ON THE SELECTION OF ALTERNATIVE LAG MODELS

By

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ABSTRACT

An attempt is made to compare and evaluate polynomial and rational lag models, both theoretically and empirically, by estimating the same lagged relationship using both procedures. Empirical comparison is achieved by eliminating the unknown nature of the lag by using generated data with a known double lag structure.

Presented at the
1979 AAEA Summer Meetings

Pullman, Washington

July 29 - August 1, 1979

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ON THE SELECTION OF ALTERNATIVE LAG MODELS[1/]

Given the need for a distributed lag model, there is generally no unambiguous rule for deciding among alternative distributed lag specifications. Despite the large quantity of theoretical and empirical work which currently exists, opinions differ widely as to the general appropriateness of alternative lag specifications. Indeed, it is highly probable that the selection decision is based on the availability of convenient software packages for estimation and/or conventional local use rather than a rigorous comparison of alternatives. The purpose of this paper is to compare two widely used lag specifications, the polynomial and rational lag models, by estimating the identical lag structure using both procedures. Since comparison of the two methods achieves little if the true structure remains unknown, data generated from an economic model with known lag structure is used.

METHODOLOGY OF ESTIMATION

A general specification of the linear regression model in which the current dependent variable is a function of both current and past exogenous variables may be given as:

$$(1) \quad Y_t = \alpha + B_1\,(L)\,X_{1t} + B_2\,(L)\,X_{2t} + E\,(L)\,\varepsilon_t$$

where: Y_t = the dependent variable

X_{1t} and X_{2t} = exogenous variables

ε_t = stochastic residual assumed to be randomly distributed

α = an unknown intercept

[1/] This paper complements a previous paper by Thompson and Mount.

$B_s(L) \quad = \sum_{i=0}^{\infty} \beta_{si} L^i$; $s = 1, 2$ is an unknown lag structure of the exogenous variables and L^i is a lag operator where $L^i X_{st} = X_{st-i}$

$E(L) \quad = \sum_{i=0}^{\infty} \eta_i L^i$ lag structure of the residual

Since the lag structures are assumed to be infinite, then it is necessary to reduce the number of unknown parameters in order to estimate the structure. This can be achieved in two ways. First, it may be possible to truncate the lag structures after N periods such that $\beta_{si} = 0$ if $i > N$. It is this type of truncation which underlies polynomial distributed lags. A second procedure is to approximate each infinite lag structure by the ratio of two finite lags; this procedure underlies the rational lag.

Polynomial Distributed Lags

Generally, the polynomial lag can be estimated by ordinary least squares (OLS) methods, but the OLS estimator will be inefficient unless $E(L) = \eta_0$, hence, $\eta_i = 0$ for $i > 0$. The general method of estimating N would appear to be to fit successively longer lags until ideally the lag approaches zero. A change in sign of the last added parameter will often indicate the end of the lag structure. Such an exercise, however, may be hazardous when the nature of the true lag structure is unknown.

As is well known, the introduction of successive lags of exogenous variables often introduces a high degree of collinearity among the regressors, making the OLS estimators inefficient. In such a case, further constraints may be imposed on the structure. When faced with a potentially serious multicollinearity problem (especially with long lag structures), some additional constraints may be imposed to further reduce the number of parameters requiring estimation. For instance, the truncated lag structures in equation (1) can be expressed as:

(2) $\beta_{si} = \Sigma_{j=0}^{M_s} \gamma_{sj} W_{sj}(i)$ for s = 1, 2, and i = 0, 1, 2, ... N_s,

where: M_s = the degree of the polynomial of the lag structure

γ_{sj} = unknown parameters

$W_{sj}(i)$ = known functions of i which define weights for i = 0, 1, ... N_s

Equation (2) can then be substituted into equation (1) and the parameter estimates can be obtained by OLS regression.

There are two recognized procedures for specifying the weights of the unknown lag structure in equation (2). The first procedure, adopted by Chen et al. (1972), may be termed the direct polynomial (DP) method, where the weights are given by:

(3) $W_j(i) = i^j$

The alternative is the procedure suggested by Almon using Lagrangian interpolation polynomials (LIP) where the weights are given by:

(4) $W_j(i) = [\pi_{k \neq j} \frac{i - m_k}{m_j - m_k}]$; k = 1, 2 --- M

where: m_0, m_1 --- m_M are scalars chosen in the interval [0, N].

While both methods yield equivalent lag structure estimates, the regression coefficients will be different under each method. It appears that a decision as to which weighting procedure to use is directly related to the importance of the selection of N (the appropriate lag length) and M (the degree of polynomial). If it is considered that the selection of M is of paramount importance, then DP weights may be preferred. When equation (3) is adopted, each W_j corresponds to the j^{th} power of the polynomial. If M is to be determined, it can be tested by the recognition of one regression coefficient being equal to zero.

On the other hand, the properties of the LIP method make it relatively easy to test for N (lag length). It is generally found that the lag length is often more difficult to determine than the degree of polynomial. It is rare that anything more than a 4^{th} degree polynomial is required for the regression model, and generally, a 3^{rd} degree polynomial provides sufficient flexibility to pick up most lags. Moreover, although the LIP weights are theoretically more difficult to estimate, they do not have as great of variability as do the DP weights. For instance, DP weights with M = 3 and N = 10 give weights W_3 (0) = 0 and W_3 (10) = 1000. LIP weights, on the other hand, fall within a relatively narrow range of -1.0 to 2.0. In addition, it would appear that regressions based on DP weights tend to be more highly collinear than the LIP weight regressors (Yon and Mount, 1975). Therefore, LIP weighting procedures seem to have a relative advantage over the alternative DP weights.

While the theoretical problems associated with the polynomial lag specification do not appear to be serious, the selection of N and M has remained problematic and their selection generally involves ad hoc methods. In addition, it would appear that the selection methodology can be manipulated to obtain desired results. The importance of selecting the correct lag length and degree of polynomial is exemplified by comparing the estimated short-run, β_0, and long-run effects, $\sum_{i=0}^{N} \beta_i$, for different specifications of the lag structure, e.g., see Chen _et al_. (1972).

Although the problem is recognized by most researchers who work with polynomial lags, relatively little in the way of objective testing of N and M is available. Harper (1977) has suggested the use of specification error tests to empirically determine if incorrect length of lag and/or degree of polynomial has been chosen in Almons method. The tests which Harper suggests involve the use of RASET and RESET developed by Ramsey (1969). Basically, the testing procedure involves the determination of specification errors through location of non-zero mean of residuals, using either F-tests or Spearman's rank correlation test. Harper argues that these tests can be used to detect errors in lag length or degree of polynomial or both, despite the many other possibilities of model misspecification.

Rational Lags

The infinite lag structure in (1) can also be approximated by the ratio of two finite lag structures (Jorgenson). Direct estimation, however, includes nonlinear regression, and the model is usually reformulated in a simplified linear form such as:

$$(5)\quad D(L)\, Y_t = D(L)\, \alpha + C_1(L)\, X_{it} + C_2(L)\, X_{2t} + D(L)\, E(L)\, \varepsilon_t$$

where: $D_s(L) = \sum_{i=0}^{M} \omega_{si} L^i$ is the denominator lag structure where

$$D_1(L) = D_2(L) = D(L) = 1 - \sum_{i=1}^{M} \omega_i L^i$$

$C_s(L) = \sum_{i=0}^{N} \gamma_{si} L^i$ is the numerator lag structure, for s = 1, 2

and:

$$\gamma_{s0} = \beta_0 \omega_{s0}$$

$$\gamma_{s1} = \beta_0 \omega_{s1} + \beta_1 \omega_{s0}$$

$$\gamma_{s2} = \beta_0 \omega_{s2} + \beta_1 \omega_{s1} + \beta_2 \omega_{s2}$$

$$\vdots$$

$$\gamma_{sk} = \beta_0 \omega_{sk} + \beta_1 \omega_{sk-1} \cdots \beta_k \omega_{s0}$$

(ω_{s0} is assumed to be 1 by convention)

While it is possible to apply OLS methods to this model, there are two basic serious problems:

1. the model may be underidentified if there are no constraints imposed; and
2. the residuals are serially correlated, thus causing OLS estimates to be biased and inconsistent.

The solution to these problems have been varied in the literature. With respect to the underidentification problem, some nonlinear constraint is usually imposed. Unfortunately, this limits the flexibility of the lag structure, but seems to be the most tractable alternative. Several solutions exist for the autocorrelation problem. One is to assume an M^{th} order Markov process for the residuals, but this is extremely restrictive, usually requiring nonlinear regression techniques if the assumption is modified. Another solution is to assume that the residuals are uncorrelated, but this is extremely hard to justify. The final solution, and one which is adopted in this paper, assumes the residuals exhibit M^{th} order serial correlation, thus requiring the use of an autoregressive scheme.

Despite, again, the theoretical underpinnings, the problem still remains to select M and N. Griliches has shown that even with N = 0 and M = 2, the potential shape of the lag structure can vary widely. This is supported by the findings in this paper. Since the transformation process of C_s (L) and D (L) into B_s (L) is nonlinear, it is difficult to identify the statistical properties of the estimators. This remains a serious problem in rational lags.

<u>Lags in General</u>

In summary, while both polynomial and rational lags involve some restrictive assumption, the most serious practical problem is the selection of N and M. In both cases, it would seem that the most serious of these two problems is the selection of lag length (N), since it would seem to be rare to find lag structures of more than 4^{th} degree polynomial and usually 3^{rd} degree allows sufficient flexibility for the relevant shapes. Deriving the relevant lengths and shapes of lag structures implies considerable knowledge of the specific problem of concern, and this is perhaps the major point to be made. Where knowledge of the decision-making strategies in an industry are unknown, the true structure of the lag remains obscure.

EMPIRICAL ESTIMATION

Previous attempts to compare polynomial and rational lags have involved the use of real world data, implying that the real structure of the lag model is known. In this rather simple experiment, however, any uncertainty as to the true lag structure is eliminated by generating the data from a simple economic model with known lag shape and length. In this way, an attempt is made to show:

1. the essential differences between the two estimation methods;
2. the ability of each method to describe the lag structure;
3. the overall "predictive ability" of the estimated model; and
4. some simple, yet sensible, rules for the use of polynomial and rational lags.

The economic model used is a simple three variable linear function such as:

(6) $Q = f(P, I)$

where: Q = quantity
P = price
I = inventory

The data used to generate Q is from the U.S. Beef Industry and is generated with the following lag structures:

$$(7)\quad P_{t-i} = (-.5)P_0 + (-.6)P_1 + (-.45)P_2 + (-.3)P_3 + (.3)P_5 + (.45)P_6 + (.55)P_7 + (.6)P_8 + (.55)P_9 + (.45)P_{10} + (.3)P_{11}$$

Notice that this is a 12 period lag (i.e., with P_4 and $P_{12} = 0$) and that the lag is best described as a 3^{rd} degree polynomial.

$$(8)\quad I_{t-j} = (.3)I_0 + (.4)I_1 + (.45)I_2 + (.5)I_3 + (.45)I_4 + (.375)I_5 + (.3)I_6 + (.2)I_7$$

This is an eight period lag ($I_8 = 0$) and is probably best described as a 2^{nd} degree polynomial. Q is generated as:

$$(9)\quad Q_t = 300 + P_{t-i} + I_{t-j}; \qquad t = 1, 2, \ldots, 52; \quad i = 0, \ldots, 12; \quad j = 0, \ldots, 8$$

In Tables 1 and 2 the estimated lag structures for various specifications of the polynomial and rational lag models are presented. Comparison of the estimated structures, however, is facilitated by the plots of the various lag specifications as depicted in Figures 1 to 2. Each model is evaluated with respect to its ability to approximate the true and known lag structure.

As shown in Figure 1, all of the fitted polynomial lag models provide reasonable approximations to the true lag structure; the only possible exception might be the 2^{nd} degree polynomial. The major inadequacy of the 2^{nd} degree relationship in Figure 1 is in its misrepresentation of the short-run influence. Hence, given the particular nature of the true lag structure of price, a 3^{rd} degree polynomial is sufficient to effectively capture the nature of the true structure. Notwithstanding minor differences in the estimated polynomial results, perhaps the "best" estimate of the price lag would be the specification where $N = 12$, $M_1 = 3$ and $\beta_N = 0$. In Figure 2 the inventory lag structure is well approximated by all-fitted relationships with $N_2 = 8$. Selection criteria also included consideration of the model's ability to approximate the true short- and long-run effects.

Dramatically different results are obtained among the alternative rational lag specifications (see Figures 3 and 4). Substantial differences occur among the estimated structures shown in Figure 3. When a simple geometric constraint is placed on the lag structure (i.e., $M = 1$) a severe misrepresentation of the true lag structure results. Specifically, the long-run effect of the geometric price model is highly negative and indeed approaches the horizontal axis asymptotically from below. In Figure 4 similar misrepresentations are observed, especially when the geometric constraint is imposed. Given the postulated true structures of both price and inventory, the closest approximations are achieved when $M = 2$.

TABLE 1. ESTIMATED POLYNOMIAL DISTRIBUTED LAG STRUCTURES

	$N_1 = 11$						$N_1 = 12$						$N_1 = 13$						
	A[a]	B[b]	C[c]	D[d]	E[e]	F[f]	A	B	C	D	E	F	A	B	C	D	E	F	True Lag
	Unconstrained			Constrained			Unconstrained			Constrained			Unconstrained			Constrained			
β_{10}	-.81	-.52	-.47	.32	-.23	-.24	-.83	-.64	-.56	-.93	-.56	-.57	-.81	-.64	-.42	-1.96	-.79	-.91	-.50
β_{11}	-.53	-.54	-.56	.16	-.02	.06	-.52	-.55	-.54	-.57	-.47	-.49	-.48	-.50	-.49	-1.24	-.59	-.93	-.60
β_{12}	-.27	-.45	-.50	.02	.09	.21	-.25	-.40	-.42	-.25	-.33	-.35	-.20	-.34	-.43	-.60	-.39	-.80	-.45
β_{13}	-.06	-.28	-.32	-.08	.13	.23	-.03	-.21	-.25	0	-.16	-.18	.03	-.15	-.29	-.07	-.18	-.54	-.30
β_{14}	.12	-.05	-.08	-.16	.11	.15	.15	0	-.03	.21	.02	0	.21	.03	-.09	.37	.03	-.22	0
β_{15}	.27	.19	.20	-.22	.05	.02	.30	.21	.19	.37	.19	.19	.35	.20	.13	.71	.22	.14	.30
β_{16}	.39	.40	.46	-.25	-.03	-.13	.41	.39	.41	.47	.34	.36	.44	.35	.36	.95	.39	.52	.65
β_{17}	.47	.59	.66	-.25	-.10	-.27	.47	.54	.57	.53	.46	.49	.47	.47	.54	1.10	.51	.85	.55
β_{18}	.52	.68	.76	-.23	-.16	-.37	.49	.63	.67	.53	.53	.56	.46	.54	.66	1.16	.60	1.12	.60
β_{19}	.53	.67	.71	-.18	-.17	-.37	.48	.64	.67	.48	.54	.57	.40	.55	.68	1.12	.63	1.27	.55
β_{110}	.51	.51	.47	-.10	-.13	-.26	.42	.55	.55	.37	.46	.49	.29	.49	.58	.98	.60	1.27	.45
β_{111}	.46	.18	0	0	0	0	.33	.33	.27	.21	.29	.31	.12	.35	.32	.75	.49	1.08	.30
β_{112}	-	-	-	-	-	-	.19	-.03	-.17	0	0	0	-.08	.11	-.13	.42	.29	.68	0
β_{113}	-	-	-	-	-	-	-	-	-	-	-	-	-.34	-.23	-.81	0	0	0	-
Long-Run Effect	1.61	1.37	1.34	-.98	-.47	-.95	1.61	1.46	1.36	1.42	1.30	1.39	-.87	1.21	.60	3.71	1.82	3.55	1.35
	$N_2 = 7$						$N_2 = 8$						$N_2 = 9$						
β_{20}	.31	.29	.31	.26	.28	.24	.30	.29	.31	.29	.30	.31	.32	.28	.33	.32	.27	.37	.30
β_{21}	.40	.41	.40	.42	.39	.41	.40	.40	.40	.39	.40	.40	.40	.41	.40	.39	.41	.41	.40
β_{22}	.45	.46	.45	.51	.48	.51	.46	.47	.45	.45	.46	.46	.45	.48	.44	.42	.49	.43	.45
β_{23}	.48	.47	.47	.54	.53	.54	.48	.48	.47	.48	.48	.48	.46	.49	.45	.44	.49	.43	.50
β_{24}	.46	.44	.45	.50	.51	.51	.46	.45	.45	.46	.46	.46	.45	.45	.43	.43	.45	.41	.45
β_{25}	.40	.38	.40	.40	.43	.41	.40	.39	.40	.41	.40	.40	.40	.38	.39	.39	.37	.37	.38
β_{26}	.30	.30	.31	.23	.26	.24	.31	.29	.30	.31	.30	.30	.32	.28	.31	.33	.27	.31	.30
β_{27}	.17	.21	.19	0	0	0	.17	.17	.18	.17	.17	.17	.21	.17	.21	.24	.17	.23	.20
β_{28}	-	-	-	-	-	-	-.03	.03	.01	0	0	0	.06	.06	.07	.13	.07	.12	0
β_{29}	-	-	-	-	-	-	-	-	-	-	-	-	-.11	-.04	-.09	0	0	0	-
Long-Run Effect	2.98	2.97	2.97	2.85	2.88	2.87	2.98	2.98	2.98	2.98	2.97	2.97	2.95	2.97	2.96	3.09	2.99	3.06	2.98

[a] A: $M_1 = 2$, $M_2 = 2$, no restrictions
[b] B: $M_1 = 3$, $M_2 = 3$, no restrictions
[c] C: $M_1 = 3$, $M_2 = 2$, no restrictions
[d] D: $M_1 = 2$, $M_2 = 2$, $\beta_N = 0$
[e] E: $M_1 = 3$, $M_2 = 3$, $\beta_N = 0$
[f] F: $M_1 = 3$, $M_2 = 2$, $\beta_N = 0$

Long-Run Effect = $\sum_{i=0}^{N_k} \beta_{ki}$; $k = 1, 2$

Short-Run Effect = β_{k0}, $k = 1, 2$

TABLE 2. ESTIMATED RATIONAL LAG STRUCTURES

	N = 0				N = 1				N = 2				
	M = 1		M = 2		M = 1		M = 2		M = 1		M = 2		True Lag
	A	B	A	B	A	B	A	B	A	B	A	B	
β_{10}	- 1.82	- 1.06	.09	.09	- .60	- .22	- .15	- .15	-1.18	- .68	- .37	- .42	- .50
β_{11}	- 1.63	- 1.00	.14	.15	-1.44	- .73	.07	.08	- .50	- .87	- .39	- .46	- .60
β_{12}	- 1.47	- .96	.16	.15	-1.21	- .64	.21	.22	-1.13	- .75	- .02	- .02	- .45
β_{13}	- 1.32	- .91	.16	.17	-1.02	- .55	.29	.29	- .88	- .60	.19	.23	- .30
β_{14}	- 1.18	- .86	.15	.15	- .85	- .48	.31	.31	- .68	- .48	.30	.35	0
β_{15}	- 1.06	- .81	.12	.13	- .72	- .42	.28	.29	- .53	- .39	.31	.37	.30
β_{16}	- .95	- .78	.08	.09	- .61	- .36	.24	.25	- .41	- .31	.27	.33	.45
β_{17}	- .85	- .73	.06	.06	- .51	- .32	.19	.19	- .31	- .25	.21	.26	.55
β_{18}	- .76	- .70	.03	.03	- .42	- .27	.13	.13	- .25	- .19	.15	.19	.60
β_{19}	- .69	- .66	.01	.01	- .36	- .24	.07	.07	- .19	- .16	.09	.12	.55
β_{110}	- .62	- .63	0	0	- .30	- .20	.03	.03	- .15	- .13	.05	.06	.45
β_{111}	- .55	- .59	- .02	- .02	- .26	- .18	0	0	- .12	- .10	.02	.02	.30
β_{112}	- .49	- .56	- .02	- .02	- .21	- .16	- .02	- .02	- .09	- .08	- .02	0	0
Long-Run Effect	-17.71	-20.80	.89	.94	-9.66	-5.80	1.48	1.52	-6.72	-5.33	.72	.891	1.35
β_{20}	.37	.22	.28	.28	.26	.26	.26	.26	.28	.29	.27	.28	.30
β_{21}	.33	.21	.44	.44	.48	.40	.44	.44	.38	.38	.41	.40	.40
β_{22}	.30	.19	.51	.51	.41	.35	.51	.51	.56	.52	.53	.54	.45
β_{23}	.27	.18	.50	.50	.34	.30	.51	.51	.44	.41	.53	.54	.50
β_{24}	.24	.17	.45	.45	.29	.26	.45	.45	.34	.33	.46	.47	.45
β_{25}	.21	.16	.36	.36	.24	.23	.36	.36	.26	.27	.35	.36	.375
β_{26}	.19	.16	.27	.27	.20	.20	.27	.27	.20	.21	.24	.25	.30
β_{27}	.17	.15	.17	.18	.17	.17	.17	.17	.16	.17	.15	.15	.20
β_{28}	.15	.14	.09	.09	.14	.15	.09	.09	.12	.14	.07	.07	0
Long-Run Effect	3.62	4.29	2.74	2.74	3.30	3.28	2.75	2.75	3.16	3.26	2.84	2.89	2.975
γ_{10}	- 1.824	- 1.061	0.091	0.096	-0.601	-0.215	-0.151	-0.146	-1.183	-0.682	-0.374	-0.417	
γ_{11}	-	-	-	-	-0.935	-0.565	0.309	0.309	0.414	-0.329	0.153	0.146	
γ_{12}	-	-	-	-	-	-	-	-	-0.736	-0.049	0.321	0.394	
γ_{20}	0.373	0.219	0.279	0.279	0.256	0.256	0.260	0.261	0.279	0.290	0.274	0.281	
γ_{21}	-	-	-	-	0.269	0.174	0.034	0.034	0.160	0.152	0.011	-0.003	
γ_{22}	-	-	-	-	-	-	-	-	0.268	0.207	0.109	0.121	
ω_1	0.897	0.949	1.586	1.586	0.841	0.869	1.563	1.564	0.776	0.801	1.439	1.443	
ω_2	-	-	-0.688	-0.688	-	-	-0.670	-0.671	-	-	-0.578	-0.581	

[a] A = estimated by O.L.S.

[b] B = corrected for autocorrelation

$$\text{Long-Run Effect} = \sum_{i=0}^{N_s} \gamma_{si} \Big/ \left(1 - \sum_{j=1}^{M_s} w_j\right); \quad s = 1, 2$$

$$\text{Short-Run Effect} = \beta_{s0} = \gamma_{s0}; \quad s = 1, 2$$

Figure 1

Selected Estimated Polynomial Lags (Price)

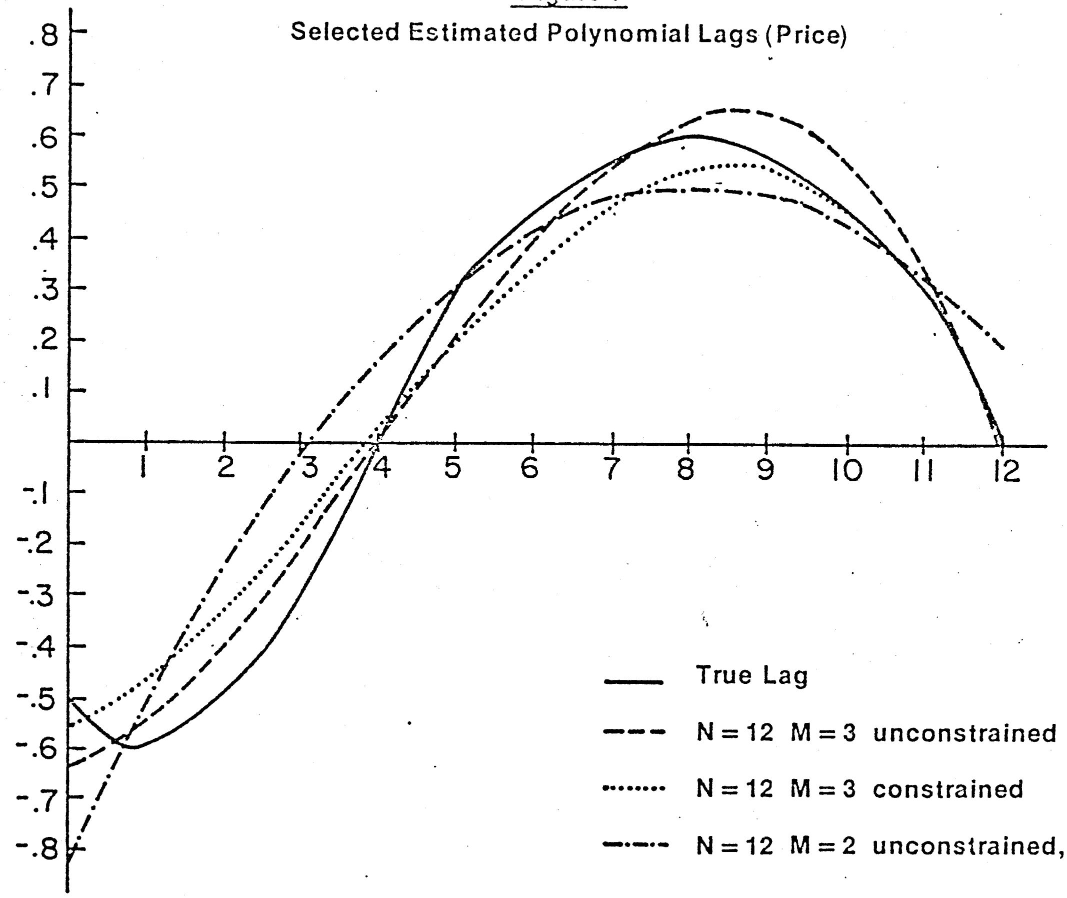


Figure 2

Selected Estimated Polynomial Lags (Inventory)

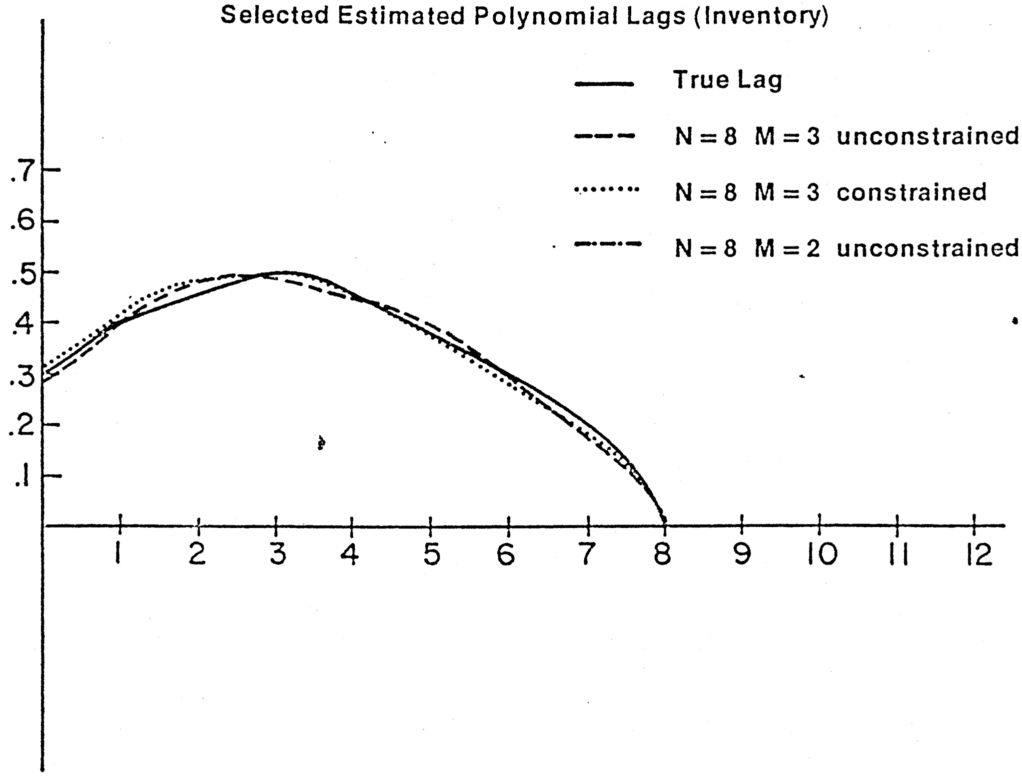


Figure 3

Selected Estimated Rational Lags (Price)

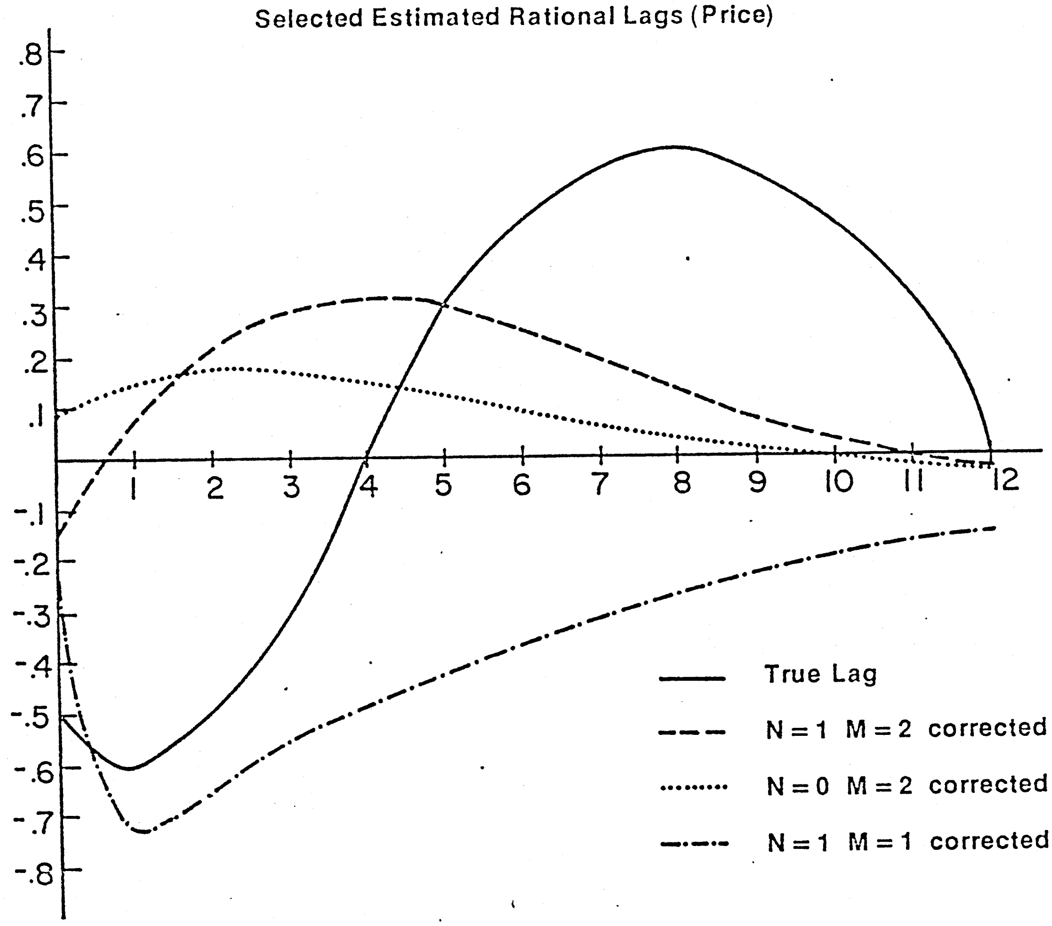
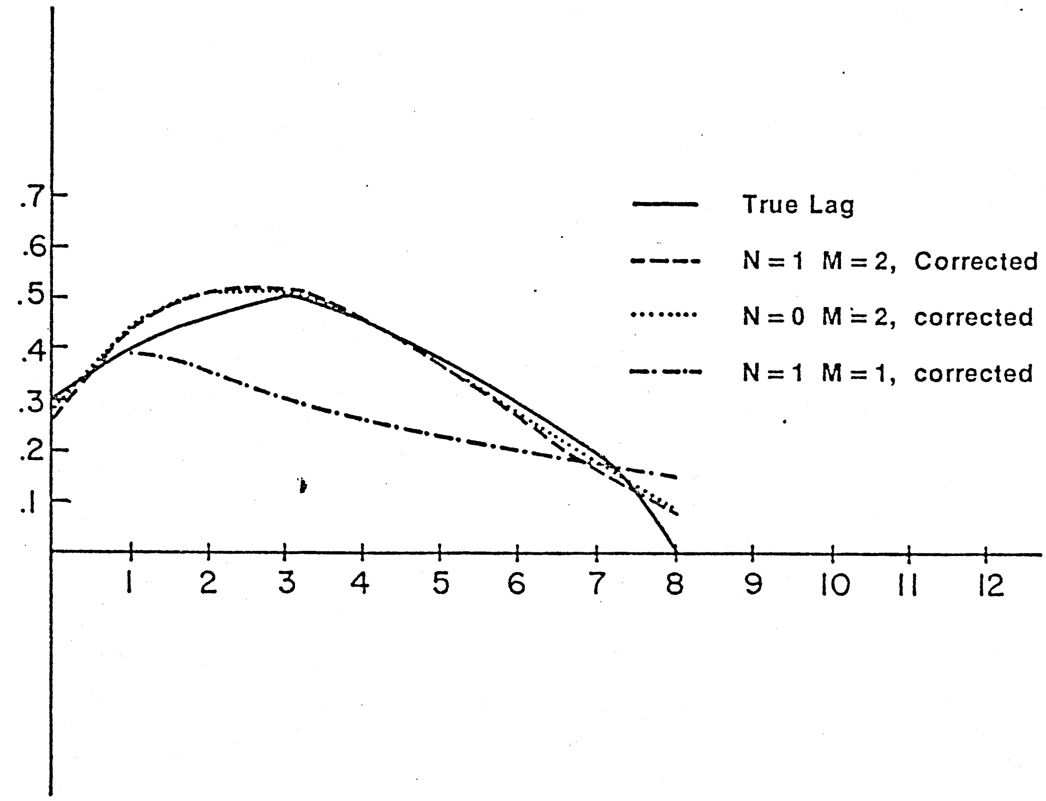


Figure 4

Selected Estimated Rational Lags (Inventory)



In comparing the results of the two alternative estimation techniques, it would appear that the polynomial model is generally a superior approximation specification. However, in order to provide a more powerful test of this conclusion, some means of testing for specification error would seem appropriate (Ramsey, 1969). Use of the LIP weights and a 3rd degree polynomial lag model appears to maintain adequate flexibility to approximate most lag structures. The length of the lag can be estimated by fitting various lag lengths (without imposing additional constraints) and then inspecting the estimated structures to determine which specification most closely conforms to economic logic and the specific problem under investigation.

Polynomial lag approximations are particularly appropriate when long lag structures are expected. Hence, estimation of the nature of lagged responses when monthly or quarterly data are used is often conducive to a polynomial approximation. On the other hand, when the number of available observations is small relative to the number of regressors, a rational lag specification may prove to be the only effective means of capturing a lagged relationship.

SUMMARY AND CONCLUSIONS

When confronted with the problem of estimating the nature of a lagged response relationship, the selection among available techniques is often difficult. To facilitate this selection decision, in this paper a comparison is made between the polynomial and rational lag models. A theoretical discussion of the two models is presented followed by an empirical illustration of the effectiveness of each model to approximate the true lag structure. An effective empirical evaluation is achieved by comparisons to the true lag structure. The data used are generated from a simple economic model with a known double lag structure.

The results support the contention that a 3rd degree polynomial lag model is a good approximation of a fairly complex lag structure; a 2nd degree polynomial was found to be adequate. In the rational lag specifications estimated here, good approximations of the true lag structure were not achieved, especially when a geometric constraint is imposed. These findings have important implications for applied price analysts interested in obtaining reliable elasticity estimates for, say, policy recommendations.

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