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Reputation and Production Standards

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This paper analyzes a monopolist's behavior when consumers cannot observe the production standards. These types of products are usually known as credence goods. The steady-state level of quality with credence goods is found to be lower than that with experience goods, and perfect information goods. The finding that only perceived quality, which is effectively a filtered version of true quality, affects reputation indicates rewards for high quality production are lower in the credence good case. Further, an increase in the level of monitoring can increase the true level of product quality in the market for credence goods.

Key words: credence goods, production standards, reputation

Introduction

Food products with claims of usage or nonusage of specific production standards have become a more important part of consumer food purchases in recent years. Eco-labeled products, which claim to use environmentally friendly production methods, have become very popular in Europe and are gaining prominence throughout the world. In Japan and Europe, food products are being sold with claims that their ingredients have not been genetically modified. There are also products with claims stating they were produced using sound animal welfare practices, such as "cruelty-free" cosmetics. As Noe and Rebello (1995) discuss, there are many "ethical" production standards, such as products with claims they were produced without child labor.

In each of the above examples, the buyer cannot determine the validity of the seller's claim (i.e., the quality) by visual inspection or by consuming the product. Products with production standard claims are fundamentally different from products where quality is observable or where quality can be determined after consumption. Such products are also called "credence goods" (Darby and Karni, 1973).[1] Asymmetric information problems occur with unobservable production standards because producers know whether they used the appropriate methods to achieve the desired quality attributes, but consumers only know with certainty what the producers' quality claims are or what the label says. If a product has unobservable-quality attributes, then its reputation becomes very important. Consumers will only be willing to pay more for claims of the use of special production standards if they trust the validity of the claim.

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[1] Darby and Karni (1973) examined the repair problem in which an expert provides both the diagnosis and the service. This results in the provision of repair services in amounts greater than would be economically efficient.

Although production standards are sometimes a binary choice (such as satisfying organic production standards or not), here we consider a continuous choice of quality, which may be more appropriate to reflect the distribution of the perceived quality levels by consumers. Further, meeting buyers' special product requirements may be better represented by a continuous choice of quality. Finally, from a product purity standpoint, the continuous choice of quality level is appropriate. For example, a recent Consumers Union-led study of government data found pesticide residues on 23% of organic produce. One sample of certified organic peaches contained 3.3 parts per million of the pesticide phosmet, suggesting the crop was sprayed shortly before harvest (Brasher, 2002). One approach for hedging against conviction is to engage in fraud while simultaneously maintaining legitimate production of the premium product (Hamilton and Zilberman, 2003). The firm that produces both a fraudulent and a true premium product is more likely to escape detection than a firm that produces only a fraudulent product. On the other hand, it may be the firm is not actively engaging in fraud, but instead not taking sufficient care to preserve the identity of the premium product.

In this paper, we analyze firm-specific reputations for goods with claims that unobservable production methods or standards were used. The analysis complements Shapiro's (1982) pioneering work on reputation and product quality. Shapiro analyzes a monopolist's behavior when consumers cannot observe quality attributes before purchase but can observe quality attributes after purchase, e.g., an experience good (Nelson, 1970). To achieve results comparable to those of Shapiro, we employ many of the same assumptions—specifically, it is assumed the firm is a monopolist that controls the quantity as well as quality of the good in question. The major difference is that in addition to the information lag which is associated with the goods analyzed by Shapiro, we include monitoring of compliance with production standards claims (product quality), assumed here to be effectively imperfect. We assume it would be too expensive for each individual consumer to monitor the compliance of each production standard. As a result, reputation is only an indirect function of *true* quality because true quality is filtered through *consumer perceptions* about quality.[2]

With unobservable production standards, only perceived quality affects reputation, suggesting that rewards (e.g., higher prices) for compliance with production standards claims are lower than the case in which quality is observable after consumption. Specifically, in Shapiro's work, the rewards for quality are discounted relative to the perfect information good owing to the lagged nature of quality discovery. In the unobservable production standards case, in addition to the discount from the lag, the rewards are further discounted because only perceived quality is observed.

The analysis presented in this paper contributes to the literature on product quality by including unobservable production standards in a reputation model, thus extending Shapiro's (1982) findings.[3] With the recent evolution of food markets, goods with unobservable quality attributes are attracting attention from economists. Ligon (2001) presents a sequence of models in which different assumptions regarding product quality

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[2] A reviewer highlighted the importance of the divergence between buyer perceptions of certified standards and actual practices. For example, Delate (2000) acknowledges that many organic farmers in Iowa use flame-burners as an additional tool in their weed management practices because herbicides are not allowed. Most organic shoppers would likely be surprised by this fact, yet remain unaware because they do not inquire about the production practices behind an organic certification.

[3] We do not review the extensive literature on product quality and reputations. For a survey of this literature, see Tirole (1988, pp. 103–131).

and its observation by consumers are made. He concludes that when a grading system (which transmits objective information about any of the quality dimensions of the product) is applied to markets in which quality is unobservable by consumers, the quality of the produced commodity is higher. Using an infinitely repeated non-cooperative game-theoretic model, McCluskey (2000) shows imperfect monitoring can support an equilibrium in which organic food (based on production standards) is available. Feddersen and Gilligan (2001) use a non-cooperative game-theoretic approach to find that an information-supplying "activist" can support equilibria in which firms differentiate their products with unobservable characteristics. Hamilton and Zilberman (2003) analyze the incentives for fraud in a dual production framework in the presence of imperfect monitoring and imperfect enforcement. The intent of the current study is to build on the contributions of these earlier works through the introduction of a model that specifically considers product reputation.

The Model

In order to focus on the consumer information issues rather than the interaction of these issues with competition, we analyze the monopolist case.[4] The industrial organization literature suggests that quality regulation (such as product safety) cannot be considered independently of market structure (Daughety and Reinganum, 1997). The monopolist case also allows for direct comparison with Shapiro's (1982) results. At each point in time, the product's reputation, R, determines the location of its (inverse) demand curve, $p(x, R)$. Reputation is a dynamic process which depends on past levels of perceived quality. The monopolist chooses the true level of quality, $q(t)$, and quantity, $x(t)$, over time to maximize profits. Production costs are given by $c(x, q)$. The variable denoting consumers' perceptions of product quality, $\tilde{q}$, is a function of both true quality and the level of quality assessment (probabilistic) or monitoring, m. The level of monitoring is normalized to be between zero and one, inclusive. If $m = 0$, then there is zero monitoring effectiveness, while if $m = 1$, then observability is perfect, and the certifiers report all fraudulent claims to consumers. If m is between zero and one, then there is imperfect monitoring, which increases in effectiveness as the level of m increases.

We are interested in imperfect monitoring, since imperfect monitoring is a better fit with reality compared to perfect monitoring. In general, not all of the imposter organic produce will be detected in the certification process. For example, as Fetter and Caswell (2002) point out, organic certifiers may believe the reputation of the entire industry depends on how consumers perceive their integrity as a group, and thus they would be reluctant to report wrongdoing by another certifier.

[4] The classic papers on quality choice by a monopolist under perfect information are Spence (1975) and Sheshinski (1976). Both conclude that the provision of quality may be over- or undersupplied relative to the socially optimal level, depending on the shape of the inverse demand curve in quality. The deviation between the socially optimal level of quality and the profit-maximizing level of quality is explained by the fact that monopolists choose based on the marginal consumer, while social welfare requires the effect on the average consumer to be considered. Therefore, at a given quantity level, the monopolist's quality is too low if and only if the effect on the marginal buyer is less than the effect on the average buyer (see Spence, 1975, p. 419; and Shapiro, 1982, p. 27). Further, the choice of quality may interact with the amount of quantity restriction if the price elasticity is dependent on quality. If quality for the monopolist under perfect information is underprovided, then the informational problems we introduce with unobservable-quality goods will aggravate the welfare problems. If quality for the perfect information monopolist exceeds the socially optimal level, then introducing information problems may improve social welfare. As Shapiro (1982) points out, this is an example of the second best with two imperfections counteracting each other.

We also presume that the level of monitoring is set outside the firm, which allows us to focus on quality and information issues. This is the most common case for food industries, rather than the exception. For example, the U.S. Department of Agriculture, rather than the individual firms, sets standards and monitoring levels for organic food. The European Union does the same for Protected Geographical Indication labels.

The relationship between perceived quality and monitoring, $\tilde{q}_m(q, m)$, is assumed to be positive up until $\tilde{q}(q, m) = q$, and then it becomes negative. The intuition is that increases in monitoring cause the level of perceived quality to approach the level of true quality. Finally, we assume $\tilde{q}(q, 0) = 0$. Based on this latter assumption, when there is no monitoring, consumers perceive quality to be the lowest level, which is normalized at zero. The rationale is that without monitoring, the consumer does not know the quality of the product even after purchase. Consequently, there is no incentive for the producer to spend money on quality when it is not possible for the consumer to punish the producer by not purchasing the product in the future in response to a false quality claim (Ligon, 2001). These assumptions about perceived quality are consistent with Grossman's (1981) paper on product quality disclosure. According to Grossman, if sellers hold back information, consumers will assume that quality is the lowest level consistent with what the sellers have disclosed. Thus, monitoring plays the role of product quality disclosure in our analysis.

In Shapiro's (1982) analysis, reputation is based on past quality levels. In contrast, reputation in our analysis is based on past levels of *perceived* quality. This is the difference between an experience good and an unobservable-quality good (or credence good). Specifically, in Shapiro's analysis, after consumption, consumers know the true quality of the product. In the current analysis, since we are dealing with product standards, true quality remains unobservable even after consumption.

Reputation changes over time as a function of perceived quality and quantity, $\dot{R} = f(x, \tilde{q}(q, m) - R)$. The rationale for including quantity in the reputation state equation is that the greater the amount produced with a deviation in perceived quality from the current reputation, the greater will be the effect on reputation. In the extreme, if no product is produced, there should be no change in reputation, $f_x(0, \tilde{q}(q, m) - R) = 0$. Following Shapiro, we assume that $f(x, 0) = 0$ and $f_x(x, 0) = 0$. There is no change in reputation when perceived quality is equal to current reputation. Further, there is no change in reputation when perceived quality is equal to current reputation, and there is a marginal change in quantity. The product's initial reputation, R_0, is taken as exogenous. We model two particular cases in which consumers' perceptions of quality do not correspond with true quality. For the first part of the analysis, it is assumed consumers underestimate true quality, and later we assume the opposite occurs, whereby consumers overestimate true quality.

Case 1: Consumers Underestimate Quality

For this first case, we assume perceived quality is below true quality, $\tilde{q}(q, m) < q$, and $0 \le \tilde{q}_q(q, m) < 1$. The monopolist faces the following control problem:

$$(1) \qquad \max_{x(t), q(t)} \int_0^\infty e^{-rt}\big[G(x, R) - c(x, q) - F\big]dt,$$

$$\text{s.t.:} \quad \dot{R} = f(x, \tilde{q}(q, m) - R) \ \text{ and } \ R(0) = R_0,$$

where $G(x, R) = xp(x, R)$ is the gross revenue function, $c(x, q)$ denotes the production costs, and F represents fixed certification costs. The associated current-value Hamiltonian is written as:

$$H = G(x,R) - c(x,q) - F + \lambda f(x, \tilde{q}(q,m) - R). \tag{2}$$

The necessary conditions for an interior solution (assuming it exists) are as follows:

$$H_q = -c_q + \lambda f_{\tilde{q}} \tilde{q}_q = 0, \tag{3}$$

$$H_x = G_x - c_x + \lambda f_x = 0, \tag{4}$$

$$H_R = G_R - \lambda f_{\tilde{q}} = r\lambda - \dot{\lambda}, \tag{5}$$

$$H_\lambda = f = \dot{R}. \tag{6}$$

The second-order conditions are assumed to be satisfied (see the appendix).

In order to achieve results comparable to Shapiro's findings, we focus our analysis on the steady-state equilibria for each classification of good. The steady state occurs when $\dot{R} = 0$, so that $\tilde{q}(q, m) = R$, and $\dot{\lambda} = 0$. Since $f_x(x, 0) = 0$, we have:

$$-c_q(x, q) + \lambda f_{\tilde{q}}(x, 0)\tilde{q}_q(q, m) = 0, \tag{7}$$

$$G_x(x, \tilde{q}(q, m)) - c_x(x, q) = 0, \tag{8}$$

$$G_R(x, \tilde{q}(q, m)) = \lambda\left(r + f_{\tilde{q}}(x, 0)\right). \tag{9}$$

Eliminating λ, a steady state is obtained in (x, q), determined by (8) and

$$G_R(x, \tilde{q}(q, m)) = c_q(x, q)\left(\frac{1}{\tilde{q}_q(q, m)} + \frac{r}{f_{\tilde{q}}(x, 0)\tilde{q}_q(q, m)}\right). \tag{10}$$

The perfect information pair (x^*, q^*) satisfies the steady-state equations, (8) and (10), with the term in parentheses on the right-hand side, $(1/(\tilde{q}_q(q, m))) + (r/(f_{\tilde{q}}(x, 0)\tilde{q}_q(q, m)))$, equal to one, and true quality (q) in place of perceived quality ($\tilde{q}$). The experience good pair (x^e, q^e) satisfies these equations with $\tilde{q}_q$ equal to one, and true quality (q) in place of perceived quality ($\tilde{q}$). If $\tilde{q}_q$ is equal to one, it means there is no transformation from true quality to perceived quality. Therefore, with $\tilde{q}_q = 1$, an unobservable-quality good is equivalent to an experience good in terms of the way quality affects reputation. If both $\tilde{q}_q = 1$ and $r/f_{\tilde{q}}$ goes to zero, then the unobservable-quality good solution converges to the perfect information case. For the experience good steady-state solution, Shapiro showed the quality level must be less than the perfect information case, q^*. We now compare the unobservable-quality good steady state with the experience good steady state.

- THEOREM 1. *So long as reputation has positive value and perceived quality is of the form $\tilde{q}(q, m) = b(m)q$, where $b(m)$ is a real-valued and continuous function, any steady-state quality level for an unobservable-quality good must lie below both the perfect information quality level and the experience good quality level.*

Proof. Since Shapiro (1982, p. 31, theorem 5) proved that any steady-state quality level of an experience good must lie below the perfect information quality level, we need to show there is a further reduction in quality when going from the experience good to the unobservable-quality good (see the appendix for proof of theorem 1).

Given Shapiro's result, our result implies the following ordering of product quality in the steady state for the three types of goods classified by the consumer's information level at the moment of the purchase and after consumption:

$$q^{ps} < q^{e} < q^{*}, \tag{11}$$

where q^{ps} is the quality level of the production standards good or credence good, q^{e} is the quality level of the experience good, and q^{*} is the perfect-information quality level. This result is consistent with Shapiro's (1982, p. 22) summation about reputation and product quality: "... since reputation adjustment can reward high quality production only with a lag, the firm will not find it profitable to provide so high a quality as under perfect information." Because only a filtered version of quality affects reputation in the case of unobservable-quality goods, the rewards for high quality production are lower than with experience goods. Specifically, with experience goods, the rewards are discounted due to lagged quality information. In the unobservable-quality good case, in addition to the discount from the lag, the rewards are further discounted because true quality is filtered, and only perceived quality is observed.

Shapiro considered the effect of a change in the "speed of consumer learning." An increase in the speed of learning diminishes the time lag of the reward for high quality production in Shapiro's model. The effect of a change in the speed of learning (or similar effects, such as randomness in learning) is somewhat similar to a change in the transformation from true quality to perceived quality—i.e., they both affect the rewards to quality. Consistent with our analysis, Shapiro reports that as the speed of learning increases, so does the steady-state quality. However, the speed-of-learning parameter and the perceived quality transformation are inherently different concepts. Next, we consider the effect of a change in monitoring on steady-state quality.

■ THEOREM 2. *If the shift in the marginal revenue curve caused by an increase in perceived quality, $G_{\tilde{q}x}$, is sufficiently large in the production-standards good case, then an increase in the level of monitoring will increase the true level of product quality in the steady state.*

Proof. The proof of theorem 2 is provided in the appendix.

This result is related to the previous result—i.e., as monitoring increases, the unobservable-quality good becomes more like an experience good. With increases in monitoring, the distortion from true quality to perceived quality diminishes.

Case 2: Consumers Overestimate Quality

The second case to be examined is one in which consumers overestimate quality by assuming that $\tilde{q}(q, m) > q$ and $\tilde{q}_q(q, m) > 1$. Under these assumptions, an increase in monitoring lowers the level of perceived quality, $\tilde{q}_m < 0$, because increased monitoring causes perceived quality to converge to true quality. There can be overestimation of quality with most credence goods. For example, there could be overestimation of quality in organic foods if consumers believe the level of pesticide residue is always zero, whereas there are sometimes positive levels of pesticide residue.

We now consider how overestimating quality affects the results stated in theorems 1 and 2. First, since $\tilde{q}_q(q, m) > 1$ now holds, the returns to quality in terms of perceived quality will be reduced when going from the quality standards or credence good to the experience good. Consequently, under this set of assumptions, the steady-state quality level is actually higher for the credence good compared with the experience good. Similarly, if the shift in the marginal revenue curve caused by an increase in perceived quality, $G_{\tilde{q}x}$, is sufficiently large in the credence good case, then an increase in the level of monitoring will now decrease the true level of product quality in the steady state. The intuition for both of these reversals of results is that the returns to true quality are now magnified by the overestimated level of perceived quality. When the returns to true quality are lowered, as they would be in going to the experience good level or if monitoring is increased, then the firm will choose a lower level of quality in the steady state. The results under the assumption of overestimation of quality are counterintuitive because, as Shapiro (1982, p. 27) writes about his analysis, "imperfect information will tend to cause a reduction in the quality of products provided." Under this alternative, improving consumers' information actually decreases the level of quality.

Directions for Future Research[5]

Alternative Approaches to Formation of Quality Perceptions

The model discussed above is structural and allows us to analyze the classic reputation model with the new information scenarios that are increasingly important in differentiated product markets, especially in food products. However, this model is agnostic about where the initial perceptions of quality come from and how they may be formed. Alternative approaches should address whether individuals believe quality claims until proven false, or conversely, whether they inherently disbelieve quality claims until they are proven true. Initial perceived quality could alternatively be modeled with a random variable. Consumers could have a statistical expectation about current quality, which would be updated as new quality information is revealed.

A second modeling issue is the distribution of beliefs among consumers. A spatial model with a distribution over beliefs about quality is likely more appropriate than a representative consumer model. Some consumers may be skeptical, and others may be willing to believe. Given these issues, in future research we expect the impact of monitoring will depend heavily on (*a*) how many consumers care about the quality attribute, and (*b*) the distribution of perceptions of quality among those who care.

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[5] We thank an anonymous reviewer for these suggestions.

Strategic Use of Third-Party Monitoring and Choice of Quality Claims

Strategic use of third-party monitoring based on consumer perceptions of quality may be an interesting area of inquiry. Not surprisingly, if monitoring is not mandatory, firms should strategically choose whether they will submit to monitoring. Profit-maximizing companies do not have an incentive to exceed their quality claims. The discovery by a monitor that a company has exceeded its quality claims will not make the evening news—i.e., monitoring is unlikely to raise quality perceptions beyond the point of the claim. However, if a company is monitored by a third party and fails to meet the quality standard, its reputation will be damaged. If the company exceeds the quality standard, its reputation will remain constant. Therefore, when consumers are optimistic about quality, the producer is best advised to avoid certification (or evade legal scrutiny of the product's claims), and hence avoid the possibility of failing to meet claims. Under pessimistic quality perceptions, the firm will choose third-party certification.

As for strategically setting the level of quality claims, Grossman (1981) concludes that if sellers are required to be truthful in any disclosures they make about product quality, they will fully disclose the true level of product quality. Extending this line of research, one could examine how firms set quality claims in a dynamic setting with imperfect monitoring and variable consumer perceptions of quality.

Conclusions

Products with claims of using special production standards that are unobservable to consumers have become an increasingly popular marketing strategy to tap into consumer demand for healthier, safer, and more environmentally friendly products. Product labels often explicitly make claims about the production methods or ingredients, which are unobservable to consumers. As a result, there are often problems of asymmetric information that can affect the markets for these products. Since consumers cannot observe true quality even after consuming the product, there must be some reward mechanism for encouraging firms to produce high quality. Outside monitoring is one approach. Monitoring can create the proper incentives for firms to provide high-quality goods with production standard claims.

Monitoring allows reputation effects to increase the incentives for quality. Monitored claims allow consumers to make repeat-purchase decisions based, to some extent, on past quality. Therefore, firms have a greater incentive to maintain high quality. Findings of this analysis reveal the steady-state level of quality with unobservable production standards is lower than that with experience goods. When only perceived quality affects reputation, the rewards for high-quality production are lower in the unobservable production standards case. Further, an increase in monitoring can increase the true level of product quality in the market for goods with unobservable production standards.

The above results are consistent with Shapiro's (1982) general finding that product quality will be lower if reputation offers reduced returns for high-quality production compared with the perfect information case. In Shapiro's work, the rewards for high-quality production are discounted due to the lagged nature of experience goods. In our analysis, in addition to the discount from the lag, the rewards are further discounted because true quality is filtered, and only perceived quality is observed.

This analysis has highlighted an important result: Because the production of quality is not fully rewarded when unobservable production standards are used, the steady-state of care provision is below that of goods with lagged-information characteristics. This finding suggests that in order to achieve a desired threshold of quality in the market of unobservable production standards (such as food safety attributes), a form of direct regulation may be required. Thus, reliance on monitoring implemented by certifiers or third-party organizations clearly may not be sufficient to assure a certain level of quality provision when unobservable production standards are used.

As products with unobservable-quality attributes are increasingly marketed, process attribute information issues and their implications for markets will continue to gain prominence. Further research is needed to more fully understand these markets and information issues and to evaluate policies.

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Appendix: Proof of Theorems 1 and 2

Second-Order Conditions

The sufficient condition for a maximum requires that the Hessian matrix of the Hamiltonian with respect to the Lagrange multiplier (λ), the control variables (q, x), and the state variable (R) is negative definite when evaluated at steady-state levels of q^*, x^*, and R^*. The Hessian matrix of the Hamiltonian is given by:

(A1)
$$\begin{bmatrix} 0 & f_{\tilde{q}}\tilde{q}_q & f_x & f_R \\ f_{\tilde{q}}\tilde{q}_q & -c_{qq} + \lambda(f_{\tilde{q}}\tilde{q}_{qq} + \tilde{q}_q^2 f_{\tilde{q}}) & -c_{xq} + \lambda f_{x\tilde{q}}\tilde{q}_q & -\lambda f_{\tilde{q}\tilde{q}}\tilde{q}_q \\ f_x & -c_{xq} + \lambda f_{x\tilde{q}}\tilde{q}_q & G_{xx} - c_{xx} + \lambda f_{xx} & G_{xR} - \lambda f_{xR} \\ f_R & -\lambda f_{\tilde{q}\tilde{q}}\tilde{q}_q & G_{xR} - \lambda f_{xR} & G_{RR} - \lambda f_{RR} \end{bmatrix}.$$

For negative definiteness, the necessary and sufficient condition is that the determinants of the principal minors must alternate in sign. We assume this holds.

Proof of Theorem 1

Since Shapiro (1982, p. 31, theorem 5) proved that any steady-state quality level of an experience good must lie below the perfect information quality level, we need to show there is a further reduction in quality when going from the experience good to the unobservable-quality good or credence good. Under the current set of assumptions, the value of $\tilde{q}_q$ ranges from zero to one (not including one) in the unobservable-quality good case, and $\tilde{q}_q = 1$ in the experience good case. To show there is a further reduction in quality when going from the experience good to the unobservable-quality good, we must consider how a change in the magnitude of the effect of true quality on perceived quality, $\tilde{q}_q$, affects steady-state quality. For this theorem, it is assumed that $\tilde{q}(q, m) = b(m)q$. By holding the level of exogenous monitoring constant, we can state that $\tilde{q}_q = b$. In moving from an unobservable-quality good to an experience good, the value of b will increase. Therefore, the sign of the derivative $((dq)/(db))$ is critical. Linearizing text equations (8) and (10) and allowing the magnitude of b to exogenously change, we obtain a system of equations in the matrix form $\mathbf{A}d\mathbf{X} = \mathbf{k}d(\tilde{q}_q)$:

(A2)
$$\begin{bmatrix} G_{xx} - c_{xx} & -c_{xq} \\ G_{\tilde{q}x} - \dfrac{c_{qx}}{\tilde{q}_q} + \dfrac{c_q r f_{\tilde{q}x}}{\tilde{q}_q f_{\tilde{q}}^2} - \dfrac{c_{qx} r}{\tilde{q}_q f_{\tilde{q}}} & G_{\tilde{q}\tilde{q}}\tilde{q}_q - c_{qq}\left(\dfrac{1}{\tilde{q}_q} + \dfrac{r}{f_{\tilde{q}}\tilde{q}_q}\right) \end{bmatrix} \begin{bmatrix} dx \\ dq \end{bmatrix} = \begin{bmatrix} 0 \\ -c_q\left(\dfrac{f_q + r}{f_q \tilde{q}_q^2}\right) \end{bmatrix} db.$$

The derivative $((dq)/(db))$ can be calculated using Cramer's Rule:

(A3)
$$\frac{dq}{db} = \frac{-(G_{xx} - c_{xx})c_q\left(\dfrac{f_q + r}{f_q b^2}\right)}{DetA}.$$

The second-order conditions ensure the numerator will be positive. The denominator can be signed as positive if the shift in the marginal revenue curve caused by an increase in perceived quality, $G_{\tilde{q}x}$, is sufficiently large. If this holds, the derivative $((dq)/(db)) > 0$, as required.

Proof of Theorem 2

By linearizing text equations (8) and (10), which describe the steady state in (x, q), a system of equations is obtained in the matrix form $\mathbf{A}d\mathbf{X} = \mathbf{k}dm$:

(A4)
$$\begin{bmatrix} G_{xx} - c_{xx} & -c_{xq} \\ G_{\tilde{q}x} = \dfrac{c_{qx}}{\tilde{q}_q} + \dfrac{c_q r f_{\tilde{q}x}}{\tilde{q}_q f_{\tilde{q}}^2} - \dfrac{c_{qx} r}{\tilde{q}_q f_{\tilde{q}}} & G_{\tilde{q}\tilde{q}}\tilde{q}_q - c_q \tilde{q}_{qq}\left(\dfrac{f_{\tilde{q}}+1}{\tilde{q}_q^2}\right) - c_{qq}\left(\dfrac{1}{\tilde{q}_q} + \dfrac{r}{f_{\tilde{q}}\tilde{q}_q}\right) \end{bmatrix} \begin{bmatrix} dx \\ dq \end{bmatrix} = \begin{bmatrix} -G_{x\tilde{q}}\tilde{q}_m \\ -G_{\tilde{q}\tilde{q}}\tilde{q}_m - \dfrac{c_q \tilde{q}_{qm}}{\tilde{q}_q^2} - \dfrac{r c_q \tilde{q}_{qm}}{f_{\tilde{q}}\tilde{q}_q^2} \end{bmatrix} dm.$$

The derivative $((dq)/(dm))$ can be calculated using Cramer's Rule:

(A5)
$$\frac{dq}{dm} = \frac{-(G_{xx} - c_{xx})\left[G_{\tilde{q}\tilde{q}}\tilde{q}_m + \dfrac{c_q \tilde{q}_{qm}}{\tilde{q}_q^2} + \dfrac{r c_q \tilde{q}_{qm}}{f_{\tilde{q}}\tilde{q}_q^2}\right] + G_{x\tilde{q}}\tilde{q}_m\left[G_{\tilde{q}x} + \dfrac{c_q r f_{\tilde{q}x}}{\tilde{q}_q f_{\tilde{q}}^2} - \dfrac{c_{qx}}{\tilde{q}_q} - \dfrac{d_{qx} r}{\tilde{q}_q f_{\tilde{q}}}\right]}{(G_{xx} - c_{xx})\left[G_{\tilde{q}\tilde{q}}\tilde{q}_q - c_q \tilde{q}_{qq}\left(\dfrac{f_{\tilde{q}}+1}{\tilde{q}_q^2}\right) - c_{qq}\left(\dfrac{1}{\tilde{q}_q} + \dfrac{r}{f_{\tilde{q}}\tilde{q}_q}\right)\right] + c_{xq}\left[G_{\tilde{q}x} + \dfrac{c_q r f_{\tilde{q}x}}{\tilde{q}_q f_{\tilde{q}}^2} - \dfrac{c_{qx}}{\tilde{q}_q} - \dfrac{c_{qx} r}{\tilde{q}_q f_{\tilde{q}}}\right]}.$$

The derivative $((dq)/(dm))$ can be signed as positive as required if the shift in the marginal revenue curve caused by an increase in perceived quality, $G_{\tilde{q}x}$, is sufficiently large.