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TECHNICAL EFFICIENCY AND FARM SIZE:
A NON-PARAMETRIC FRONTIER ANALYSIS

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ABSTRACT

This study examines the argument that farm size is associated with technical efficiency. A non-parametric, frontier analysis method is used, together with some additional statistical analysis. Although some gains in efficiency appear as we move toward larger farms, they do not seem to be significantly large. The input/output ratios are indeed smaller for larger farms and the difference is statistically significant. It can be attributed however to the fact that smaller farms operate on a smaller scale and not inside the production frontier.

INTRODUCTION

Three types of efficiency are usually distinguished in the literature: Technical efficiency, price (allocative) efficiency, and economic efficiency (Farrell, 1957). A technically efficient farm is one which produces the maximum quantity of output for given quantity of inputs, given the production function. A price efficient farm applies that quantity of inputs which maximizes profits, given of course the production function and the prices it faces. An economically efficient farm is one that is both technically and price efficient. We deal here with the argument often made, that larger farms are more technically efficient than smaller farms. If the efficiency gains of larger farms can be established as a general rule, the structural changes in the farm sector, which include changes in the farm size distribution, will have an important impact on the overall performance of agriculture. This is also important for policy makers. They can assess better the impact of policies that affect farm structure, on efficiency. It is also important to know how far a given industry, in this case agriculture, can be expected to increase its outputs by simply increasing its efficiency without utilizing more resources (Farrell, 1957).

The purpose of this study is to examine if and how technical efficiency is affected by the farm size. Some statistical evidence concerning input/output ratios is initially examined and a non-parametric frontier analysis method is used. An explanation of this type of frontier analysis and how it can be used for our own purposes is also presented. We conclude with an analysis of our results, factors that affect them, and some inferences.

It would be more accurate to measure farm size in terms of value added. Given the data limitations however, we have adopted as the best alternative the value of sales. The source of our data was the U.S. Census of Agriculture for 1987 issued by the U.S. Department of Commerce.

SOME EMPIRICAL EVIDENCE

Using Census data, we aggregated all farms into three different sales classes. Class A included farms with value of sales of more than \$100,000, Class B farms with sales of \$40,000 to \$99,999 and Class C farms with sales of less than \$40,000. This was done for 31 states of the U.S.¹ For every one of these states and for every sales class, we selected data on aggregate agricultural output and on farm production expenses for several agricultural inputs. We estimated then the corresponding input/output ratios, where both input and output were originally expressed in monetary terms. Therefore, for each input/output ratio we established three distributions (one for each size class) with 31 observations (one for each of the 31 states). Generally speaking, in every observation (state) the input-output ratio was declining as we were moving from distribution (sales class) C to A. We performed a z-test for the means of these distributions, where the observations for every state were the ratios of total farm production expenditures over the total value of agricultural products sold. The mean of distribution C was significantly higher than the mean of distribution B at 5% level of significance. The mean

¹These states were: North Dakota, Virginia, Washington, Minnesota, Utah, West Virginia, Texas, Pennsylvania, Oregon, Wyoming, Wisconsin, Ohio, Oklahoma, South Dakota, South Carolina, Tennessee, Alabama, Montana, New York, Iowa, California, Vermont, Missouri, Mississippi, Michigan, Nebraska, Arizona, Kansas, Kentucky, and Illinois. These states, coming from all regions of the U.S., were selected randomly. They include however the majority of the U.S. farms.

of distribution B was also larger than the mean of distribution A although the difference was not statistically significant at 5% level of significance. The above results are influenced by expenditures on relatively fixed factors. When disaggregated farm inputs are used in the input/output ratio, the results appear much stronger. For example, when the ratio machinery²/total sales is considered, the mean of distribution A is smaller than the one of B and the latter smaller than the mean of A at all levels of significance. The results were similar when the distributions concerned the ratio of agricultural chemicals over the total value of agricultural sales. When the ratio was fertilizer/total sales the mean of distribution B was significantly smaller than the mean of C, at 5% level of significance, and the mean of A smaller than the one of B at 10% level of significance. When the ratio, however, included physical units of fertilizer and lime (tons) instead of monetary values the results were similar and significant at all levels of significance. Hence, "on the average" (and as a general rule) the input/output ratios are significantly smaller for larger farms than for smaller ones.

There are two other reasons - besides the fixity of some inputs - that when total production expenditures are used in the input/output ratio, the above results are less significant than when individual inputs are considered. One may have to do with the way we aggregated the sales classes into the three that we used here. Another classification of small, medium, and large farms could provide perhaps stronger results. The second is related to farm labor. Total farm production expenditures include expenditures on hired and contract farm labor. Family labor on which medium and small farms rely heavily is not

²Machinery includes expenditure on repair and maintenance, customwork, machine hire, rental of machinery and equipment.

included in their production expenses. We shall discuss this issue later again, in the frontier analysis model.³ Although the input/output ratios of smaller farms were found to be significantly higher than those of the large farms in several occasions, no conclusions should be derived about their technical efficiency. Considering the scale on which they operate, small farmers can still be producing the maximum possible output for the given amount of inputs or using the minimum amount of inputs given their output. To examine the issue of technical efficiency we employ below a non-parametric frontier analysis method.

FRONTIER ANALYSIS

Background

Frontier analysis, as a method of measuring production efficiency and estimating production frontiers, increased its popularity rapidly during the 80's. Two major "types" of frontier analysis have been developed simultaneously but so far, in general, proceeded independently from each other (A.Y. Lewin, C.A. Knox Lovell, 1990). One is the econometric approach, mostly popular among the economists, and the other is the operations research and management science approach which uses mathematical programming techniques. The second approach is mostly popular among industrial engineers, managers and many others concerned with optimization behavior and the evaluation of its performance. The mathematical programming approach is also known as DEA (Data Envelopment Analysis). Both approaches attempt to obtain information from a body of data and have a common interest in extreme

³For more explanations on the data used and some statistical information, see the Appendix.

observations. Despite their different methodologies and separate developments, recent efforts have begun to reconcile them, (Sengupta 1990, Varian 1990) and combine them for empirical studies. In our analysis of the efficiency and relative efficiency of different farm sizes we use a DEA model. The DEA methodology is considered non-parametric because it does not assume and impose a particular form of the production function as parametric techniques do. Instead of estimating the production function with the assumed form and proceed to efficiency computations relative to this frontier (Lovel and Schmidt, 1988), DEA imposes the simple restriction that all firms, - Decision Making Units (DMU) - lie on, or "below" the efficient frontier. L.M. Seiford and R.M. Thrall (1990) who have tried to put together and present the last developments in the DEA, characterize it as a methodology directed to frontiers rather than central tendencies. Contrary to regression techniques that fit a plane or hyperplane through the center of the data, the DEA "floats" a piece-wise linear surface to rest on top of the observations. This characteristic, permits the DEA to discover relationships that parametric or other methodologies would not. A rapidly increasing literature has been enriching the possibilities of frontier analysis and DEA in particular. But the empirical applications are noteworthy too. An example that Seiford and Thrall (1990) emphasized, is the study of Banker, Conrad, and Strauss (1986) which estimated a production function for hospitals in North Carolina. Comparing the results of both, regression techniques and DEA, the second outperformed the first, utilizing the same information.

The Model

M.J. Farrell (1957), who presented a classic paper on the measurement of productive efficiency, argued that efforts to estimate a production frontier, usually produce careful measurements but they fail to combine the measurements of the multiple inputs into any satisfactory overall measure of efficiency and proposed an activity analysis to overcome this problem. Based on his ideas, Charnes, Cooper, and Rhodes (1978) proposed a non parametric linear programming DEA model (CCR). Later models related to that were the ones of Banker, Charnes, and Cooper (BCC), in 1984 and of Charnes, Cooper and Thrall in 1986. We use here the CCR linear programming model as presented in Seiford and Thrall (1990).

Assume that there are n firms (DMU's) to be evaluated and each of them uses m inputs to produce s outputs. Each firm j uses n_{ji} amount of input i and produces amount y_{jr} of output r . It is also assumed here that $x_{ji} \geq 0$ and $y_{jr} \geq 0$. The main characteristic of the CCR model is that it reduces the multiple-output and multiple-input situation to that of a single "virtual" output and "virtual" input. Their ratio provides a measure of efficiency as a function of multipliers. The mathematical programming problem of each firm is:

$$\max_{\{u, v\}} h_o(u, v) = \frac{\sum_r u_r y_{ro}}{\sum_i v_i x_{io}} \quad (1)$$

where the variables are the u_r 's and v_i 's while the y_{ro} 's and n_{io} 's are the values of outputs and inputs for the firm DMU_o. The objective function given by (a) is, however, unbounded. A set of normalizing constraints can be added, expressing the condition that the "virtual" output to "virtual" input ratio

for every DMU is less than or equal to 1. Hence, the problem can be expressed as:

$$\begin{aligned} & \max_{\{u, v\}} \frac{\sum_r u_r y_{r0}}{\sum_i v_i x_{i0}} \\ \text{s. t.} & \\ & \frac{\sum_r u_r y_{rj}}{\sum_i v_i x_{ij}} \leq 1 \quad \text{for } j = 0, 1, \dots, n \quad (\text{IR}) \\ & u_r, v_i \geq 0 \end{aligned}$$

The above model is called input-oriented (IR), as opposed to the output oriented model where the inputs appear in the nominator of the objective function which in this case is to be minimized. The (IR) model yields infinite solutions. For a solution (u^*, v^*) , $(\alpha u^*, \alpha v^*)$ is a solution too, as long as $\alpha > 0$. Charnes and Cooper (1962), proposed a transformation for linear fractional programming which selects a representative solution (i.e., the (u^*, v^*) for which $u^T X_0 = 1$ where X_0 is the vector of inputs for DMU_0). The selection occurs for every equivalence class into which the set of feasible solutions is partitioned.

The transformation results into the problem:

$$\begin{aligned} & \max_{\{\mu, v\}} z = \mu^T Y_0 \\ \text{s. t.} & \\ & v^T X_0 = 1 \quad (\text{DI}_0) \\ & \mu^T Y - v^T X^0 \leq 0 \quad (\text{Multiplier problem}) \\ & \mu^T \geq 0 \\ & v^T \geq 0 \end{aligned}$$

The dual problem of the above is:

$$\begin{aligned}
 & \min \theta \\
 & \{ \theta, \lambda \} \\
 & \text{s.t.} \\
 & Y\lambda \geq Y_0 \quad (PI_0) \\
 & \theta X_0 - X\lambda \geq 0 \quad (\text{Envelopment problem}) \\
 & \theta \text{ free, } \lambda \geq 0
 \end{aligned}$$

where λ is a vector of multipliers with dimension $(1 \times n)$ where n is the number of DMU's. X is a matrix of elements x_{ij} where j (column) indicates the j^{th} DMU and i the i^{th} input. Similarly Y is a matrix of outputs y_{ij} where j refers to the DMU and i to output. Similarly to X_0 , Y_0 is a vector of outputs for DMU₀. θ is a scalar, while θ, λ are the variables⁴. One can solve either (DI₀) or (PI₀) for DMU₀, and similarly for every other DMU_j by replacing Y_0 and X_0 with Y_j and X_j respectively. There are alternative structures however for

⁴If we had chosen to use the output oriented DEA, (CCR) model, the corresponding models to (IR), (DI₀), and (PI₀) would have been

$$\begin{array}{lll}
 \min \frac{v^T X_0}{u^T Y_0} & \min \sigma = v^T X_0 & \max \phi \\
 \{u, v\} & \{\mu, v\} & \{\theta, \lambda\} \\
 \text{s.t.} & \text{s.t.} & \text{s.t.} \\
 \frac{v^T X_j}{u^T Y_j} \geq 1 & \mu^T Y_0 = 1 & X\lambda \leq X_0 \\
 j = 1, \dots, n & -\mu^T Y + v^T X \geq 0 & \phi Y_0 - Y\lambda \leq 0 \\
 u \geq 0 & \mu^T \geq 0 & \phi \text{ free} \\
 v \geq 0 & v^T \geq 0 & \lambda \geq 0
 \end{array}$$

These are symbolized as OR, DO₀, PO₀ respectively.

both (DI_0) and (PI_0) . (Note that the subscript is not referring now to DMU_0 as is the case with X_0 and Y_0 but to the first alternative structure for DI_p and PI_p , where $p = 0, 1, 2, 3$). In particular, if e^T is a row vector of ones with dimension $(n \times 1)$, we have the following alternative forms of the envelopment problem (PI_p) :

$$\begin{array}{ll}
 \min \theta & (PI_p) \\
 \{\theta, \lambda\} & \text{For } PI_0, \text{ append nothing} \\
 s. t. & \text{For } PI_1, \text{ append } e^T \lambda \leq 1 \\
 Y\lambda \geq Y_0 & \text{For } PI_2, \text{ append } e^T \lambda \geq 1 \\
 \theta X_0 - X\lambda \geq 0 & \text{For } PI_3, \text{ append } e^T \lambda = 1 \\
 \theta \text{ free, } \lambda \geq 0 &
 \end{array}$$

and of the multiplier problem (DI_p) :

$$\begin{array}{l}
 \max z = \mu^T Y_0 + u_* \\
 \{\mu, v\} \\
 s. t. \\
 v^T X_0 = 1 \\
 u_* e^T + \mu^T Y - v^T X \leq 0 \\
 \mu^T \geq 0 \\
 v^T \geq 0
 \end{array}$$

where $u_* = 0$ in DI_0 , u_* is free in DI_3 , u_* is smaller than equal to zero in DI_1 and u_* is larger than equal to zero in DI_2 . We use in this study the

envelopment model (PI_p) of the input oriented model.⁵ The constraints concerning $e^T\lambda$ are associated with the assumptions about the returns to scale.

If we add:

- 1) $e^T\lambda \leq 1$: increasing returns are not allowed (p = 1)
- 2) $e^T\lambda \geq 1$: decreasing returns are not allowed (p = 2)
- 3) $e^T\lambda = 1$: no assumption is made (p = 3)
- 4) nothing: only constant returns allowed (p = 4)

When applying the model we use (p = 3) and (p = 2) primarily, but we also examine the other assumptions, as well. Solving (PI_p) with respect to θ and λ , for each DMU, the optimal solution θ^* provides an efficiency score. By its structure the model always provides $\theta^* \leq 1$. When $\theta^* = 1$ the firm (DMU) is operating on the frontier. If for the optimal λ^* , $X_0 = X\lambda^*$ and $Y_0 = Y\lambda^*$, then $\theta^* = 1$ guarantees that the boundary point is efficient. If $\theta^* < 1$ the DMU does not operate on the frontier and is inefficient. The smaller θ^* is from 1, the larger the distance of the operating point from the frontier. (θ^* can also be used to project an inefficient point on the frontier.)

⁵The envelopment and multiplier problems of the output-oriented model can also be used in different forms depending on the constraints added. Thus, we have the envelopment problems (PO_p) and the multiplier problems (DO_p) as in the input oriented case:

$\begin{array}{l} (PO_p) \\ \max \phi \\ \{\theta, \lambda\} \\ s. t. \\ X\lambda \leq X_0 \\ \phi Y_0 - Y\lambda \leq 0 \\ \phi \text{ free}, \lambda \geq 0 \end{array}$	$\begin{array}{l} (DO_p) \\ \min q = v^T X_0 + v_* \\ \{\mu, v\} \\ s. t. \\ \mu^T Y_0 = 1 \\ v_* e^T \mu^T + v^T X \geq 0 \\ \mu^T, v^T \geq 0 \end{array}$
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where in the PO_p model we append nothing if p = 0, $e^T\lambda \leq 1$ if p = 1, $e^T\lambda \geq 1$ if p = 2, and $e^T\lambda = 1$ if p = 3. In the DO_p model, $v_* = 0$ if p = 0, $v_* \geq 0$ if p = 1, $v_* \leq 0$ if p = 2 and v_* is free if p = 3.

For the purposes of our analysis, we have considered 12 different farm sizes - for the whole U.S. - the efficiency and relative efficiency of which, we wish to investigate. The twelve farm sizes (sales classes) are:

- Class 1: Values of sales, less than \$2,500
- Class 2: Values of sales, \$2,500 to \$4,999
- Class 3: Values of sales, \$5,500 to \$9,999
- Class 4: Values of sales, \$10,000 to \$19,999
- Class 5: Values of sales, \$20,000 to \$24,999
- Class 6: Values of sales, \$25,000 to \$39,999
- Class 7: Values of sales, \$40,000 to \$49,999
- Class 8: Values of sales, \$50,000 to \$99,999
- Class 9: Values of sales, \$100,000 to \$249,999
- Class 10: Values of sales, \$250,000 to \$499,999
- Class 11: Values of sales, \$500,000 to \$999,999
- Class 12: Values of sales, \$1,000,000 or more

Although one would not expect substantial efficiency differences for all those classes, especially the small and neighboring ones, we kept the number of sales classes large for two reasons: First, not to avoid noticing efficiency differences where they exist because of unfavorable aggregation into few classes. Second, because the sales classes constitute our observations and the larger the number of observations relative to the number of inputs plus the number of outputs, the larger the discriminatory power of the model. These 12 classes are all for which the U.S. Census of Agriculture provides data on inputs and outputs. Initially, we selected seven inputs accounting for all production expenses and two outputs accounting for all the value of agricultural sales (all inputs and outputs are included in the DEA model).

Dividing the expenditures for each input and sales of each output by the number of farms of each farm size, we estimated what an "average" farm of a particular farm size, uses, and produces, in the U.S. The two aggregate outputs were (1) all crops and (2) all livestock and dairy. The seven aggregate inputs were: (1) livestock, poultry, and feed purchased, (2) agricultural chemicals, seeds, bulbs, plants, and trees, (3) commercial fertilizer, (4) energy products (includes petroleum products and electricity), (5) farm labor (includes hired and contract labor), (6) machinery (includes repair, maintenance, customwork, machine hire, and rental of machinery and equipment, and (7) other production expenditures (includes interest expenses, cash rent, property taxes, and all other production expenditures). Every class is represented in the model by its "average" farm.

Empirical Results

Based on what we said before about the DEA model, and making no restrictive assumptions about returns to scale, the envelopment problem of the input-oriented model that we apply can be expressed in matrix form as follows:

$\begin{aligned} & \min \theta \\ & \{\theta, \lambda\} \\ & s. t. \\ & Y\lambda \geq Y_0 \\ & \theta X_0 - X\lambda \geq 0 \\ & \theta \text{ free}, \lambda \geq 0 \\ & e^T \lambda = 1 \end{aligned}$	or	$\begin{aligned} & \min \theta \\ & \{\theta, \lambda_1, \dots, \lambda_{12}\} \\ & s. t. \\ & \begin{bmatrix} y_{1,1} & \dots & y_{1,12} \\ y_{2,1} & \dots & y_{2,12} \end{bmatrix} \begin{bmatrix} \lambda_1^0 \\ \vdots \\ \lambda_{12}^0 \end{bmatrix} \geq \begin{bmatrix} y_1^0 \\ y_2^0 \end{bmatrix} \\ & \theta \begin{bmatrix} x_1^0 \\ \vdots \\ x_7^0 \end{bmatrix} - \begin{bmatrix} x_{1,1} & \dots & x_{1,12} \\ \vdots & \dots & \vdots \\ x_{7,1} & \dots & x_{7,12} \end{bmatrix} \begin{bmatrix} \lambda_1^0 \\ \vdots \\ \lambda_{12}^0 \end{bmatrix} \geq 0 \\ & \theta \text{ free}, \lambda \geq 0 \text{ and } \sum_i \lambda_i = 1 \end{aligned}$
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Rotating Y_0 and X_0 for each farm size, we solved the model 12 times (the program used was LP88). The results of this model provide optimal values of $\theta^* = 1$ for all sales classes from 6 to 12, (farms with more than \$25,000 value of sales). Moreover the first two constraints for $\theta^* = 1$, hold with equality suggesting that these farms are both, operating on the frontier and efficient. The first signs of inefficiency appear in class 5 (\$20,000 to \$24,999) where $\theta^* = 0.957$. The result, however, suggests that despite this inefficiency, these farms operate very close to the frontier. The small inefficiency exists in the smaller class 4 (\$10,000 to \$19,999) too, where $\theta^* = 0.967$ presenting actually a small increase from the previous inefficient case. After that, even smaller farms (less than \$10,000) appear to be efficient with $\theta^* = 1$ and the first two constraints holding with equality. Hence, large farms appear to be efficient and as we move towards smaller farms a small inefficiency appears only for farms with value of sales \$10,000 to \$24,999, while even smaller farms appear efficient again. Before we discuss these results further, we should note that 12 observations with seven inputs and two outputs substantially reduce the discriminatory power of our model and in every one of the 12 linear programming models tend to push the 13th variable (θ) close to 1. To increase the discriminatory power of the model we aggregated all outputs to one ("agricultural output") and all inputs to one ("bundle of agricultural inputs"). The model then becomes⁶:

⁶Charnes, et al. (1990) presented the "Cone Ratio Model" to restrict the set of efficient DMU's. Also, Thompson et al. (1990) proposed for this purpose the Assurance Region method (A/R).

$$\begin{aligned}
& \min \theta \\
& \{\theta, \lambda_1, \dots, \lambda_{12}\} \\
& \text{s. t.} \\
& [y_1, \dots, y_{12}] \begin{bmatrix} \lambda_1^0 \\ \vdots \\ \lambda_{12}^0 \end{bmatrix} \geq y_0 \\
& \theta x_0 - [x_1, \dots, x_{12}] \begin{bmatrix} \lambda_1^0 \\ \vdots \\ \lambda_{12}^0 \end{bmatrix} \geq 0 \\
& \theta \text{ free, } \lambda \geq 0 \text{ and } \sum \lambda_i = 1
\end{aligned}$$

The results suggest again that classes 9, 10, 11, and 12 (more than \$100,000 value of sales) are efficient. Again $\theta^* = 1$ and the constraints hold with equality. For class 8 (\$50,000 to \$99,999), a small inefficiency appears as $\theta^* = 0.975$. This continues as we go to class 7 (\$40,000 to \$49,000) where $\theta^* = 0.920$. Classes 5 (\$20,000 to \$24,999) and 4 (\$10,000 to \$19,999) continue to exhibit a small inefficiency but θ^* rises to 0.950 and 0.960 respectively. After that, very small farms become efficient again (less than \$10,000).

Conclusions

Our results indicate that large farms (\$100,000 or more) are efficient. A small and rising inefficiency as we go to smaller sizes prevails for farms with sales of \$25,000 to \$100,000. Then, the inefficiency continues for even smaller farms but at a declining rate and finally efficiency prevails again for very small farms. This upward and then downward trend of inefficiency as we move towards smaller sizes is interesting. A reason for that, discussed before, has to do with farm labor. Those farms rely on family labor more than the larger farms but only expenditures on hired and contract farm labor are

published and considered here. Therefore, small farms "appear" more labor efficient and this affects the overall evaluation of their performance. Labor costs constitute 16.3% of all production expenditures, in farms with more than \$1,000,000 of sales and 13.2% for farms with sales of \$500,000 to \$1,000,000. For farms with values of sales less than \$10,000 this percent is 1-2%. In any case, the losses of efficiency that occur as we go from large farms to medium and smaller do not appear to be significant. The higher input/output ratios for the smaller farms could then be attributed to the fact that they operate on a different scale of the frontier, instead of its interior.

We should note also that for classes 1-10 the assumption of increasing (and in some cases constant) returns to scale, yields the same results as no restriction at all ($e^T \lambda = 1$). However, in the large classes 11 and 12, it is the declining returns to scale assumption, that seems to "prevail" and yields the same results with no restriction at all.

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APPENDIX

FARM PRODUCTION EXPENSES

According to the U.S. Census of Agriculture, data on production expenses are limited to those incurred in the operation of farm business. They include the share of expenditures provided by landlords, contractors, and partners. Property taxes paid by landlords however, are excluded. Expenditures for non-farm activities are excluded. Expenditures for agricultural activities outside the farm are also excluded.

The data on fertilizer include the cost of custom application. The same is true for agricultural chemicals. This category of inputs includes the cost of all insecticides, herbicides, fungicides, and other pesticides. The data on interest which were collected separately for 1987, include interest paid on debts secured by real estate and on those which were not.

MARKET VALUE OF AGRICULTURAL PRODUCTS SOLD

This value is estimated before taxes and production expenses. It includes sales by operator as well as the value of any shares received by partners, landlords, contractors, or others associated with the operation. It also includes receipts from placing commodities in the Commodity Credit Corporation loan program in 1987. It does not include payments received for participation in federal programs, income from farm related sources such as customwork and other agricultural services, or income from non-farm sources. A part of this value may be coming however from products harvested and stored before 1987.

Summary by Value of Agricultural Products Sold - 1987			
Sales Classes	All Farms	\$500,000 or more	\$250,000 to \$499,999
Number of farms	2087759	32023	61148
VALUE OF SALES (\$1,000)			
Grains	28340524	2764690	4747732
Cotton and cottonseed	4207891	1601045	920298
Tobacco	1745417	124169	195972
Cotton, cottonseed and tobacco.	5953308	1725214	1116270
Hay, silage and field seeds.	2598615	604574	324054
Vegetables, sweet corn and melons	4698083	3296294	499055
Fruits, nuts and berries	7084018	3882863	929457
Nursery and greenhouse crops	5774391	4043164	653418
Other crops	4482146	2373309	873972
Vegetables, sweet corn, melons, fruits, nuts, berries, nursery and greenhouse crops.	22038638	13595630	2955902
Poultry and poultry products.	12758270	7588179	2844049
Dairy products	16029195	3766489	2664002
Cattle and calves	35876720	18441285	3623898
Hogs and pigs.	9890644	2301045	2170293
Sheep, lambs and wool.	791219	276656	102024
Other livestock and livestock products.	1771382	888518	191456
Cattle, calves and dairy products.	51905915	22207774	6287900

\$100,000 to \$249,000	\$50,000 to \$99,999	\$40,000 to \$49,999	\$25,000 to \$39,999	\$20,000 to \$24,999
202550	218050	69537	150099	75572
10016703	5689702	1160752	1746007	566899
1101506	371194	58847	77157	22466
420264	296062	81860	158359	70632
1521770	667256	140707	235516	93098
505181	331947	93329	175194	77015
436179	201189	47590	77860	27682
1084162	546538	122456	196899	72338
579990	228157	48895	84050	31916
779390	266816	48692	66589	21406
2879721	1242700	267633	425398	153342
1914736	313296	28748	33270	8346
5768010	2869319	373974	394322	85080
5090603	3033197	742530	1325040	545240
3133987	1289820	233305	324096	104582
143165	84321	23289	39839	16365
204370	139821	38036	67673	31739
10858613	5902516	1116504	1719362	630320

\$10,000 to \$19,999	\$5,000 to \$9,999	\$2,500 to \$4,999	Less than \$2,500
250594	274972	262918	490296
1039762	410695	140051	57532
37940	12773	3664	1001
184584	119056	61361	33099
222524	131829	65025	34100
208906	148656	81604	48156
60515	32668	13847	5205
134379	65498	31023	18404
59558	28542	11477	5224
34044	12590	4012	1326
288496	139298	60359	30159
12112	6505	4563	4467
92051	12861	2355	730
1390210	936060	495447	253210
192134	86698	37041	17644
39050	29030	20141	17339
77466	58404	39280	34619
1482261	948921	497802	253940

Summary by Value of Agricultural Products Sold - 1987

Sales Classes	All Farms	\$500,000 or more	\$250,000 to \$499,999
Number of farms	2087759	32023	61148
PRODUCTION EXPENSES (\$1,000)			
Livestock and poultry purchased	19344645	12550164	2069061
Feed for livestock and poultry.	19163364	9327603	3311775
Seeds, bulbs, plants and trees.	3390762	753033	488123
Commercial fertilizer	6684944	1196709	1005283
Agricultural chemicals	4690243	1150020	700263
Petroleum products	5277227	920825	679628
Electricity	2225206	667347	299165
Energy (petroleum products and electricity).	7502433	1588172	978793
Hired farm labor	10866236	5943280	1768654
Contract labor	1842984	953963	235551
Farm labor (hired and contract).	12709220	6897243	2004205
Repair and maintenance	6361980	1359751	800651
Customwork, machine hire and rent.	2176467	558703	255269
Machinery (repair, maintenance, customwork, hired and rented).	8538447	1918454	1055920
Interest expense	8158268	1812953	1170784
Cash rent	4689455	967247	770621
Property taxes	3120405	385392	291432
All other production expenditures.	10145866	3859223	1275710

\$100,000 to \$249,000	\$50,000 to \$99,999	\$40,000 to \$49,999	\$25,000 to \$39,999	\$20,000 to \$24,999
202550	218050	69537	150099	75572
2253789	1055704	207571	340083	133823
3522651	1469258	243373	352997	127468
955283	559691	123404	184970	68375
1962006	1119679	247784	354719	136213
1269991	717344	155028	242430	89545
1387294	882336	210497	324791	129065
515632	298053	64506	97525	38081
1902926	1180389	275003	422316	167146
1842149	662538	126271	176544	66063
274364	138843	30827	56707	23857
2116513	801381	157098	233251	89920
1578440	971893	221228	351410	142762
497573	327334	78769	138140	55794
2076013	1299227	299997	489550	198556
2167266	1187936	254966	377836	148154
1459568	784626	159461	218683	73812
644793	469137	120097	215339	93867
2134361	1213244	260846	391983	153527

\$10,000 to \$19,999	\$5,000 to \$9,999	\$2,500 to \$4,999	Less than \$2,500
250594	274972	262918	490296
306805	177181	114108	136355
294951	180549	128111	204628
134597	66439	32635	24212
295268	175991	98677	92616
180473	89610	44600	50939
288244	189549	124794	140206
86140	60868	42466	55424
374384	250417	167260	195630
125801	66726	37801	50410
53617	32401	18620	24233
179418	99127	56421	74643
328239	231776	159511	216317
122063	67918	39176	35726
450302	299694	198687	252043
342133	231118	170528	294595
136116	60064	30074	29183
247668	204149	163412	285119
347088	199234	133203	177448