Production Insurance and Input Use: An Analytical Framework

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WP 99-03

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Duality applies under uncertainty. In particular, Chambers and Quiggin (1998) have shown that dual cost structures exist for the continuous, stochastic technologies most familiar to agricultural economists. Beyond merely demonstrating existence, however, this finding has important implications for the analysis of stochastic decisionmaking. For years now, agricultural economists have intensively studied decisionmaking by producers facing stochastic technologies. And yet, no commonly accepted body of 'stylized facts' exists for most truly interesting formulations of this problem. Some have even questioned the relevance of the cost minimization hypothesis for risk-averse decisionmakers (Pope and Chavas). More generally, apart from a number of results that have been established for trivially stochastic situations, e.g., price but not production uncertainty, there is no common agreement as to what one can expect from a risk-averse producer facing a stochastic world.

A particular case in point is the literature on the economic implications of the public and private provision of crop insurance. The basic theory of crop insurance has been studied by several authors (Nelson and Loehman, Chambers). However, few really new results have emerged. For example, under the expected-utility hypothesis, a primary result is that optimality requires that a risk-neutral insurer offer full insurance to risk-averse farmers producing a single stochastic output so long as no informational asymmetries exist between the insurer and the farmer (Nelson and Loehman). But it is easy to recognize this result as a direct consequence of basic re-insurance results established much earlier by Borch.

Other authors have studied the related and potentially more important question of how input utilization is affected by the provision of crop insurance or income support (Quiggin 1992, Ramaswami, Hennessey). Intuitively, one expects that providing insurance encourages producers to undertake risky activities that carry with them the promise of higher expected returns. Reasoning thus, one also expects that inputs which might be perceived as enhancing the riskiness of the production outcome would be used more intensively in the presence of insurance than in its absence. Conversely, inputs which do little to enhance productivity, but which act as damage-control agents, would be used less intensively in the presence of insurance than in its absence. When stated in this fashion, these facts seem to be self-evident. However, the existing literature suggests that this is not generally the case even if attention is restricted to single-output, single-input technologies (Ramaswami; Horowitz and Lichtenberg). Because such technologies are highly restrictive, the natural implication seems to be that little, if anything, can be said for more realistic technologies.

The goal of this paper is to demonstrate, by way of theoretical example, the importance of the duality between cost and stochastic technologies by studying the impact of crop insurance upon input utilization. The basic model is a state-contingent formulation of the problem, which encompasses both production and price uncertainty, and that allows full exploitation of the duality between the technology and the cost structure in comparative-static analyses. In particular, we can rely on this duality and a stochastic version of Shephard's lemma to allow us to examine input responsiveness to the provision of crop insurance in a new and informative manner that does not rely on the single-input, single-output stochastic production function model that has dominated most previous studies. Moreover, this formulation carries it with the additional benefit that it also allows us to consider preference structures that are far more general than the expected-utility preference structures that have been used in the existing literature.

Using this formulation of the problem, we show that it is straightforward to develop a complete, analytical framework for analyzing the impact of crop insurance (and more generally any other comparative static problem regarding input utilization) that can be usefully illustrated with graphical techniques that should be familiar to virtually all economists. In particular, the analytical framework presents a decomposition of the problem reminiscent of the classic Hicks-Allen decomposition of the Slutsky effect familiar from rudimentary consumer theory.

1. Model and Assumptions

1.1. A State-Contingent Technology

Following Chambers and Quiggin (1996, 1997, 1999), the stochastic technology is represented by a multi-product, state-contingent input correspondence. To make this explicit, suppose that the states of nature are given by the set $\Omega = \{1, 2, ..., S\}$, let $\mathbf{x} \in \Re_+^N$ be a vector of inputs committed prior to the resolution of uncertainty, and let $\mathbf{z} \in \Re_+^{M \times S}$ be a vector of state-contingent outputs. So, if state $s \in \Omega$ is realized (picked by 'Nature'), and the producer has chosen the *ex ante* input-output combination $(\mathbf{x}, \mathbf{z})$, then the realized or *ex post* output vector is $\mathbf{z}^s$ corresponding to the sth column of $\mathbf{z}$. In other words, the observed output is an M-dimensional vector $\mathbf{z}^s$ where z_m^s corresponds to the mth output that would be produced in state s.

The input correspondence, $X : \Re_+^M \to \Re_+^N$, maps matrices of state-contingent outputs into input sets that are capable of producing that state-contingent output matrix. Formally, it is defined by

$$X(\mathbf{z}) = \{\mathbf{x} \in \Re_+^N : \mathbf{x} \text{ can produce } \mathbf{z} \in \Re_+^{MxS}\}.$$

We impose the following axioms on $X(\mathbf{z})$:

X.1 $X(\mathbf{0}_{MxS}) = \Re_+^N$ (no fixed costs), and $\mathbf{0}_N \in X(\mathbf{z})$ for $\mathbf{z} \geq \mathbf{0}_{MxS}$ and $\mathbf{z} \neq \mathbf{0}_{MxS}$ (no free lunch).

X.2 $\mathbf{z}' \leq \mathbf{z} \Rightarrow X(\mathbf{z}) \subset X(\mathbf{z}')$.

X.3 $\mathbf{x}' \geq \mathbf{x} \in X(\mathbf{z}) \Rightarrow \mathbf{x}' \in X(\mathbf{z})$.

X.4 $\lambda X(\mathbf{z}) + (1-\lambda)X(\mathbf{z}') \subset X(\lambda\mathbf{z} + (1-\lambda)\mathbf{z}') \qquad 0 \leq \lambda \leq 1.$

X.5 $X(\mathbf{z})$ is closed for all $\mathbf{z} \in \Re_+^{MxS}$.

The first part of X.1 says that doing nothing is always feasible, while the second part of X.1 says that realizing a positive output in any state of nature requires the committal of some inputs. X.2, free disposability of state-contingent outputs, says that if an input combination can produce a particular matrix of state-contingent outputs then it can always be used to produce a smaller matrix of state-contingent outputs. X.3 implies that inputs have non-negative marginal productivity. X.4 tells us that the state-contingent technology is convex, and intuitively it leads to diminishing marginal productivity of inputs. X.5 is a technical assumption that ensures the existence of the revenue-cost function that we develop next.

1.2. The revenue-cost function

Denote by $\mathbf{p} \in \Re_{++}^{M \times S}$ the matrix of state-contingent output prices corresponding to the matrix of state-contingent outputs. The interpretation of $\mathbf{p}$ is basically the same as $\mathbf{z}$. If 'Nature' picks $s \in \Omega$, then the vector of realized spot prices is $\mathbf{p}^s \in \Re_{++}^M$. We assume that the producers in question are competitive in the sense that they take these state-contingent output prices and the prices of all inputs as given. The state-contingent revenue vector $\mathbf{r} = \mathbf{pz} \in \Re^S_+$ has typical elements of the form $r_{s=}\mathbf{p}^s \bullet \mathbf{z}^s$.

In all cases we consider, producers will be concerned with state-contingent revenue rather than output *per se*, and it is useful to consider the *revenue-cost function* defined as

$$C(\mathbf{w},\mathbf{r},\mathbf{p}) = \min\left\{\mathbf{w}\cdot\mathbf{x} : \mathbf{x} \in X(\mathbf{z}), \sum_m p_{ms}z_{ms} \geq r_s, s \in \Omega\right\}$$

if there exists a feasible state-contingent output array capable of producing $\mathbf{r}$ and ∞ otherwise. Here $\mathbf{w} \in \Re^N_{++}$ represents a strictly positive vector of input prices that the producer takes as given. The properties of $C(\mathbf{w},\mathbf{r},\mathbf{p})$ that follow from X.1-X.5 (Chambers and Quiggin, 1999):

Properties of the Revenue-Cost Function (CR):

CR.1$C(\mathbf{w}, \mathbf{r}, \mathbf{p})$ is positively linearly homogeneous, non-decreasing, concave, and continuous in $\mathbf{w} \in \Re^N_{++}$.

CR.2 Shephard's Lemma.

CR.3 $C(\mathbf{w},\mathbf{r},\mathbf{p}) \geq \mathbf{0}$ with equality if and only if $\mathbf{r} = 0$.

CR.4 $\mathbf{r}' \geq \mathbf{r} \Rightarrow C(\mathbf{w},\mathbf{r}',\mathbf{p}) \geq C(\mathbf{w},\mathbf{r},\mathbf{p})$.

CR.5 $\mathbf{p}' \geq \mathbf{p} \Rightarrow C(\mathbf{w},\mathbf{r},\mathbf{p}') \leq C(\mathbf{w},\mathbf{r},\mathbf{p})$.

CR.6 $C(\mathbf{w},\mathbf{r}_{-s},\theta r_{s,},\mathbf{p}_{-s},\theta\mathbf{p}_s) = C(\mathbf{w},\mathbf{r}_{-s},\theta r_{s,},\mathbf{p}_{-s},\theta\mathbf{p}_s), \quad \theta > 0$.

CR.7$C(\mathbf{w},\mathbf{r},\mathbf{p}) = C(\mathbf{w},\mathbf{r}/k,\mathbf{p}/k), \quad k > 0$.

CR.8 $C(\mathbf{w},\mathbf{r},\mathbf{p})$ is convex in $\mathbf{r}$.

For analytic simplicity, we shall typically assume that $C(\mathbf{w},\mathbf{r},\mathbf{p})$ is smoothly differentiable in all state-contingent revenues and input prices. By assuming a differentiable in revenues cost structure, we, therefore, rule out the stochastic-revenue function approach and the non-stochastic production approach of Sandmo (Chambers and Quiggin, 1998, 1999).[1]

[1] If production is non-stochastic, then it follows immediately that

$$C(\mathbf{w},\mathbf{r},\mathbf{p}) = \mathbf{Max}_{1,2,\ldots S}\left\{C^f(\mathbf{w},r_s,p_s)\right\}$$

where

$$C^f(\mathbf{w},r_s,p_s) = Min\left\{\mathbf{wx} : p_s f(\mathbf{x}) \geq r_s\right\}.$$

Generally, neither this function or the one corresponding to the stochastic-revenue function will be everywhere

1.3. Preferences

The producer's preferences are represented by an increasing function of his vector of state contingent net returns.

$$\mathbf{y} = \mathbf{r} - (\mathbf{w} \cdot \mathbf{x})\,\mathbf{1}_s,$$

where $\mathbf{1}_S$ is the S-dimensional unit vector. As Chambers and Quiggin (1998) demonstrate, without loss of generality, the producer's preferences can thus be expressed in terms of the revenue-cost function as

$$\mathbf{y} = \mathbf{r} - \mathbf{C}(\mathbf{w}, \mathbf{r}, \mathbf{p})\mathbf{1}_s.$$

Following Quiggin and Chambers (1998) and Chambers and Quiggin (1999), these preferences over state-contingent net returns will be represented in terms of a continuous, strictly increasing preference function $W : \Re^s \rightarrow \Re$. A producer is said to be *risk-averse with respect to the probability vector* $\boldsymbol{\pi} \in \Pi$ *if*

$$W(\bar{y}\mathbf{1}^S) \geq W(\mathbf{y}), \forall \mathbf{y}$$

where $\bar{y}\mathbf{1}^S$ is the state-contingent outcome vector with $\bar{y} = \sum_{s\in\Omega} \pi_s y_s$ occurring in every state of nature (Yaari, 1969; Quiggin and Chambers, 1998).

If preferences are smoothly differentiable, the vector of probabilities is unique and proportional to the marginal rate of substitution between state-contingent incomes along the equal-incomes vector. More concretely, without loss of generality, if preferences are smoothly differentiable

$$\pi_s = \frac{W_s(c\mathbf{1}^S)}{\sum_{t\in\Omega} W_t(c\mathbf{1}^S)}, \quad s \in \Omega, \quad c \in \Re.$$

Pictorially, therefore, the *fair-odds line*, which gives the locus of points having the same expected value and whose slope is given by minus the relative probabilities is given by the slope of the tangent to the producer's indifference curve at the bisector. Figure 1 illustrates.

In order to impose some structure upon preferences other than simple aversion to risk, consider the partial ordering $\preceq_\pi$ of risky outcomes which possess a common mean for the probability vector $\boldsymbol{\pi}$. This partial ordering is defined by

$$\mathbf{y} \preceq_\pi \mathbf{y}'$$

smoothly differentiable in revenues or outputs respectively.

if and only if $\mathbf{y}$ and $\mathbf{y}'$ have the same mean and $\mathbf{y}$ is less risky than $\mathbf{y}'$ in the sense of Rothschild and Stiglitz. Chambers and Quiggin (1997) define a function $W : \Re^S \to \Re$ to be *generalized Schur-concave* for π if $\mathbf{y} \preceq_\pi \mathbf{y}' \Rightarrow W(\mathbf{y}) \geq W(\mathbf{y}')$.

A comment about generalized Schur concavity is worthwhile. Unlike the assumption of expected-utility maximization, generalized Schur concavity doesn't impose additive separability across states of nature. Consequently, it does not rely upon the independence axiom which has proved vulnerable to a variety of criticisms. Even so, the expected-utility functional with concave u is generalized Schur-concave as can be recognized from the result due to Rothschild and Stiglitz that if $\mathbf{y} \preceq_\pi \mathbf{y}'$ then $\mathbf{y}$ would be preferred to $\mathbf{y}'$ by all individuals with risk-averse expected-utility preferences. More generally, generalized Schur concavity characterizes a number of preference classes, which are consistent with risk-aversion in our sense, but which are not consistent with expected utility. An obvious example is given by individuals with maximin preferences

$$W(\mathbf{y}) = \min\{y_1, \dots, y_S\}.$$

This class of preferences is risk-averse in our sense for all possible probability vectors (note it is not differentiable), and it is also generalized Schur concave. Another obvious class of generalized Schur concave preferences is the mean-variance class. More generally, virtually all preference functions currently in use, including the rank-dependent models (Quiggin 1982, Yaari 1987) and weighted-utility models (Chew 1983) are consistent with generalized Schur concavity. The main result of Machina (1982) may be restated in our terminology as saying that preferences are generalized Schur concave if and only if the local utility function is everywhere concave.

In what follows, we shall frequently restrict attention to the case where W is smoothly differentiable. In that case, a basic result due to Chambers and Quiggin (1997), which we state in lemma form for future use, will prove useful:

Lemma 1 If $W : \Re^S \to \Re$ is generalized Schur-concave and once continuously differentiable everywhere on its domain, then :

$$\left(\frac{W_s(\mathbf{y})}{\pi_s} - \frac{W_r(\mathbf{y})}{\pi_r}\right)(y_s - y_r) \leq 0,$$

for all s and r.

2. Production Equilibrium in the Absence of Insurance

As a point of comparison, we first present some basic results on the production choices of risk-neutral and risk-averse producers in the absence of insurance. Suppose the risk-neutral producer's subjective probabilities are given by the vector $\boldsymbol{\pi}$. Then her first-order conditions on $\mathbf{r}$ may be written in the notation of complementary slackness as

$$\pi_s - C_s(\mathbf{w}, \mathbf{r}, \mathbf{p}) \leq 0, \quad r_s \geq 0, \quad s \in \Omega$$

where

$$C_s(\mathbf{w}, \mathbf{r}, \mathbf{p}) = \frac{\partial C(\mathbf{w}, \mathbf{r}, \mathbf{p})}{\partial r_s}.$$

That is, the marginal cost of increasing revenue in any state is at least equal to the subjective probability of that state. Pictorially, therefore, we represent the producer equilibrium by a hyperplane being tangent to an isocost curve of the producer. Figure 2 illustrates. Here the slope of the hyperplane is determined by the ratio of the producer's subjective probabilities, *the fair-odds line*, and the isocost curve is determined by the equilibrium level of revenue-cost. This is exactly analogous to the representation of production equilibrium in the non-stochastic, multi-product case. Instead of determining an optimal mix of outputs as in the non-stochastic multi-product case, however, the producer equilibrium now determines the optimal mix of state-contingent revenues. This analogy naturally suggests interpreting the producer's subjective probabilities as the producer's subjective prices of the state-contingent revenues.

Summing the first-order conditions on $\mathbf{r}$ yields an *arbitrage condition*

$$\sum_{s \in \Omega} C_s(\mathbf{w}, \mathbf{r}, \mathbf{p}) \geq \sum_{s \in \Omega} \pi_s = 1. \qquad (2.1)$$

Intuitively speaking, $\sum_{s \in \Omega} C_s(\mathbf{w}, \mathbf{r}, \mathbf{p})$ is the marginal cost of increasing all state-contingent revenues by the same small amount in each state of nature, i.e., it is the marginal cost of a sure increase in revenue of one unit. Hence, (2.1) simply requires that this cost be at least as large as the associated sure increase in returns. If it were not, the decisionmaker could increase profit with probability 1, and she would thus have an incentive to continue expanding all revenues equally. For an interior solution, (2.1) must hold as an equality.

We shall refer to the set of revenue vectors $\mathbf{r}$ satisfying (2.1) for given $\mathbf{w}$,$\mathbf{p}$ as the *efficient set*, denoted $\Xi(\mathbf{w},\mathbf{p})$,

$$\Xi(\mathbf{w},\mathbf{p}) = \left\{ \mathbf{r} : \sum_{s\in\Omega} C_s(\mathbf{w},\mathbf{r},\mathbf{p}) \geq 1 \right\}.$$

We call the boundary of $\Xi(\mathbf{w},\mathbf{p})$ the *efficient frontier* and note that its elements are given by:

$$\bar{\Xi}(\mathbf{w},\mathbf{p}) = \left\{ \mathbf{r} : \sum_{s\in\Omega} C_s(\mathbf{w},\mathbf{r},\mathbf{p}) = 1 \right\}.$$

By the homogeneity properties of the revenue-cost function, we can conclude that: $\Xi(\theta\mathbf{w},\theta\mathbf{p}) = \theta\Xi(\mathbf{w},\mathbf{p})$ and $\bar{\Xi}(\theta\mathbf{w},\theta\mathbf{p}) = \theta\bar{\Xi}(\mathbf{w},\mathbf{p})$, $\theta > 0$ (Chambers and Quiggin, 1999). That is, the efficient set and the efficient frontier are positively linearly homogeneous in input and output prices.

Different risk-neutral producers may hold different subjective probabilities. Regardless of the individual's subjective probabilities, however, a revenue vector $\mathbf{r}$ is potentially optimal for some risk-neutral decision-maker only if (2.1) holds. Hence, $\bar{\Xi}(\mathbf{w},\mathbf{p})$ can be interpreted naturally as the collection of state-contingent revenues that are potentially expected-profit maximizing. To see why, suppose that (2.1) holds for an arbitrary revenue vector, call it $\hat{\mathbf{r}}$. Now construct a set of probabilities by setting $\hat{\pi}_s = C_s(\mathbf{w},\hat{\mathbf{r}},\mathbf{p})$ for all s. Because they belong to the efficient set and are derived from a non-decreasing revenue-cost function, these probabilities are positive and sum to one. Moreover, a risk-neutral individual having such probabilities would choose $\hat{\mathbf{r}}$ as the expected-profit maximizing vector of state-contingent revenues. The correspondence of the producer's subjective probabilities with these state-contingent marginal costs then determines the optimal point on the efficient set.

Now let us turn to the case where the producer is not risk-neutral but risk-averse with generalized Schur-concave preferences[2]. The producer chooses state-contingent revenues to maximize:

$$W(\mathbf{y}) = W(\mathbf{r} - C(\mathbf{w},\mathbf{r},\mathbf{p})\mathbf{1}_S).$$

So long as the preference function is smoothly differentiable in state-contingent revenues,

[2]Risk-neutral preferences are trivially generalized Schur-concave.