average month of a fiscal year. In this report, we develop a new measure—an annual SNAP participation rate, which uses participation and eligibility estimates measured “at some time during the year.”

The annual SNAP participation rate is useful for understanding how well the program serves all individuals who were eligible during the year, counting everyone equally regardless of whether they were eligible for many months or only a few. In contrast, the monthly SNAP participation rate captures how well the program serves those individuals who are eligible in an average month and, accordingly, are more likely to be eligible for many months. The annual and monthly SNAP participation rates are complementary, assessing program performance for different target populations—annual (ever-during-the-year) eligibles versus monthly (average month) eligibles.

This report provides 2012 estimates of an annual SNAP participation rate for the United States and for selected demographic subgroups. In addition, using State-level SNAP administrative records, our study estimated the 2012 annual SNAP participation rate for the State of New York. We compare the estimated U.S. and New York annual participation rates to their corresponding monthly rates to provide a more comprehensive view of SNAP’s reach. New York State’s administrative records also allow us to assess the extent to which error in the measurement of participation in self-reported survey data can affect estimates of an annual participation rate.
Complementary measures of SNAP participation

To evaluate SNAP’s performance, the question “How effectively does SNAP serve those who were eligible in an average month of the year?” differs from “How effectively does SNAP serve those who are eligible at some time during the year?” The answer to each question involves a distinct statistical tool:

(1) Monthly SNAP Participation Rate: Each person-month of eligibility receives equal weight and people with more months of eligibility receive more weight.

* Monthly participants = Number of individuals who participate in SNAP in an average month of the year.
* Monthly eligibles = Number of individuals who are eligible for SNAP in an average month of the year.
* Person-month = One month of eligibility for one person; a person who is eligible for 8 months has 8 person-months of eligibility, as do 2 people each with 4 months of eligibility.

\[
\text{Monthly SNAP Participation Rate} = \frac{\text{monthly participants}}{\text{monthly eligibles}}
\]

(2) Annual SNAP Participation Rate: Each person (annual eligible) receives equal weight.

* Annual participants = Number of individuals who participate in SNAP “at some time during the year” (i.e., in 1 or more months).
* Annual eligibles = Number of individuals who are eligible for SNAP “at some time during the year” (i.e., in 1 or more months).

\[
\text{Annual SNAP Participation Rate} = \frac{\text{annual participants}}{\text{annual eligibles}}
\]

The monthly SNAP participation rate is weighted toward people who are eligible for a relatively large number of months—a person’s probability of being identified as eligible in an average month increases in proportion to the number of months of eligibility. For example, a person who is eligible for 6 months carries double the statistical weight of the 3-month eligible, while a 12-month eligible carries four times the statistical weight of the 3-month eligible. People who have greater eligibility intensity (the number of months a person is eligible during a year) may have a greater need for SNAP, so the monthly SNAP participation rate especially captures how well SNAP serves individuals whose eligibility for SNAP lasts for more months. In contrast, the annual SNAP participation rate especially captures how well SNAP reaches all individuals who are eligible, regardless of how many months eligibility lasted that year.

SNAP examines a household’s composition, income, and expenditures for a month-long accounting period when making an official determination of eligibility for SNAP benefits.¹ FNS regularly publishes estimated SNAP participation rates developed for an average month of the fiscal year (what

¹ Details on eligibility requirements for fiscal 2012 are provided in Gray and Eslami, 2014. The term accounting period can be found in studies such as Ohls and Beebout, 1993, and Citro and Michael, 1995.
we call monthly participation rates). However, an accounting period of one month does not imply that it is necessarily better to examine SNAP’s reach using the monthly participation rate instead of the annual rate. Program officials can be interested in both. Which rate is a better tool for examining program performance depends on the motivation for estimating a SNAP participation rate.

The target populations of annual and monthly eligibles differ by:

- **Size.** Given that some people are eligible for only part of the year, the people who are eligible at some time during the year necessarily outnumber those who are eligible in an average month.

- **Composition.** The two populations of eligibles comprise different mixes of people who exhibit different patterns of SNAP participation and other characteristics.

In addition to SNAP participation itself, other variables and characteristics correlated with length of eligibility include age, disability status, and work status. Individuals who are retired and have low income or who have work-limiting disabilities are more likely to be eligible for all 12 months of the year. In contrast, individuals who have relatively stronger connections to the labor market are likely to be eligible for just part of the year; variability in monthly income is a major factor that moves people into or out of eligibility over the course of a year.

The distinction between annual and monthly perspectives is pertinent not only to analyses of participation rates but also to studies of the characteristics of participants. Distributions and averages of characteristics depend on whether the population under study receives SNAP in an average month or ever during the year. Each year, FNS publishes a report in its *SNAP Characteristics* series that provides demographic and economic information about official SNAP units and the individuals who live in them. The chosen target universe of the report is the population of SNAP participants who participate in an average month of the year. The population of SNAP participants who participate at some time during the year could be a second target universe, but it would require a different sample design. The data used by the *SNAP Characteristics* reports capture the SNAP units that are especially reliant on SNAP, as reflected in relatively more months of participation. Because there can be programmatic interest in those units, the *SNAP Characteristics* reports are a key source of information and analysis.

At the same time, an important target population for SNAP is the working poor and SNAP’s support for them may not be portrayed fully in the *SNAP Characteristics* reports. A relatively large number of the working poor, with transitorily low income, may receive SNAP benefits for only part of the year.

---

2 The latest Food and Nutrition Service (FNS) reports in the annual series on national and State Supplemental Nutrition Assistance Program (SNAP) participation rates are from Eslami, 2014, and Cunnyngham, 2015. The methodology used to estimate State SNAP participation rates is described in Cunyngham et al., 2015. Based on available data, our estimates for annual SNAP participation rates are developed for calendar year 2012. In contrast, FNS estimates are developed on a fiscal-year basis. Due to this disparity, the two sets of estimates are not fully comparable.

3 Official Supplemental Nutrition Assistance Program (SNAP) units in SNAP administrative data are groups of people who are deemed by the SNAP office to purchase and prepare food together. The SNAP Characteristics reports and others frequently refer to SNAP units as SNAP households. Here, we retain the term unit to maintain a distinction between these SNAP units and the household, which is defined as a residence in the U.S. Census Bureau survey data used for our study.

4 The data source is a sample of Supplemental Nutrition Assistance Program (SNAP) units selected to determine the quality of State administration of SNAP and the sample design is appropriate for this purpose.

5 Here, the phrase *target population* is used in a programmatic sense of striving to serve a group more fully and effectively, rather than the statistical sense of a universe from which to obtain a sample.
year. Because SNAP’s support for the working poor is portrayed differently in the monthly and annual timeframes, findings based on each timeframe can be useful complements.

The statistical phenomenon by which a monthly timeframe shifts the distribution of months of participation (or eligibility) toward those with relatively more months of participation (or eligibility) is an example of length-biased sampling. Bane and Ellwood (1986, 1994) examined the effects of length-biased sampling on distributions of poverty spells and spells of welfare participation. Recognizing the important role of different timeframes, recent studies of SNAP dynamics estimated durations of SNAP participation for two groups of SNAP participants—those who participated at a specified point in time (a cross-section analysis) and those who were new entrants to SNAP over some period (a cohort analysis) (see Leftin et al., 2014; Mabli et al., 2011; and Cody et al., 2007).

A study’s timeframe can depend on both the length of the item’s reference period and on the methodology chosen for the study. For example, the Survey of Income and Program Participation (SIPP) has historically interviewed SIPP households every 4 months and collected data on SNAP participation for each of the 4 months. Using SIPP’s month-by-month data, it is feasible for a study to examine SNAP eligibility (and participation among eligibles) using a single specific calendar month for a timeframe, an average-month timeframe, or an annual timeframe (among other possibilities). To examine SNAP participation and estimate hazard models, Blank and Ruggles (1996) used SIPP and adopted a timeframe of ever-during-4-months that coincided with the periodicity of SIPP’s interview. Additionally, Rank and Hirschl (2005) merged 30 waves (1968 to 1997) of data from the Panel Study of Income Dynamics (PSID) to develop ever-during-adulthood estimates for the probability of food stamp receipt at some point between 25 to 65 years of age—a 40-year timeframe that is beyond what is reflected in any single wave of PSID data.

Additionally, we found that Federal survey data on SNAP participation frequently pertain to an annual timeframe. Table 1 provides the wording and the reference periods of items that collect SNAP participation data on several nationally representative surveys. In most cases reviewed, survey data do not represent SNAP participants in an average month. When examining characteristics of SNAP participants using Federal survey data, results may differ from those published in SNAP Characteristics reports, partly because the reports make use of administrative data and partly because of the different timeframes.

6 The term length biased is unfortunate, albeit conventional in statistical theory, because the term suggests that the monthly eligibles and their monthly Supplemental Nutrition Assistance Program (SNAP) participation rate are the wrong group and rate to examine. For this report, it is best to interpret the term bias only in a relative sense: the monthly SNAP participation rate is a biased overestimate of the annual rate, but it is equally true that the annual rate is a biased underestimate of the monthly rate. A bias, in this context, occurs when there is a mismatch between the statistical population of interest—annual versus monthly eligibles—and the type of SNAP participation rate measured.
<table>
<thead>
<tr>
<th>Survey</th>
<th>Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>American Community Survey, 2014</td>
<td>15. <strong>In the past 12 months</strong>, did you or any member of this household receive benefits from the Food Stamp Program or SNAP (the Supplemental Nutrition Assistance Program)?</td>
</tr>
</tbody>
</table>
| Annual Social and Economic (ASEC) Supplement to the Current Population Survey, 2013 | Q87. Did (you/ anyone in this household) get SNAP (Supplemental Nutrition Assistance Program), food stamps or a food stamp benefit card **at any time during 2012**?  
Q902. **How many months** was food assistance received in 2012? |
| Food Security Supplement (FSS) to the Current Population Survey, 2012 | **HESP1. In the past 12 months**, since December of last year, did (you/anyone in this household) get SNAP/Supplemental Nutrition Assistance Program or food stamp benefits?  
**HESP21-HESP212. In which months** of 2012 were SNAP or food stamp benefits received? January? . . . |
| National Health and Nutrition Examination Survey (NHANES), Food Security Module of Family Questionnaire, 2013-2014 | **FSQ.165.** Have (you/you or anyone in your household) **ever** received SNAP or food stamp benefits?  
**FSQ.012. In the last 12 months**, did (you/you or anyone who lives here) receive SNAP or food stamp benefits?  
**FSQ.230.** (Do you/Does any member of your household) **currently receive** SNAP or food stamp benefits? |
| National Health Interview Survey, Family Questionnaire, Draft 2014 | **FSNAP.** At any time during [last calendar year X], did (you/any family members living here) receive (food stamp benefits/SNAPNAME or food stamp benefits)? |
| Panel Study of Income Dynamics, 2013 | **F8.** Did [you or anyone else in your family] receive food stamp benefits, that is, either food stamps or a food stamp benefit card, **at any time in [calendar year X], 2 years ago**?  
**F11.** Did [you or anyone else in your family] receive food stamp benefits, (that is, either food stamps or a food stamp benefit card,) **at any time last year**, in [calendar year X]?  
**F13.** During **which months** was that?  
**F14.** Did [you or anyone else now living in your family] use food stamp benefits **last month**? |
| Survey of Income and Program Participation, 2008 Panel | **FSMTHYM.** In **which of the last four months** -- [month 1], [month 2], [month 3], or [month 4],-- did [you/she/he] receive food stamps?” |

Note: Highlighting identifies the item reference period. SNAP refers to the Supplemental Nutrition Assistance Program.

The relationship between annual and monthly SNAP participation rates

The mathematical relationship between the annual and monthly SNAP participation rates depends on two types of turnover: turnover among participants and turnover among eligibles. The annual SNAP participation rate is strictly less than the monthly SNAP participation rate when participation turnover is less than eligibility turnover—a condition that is essentially certain to be met in practice for SNAP. Eligibility and participation turnover are also related to the average number of months of eligibility and of participation.

The monthly SNAP participation rate, represented by $\pi^M$, is:

$$ (1) \quad \pi^M = 100 \times \frac{P^M}{E^M} $$

where $P^M$ is the number of monthly participants and $E^M$ is the number of monthly SNAP eligibles, which are defined by:

$$ (2) \quad P^M = \frac{TPM^P}{12} \quad E^M = \frac{TPM^E}{12} $$

where $TPM^P$ is the year’s total person-months of participation summed across all participants and all 12 months, and $TPM^E$ is the year’s total person-months of eligibility summed across all eligibles and all 12 months. The annual SNAP participation rate, represented by $\pi^A$, is:

$$ (3) \quad \pi^A = 100 \times \frac{P^A}{E^A} $$

where $P^A$ and $E^A$ are the numbers of annual participants and eligibles.

The phenomenon of participation turnover refers to the number of different people who receive support from SNAP at some time during the year ($P^A$) relative to the number of participants in a typical month ($P^M$), reflecting participation entry and exit during the year. The participation turnover rate and the eligibility turnover rate, represented by $T^P$ and $T^E$, are:

$$ (4) \quad T^P = \frac{P^A}{P^M} \quad T^E = \frac{E^A}{E^M} $$

We use the terms "turnover" and "turnover rate" interchangeably.

Participation turnover is an important concept examined in several previous SNAP studies. Leftin et al. (2014) explained the implications of participation turnover for State SNAP offices:

"With a low turnover rate, the program will handle the same participants over long periods of time with few participants entering or exiting in a given month. With a high turnover rate, by contrast, the program will process applications for a large numbers of individuals, even if the number of cases actually participating remains steady. In any given month, there will be many new faces in the SNAP office, and many others who had participated in the past will no longer participate" (p. 143).
We extend and apply the concept of turnover to eligibility turnover using equation (4). To our knowledge, the only previous estimate for the turnover rate of SNAP eligibles is provided by Carr et al. (1984) for 1979. Our study’s estimates of eligibility turnover for 2012 adds current understanding to the dynamics of SNAP eligibility and participation.

Using equation (4), the relationship between the annual SNAP participation rate and the monthly SNAP participation rate can be expressed as:

\[
(5) \quad \pi^A = \left( \frac{T^P}{T^E} \right) \pi^M
\]

The annual SNAP participation rate \(\pi^A\) is less than the monthly SNAP participation rate \(\pi^M\) because participation turnover \(T^P\) is less than eligibility turnover \(T^E\). Thus, the disparity between the turnover rates of participation and eligibility is central to our analysis.

We refer to the number of months of SNAP participation during the year as participation intensity, represented by \(IP\) for any one individual, which takes on a value between 1 and 12 months for each annual participant. If a person has multiple participation spells within a year, participation intensity adds the number of months of participation across spells (but counts only months of participation occurring within the year). A person’s eligibility intensity (the number of months of eligibility during the year) is represented by \(IE\) and lies between 1 and 12 months.

Averaged across annual participants and annual eligibles, respectively, average intensities of participation and of eligibility can be defined by:

\[
(6) \quad \bar{I}^P = \frac{TPM^P}{PA}; \quad \bar{I}^E = \frac{TPM^E}{EA}
\]

Burstein (1993, p. 96) provides a useful inverse relationship between turnover and average participation intensity (although the term “months of receipt” was used rather than participation intensity). An analogous relationship emerges for eligibility. It follows definitions in equations (2) and (4) that average participation intensity and average eligibility intensity can also be expressed as:

\[
(7) \quad \bar{I}^P = \frac{12}{T^P}; \quad \bar{I}^E = \frac{12}{T^E}
\]

The average intensities of participation and eligibility differ because participation turnover \(T^P\) and eligibility turnover \(T^E\) also differ. For example, eligibility turnover of 1.50 implies that an eligible person is eligible, on average, for 8 months of the year \(IE = 12/1.5 = 8\). Participation turnover of 1.30 implies that, on average, a SNAP participant receives SNAP benefits for more than 9 months of the year \(IP = 12/1.30 = 9.2\).

---

7 Our measure of participation intensity relates to another measure in the literature. In a study of the Aid to Families with Dependent Children program, Moffitt (2002) defined Total Time On (TTO) as “the total amount of time within a fixed calendar time interval that the individual has received welfare” (pp. 474-75), up to 120 months in Moffitt’s study, and noted that the TTO measure had been used many times. Our measure of participation intensity is simply the TTO measure restricted to a period of 12 months.

8 In contrast to intensity, the duration of a participation spell is the full (non-censored) length of a single continuous spell of Supplemental Nutrition Assistance Program receipt, which can begin before or last beyond the year under study.
The study’s central relationship between $\pi^A$ and $\pi^M$ was shown in equation (5) in terms of the relative turnover rates ($TP/TE)$. Using equation (7), the relationship can be re-expressed as:

$$\pi^A = \left( \frac{\bar{P}^E}{\bar{P}^P} \right) \pi^M$$

(8) $\pi^A = \left( \frac{\bar{P}^E}{\bar{P}^P} \right) \pi^M$

The annual SNAP participation rate ($\pi^A$) is strictly less than the monthly rate ($\pi^M$) because the average intensity of participation ($\bar{P}^P$) exceeds the average intensity of eligibility ($\bar{P}^E$)—i.e., the average SNAP participant receives SNAP for more months of the year than the average number of months of eligibility. Thus, by equations (8) and (5), the relationship between annual and monthly SNAP participation rates can be analyzed in terms of either turnover rates or intensities.
Bane and Ellwood (1986, 1994) used a hospital stay as a metaphor for a poverty spell—the length of time for which annual income is below poverty. The hospital metaphor considers two different groups of patients: the flow (over some period) of all newly admitted patients and the stock of patients who occupy hospital beds at any one time. Patients who are chronically ill are a small proportion of new patients, but a large proportion of patients on any given day. This can also be an effective way to represent a period of SNAP eligibility. The two groups are the flow of people newly entering SNAP eligibility (over time) versus the stock of people who are eligible for SNAP at a moment in time (in a given month). The distribution of durations in the stock sample contains relatively longer term durations and, therefore, has a higher average duration compared to the distribution and mean of durations in the flow sample.

The relationship between stocks and flows resembles, but is not identical to, the relationship between the monthly eligibles and annual eligibles in our study. That is, the monthly and annual timeframes give rise to another real-world example of length-biased sampling that is similar to the classic version of stock versus flow, but is not mathematically equivalent. In this new version, average-month sampling is a variation of stock sampling that simply averages 12 stock samples. Ever-during-the-year sampling reflects both a stock and a flow. All people eligible in January 2012 constitute a stock sample for the month. In the remaining 11 months of 2012, new people enter SNAP eligibility. By combining a stock sample for January with 11 months’ worth of a flow sample, ever-during-year sampling is a hybrid of both stock and flow sampling.

Table 2 is an example of the effect of censoring and how annual eligibles and monthly eligibles are estimated for our study.

The table shows eligibility intensities ($I^E$) for 10 individuals (A through J), each of whom are eligible at some time during the year, along with individuals’ eligibility intensities ranging from 3 to 12 months. Suppose that August is a single month selected for studying the whole set of 10 annual eligibles. In August, six people happen to be eligible. Four people are not eligible in August because their spells either start too early or too late or are interrupted. Person A happens to be included in an August sample, but person A would not necessarily be in the sample if another month was chosen. Given that the eligibility intensity of person A is 3 months, the person has a 3/12 probability of being in a sample for a randomly chosen month. In contrast, people with eligibility intensities of 12 months have a 12/12 probability—a certainty—of being eligible in an August sample (or a sample for any other given month).

Total person-months of eligibility (TPM$^E$)—the sum of eligibility intensity across 10 annual eligibles—is 70 person-months (table 2). By equation (2), the number of monthly eligibles ($E^M$) is calculated as 5.8 people (70 person-months/12 months = 5.8 people). Given that at least one of the

---

9 It is possible, even likely, that a majority of those who are eligible in the first or last month of the year are also eligible in months prior to or after the year. If so, then the number of months of (intrayear) eligibility intensity does not equal the number of months of eligibility duration of the full (nonsensored) eligibility spell; it is for this reason that we introduced the term intensity to avoid using the term duration for our intrayear context. Analysis of full-length spell durations can be important. However, neither the annual nor monthly Supplemental Nutrition Assistance Program participation rates involves months outside of the year—each measure relies only on intrayear eligibility and participation patterns considered in the example.
annual eligibles \((E^A)\) has less than 12 months of eligibility, it necessarily follows that the number of monthly eligibles exceeds annual eligibles \((E^M > E^A)\).\(^{10}\)

Appendix 1 establishes new results applicable for our study; the key result is that a person who is eligible for more months carries relatively more statistical weight (has a higher probability of selection into the average-month sample) than a person with fewer months.

Our study used data for calendar 2012 from the 2008 panel of the SIPP. As a nationally representative survey providing data with monthly resolution, SIPP is well suited for the study’s examination of annual and monthly SNAP participation rates (see box, "Modeling SNAP Eligibility Using the Survey of Income and Program Participation").

When examining distributions of eligibility intensity and eligibility turnover, it can be difficult to interpret results that include observations providing data for less than all 12 months of 2012.\(^{11}\) To

---

\(^{10}\) While each annual eligible can be identified as an actual person, there is no correspondence between the 5.8 statistical people who are monthly eligibles and any of the example’s 10 actual people who are annual eligibles. In fact, for most sets of data, it is likely that (similar to the example) the number of monthly eligibles is a non-integer number of people—a result that is statistically meaningful albeit biologically impossible.

\(^{11}\) For example, if an observation provided 8 months of data in the U.S. Census Bureau’s Survey of Income and Program Participation (SIPP) and was modeled to be eligible for 4 of those 8 months, the distribution of months of eligibility could either: (a) treat the observation’s eligibility intensity as 4 months, or (b) factor the measured intensity upward to 6 months to reflect that the observation was modeled to be eligible for half of the 8 months the observation provided data in SIPP.
Box: Modeling SNAP Eligibility Using the Survey of Income and Program Participation

The U.S. Census Bureau's Survey of Income and Program Participation (SIPP) is a nationally representative longitudinal survey that collects detailed information about individuals and households over time, with a focus on income, labor force activity, and program participation. SIPP is structured in short panels, with each panel covering 2 to 6 years. This report uses calendar 2012 data from the 2008 panel, which started in May 2008 and ended in July 2013.

Our model of Supplemental Nutrition Assistance Program (SNAP) eligibility integrates many of the Federal rules of eligibility with survey information about household relationships, incomes by source, participation in programs besides SNAP, and many other types of data (income and participation in programs besides SNAP are subject to underreporting to SIPP). While SIPP does not contain all of the data used by a local SNAP office to make an official determination of eligibility, for research purposes, SIPP has more data for modeling (or simulating) SNAP eligibility than any other household survey. Another benefit of using SIPP is that it provides data for each month of calendar 2012, and the accounting period for official determination of SNAP eligibility is based on monthly income (among other factors) rather than annual income.

To be determined to be eligible at a SNAP office, an individual applies to the program along with others with whom the applicant purchases and prepares food. Importantly, this group of people can differ from the group of people living in the household. In SIPP and other Census Bureau surveys, a household refers to a residential address, which can include more people than belong in any one administrative SNAP unit and can have two (or more) different administrative SNAP units participating within it at the same time. As a result, researchers using survey data have regrouped people into smaller units, such as a subfamily within a household, as estimates of SNAP units (Newman and Scherpf, 2013 and Scherpf et al., 2015). Because SIPP data do not contain all information used by SNAP offices, we refer to the subgroups of people developed in survey data as constructed SNAP units (CSUs) as distinct from people officially determined to be in a SNAP unit.

Individuals within CSUs may or may not be considered eligible for SNAP within any particular calendar month in our model. The model sums the incomes of individuals who form a CSU and calculates a new income-to-poverty ratio based on CSU size. SNAP has two income-eligibility criteria for gross income and net income. Following SNAP rules, the model subtracts various deductions from gross income to calculate net income (including shelter expenses, medical expenses for elderly or disabled members, dependent care expenditures, and child support payments). The resulting (estimated) net income must be lower than the poverty guideline that applies to the CSU to meet the net income test for simulated eligibility. Gross income, the total income of the CSU, must be less than 130 percent of the applicable poverty line for all CSUs except those that have an elderly or a disabled member, which our model (following SNAP rules) exempts from the gross income test. The model does not make State-by-State adjustments —continued
facilitate interpretation, we created a balanced panel of individuals who had 12 months of data (in addition to having a positive calendar-year statistical weight in the SIPP for 2012). Nearly all observations with a positive calendar-year weight for 2012 provided a full 12 months of data.

The estimated number of 2012 annual eligibles (in the balanced panel) is about 76.6 million (table 3); using all 2012 observation results in an estimate of about 77.2 million annual eligibles, which exceeds the balanced panel estimate by less than 0.6 million or about 0.7 of 1 percent. Given that the balanced panel drops so few observations, it is not surprising that the estimate of monthly SNAP eligibles is so similar to the estimate based on all 2012 observations (53.0 versus 53.2 million). For our study, an important statistical issue is whether sampled people retained in the 2012 balanced panel exhibit systematically different dynamics compared to the 2012 full sample. Eligibility turnover in the balanced sample is estimated to be 1.446. The corresponding eligibility turnover based on all 2012 observations is 1.450. The closeness of the two SNAP turnover rates means that the eligibility dynamics of the balanced panel and the full 2012 sample are essentially the same, which we round to 1.45. Based on SIPP data for 2012, our study finds that about 45 percent more people are eligible for SNAP at some time during the year (for one or more months) than are eligible for SNAP in an average month of the year.

Eslami (2014) provides estimates of eligible individuals and participating individuals for an average month of (fiscal) 2012, which are used for the monthly SNAP participation rate published by FNS. The table shows the number of estimated monthly eligibles of 50.7 million is about 2.5 million fewer (about 4.9 percent) than what our study estimated (based on all 2012 SIPP observations). Some differences are to be expected. First, our study’s estimate pertains to calendar 2012 while the estimate in Eslami (2014) is for fiscal 2012. Additionally, the latter is largely based on a different survey—the

---

12 A person who spent part of the year in an institution could have a positive calendar-year weight and yet provide less than 12 months of data (U.S. Census Bureau, 2001, pp. 8-25).
Annual Social and Economic Supplement (ASEC) to the Current Population Survey—so eligibles are estimated based on methodologies that differ from ours for forming simulated SNAP units and modeling which ones satisfy which eligibility criteria. In the end, while the difference of 2.5 million people might not be considered negligible, neither are the two estimates so different that our study’s SIPP-based model of eligibility seems unreasonable in comparison to the ASEC-based model of eligibility in Eslami (2014).

The frequencies by which the 76.6 million annual eligibles (estimated in the balanced panel) are distributed across various eligibility intensities are shown in table 4. About 5.0 million people are estimated to be eligible for SNAP just 1 month of the year, and about 20.1 million were eligible for 1 to 4 months when combined as a set of highly transitory eligibles. About 32.8 million people are estimated to be eligible for all 12 months.

Table 4 also shows the derivation of the study’s estimate of monthly eligibles for 2012. Each annual eligible who is estimated to be eligible for n months contributes n person-months’ worth of eligibility to the estimated total of about 636.0 million person-months. In an average month of 2012, there are an estimated 53.0 million people who are monthly eligibles (636.0 million person-months/12 months) (table 4). Figure 1 depicts the two percentage distributions of eligibility intensities for annual eligibles and monthly eligibles.

The distribution of eligibility intensity is skewed heavily to the right for both annual and monthly eligibles. In fact, the mode of each distribution is 12 months—more people are eligible for 12 months than for any other single number. The typical annual eligible is eligible for about 8.3 months, and the typical monthly eligible has a longer estimated average intensity of 10.2 months.

A strong contrast between the two distributions emerges when considering those with 1 to 4 months of eligibility. Of all annual eligibles, 26.3 percent (the sum of 6.6 through 8.2 percent of months 1-4)
Table 4

Annual SNAP eligibles and monthly SNAP eligibles, by eligibility intensity, 2012 estimates

<table>
<thead>
<tr>
<th>Eligibility intensity (months)</th>
<th>Annual eligibles, by eligibility intensity (people)</th>
<th>As percent of total annual eligibles</th>
<th>Person-months of eligibility intensity</th>
<th>As percent of total person-months of eligibility</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3) = (1)(\times) (2)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>5,043,081</td>
<td>6.6</td>
<td>5,043,081</td>
<td>0.8</td>
</tr>
<tr>
<td>2</td>
<td>4,237,430</td>
<td>5.5</td>
<td>8,474,860</td>
<td>1.3</td>
</tr>
<tr>
<td>3</td>
<td>4,572,925</td>
<td>6.0</td>
<td>13,718,775</td>
<td>2.2</td>
</tr>
<tr>
<td>4</td>
<td>6,264,171</td>
<td>8.2</td>
<td>25,056,684</td>
<td>3.9</td>
</tr>
<tr>
<td>5</td>
<td>3,136,614</td>
<td>4.1</td>
<td>15,683,069</td>
<td>2.5</td>
</tr>
<tr>
<td>6</td>
<td>3,178,136</td>
<td>4.1</td>
<td>19,068,817</td>
<td>3.0</td>
</tr>
<tr>
<td>7</td>
<td>3,140,723</td>
<td>4.1</td>
<td>21,985,064</td>
<td>3.5</td>
</tr>
<tr>
<td>8</td>
<td>5,038,471</td>
<td>6.6</td>
<td>40,307,766</td>
<td>6.3</td>
</tr>
<tr>
<td>9</td>
<td>2,733,014</td>
<td>3.6</td>
<td>24,597,127</td>
<td>3.9</td>
</tr>
<tr>
<td>10</td>
<td>2,738,238</td>
<td>3.6</td>
<td>27,382,376</td>
<td>4.3</td>
</tr>
<tr>
<td>11</td>
<td>3,708,994</td>
<td>4.8</td>
<td>40,798,937</td>
<td>6.4</td>
</tr>
<tr>
<td>12</td>
<td>32,824,325</td>
<td>42.8</td>
<td>393,891,898</td>
<td>61.9</td>
</tr>
<tr>
<td>Totals</td>
<td>76,616,122</td>
<td>100.0</td>
<td>636,008,453</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Note: Annual Supplemental Nutrition Assistance Program (SNAP) eligibles are people who are estimated to be eligible at some time during the year (for one or more months). Monthly SNAP eligibles are those who are estimated to be eligible in an average month of the year. Eligibility intensity is the number of months of a person’s estimated eligibility within the year. Figures in column (3) may not exactly equal the product of figures in columns (1) and (2) due to rounding error.


have shorter term eligibility lasting 4 months or less, while just 8.2 percent (the sum of 0.8 through 3.9 percent of months 1-4) of monthly eligibles have eligibility intensities between 1 and 4 months. The difference of 18.1 percentage points between the shares of annual and monthly eligibles in the range of 1 to 4 months mostly counterbalances the difference between the two 12-month spikes of about 19.1 percent (61.9 versus 42.8 percent).

It is helpful to divide annual eligibles into full-year eligibles (eligible for all 12 months) and part-year eligibles (eligibility intensities from 1 to 11 months). Part-year eligibles are estimated to be the majority—about 57.2 percent—of annual eligibles in 2012. When examining the distribution of eligibility intensity for monthly eligibles, the proportions between full-year and part-year eligibles are reversed from what they are estimated to be for annual eligibles. Among the 53.0 million monthly eligibles, 61.9 percent are estimated to be full-year eligibles, while only 38.1 percent are part-year eligibles.
Figure 1
Eligibility intensities for annual SNAP eligibles and monthly SNAP eligibles, 2012 estimates

Note: Annual Supplemental Nutrition Assistance Program (SNAP) eligibles are people who are estimated to be eligible at some time during the year (for 1 or more months). Monthly SNAP eligibles are those estimated to be eligible in an average month of the year. Eligibility intensity is the number of months of a person's estimated eligibility within the year.

National annual and monthly SNAP participation rates

A disparity between the turnover rates of SNAP eligibility and SNAP participation creates a wedge between annual and monthly SNAP participation rates, as shown in equation (5). We reported our 2012 estimate for eligibility turnover of 1.45—i.e., there are about 45 percent more people who are eligible at some time in 2012 than are eligible in an average month. We provide estimates of participation turnover, examine the estimated 2012 annual and monthly SNAP participation rates, and consider the program participation choices made by individuals that may explain our results.

SNAP participation turnover. When using a balanced panel of SIPP observations with data reported for all 12 months of 2012, an estimated 54.1 million people receive SNAP at some time during the year (table 5). If all observations with a positive calendar-year weight are included, an estimated 54.5 million people are annual participants. The difference between the two figures of about 0.4 million is small in our estimation. Our estimate based on all 2012 observations can be compared to the estimate of about 49.4 million annual participants reported in Leftin et al. (2014), who also used SIPP data for 2012. The difference between the two figures reflects that our study and Leftin et al. (2014) used different methodologies to form simulated SNAP units within households. The difference in the levels of annual participants is 5.1 million, or about 10.5 percent of the estimate by Leftin et al. (2014). Differences of that magnitude are not small, and they point to the challenges inherent in simulating SNAP units using survey data.

Our 2012 estimate of monthly participants is about 42.3 million based on the balanced panel, which differs by about 0.1 million from the estimate based on all 2012 observations, which in turn differs from the estimate of 38.6 million by Leftin et al., 2014 by about 3.8 million or 9.9 percent. Based on the balanced panel, estimated SNAP participation turnover—the ratio of annual participants to monthly participants—is 1.282, that is, in 2012, about 28 percent more people participated in SNAP at some time during the year than participated in an average month. Our use of the balanced panel estimates has little effect on participation turnover: the estimate based on all 2012 observations is 1.286, which we consider to be essentially the same.

Table 5
Annual and monthly SNAP participants and participation turnover, 2012 estimates

<table>
<thead>
<tr>
<th></th>
<th>Annual SNAP participants</th>
<th>Monthly SNAP participants</th>
<th>SNAP participation turnover</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>ERS-constructed SNAP units, full-year 2012 observations (balanced panel)</td>
<td>54,152,413</td>
<td>42,252,536</td>
</tr>
<tr>
<td>B</td>
<td>ERS-constructed SNAP units, all 2012 observations (includes part-year observations)</td>
<td>54,546,560</td>
<td>42,410,768</td>
</tr>
<tr>
<td>C</td>
<td>Leftin et al. (2014)</td>
<td>49,375,206</td>
<td>38,586,703</td>
</tr>
<tr>
<td>D</td>
<td>Difference (B less C)</td>
<td>5,171,354</td>
<td>3,824,065</td>
</tr>
<tr>
<td>E</td>
<td>Percentage Difference (D/C)</td>
<td>10.5</td>
<td>9.9</td>
</tr>
</tbody>
</table>

Note: Annual Supplemental Nutrition Assistance Program (SNAP) participants are people who are estimated to participate in SNAP at some time during the year (for 1 or more months). Monthly SNAP participants are those who are estimated to participate in SNAP in an average month of the year.

Our study’s estimate of participation turnover also essentially matches the 1.280 estimate based on the annual and monthly participants reported in Leftin et al., 2014. Thus, we can conclude that for the purposes of comparing annual versus monthly SNAP participation rates, it makes little difference whether participation turnover is estimated based on our study’s constructed SNAP units or on the methodology of Leftin et al., 2014.

A SNAP participation turnover of 1.28 means that participation intensity is about 9.4 months (12 months/1.28). At the same time, a SNAP eligibility turnover of 1.45 means that eligibility intensity is about 8.3 months (12 months/1.45). This result (that 9.4 months of participation intensity exceeds 8.3 months of eligibility intensity) re-expresses how eligibility turnover exceeds participation turnover; behavioral factors that contribute to these results are considered in the section that follows.

Among the major surveys examined by Meyer et al. (2009), SIPP has the least amount of underreporting for SNAP participation (e.g., capturing 82.9 percent of SNAP participants in 2005). Agreement between SIPP self-reported data on SNAP participation and data from SNAP administrative records seems to be especially good for 2012. As estimated for the monthly SNAP participation rate published by FNS, the number of individuals participating in SNAP in an average month in (fiscal) 2012 is 42,129,048 (Eslami, 2014). That figure, which relies heavily on administrative SNAP data, is quite close to our study’s estimates.

**Annual versus monthly SNAP participation rates.** Following the definition of the annual SNAP participation rate, our (calendar) 2012 estimate is:

\[
\pi^A = 100 \times \frac{P^A}{E^A} = 100 \times \frac{54,152,413 \text{ annual participants}}{76,616,122 \text{ annual eligibles}} = 70.7 \text{ percent}
\]

For (fiscal) 2012, the monthly SNAP participation rate published by FNS is:

\[
\pi^M = 100 \times \frac{P^M}{E^M} = 100 \times \frac{42,129,048 \text{ monthly participants}}{50,708,090 \text{ monthly eligibles}} = 83.1 \text{ percent}
\]

At the national level, the estimated annual SNAP participation rate for (calendar) 2012 is 70.7 percent—12.4 percentage points below the FNS estimate of the monthly SNAP participation rate of 83.1 percent for (fiscal) 2012. These two national estimates for annual and monthly SNAP participation rates are in figure 2. Both annual participants and annual eligibles exceed their monthly counterparts due to participation turnover and eligibility turnover, respectively. While estimates of annual participants exceed monthly participants (54.1 versus 42.1 million, a difference of about 29 percent) due to participation turnover, annual eligibles exceed monthly eligibles by an even larger proportion (76.6 versus 50.7 million, a difference of about 51 percent). It is the disparity between participation turnover and eligibility turnover that makes the annual SNAP participation rate lower than the monthly rate.

---

13 The sources of the estimated 54.1 million annual participants and 42.1 million monthly participants are, respectively, the Survey of Income and Program Participation (SIPP) and a report published by the Food and Nutrition Service (Eslami, 2014). When that estimate of annual participants is compared to a SIPP-based estimate of monthly participants of about 42.2 million, the percentage difference is about 28 percent (as reported in table 5) rather than 29 percent. The sources of the estimated 76.6 million annual eligibles and the 50.7 million monthly eligibles are also SIPP and Eslami, 2014. When that estimate of annual eligibles is compared to a SIPP-based estimate of monthly eligibles of about 53.0 million, the percentage difference is about 45 percent (as reported in table 3) rather than 51 percent.
Discussion of turnover rates (or of intensities) is inseparable from the explanation for why the estimate of \( \pi_A \) is below the estimate of \( \pi_M \), as depicted in equation (5) (or in equation (8)). First, annual SNAP participation exceeds monthly SNAP participation due to participation turnover. According to an estimated \( \frac{TP}{TE} \) (table 5) of 1.282, there are about 28 percent more annual participants than monthly participants. However, even though participation turnover is in the numerator of the \( \frac{TP}{TE} \) ratio in equation (5), participation turnover is relatively sluggish compared to the even higher estimate for eligibility turnover of 1.45 in the denominator. Thus, the ratio \( \frac{TP}{TE} \) is less than 1.0 and, as a result, the annual SNAP participation rate is less than the monthly SNAP participation rate.

We cannot conclude that the disparity between the annual SNAP participation rate (\( \pi_A \)) and the substantially larger monthly SNAP participation rate (\( \pi_M \)) shows a dramatic shortcoming of the program. From one perspective, it can be a desirable aspect of program performance that SNAP is serving relatively more people who typically have relatively more months of eligibility. If greater eligibility intensity is taken as a measure of greater need, it could be argued that the relatively high \( \pi_M \) shows that the program’s reach is greatest in the target population—the monthly eligibles—where the need is greatest.

The estimates presented above, including 1.282 for participation turnover \( TP \) and 1.466 for eligibility turnover \( TE \), do not quite fit the relationship shown in equation (5). We opted to compare our study’s estimate for \( \pi_A \) of 70.7 percent with the estimate published by FNS for \( \pi_M \) of 83.1 percent, which we take to be the standard of interest for the rate of participation in an average month. If instead SIPP-
based estimates were used for monthly participants and monthly eligibles, resulting in an estimated \( \pi^M \) of 79.7 percent (along with SIPP-based estimates for the two turnover rates), then equation (5) would hold as a mathematical identity:

\[
\frac{54,152,413 \text{ annual participants}}{76,616,122 \text{ annual eligibles}} = \left( \frac{54,152,413 \text{ annual participants}}{42,242,536 \text{ monthly participants}} \right) \cdot \left( \frac{76,616,122 \text{ annual eligibles}}{53,000,704 \text{ monthly eligibles}} \right)
\]

Because the \( \pi^M \) published by FNS of 83.1 percent is obtained using administrative data (for the numerator of average-month participants) and ASEC (for the denominator of average-month eligibles), it is too much to expect that the estimate would be identical to the SIPP-based estimate for \( \pi^M \) of 79.7.
Behavioral explanations for intrayear dynamics of eligibility and participation

The ratio of participation and eligibility turnover rates drives a wedge between the annual and monthly SNAP participation rates. Economic theory suggests that the disparity between the participation and eligibility turnover rates reflects incentives for individuals’ program participation choices. Consistent with this, our study found that people who are estimated to be eligible for more (fewer) months of the year tend to apply for and participate in SNAP at higher (lower) rates.

SNAP participation can be expected to be positively correlated with eligibility intensity. People who are eligible for SNAP for only a shorter term period (e.g., a couple of months) have, on average, a relatively small incentive to locate a SNAP office, apply for SNAP, learn that they are eligible, and receive SNAP benefits for that relatively short-term period of eligibility. In principle, people with relatively few months of SNAP eligibility should exhibit a relatively low SNAP participation rate. In contrast, we expect people who are eligible for SNAP for many months, and who thus have a higher incentive to apply for SNAP, to exhibit a relatively high SNAP participation rate. Thus, differences across people in their (expected) number of months of eligibility should help explain different behavioral choices that result in only some eligibles applying for and receiving SNAP benefits.14

Previous studies report findings that are consistent with the predicted behavior. Blank and Ruggles (1996) used SIPP’s 1986 and 1987 panel files to examine eligibility and participation patterns in receipt of food stamps and Aid to Families with Dependent Children (AFDC, renamed Temporary Assistance for Needy Families in 1996) by single mothers and their children. They found a large number of eligibility spells for AFDC and food stamps that were relatively short and that did not result in participation. A study by Farrell et al. (2003), which used the 1996 SIPP panel, found that:

“Non-participant households experienced substantially more variability in their monthly income and earnings than participant households. In particular, before the months leading up to the reference month, mean income of non-participating food-stamp eligible households fell by much more than mean income of participant households; similarly, their mean income grew much more rapidly after the reference month … This is consistent with the premise that expectations of higher future income explain why some non-participant households do not participate.” (p. ES-2, emphasis added)

The study by Blank and Ruggles (1996) focused on families headed by single mothers and was developed to analyze conditions and behaviors in the Food Stamp Program in the 1980s. Our study pertains to SNAP eligibles for all individuals living in all types of SNAP units in 2012. The study by Farrell et al. (2003) was designed to examine longitudinal patterns of income, while our study is concerned with eligibility intensity (number of months of eligibility) within a given calendar year as a factor that influences participation decisions.

The estimated percentages $100 \times \frac{PC(t)}{EA(t)}$ shown in figure 3 are the numbers of conditional SNAP participants relative to the numbers of eligibles, by months of eligibility intensity in 2012; the appen-

---

14 Strictly speaking, economic theory suggests that the decision of whether or not to apply for the Supplemental Nutrition Assistance Program (SNAP) at a given time is based (in part) on the person’s expected future length of SNAP eligibility, which can differ from the actual length of SNAP eligibility that results only after time goes by. Our study’s estimation of eligibility is designed to be an approximation of the unobservable expectation that a person has about future conditions.
dix 2 table provides estimates of these percentages and their separate denominators and numerators. The denominators of the percentages are the numbers of annual eligibles with \( t \) months of eligibility represented by \( E^A(t) \), \( t = 1, 2, ..., 12 \). For example, \( E^A(3) \) is the number of people who are estimated to have 3 months of eligibility in 2012. The conditional SNAP participants in a numerator is the subgroup of \( E^A(t) \) who received SNAP at some time during the year or, equivalently, the subgroup of annual participants who have \( t \) months of eligibility. For example, \( P^C(3) \) is the number of 3-month eligibles who ever participated during the year.

We interpret these 12 percentages \( 100 \times \frac{P^C(t)}{E^A(t)} \) as a set SNAP participation rates, by months of eligibility intensity. Relatedly, the 12 percentages given by \( 100 \times [1 - \frac{P^C(t)}{E^A(t)}] \), which are not shown in the figure, would be nonparticipation rates—people who are eligible for \( t \) months and who did not report any SNAP participation during 2012.

As shown in the figure, the estimated conditional participants-to-eligibles percentages exhibit a strongly increasing pattern, ranging from 25.9 percent (for \( t = 1 \)) to 64.6 percent (for \( t = 12 \)). A regression of the conditional participants-to-eligibles percentages against time has an estimated slope of 3.8 percent per month with a statistically significant t-statistic of 10.9.\(^{15}\) These findings provide statistical evidence that people who are eligible for more (fewer) months tend to participate in SNAP at higher (lower) rates. It is this behavior that drives the disparity between the participation and eligibility turnover rates in equation (5), which results in the difference between the annual and monthly SNAP participation rates (see appendix 2).

\(^{15}\) It so happens that the unconditional ratios of \( P^t(t)/E^t(t) \), based on (unconditioned) annual participants with \( t \) months of participation, also exhibit increases with eligibility intensity, as shown by estimates in the appendix 2 table. However, these ratios are not true proportions because the numerators include people who are not in the denominators. For example, \( P^t(3)/E^t(3) \) is the estimated ratio of 3-month participants to 3-month eligibles. Some 3-month participants were estimated to be eligible for 4 or more months, and accordingly, these people are not in the ratio’s denominator.
Contrasting annual versus monthly eligibles by selected demographic subgroups

To better understand how annual and monthly eligible populations differ, we compare their demographic compositions and examine differences by part-year versus full-year patterns of eligibility. Are some demographic subgroups more prevalent among the annual eligibles than among the monthly eligibles? How do the proportions of part-year and full-year eligibles differ across demographic subgroups? Do some subgroups have eligibility turnover rates—and, accordingly, eligibility intensities—that differ from the national average?

Some demographic subgroups are of programmatic interest because they are especially vulnerable and represent an important subpopulation served by SNAP (e.g., the elderly, the disabled, and children). These same subgroups potentially exhibit eligibility dynamics that distinguish them from the general population of annual eligibles. For example, the elderly often receive pensions, Social Security, and other fixed-income streams that are relatively steady compared to the more volatile income streams of people with stronger labor market connections. While not all elderly are low income, those who are low income and eligible for SNAP for part of the year might be expected to be eligible for most or all of the year. We hypothesized that the elderly would have an eligibility turnover rate that is lower than the national rate estimated for all annual eligibles. Similarly, while not all disabled people have work-limiting disabilities and qualify for social insurance programs, we hypothesized that the disabled would rely on fixed incomes relatively more than annual eligibles in general and, as a result, exhibit lower eligibility turnover.

While the numbers in table 6 are based on individuals and their eligibility, much of the discussion in this section involves the constructed SNAP unit (CSU) as whole because demographic subgroups of individuals are often defined based on a unit-level characteristic (such as the presence of an elderly person in the unit).

The first subgroup in row 1 is composed of individuals who reside in a CSU that includes at least one elderly person (66 years of age or older). We chose this age category based on criteria for age eligibility for Social Security benefits, which create an incentive to exit the labor force and potentially have income below SNAP income limits on a long-term basis. The estimated 7.8 million individuals in this subgroup represent about 10.4 percent of all 76.6 million annual eligibles ($E_1^A/E^A = 0.104$). Among the subgroup, a proportion of about 46.6 percent are estimated to have part-year eligibility lasting between 1 and 11 months, while the remaining 53.4 percent are estimated to be eligible all 12 months. This full-year proportion exceeds the proportion of full-year eligibles among all annual eligibles (42.8 percent). In the monthly timeframe, the 6.0 million people living with an elderly person represent about 11.4 percent of all monthly eligibles ($E_1^M/E^M = 0.114$).

The estimated eligibility turnover rate for subgroup 1 is 1.30, meaning that there are 30 percent more people in subgroup 1 who are eligible at some time during the year than are eligible in an average month. That turnover rate is lower than the national eligibility turnover rate of 1.45, indicating that there is slower turnover among this subgroup (as expected based on the hypothesized labor market exits of the elderly).

A subgroup with relatively low eligibility turnover (compared to the national average) necessarily constitutes a smaller share of annual eligibles than the group’s share among monthly eligibles. Let $T_k^* = E_k^A/E_k^M$ represent eligibility turnover for subgroup $k = 1, 2, ..., 6$. Then:
A subgroup that is slow in turning over its eligibles will end up with more of its annual eligibles observed in an average month, thus increasing the share of that subgroup among monthly eligibles. It is the relationship shown in equation (9) that ties the (relatively low) eligibility turnover of subgroup 1 to the difference between the subgroup’s share of monthly eligibles ($E_1^M/E^M = 0.114$) and its lower share of annual eligibles ($E_1^A/E^A = 0.104$).

The labor market behaviors of the elderly are diluted because results for subgroup 1 reflect the incomes and labor market behaviors of all individuals in the CSU—not just the elderly. To isolate the...
labor market behaviors of the elderly, the study defined subgroup 2 to be annual eligibles living in CSUs in which all members are elderly. Subgroup 2 is 5.4 percent of annual eligibles and 6.5 percent of monthly eligibles. The proportion of the subgroup who are full-year eligibles is about 66.2 percent—exceeding the full-year proportion for subgroup 1 and, even more so, the full-year proportion for all annual eligibles. The subgroup’s eligibility turnover of 1.20 shows that only 20 percent more elderly are eligible at some time during the year than are eligible in an average month.

Subgroup 3 is defined for individuals living with a disabled person in the CSU. This subgroup represents an estimated 20.8 percent of the annual eligibles and 22.7 percent of monthly eligibles. To isolate further the eligibility dynamics of disabled people, we examined subgroup 4 for individuals living in households where all members are disabled. The shares of subgroup 4 among annual and monthly eligibles are 6.8 and 9.3 percent, respectively. Among the annual eligibles who are disabled, 77.6 percent are eligible for all 12 months—the largest proportion of any subgroup we examined. Their eligibility turnover rate of just 1.06 is the lowest rate of any of these subgroups.

Subgroup 5 is composed of people living in households in which at least one member is a child (less than 15 years old). This relatively large subgroup is estimated to include 58.8 percent of annual eligibles and 61.6 percent of monthly eligibles. Within the subgroup, 53.5 percent have part-year eligibility—nearly as high as the proportion of part-year eligibles among all annual eligibles. Thus, intrayear transitions in eligibility status are especially important for people living with children (compared with the other subgroups).

At first, some cross-subgroup comparisons may not seem to fit together. Consider the difference between subgroup shares among monthly eligibles and annual eligibles for people living with children (subgroup 5): 2.8 percentage points. Meanwhile, for annual eligibles who are disabled (subgroup 4), the corresponding difference in shares is smaller: 2.5 percentage points. It might be logical to assume that the disabled should exhibit a larger effect in terms of this subgroup’s shares among annual and monthly eligibles. After all, the disabled have especially low eligibility turnover (1.06) and their proportion of full-year eligibles is especially high (77.6 percent). However, the relative differences can be a better measure of effects than percentage-point differences. When shifting to a monthly timeframe, the subgroup share for people living with children increases by a relative difference of about 5 percent (61.6 percent/58.8 percent), which is much less, as expected, than the relative difference for the disabled increases of about 37 percent (9.3 percent/6.8 percent).16 Thus, the especially low eligibility turnover of the subgroup of disabled people—just 1.06—does have a substantial effect on their shares of annual eligibles and monthly eligibles, but that effect is captured by the relative difference (rather than the percentage-point difference) of those shares.

To further explore economic factors affecting intra-year dynamics of eligibility, we defined a subgroup of people with strong labor market connections. For sampled people ages 15 and older (whom we refer to as adults), one of SIPP’s variables (RMESR) records labor market behavior during each month using one of eight categories. We defined the variable fully employed as a zero-one variable that combined two labor-market categories: we treated an adult as fully employed if the person was “with a job entire month, worked all weeks” or was “with a job entire month, absent from work without pay 1+ weeks, absence not due to layoff.” In the balanced panel, we have 12 observations for each adult for the variable fully employed. To identify (adult) annual eligibles with a strong labor market connection, we looked at each annual eligible in the first calendar month in 2012 in which

---

16 The ratio of the two subgroups’ relative differences in shares equals the ratio of their eligibility turnover rates.
the adult was estimated to be eligible for SNAP, and examined if that person was fully employed in that month (subgroup 6). Because SIPP’s labor market questions are posed about adults, the universe to which subgroup 6 belongs is the group of about 53.4 million annual eligibles who are adults.

Individuals in subgroup 6 are about 40.9 percent of adult annual eligibles and 33.4 percent of adult monthly eligibles. About 74.9 percent are eligible for just part of the year, reflecting the relatively strong labor market connections of this subgroup and the month-to-month fluctuations in income that can be experienced by the working poor. The proportions of part-year versus full-year eligibles for subgroup 6 are essentially the opposite of the proportions for subgroup 4 of disabled people. While only about 25.1 percent of fully employed adults are estimated to be eligible for all 12 months, about 77.6 percent of disabled people are full-year eligibles. By equation (9), the share of subgroup 6 among monthly eligibles is lower than its share among annual eligibles—unlike all other subgroups in the table—because the subgroup’s estimated eligibility turnover is so high at 1.84, which is greater than the 1.50 eligibility turnover for adult annual eligibles as a whole, that is, 84 percent more fully employed adults are estimated to be eligible at some time during the year than in an average month.

The analysis of the subgroups points to how the annual and monthly timeframes are useful complements. For each of the six subgroups examined, members of the subgroups are in each of SNAP’s statistical populations—the annual eligibles and the monthly eligibles—but in different proportions. Some subgroups involving the elderly, disabled, and children have estimated eligibility turnover rates that are low relative to the national average, while the eligibility turnover of fully employed adults is relatively high. These differences affect the composition of annual and monthly eligibles.

---

17 Because many in the sample are estimated to be eligible when the year first begins, January 2012 is the relevant month for many adult annual eligibles. For an adult who first enters eligibility in a later month of 2012, we examined the variable fully employed for that same month.
New York State case study

Survey data are often used exclusively when administrative data are either not available or not well suited for the research and policy issue. A motivation for measuring participation rates using administrative data is that such data are known for their completeness and accuracy. In contrast, household survey data are known to exhibit underreporting for SNAP and other Federal assistance programs (Meyer et al., 2009). Our study used an extract of New York State’s SNAP administrative records to estimate their 2012 annual SNAP participation rates, which we contrast with New York State’s monthly participation rate as published by FNS.

To estimate the 2012 annual SNAP participation rate for the United States, we relied on SIPP-based estimates for both annual participants and annual eligibles. We developed a survey-based methodology because SNAP administrative micro-data on participants do not exist at the national level; each State maintains its own SNAP administrative records. In contrast to our national results, we were able to obtain results for New York State by calculating 2012 annual participants directly using data from an extract of New York State SNAP administrative records. At the same time, estimates of eligibles must rely on survey-based methodologies because SNAP records include only participants, omitting those who do not participate but are eligible.

The State-level monthly participation rates published by FNS use administrative data in an average month (of the fiscal year) with some further adjustments (see Cunyngham et al., 2015). While these FNS rates use administrative data at the aggregate (State-wide) level to measure a State’s monthly rate, our study used administrative data at the individual level to obtain the number of different people who received SNAP benefits in New York State at some time during the year. We followed each individual in the administrative panel data longitudinally and obtained the unduplicated count of the full universe of all individuals in New York State who received SNAP benefits in one or more months of calendar 2012.

To obtain an estimate of annual eligibles for the denominator of New York State’s annual SNAP participation rate, we combined a previously published estimate of the number of people eligible for SNAP in New York State in (fiscal) 2012 with our study’s estimate of (national) eligibility turnover for (calendar) 2012 to obtain the number of monthly eligibles in New York State as the product (which we interpret as an estimate for calendar 2012). Our study did not pursue developing an estimate of 2012 eligibility turnover tailored specifically for New York State; for our main purpose of ascertaining the effect of using administrative records in lieu of ACS data, the comparison can proceed with a given denominator based simply on the national estimate of eligibility turnover (although a different denominator based on an eligibility turnover specific to New York State would affect the comparison to some extent).

---

18 Some people who receive benefits from a public assistance program do not report that receipt to a survey, a phenomenon known as underreporting.

19 The estimate of the monthly Supplemental Nutrition Assistance Program participation rate for the United States makes use of administrative data for the number of U.S. participants in an average month (see Gray and Eslami, 2014, for details).

20 The figure we use for the number of monthly eligibles is the final shrinkage estimate of the number of people eligible for the Supplemental Nutrition Assistance Program by Cunyngham et al., 2015, p. 70.
Our New York State estimate of the 2012 annual SNAP participation rate is:

\[
\begin{align*}
\pi^A &= 74.5 \text{ percent } = \frac{3,786,278 \text{ annual participants}}{5,080,487 \text{ annual eligibles}} \\
&= \frac{3,786,278 \text{ annual participants}}{1.45 \times 3,503,784 \text{ monthly eligibles}}
\end{align*}
\]

For New York State, the estimated annual SNAP participation rate \(\pi^A\) of 74.5 percent is about 5.2 percentage points less than the (fiscal 2012) monthly SNAP participation rate \(\pi^M\) of 79.7 percent published by FNS (Cunnyngham et al., 2015, p. 68). This finding for New York State qualitatively matches our national findings—the annual rate is less than the monthly rate for both the United States and New York State. However, the difference between the annual and monthly rates is greater for the United States than for New York State—12.4 percent rather than 5.2 percent. Inasmuch as monthly SNAP participation rates in fiscal 2012 ranged from 62.8 percent for California to 100.0 percent for Maine and Oregon (Cunnyngham et al., 2015, p. 68), annual SNAP participation rates are also likely to vary widely across States.

We developed a comparative estimate of the annual rate using American Community Survey (ACS) data. Examining the implications of using survey data in lieu of administrative data fosters understanding of the strengths and limitations of both sources of data, and points to the benefits of potential improvements in survey data on SNAP participation as well as fuller use of State-level administrative data or linked administrative-survey data. A growing body of knowledge on the properties, strengths, and limitations of self-reported survey data supports the many studies that use survey data to recognize measurement issues, develop research models, and interpret statistical findings. To add to the limited knowledge of underreporting for SNAP in ACS data at the State level, we examined ACS data for New York State collected in survey year (calendar) 2012.

ACS samples households in each State each month. If the survey respondent affirmed that someone in the household (the residence) received SNAP benefits in the past 12 months (see table 1), we treated all residents of the household as ever-in-12-month participants.\(^{21}\) This approach yields an estimate that reflects the combined effects of an underreporting of SNAP participation to the ACS, overreporting of SNAP participation, and an overcounting of all households members as SNAP participants when (for at least some households) only a subgroup of household members are the official SNAP unit that receives SNAP benefits. The resulting ACS-based 2012 estimate of ever-in-12-month participants in New York State is 3,476,417 people. Because that estimate is less than the 3,786,278 estimate of annual participants that we calculated using the extract of 2012 New York State SNAP administrative records, the overcounting effect of treating all household members as SNAP participants does not outweigh the undercounting effect due to underreporting of SNAP receipt by households.

Comparisons between estimates of SNAP participation based on ACS data to participation based on SNAP administrative data are complicated by the issue of reference period. ACS data collected in any one period, such as survey year 2012, pertain to (reported) SNAP participation that spans more

\(^{21}\) We introduce the term ever-in-12-month because elsewhere in the report the term annual refers to a calendar year, while in the American Community Survey (ACS) context, a household that affirms in March 2012 that it participated in the Supplemental Nutrition Assistance Program in the past 12 months may have participated in 2012 or 2011 or both. Moreover, ACS households interviewed in March 2012 have a 12-month reference period that differs in calendar time from the 12-month reference period for ACS households interviewed in July 2012.
than that period. As noted, households sampled by the ACS are interviewed throughout the year—a rolling sample design—and the item reference period for SNAP participation is the past 12 months. As a result, our ACS-based estimate of ever-during-12-month SNAP participants pertains to a data period that spans 2 years (calendar 2011-2012).

To obtain a SNAP ever-in-12-month participation rate for New York State, our estimate of ever-in-12-month eligibles spans fiscal years 2011-12, which we treat as a proxy for calendar 2011-12. We obtain our estimate of ever-in-12-month eligibles as the product of: (a) the study’s estimated national turnover rate (for calendar 2012), and (b) the average for fiscal 2011 and 2012 of the numbers of people in New York State estimated to be eligible in an average month. Our ACS-based New York State estimate of the ever-in-12-month SNAP participation rate for ACS households interviewed in 2012 is:

\[
\pi_{12m}^{11} = 68.5 \text{ percent} = \frac{3,476,417 \text{ ever -- in -- 12 -- month participants}}{5,073,649 \text{ ever -- in -- 12 -- month eligibles}}
\]

In equation (11), the ACS-based estimate for New York State’s annual SNAP participation rate of 68.5 percent is less than the rate obtained using New York State SNAP administrative data of 74.5 percent in equation (10). The difference can be attributed primarily to the use of a lower, survey-based estimate of annual participants in lieu of a figure obtained from administrative data; the denominators of the two rates in equations (10) and (11) differ relatively little because equation (10) uses the fiscal 2012 estimate of New York State monthly eligibles and equation (11) uses the average of the State’s fiscal 2011 and 2012 estimates.

One factor that partly accounts for the qualitative similarity of the results of New York State and the United States is that both estimates used the same (national) figure of 1.45 for eligibility turnover. At the same time, the similarity of the results is not attributable only to the use of a single value of eligibility turnover. The estimated annual rate for New York State used a figure for annual participants that we obtained from highly accurate administrative records for the State. A topic for future research could be to develop separate national and State-level estimates of eligibility turnover that would refine the national estimate of 1.45 we used for our study; a statistical issue that would emerge is whether sample size is sufficient for the purpose.

22 As reported by Cunnyngham et al. (2015), the final shrinkage estimates of the number of people in New York State eligible for the Supplemental Nutrition Assistance Program in fiscal 2011 and 2012 are 3,494,352 and 3,503,784, respectively.
Conclusion

This report offers a new annual SNAP participation rate that gives greater weight to shorter term eligibles (compared to the conventional monthly SNAP participation rate) and matches the way data on SNAP are often collected on Federal surveys when respondents are asked whether they (or their household) participated in SNAP at any time in the last year.

Both the annual and monthly SNAP participation rates provide information that is relevant for SNAP policymakers, program managers, and stakeholders who are concerned about the reach of SNAP across demographic subgroups. The populations of monthly eligibles and of annual eligibles consist of different mixes of people. The monthly eligibles have relatively higher shares of the elderly and the disabled, most of whom are eligible for all 12 months of the year if they are eligible at all. In contrast, the working poor tend to have fluctuating monthly earnings and more transitory periods of eligibility compared to the poor who do not work. The working poor are present among both monthly eligibles and annual eligibles, but they are a larger share of annual eligibles. The different compositions of the monthly eligibles and the annual eligibles are reflected in their monthly and annual SNAP participation rates. Each of the complementary rates provides a means for assessing the key issue of the extent to which SNAP is reaching people in need.

Although SNAP requires using monthly circumstances to determine program eligibility and the benefit allotment, the monthly participation rate is not necessarily a better tool to examine SNAP’s reach than the annual participation rate. Program officials can be interested in both—which rate is a better tool for examining program performance depends on the motivation for estimating a SNAP participation rate. People who are eligible for more months of the year may have a greater need for SNAP, so the monthly SNAP participation rate is a good tool because it measures how well SNAP serves people who are eligible in an average month (and thus have relatively more months of eligibility). On the other hand, the annual SNAP participation rate measures program performance among needy people regardless of how many months of eligibility they have during the year (although longitudinal data are required to estimate annual eligibles).

When examining the characteristics of SNAP participants using Federal survey data collected using an annual timeframe, results will differ from those published in SNAP Characteristics reports, which use a monthly timeframe. Such differences may be partly due to the use of SNAP administrative data for SNAP Characteristics reports, but also partly due to how different timeframes represent different populations of annual SNAP participants and monthly SNAP participants. The annual timeframe of SNAP-related data in many Federal surveys matches with the timeframe of the annual SNAP participation rate examined in this report.

Our study found that SNAP reaches proportionately fewer annual eligibles than monthly eligibles, although it reaches more people over the year than in an average month. About 54.1 million people were estimated to have participated in SNAP at some time during calendar 2012 in the United States, while individuals who were eligible at some time during the year were estimated to be about 76.6 million people. The resulting annual SNAP participation rate of 70.7 percent is below the FNS estimate of the monthly SNAP participation rate. According to FNS, in an average month of fiscal 2012, about 42.1 million people participated and 50.7 million people were estimated to be eligible, resulting in a monthly SNAP participation rate of 83.1 percent. The differences between annual and monthly SNAP participation rates are due to differences in participation and eligibility turnover,
which reflect the different rates at which people participate based on the number of months they are estimated to be eligible.

The elderly and the disabled each constituted a larger share among monthly eligibles than among annual eligibles. These demographic subgroups have eligibility turnover rates that are low (relative to the national average) and exhibit a higher proportion of members who are eligible for all 12 months. In contrast, adults who are fully employed during the month in which they are first estimated to be eligible constitute a smaller share of the monthly eligibles than of the annual eligibles. This converse pattern reflects that the working poor have eligibility turnover that is high (relative to the national average) and exhibit a higher proportion of members who are eligible for only part of the year (1 to 11 months).

We also used an extract of New York State SNAP administrative data to calculate the number of people who receive SNAP benefits in the State at some time during (calendar) 2012 (administrative data are known for their completeness and accuracy). The estimated annual SNAP participation rate in New York State was about 74.5 percent (about 3.8 million annual participants out of 5.1 million estimated annual eligibles). Similar to results found for the United States, New York State’s annual SNAP participation rate was below the New York State monthly SNAP participation of 79.7 percent published by FNS (for fiscal 2012).


Appendix 1—A new version of length-biased sampling

This appendix establishes that the length-biased sampling of the monthly timeframe:

1. Gives relatively more weight to people who have more months of eligibility;
2. Makes annual eligibles outnumber monthly eligibles;
3. Skews the distribution of eligibility intensity among monthly eligibles (compared to the distribution among annual eligibles) towards those with more months of eligibility;
4. Increases the estimated mean of eligibility intensity among monthly eligibles (compared to the mean among annual eligibles); and
5. Increases (decreases) the estimated mean of a variable positively (negatively) correlated with months of eligibility among annual eligibles.

While the first four results parallel results found in the classic version of length-biased sampling (which compares stock sampling with flow sampling), the fifth result is not well known and is fundamental for our study of SNAP participation among eligibles. These results resemble those provided in previous literature on length-biased sampling but they are new because comparing monthly and annual timeframes is a new version of length-biased sampling. The results here are applicable regardless of whether changes occur during the year in economic conditions or SNAP program policies or practices.\(^{23}\)

(1) Statistical weights. An annual timeframe gives each person who is eligible at some time during the year an equal weight regardless of the number of months of eligibility. That is, the number of annual eligibles \(E_A\) can be expressed by adding up each person \(k, k = 1, 2, \ldots, E_A\), with a weight of 1.0:

\[
(A1.1) \quad E_A^A = \sum_{k=1}^{E_A} 1.0
\]

In contrast, a monthly timeframe gives relatively greater weight to people who are eligible for more months of the year. Monthly eligibles \(E_M\) were defined as \(TPM^E/12\) in equation (2) in the report. Recognizing that \(TPM^E\) is the sum across annual eligibles of each person’s eligibility intensity gives:

\[
(A1.2) \quad E_M = \frac{1}{12} TPM^E = \frac{1}{12} \sum_{k=1}^{E_A} I_k^E = \sum_{k=1}^{E_A} \frac{I_k^E}{12}
\]

Thus, to derive the number of monthly eligibles in equation (A1.2), each annual eligible is counted up using a statistical weight of \(I_k^E/12\) (versus the weight of 1.0 for each annual eligible in equation (A1.1)).

Another related approach to deriving \(E_M\) provides additional intuition. Each annual eligible has a set of 12 zero-one indicator variables \(e_{k,s}\), \(s = 1, 2, \ldots, 12\) where \(s\) represents a month of the year.

---

\(^{23}\) Analysis of the length-biased sampling between the monthly and annual timeframes does not require the assumption that steady-state conditions must hold. In contrast, the classic version of length-biased sampling requires the assumption that conditions are in a steady state to show that a stock sample is length-biased compared to the flow sample.
and \(e_{k,s} = 1\) if person \(k\) is eligible in month \(s\) and equals zero if person \(k\) is ineligible in month \(s\). The number of eligibles in month \(s\) is a count of (actual) people who are eligible in that particular month given by \(E^A_s = \sum_k e_{k,s}\). The (integer or non-integer) number of (statistical) people who are eligible in an average month of the year—the monthly eligibles \(E^M\)—can be defined as the average of \(E^A_s\) across the 12 months of the year:

\[
(A1.3) \quad E^M = \frac{1}{12} \sum_{s=1}^{12} E^A_s = \frac{1}{12} \sum_{s=1}^{12} \left[ \sum_{k=1}^{E^A} e_{k,s} \right] = \frac{1}{12} \text{TPME}^E
\]

The result that \((1/12) \sum_s E^A_s\) in (A1.3) and \((1/12) \sum_k I^E_k\) in (A1.2) each equal \((1/12) \text{TPME}^E\) simply reflects that the sum of the elements in the matrix \([e_{k,s}]\) can be obtained as the sum across columns of row-sums or the sum across rows of column-sums.

(2) Size difference between monthly and annual eligibles. By equation (A1.2), \(E^M\) counts an annual eligible with a weight of 1.0 if and only if that person is a full-year eligible with eligibility intensity of 12 months (in which case \(I^E_k/12 = 1.0\) for that person); otherwise, for any part-year eligible, \(I^E_k/12 < 1.0\) and the person counts strictly less than that same person counts in the annual timeframe. As a result, \(E^M < E^A\). The monthly and annual timeframes treat full-year eligibles equivalently (with a weight of 1.0) but treat part-year eligibles differently.

(3) Skewed distribution of eligibility intensity for monthly eligibles. The probability distribution of eligibility intensity \(I^E\) among annual eligibles is \(a(I^E), I^E = 1, 2, \ldots, 12\). The number of annual eligibles in the subgroup of people with eligibility intensity \(I^E\) is \(E^A(I^E)\). The total number of annual eligibles is \(E^A = \sum I^E(E^A)\), where the summation (and others that follow) are taken over \(I^E\). The distribution \(a(I^E)\) is given by \(E^A(I^E) / \sum E^A(I^E)\) for each value of \(I^E\), from which it follows that \(E^A(I^E) = E^A \ast a(I^E)\)—a relationship that is used below. The probability distribution of eligibility intensity \(I^E\) among monthly eligibles is \(m(I^E), I^E = 1, 2, \ldots, 12\). The number of person-months of eligibility held in the subgroup of annual eligibles with eligibility intensity \(I^E\) is \(E^M(I^E), \text{ where } E^M(I^E) = I^E \ast E^A(I^E)\). Total person-months of eligibility is \(\text{TPME}^E = \sum E^M(I^E)\). The distribution of \(m(I^E)\) is given by \(E^M(I^E) / \sum E^M(I^E)\). The relationship between \(m(I^E)\) and \(a(I^E)\)—the two distributions of \(I^E\) in the monthly and annual timeframes—is:

\[
(A1.4) \quad m(I^E) = \frac{E^M(I^E)}{\text{TPME}^E} = \frac{E^M(I^E)}{\sum E^M(I^E)} = \frac{I^E \ast E^A(I^E)}{\sum I^E \ast E^A(I^E)} = \frac{I^E \ast [E^A \ast a(I^E)]}{\sum I^E \ast [E^A \ast a(I^E)]} = \frac{I^E \ast a(I^E)}{\sum I^E \ast a(I^E)} = \left(\frac{I^E}{E^A}\right) \ast a(I^E)
\]

where \(I^E_a\) is the mean of \(a(I^E)\). From equation (A1.4), \(m(I^E) < a(I^E)\) for \(I^E < I^E_a\) and \(m(I^E) > a(I^E)\) for \(I^E > I^E_a\); if \(I^E_a\) happens to be an integer, then \(m(I^E) = a(I^E)\) at \(I^E_a\). More of the probability of \(m(I^E)\) lies to the right of \(I^E_a\), than can be found there for \(a(I^E)\). The essence of length-biased sampling is that \(m(I^E)\) is skewed to the right compared to \(a(I^E)\).

\[\text{24}\] The derivation of this result resembles most closely the derivation in Freeman and Hutchison (1980) of the steady-state relationship between prevalence and incidence. Salant (1977) and Lancaster (1992) provide derivations of that steady-state relationship (using the terms flow sampling and stock sampling) that differ from Freeman and Hutchison (1980).
(4) Effects on mean eligibility intensity. An implication of the eligibility intensity distribution in equation (A1.4) is that the estimated mean of $I^E$ among monthly eligibles ($I^E_m$) exceeds the estimated mean among annual eligibles ($I^E_a$). Specifically:

$$\text{(A1.5)} I^E_m = \sum_{i=1}^{12} I^E \ast m(I^E) = \sum_{i=1}^{12} I^E \ast \left( \frac{I^E}{I^E_a} \right) \ast a(I^E) = \left( \frac{1}{I^E_a} \right) \left[ \sum_{i=1}^{12} (I^E)^2 \ast a(I^E) \right]$$

$$= \left( \frac{1}{I^E_a} \right) [(I^E_a)^2 + \sigma_a^2] = I^E_a + \sigma_a^2 \frac{1}{I^E_a}$$

where $\sigma_a^2$ is the variance of the $a(I^E)$ distribution, using the relationship that the variance is the difference between the expected value of $(I^E)^2$ and the squared mean. Thus, $I^E_m > I^E_a$ so long as there is any variation in months of eligibility among the annual eligibles; for $I^E_m = I^E_a$, it must be that eligibility intensity is identical for everyone ($\sigma_a^2 = 0$).

(5) Effects on means of correlated (time-invariant) variables. Let $a(X, I^E)$ be the joint probability distribution in an annual timeframe for $I^E$ and a discrete-valued variable $X$ that is time invariant, meaning that each person has a single value of $X$ for the year under study. Examples of variables that can considered to be time invariant include gender, education, elderly status, and disability status.

Another important example is annual income, measured either in terms of dollars or income relative to poverty. For any one person, annual income for the year is a constant within the year under study (although the person's annual income can vary from one year to another). The frequency $a(X, I^E)$ is given by the proportion $E^A(X, I^E) / E^A$, where $E^A(X, I^E)$ is the number of annual eligibles that exhibit the combination of values $(X, I^E)$. The distribution $a(I^E)$ considered above is one of the marginal distributions for $a(X, I^E)$ obtained by summing the joint distribution over values of $X$, that is, $a(I^E) = \Sigma_X a(X, I^E)$. The other marginal distribution in an annual timeframe is $a(X)$ obtained by:

$$\text{(A1.6)} a(X) = \sum I^E a(X, I^E)$$

In the annual timeframe, the estimated mean of $X$ is given by

$$\text{(A1.7)} \bar{X}_a = \sum_X X a(X) = \sum_X \sum_I X a(X, I^E)$$

In a monthly timeframe the joint distribution is $m(X, I^E)$. The number of months of eligibility exhibited by people who have the particular combination of values $(X, I^E)$ is $E^M(X, I^E)$ where $E^M(X, I^E) = I^E \ast E^A(X, I^E)$. $TPM^E$ is obtained, as in the univariate case, by summing $E^M(I^E)$ across $I^E$. Similar to the derivation for equation (A1.4), the relationship between $m(X, I^E)$ and $a(X, I^E)$ is:

$$\text{(A1.8)} m(X, I^E) = \frac{E^M(X, I^E)}{TPM^E} = \frac{E^M(X, I^E)}{\sum_I E^M(I^E)} = \frac{I^E \ast E^A(X, I^E)}{\sum_I I^E \ast E^A(I^E)} = \frac{I^E \ast \left[ E^A \ast a(X, I^E) \right]}{\sum_I I^E \ast \left[ E^A \ast a(I^E) \right]}$$

$$= \frac{I^E \ast a(X, I^E)}{\sum_I I^E \ast \left[ E^A \ast a(I^E) \right]} = \left( \frac{I^E}{I^E_a} \right) \ast a(X, I^E)$$

The marginal distribution for $X$ in a monthly timeframe is $m(X)$ obtained using (A1.8) as:

---

Annual and Monthly SNAP Participation Rates, ERR-192
Economic Research Service/USDA

36
\[
(A1.9) \ m(X) = \sum_{I^E} m(X, I^E) = \sum_{I^E} \left( \frac{I^E}{I^E_a} \right) * a(X, I^E)
\]

Even though the length-biased sampling occurred with respect to \(I^E\), there is a spillover effect on the distribution of \(X\). The term \((I^E / I^E_a)\) appears in \(m(X)\), creating systematic disparities between \(m(X)\) in equation (A1.9) and \(a(X)\) in equation (A1.6)—just as \((I^E / I^E_a)\) creates disparities between the distributions of \(m(I^E)\) and \(a(I^E)\). In \(m(X)\), the values of \(X\) associated with people who have relatively more months of eligibility will be over-represented compared to \(a(X)\).

The covariance (among the annual eligibles) between \(X\) and \(I^E\) is:
\[
(A1.10) \ COV(X, I^E) = \sum_X \sum_{I^E} (I^E - \bar{I}^E_a) * (X - \bar{X}_a) * a(X, I^E)
\]
\[
= \sum_X \sum_{I^E} I^E * X * a(X, I^E) - \bar{I}^E_a * \bar{X}_a
\]

The estimated mean of \(X\) in the monthly timeframe is given by:
\[
(A1.11) \ \bar{X}_m = \sum_X X \ m(X) = \sum_X X \ \sum_{I^E} m(X, I^E) = \sum_X \sum_{I^E} \left( \frac{I^E}{I^E_a} \right) * X * a(X, I^E)
\]
\[
= \left( \frac{1}{I^E_a} \right) \sum_X \sum_{I^E} I^E * X * a(X, I^E) = \left( \frac{1}{I^E_a} \right) \left[ \left( I^E_a * \bar{X}_a \right) + COV(X, I^E) \right]
\]
\[
= \bar{X}_a + \frac{COV(X, I^E)}{I^E_a}
\]

A non-zero \(COV(X, I^E)\) creates a difference between \(\bar{X}_m\) and \(\bar{X}_a\). The skewed distribution for \(m(X)\) that results from length-biased sampling of \(I^E\) implies that \(\bar{X}_m\) exceeds \(\bar{X}_a\) when \(X\) and \(I^E\) have a positive covariance among annual eligibles (or, equivalently, positive correlation), while \(\bar{X}_m\) is less than \(\bar{X}_a\) when the covariance (or correlation) is negative.
Appendix 2—Intrayear dynamics of eligibility and participation determine SNAP participation rates

The mathematical relationship between the annual and monthly SNAP participation rates can be examined in terms of participation and eligibility turnovers (in equation (5)) or, equivalently, in terms of participation and eligibility intensities (in equation (8)), according to:

\[(A2.1) \pi^A = \left( \frac{T^P}{T^E} \right) \pi^M = \left( \frac{E^M}{T^P} \right) \pi^M\]

This appendix shows how the intrayear dynamics of participation and eligibility determine the difference between intensities (or between turnovers) and thus, by equation (A2.1) determine whether the annual SNAP participation rate is greater than or less than the monthly SNAP participation rate.

Let \(E^A(t)\) be the subgroup of annual eligibles who have \(t\) months of eligibility, and \(P^A(t)\) be the subgroup of annual participants who have \(t\) months of participation, \(t = 1, 2, \ldots, 12\).\(^{25}\) The appendix 2 table shows the two frequency distributions (using the balanced panel for 2012). The appendix 2 figure shows the set of 12 \((E^A(t), P^A(t))\) data points for 2012. For example, for \(t = 12\), the \((E^A(12), P^A(12))\) data point at (32.8, 30.8) lies to the northeast, reflecting how most annual eligibles and most annual participants have 12 months of eligibility or participation, respectively. The \((E^A(1), P^A(1))\) point at (5.0, 2.0) lies to the right of most other data points, reflecting that \(E^A(1)\) exceeds most other values of \(E^A(t)\). Using the 12 \(P^A(t)\) and 12 \(E^A(t)\), the ratio estimator for the overall or average proportion of annual participants among the population of annual eligibles is \(\sum P^A(t) / \sum E^A(t)\).\(^{26}\) Inasmuch as the annual SNAP participation rate is \(\pi^A = P^A/E^A = \sum P^A(t) / \sum E^A(t)\), the ratio estimator for our study is itself the annual SNAP participation rate.

**Analysis using deviations.** For each \(t\), the value of \(P^A(t)\) that could be estimated or predicted using \(\pi^A\) and \(E^A(t)\) lies along what we call the ratio-estimator line defined by the equation \(y(t) = \pi^A \ast E^A(t)\).\(^{27}\) The deviation \(Z(t)\) is the difference between the actual \(P^A(t)\) value and the predicted value:

\[(A2.2) Z(t) = P^A(t) - y(t) = P^A(t) - \pi^A \ast E^A(t)\]

The figure shows a positive deviation \(Z(12) = P^A(12) - \pi^A \ast E^A(12)\) by the vertical arrow between the ratio-estimator line and \(P^A(12)\) (which is above the line). The negative deviation \(Z(1) = P^A(1) - \pi^A \ast E^A(1)\) is not shown but is the distance between ratio-estimator line and \(P^A(1)\) (which is below the line).

\(^{25}\) In the appendix, it is convenient to have a single term \(t\) represent both participation intensity and eligibility intensity depending on whether \(t\) is an argument of \(P^A(t)\) or \(E^A(t)\).

\(^{26}\) The ratio estimator is a particular type of average widely used in statistical analysis. See Hansen et al., 1953 or Cochran, 1963 for discussion.

\(^{27}\) What we refer to as the ratio-estimator line is equivalent to a regression line that (a) is constrained to intersect the origin, and (b) includes an adjustment for heteroskedasticity under the assumption that the regression error has a variance in proportion to the independent variable \(E^A(t)\). These assumptions make the estimated slope of the regression line equal to \(\pi^A\); see Cochran, 1963, pp. 166-67.
In previous statistical studies, the deviations $Z(t)$ are important because they facilitate study of the variance of the ratio estimator. For our study, the importance of the deviations stems from the covariance between $Z(t)$ and $t$, which is given by:

\[(A2.3) \text{Cov}(Z(t), t) = \frac{1}{12} \sum Z(t) \ast t - \bar{Z}(t) \ast \bar{t} = \frac{1}{12} \sum Z(t) \ast t\]

where the second equality follows because the average of the deviations is zero. Because the mean deviation is zero, a positive $\text{Cov}(Z(t), t)$ indicates that (below mean) negative $Z(t)$ values (such as $Z(1)$) tend to occur for lower $t$, and (above mean) positive $Z(t)$ values (such as $Z(12)$) tend to occur for higher $t$; a negative $\text{Cov}(Z(t), t)$ would reverse that pattern. By equation (A2.3), the sign of the covariance between $Z(t)$ and $t$ is the same as the sign of $\Sigma Z(t) \ast t$, where $\Sigma Z(t) \ast t$ is:

\[(A2.4) \sum Z(t) \ast t = \sum [P^A(t) - \pi^A \ast E^A(t)] \ast t = \sum [P^A(t) - \left(\frac{P^A}{E^A}\right) \ast E^A(t)] \ast t = \left[\frac{\sum P^A(t) \ast t}{P^A} - \frac{\Sigma E^A(t) \ast t}{E^A}\right] P^A = \left[\frac{TPM^P}{P^A} - \frac{TPM^E}{E^A}\right] P^A = \left[\bar{P}^P - \bar{E}^E\right] P^A\]

Thus, a positive (negative) $\text{Cov}(Z(t), t)$ results in a positive (negative) difference between the average participation intensity $\bar{P}^P$ and the average eligibility intensity $\bar{E}^E$ (each in an annual timeframe) by equations (A2.3) and (A2.4). In turn, when $\bar{P}^P$ is greater than (less than) $\bar{E}^E$, then the annual SNAP

---

28 For examination of variance, see Hansen et al.,1953, pp. 165-66 for a diagram similar to our figure depicting the ratio-estimator line.
In conclusion, the intrayear patterns of eligibility and participation, as contained in and summarized by $\text{Cov}(Z(t), t)$, ultimately determine the difference between the two SNAP participation rates. When negative $Z(t)$ tend to occur for low $t$ and positive $Z(t)$ tend to occur for high $t$ (as illustrated in the figure), making $\text{Cov}(Z(t), t)$ positive, the key result is that the annual SNAP participation rate is less than the monthly rate.
Analysis using participants-to-eligibles ratios. The intrayear patterns of eligibility and participation that fundamentally determine whether the annual rate is less than or greater than the monthly rate can be expressed in terms of \( Z(t) \) deviations, as in equation (A2.4), or, equivalently, in terms of a set of participants-to-eligibles ratios defined by \( R(t) = 100 \times \frac{P^A(t)}{E^A(t)} \). The 12 participants-to-eligibles ratios are not a set of participation rates (which prompts the use of the term ratios) because a true participation rate must be a proportion for which the numerator is a subgroup of the denominator. In contrast, while the numerator of \( P^A(3)/E^A(3) \) can include people who were estimated to be eligible for 3 months and who reported participation for 3 months, that numerator includes all 3-month participants, including some who are not in the denominator—specifically, those who are estimated to be eligible for 4 months or more.

A given participants-to-eligibles ratio \( \frac{P^A(t)}{E^A(t)} \) is greater than (less than) the ratio-estimator \( \pi^A \) when \( Z(t) \) is positive (negative), by equation (A2.2). The figure illustrates the connection between the sign of \( Z(t) \) and whether \( \frac{P^A(t)}{E^A(t)} \) is greater than or less than \( \pi^A \). The slope of a line from the origin to a data point \((E^A(t), P^A(t))\) is itself the ratio \( \frac{P^A(t)}{E^A(t)} \). The figure shows two dashed lines for the \( t = 1 \) and \( t = 12 \) data points. The \( t = 12 \) data point has \( Z(12) > 0 \) and a slope of \( \frac{P^A(12)}{E^A(12)} \) that is steeper than the slope of the ratio-estimator line, i.e., \( R(12) > \pi^A \). The data point for \( t = 1 \) has \( Z(1) < 0 \) and a slope of \( \frac{P^A(1)}{E^A(1)} \) that is less steep than the slope of the ratio-estimator line, i.e., \( R(1) < \pi^A \). In fact, the ratio-estimator line has a slope that is a weighted average of the 12 \( R(t) \) given by:

\[
(A2.5) \quad \pi^A = 100 \times \frac{P^A}{E^A} = 100 \times \sum_t \frac{P^A(t)}{E^A(k)} = 100 \times \sum_t \left( \frac{E^A(t)}{\sum_k E^A(k)} \right) \frac{P^A(t)}{E^A(t)} \\
= 100 \times \sum \omega(t) R(t)
\]

where the weight \( \omega(t) \) is the share of annual eligibles that have \( t \) months’ worth of eligibility.

Let \( Q(t) \) be a slope-deviation defined by the difference between the \( R(t) \) slope to any particular data point and the slope of the ratio-estimator line: \( Q(t) = R(t) - \pi^A \). From the analysis above, the sign of each slope-deviation \( Q(t) \) corresponds to the sign of each \( Z(t) \) deviation.\(^{29}\) Using the result in equation (A2.4) for \( \Sigma Z(t) \), equation (A2.4) can be expressed in terms of \( Q(t) \):

\[
(A2.6) \quad \sum Z(t) \times t = \sum \left[ P^A(t) - \pi^A \times E^A(t) \right] \times t = \sum \left[ \frac{P^A(t)}{E^A(t)} - \pi^A \right] \times (E^A(t) \times t) \\
= \sum Q(t) \times E^M(t)
\]

where the last equality follows using the relationship \( E^M(t) = E^A(t) \times t \) from appendix 1 (after revising notation to use \( t \) in lieu of \( t^E \)), where \( E^M(t) \) is the number of person-months of eligibility exhibited by the subgroup of people with \( t \) months of eligibility; \( E^M(t) \) takes on a relatively low (high) value when either \( t \) or \( E^A(t) \) or both are relatively low (high). By equations (A2.6) and (A2.4):

---

\(^{29}\) We note that the average of \( Q(t) \) across \( t \) is not zero because: (a) the slope-deviations are defined for differences between \( R(t) \) and \( \pi^A \), and (b) the ratio estimator \( \pi^A = \frac{\Sigma P^A(t)}{\Sigma E^A(t)} \) is a statistically useful average participation rate among annual eligibles, but \( \pi^A \) is not mathematically equivalent to the average given by \( \overline{R} = (1/12) \Sigma R(t) \). (Deviations defined by \( R(t) - \overline{R} \) do sum to zero.)
Thus, if negative values of $Q(t)$ (reflecting $R(t) < \pi^A$) tend to occur when $E^M(t)$ is low, and positive values of $Q(t)$ (reflecting $R(t) > \pi^A$) tend to occur when $E^M(t)$ is high, then $\sum Q(t)E^M(t)$ is positive and average participation intensity ($\bar{P}$) exceeds the average eligibility intensity ($\bar{E}$). In turn, by equation (A2.1), when $\bar{P} > \bar{E}$ then the annual SNAP participation rate is less than the monthly rate.

In summary, the annual SNAP participation rate is less than the monthly rate when either: (a) $\sum Z(t) > 0$ or, equivalently, (b) $\sum Q(t)E^M(t) > 0$. This condition is essentially certain to be met in practice for SNAP.