

A PRODUCTION FUNCTION FOR FLORIDA FOLIAGE NURSERIES FROM TIME-SERIES AND CROSS-SECTION DATA*

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INTRODUCTION

The foliage industry is the most rapidly expanding segment of commercial agriculture in Florida [1]. The industry accounted for about \$13 million of the agricultural income in 1966 and over \$187 million in 1975. The area in production in the state has more than doubled in the last ten years; it was increased from about 26 million square feet in 1966 to just over 65 million square feet in 1975. Nurserymen were expected to expand their production area by about 8.6 million square feet during 1976 [14].

This rapid increase in production area has been from expansion of established producers and entry of new growers into the industry. The producers increased from 163 in 1966 to 262 in 1975. The average foliage nurseryman participating in the Florida Cooperative Extension nursery business analysis program expanded employment from 23 employees in 1970 to 30 in 1975. During the same period, the average capital investment for these nurseries increased from \$160,691 to \$428,469.

New nurserymen as well as expanding nurserymen are attempting to adjust capital-labor combinations to achieve efficient production levels and adjust nursery size to take advantage of economies of scale suspected to be associated with foliage nurseries. A production function is estimated providing nurserymen information on the optimal capital-labor combinations as well as economies of scale.

ECONOMIC MODEL

The foliage nurseries are assumed to be profit maximizers operating within competitive factor and product markets. The objective function is thus:

$$\text{Max } \Pi = P\alpha L^{\beta_1} K^{\beta_2} - wL - rK \quad (1)$$

L,K

where the Cobb-Douglas production function is assumed. Maximization of equation (1) yields the following three equation system:¹

$$\begin{aligned} \ln Y &= \ln \alpha + \beta_1 \ln L + \beta_2 \ln K \\ \ln \beta_1 &= \ln(wL) - \ln(PY) \\ \ln \beta_2 &= \ln(rK) - \ln(PY) \end{aligned} \quad (2)$$

Equations (2) may be solved for the equilibrium values of labor, capital and output given values for the coefficients. In a deterministic framework, the obvious result for a cross-section of firms is that given the same prices, all firms should be at the same point; they will have identical values for output and factor levels. The introduction of stochastic terms does not alter this; it only suggests that what one observes are random movements rather than systematic effects.

Alternative developments have been set forth to counter this difficulty. They generally impose the assumption that maximization takes place over

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¹See Nerlove (Chapter 1) on this point. Throughout the remainder of the paper, labor (L) and capital (K) are measured as expenditures corresponding with our data source. Similarly, output (Y) is value added. Nerlove demonstrates that under the competitive assumptions we maintain, we can still identify β_1 and β_2 while using values rather than quantities. Only labor and capital are considered as substitutable inputs (other inputs are subtracted out of value added) since they are the major inputs the operator can vary.

expected or anticipated profits rather than observed profits [11, 16, 6]. This permits inputs in the production function to be taken as given under suitable conditions, and effectively satisfies the requirement of zero correlation between the inputs and the production function disturbance. The classical example of conditions under which this argument is assumed valid is in agricultural applications where inputs are, to a large extent, chosen with regard to expected output, and disturbance reflects random uncontrollable events between time of input selection and realization of output; e.g. weather.

Directly obtainable from this model is an estimate of the returns to scale measured as $\beta_1 + \beta_2$. In addition, the latter two equations of (2) represent the marginal conditions for profit maximization. An informative way of writing these equations is:

$$\begin{aligned}\beta_1 \frac{PY}{rK} &= R_1 \\ \beta_2 \frac{PY}{wL} &= R_2\end{aligned}\quad (3)$$

where the R_j represents what Hoch calls a systematic deviation from the optimum point for reasons arising either from a restrictive environment or a systematic lack of profit maximization. In equilibrium with profit maximization, R_j should, of course be unity, so that the point of interest is the deviation from unity of the R_j .

THE STATISTICAL MODEL

Data for the estimates are from production and accounting records of foliage nurserymen participating in the Florida Cooperative Extension nursery business analysis program [5]. Data from 11 nurseries participating in the program from 1970-75 were analyzed. A number of techniques have been developed for analysis of this type data, among the earliest of which was the analysis of covariance. One of the earliest applications of this procedure to an economic problem is in a much neglected paper by Hoch [6] who analyzed a set of farm management data within much the same framework as in this paper.² Since the procedure is well developed, only pertinent features will be summarized.

Although the historical objective of covariance analysis (in the biological sciences) was to determine estimates of "control" factors in alternative experiments, the objective within economic applications is to improve estimates of common factors by controlling for the "design" features—in our case, firm or time effects. As previously noted, a single equation

procedure for directly estimating the production function parameters is adopted under the assumption the firms are maximizing anticipated or expected profits. It then follows that least squares estimates will be optimal as long as the assumption of zero correlation with the disturbance term can be maintained. This, however, is a basic difficulty with models such as that in equations (2).

One basic ingredient of the theory of the firm is what is often referred to as entrepreneurial capacity [4]. Although it is, at best, an elusive "factor of production" to measure, it is nonetheless an important variable distinguishing one firm from another. Since it is an unobservable factor, it is often left out of the analysis; but, as Hoch correctly pointed out, this will lead to biased estimates of the parameters since it is clearly a case of an omitted variable which is correlated with the labor and capital variables. The correlation follows from the theory of the firm recognizing that factors of production (labor, capital and entrepreneurial capacity) are jointly determined.

The essential feature of the analysis of covariance is that differences between firms (controlling for other variables such as labor and capital) can be isolated so that correlation between the disturbance and the other two inputs is removed. These are typically referred to as "firm effects." Although entrepreneurial capacity will be included in this firm effect, the latter will typically capture certain other systematic differences between firms.

Complete treatment of a time-series of cross-sections in the analysis of covariance framework requires consideration of other variations. The most general model incorporates both the possibility of firm effects and time effects and the non-homogeneity of the output elasticities between firms. In principle, this requires estimating an equation for each firm, including time effects. Obviously, there will always be too few observations to accomplish this. There are thus two alternative paths: (1) assume homogeneity of the output elasticities and estimate firm and time effects, or (2) assume there are no systematic time effects and determine the homogeneity of the output elasticities.

The statistical model corresponding to the first alternative is specified in equation (4).

$$\begin{aligned}\ln Z_{it} &= \alpha_i + \tau_t + \beta_1 \ln x_{1it} + \beta_2 \ln x_{2it} + \mu_{it} \\ i &= 1, \dots, n; t = 1, \dots, T.\end{aligned}\quad (4)$$

where inputs and outputs are measured in value terms, α_i represents the effect specific to firm i , τ_t

²Recent work in a similar framework, but aggregate context, has been reported by Bauer and Lu.

represents the effect specific to the t^{th} year, and μ_{it} is the random disturbance. We maintain that

$$E(\alpha_i \mu_{it}) = E(\tau_t \mu_{it}) = E(\ln X_{jit} \mu_{it}) = 0$$

$$E(\mu_{it}) = 0 \quad \text{Var}(\mu) = \sigma^2 I$$

for all i, j , and t . That least squares estimates of β_1 and β_2 will be best linear unbiased estimates in this case.

In the context of a Cobb-Douglas production function as in (4), the year effect is typically assumed to represent shifts in technology common to all firms. Given a relatively short duration of time (6 years for the data under consideration) such effects could be argued to be of less *a priori* significance than variations between firms. In this case it might be preferable to concentrate on the second approach as we do. The statistical model in this case is:

$$\ln Z_{it} = \alpha_i + \beta_1 \ln X_{1it} + \beta_2 \ln X_{2it} + \mu_{it} \quad (5)$$

where

$$E(\alpha_i \mu_{it}) = E(\ln X_{jit} \mu_{it}) = E(\mu_{it}) = 0$$

$$\text{Var}(\mu) = \sigma^2 I$$

In this case, not only does the intercept shift from firm to firm, but the output elasticities also vary from firm to firm. The assumption is maintained that variations from year to year not accounted for by the inputs are not systematic in the context of this model. A special case of this model is when all firms have the same output elasticities, but differing intercepts. A further restriction, that all firms have the same intercept, would correspond to there being no difference between firms. The obvious advantage to this formulation is that one can statistically test for differences between firms, and these differences are properly accounted for within the model.

Before proceeding to the estimates, some alternative procedures for treating a time-series of cross-sections should be considered. The one discussed above will be referred to as a "fixed effects" model; the isolated time and firm variations are non-random. A competing and widely adopted model is the "random effects" model (also referred to as error components models) [2, 9, 15]. The random effects framework assumes a distribution associated with variations between firms or over time. The objective is to estimate the mean and variance of that distri-

bution and incorporate that into the estimation procedure. The model is typically cast as:

$$\ln Z_{it} = \alpha + \beta_1 \ln X_{1it} + \beta_2 \ln X_{2it} + \mu_{it} \quad (6)$$

$$\mu_{it} = u_i + v_t + w_{it}$$

where

u_i = error component corresponding to firm specific variations

v_t = error component for time specific variations and

w_{it} = non-specific error.

Generally, a two-step estimation procedure is utilized to obtain generalized least square estimates of the coefficients. A recent paper by Mundlak [10], however, raises a serious question with respect to consistency of parameter estimates so obtained. In particular, he argues that for a model such as this one, the essential feature of covariance analysis is to eliminate the non-zero input correlation with the disturbance. When firm effects are treated as random, inputs will typically be correlated with the firm specific error component (representing, in part, entrepreneurial capacity) and the estimates will be inconsistent. Although estimates based on the Balestra-Nerlove procedure are illustrated for comparative purposes, these are discounted due to Mundlak's rather cogent argument.

A third alternative is the random coefficients model. This estimation procedure is set forth in Swamy. In this case, output elasticities and intercept are assumed to be random between firms, possessing a distribution for which the mean and variance are estimated. Again estimates based on this procedure are presented, but are discounted for the same reason cited in the previous random effects model.

EMPIRICAL RESULTS

Alternative estimates based on the fixed effects model are in Table 1. Looking first at column 3 (corresponding to equation (4)), the test for significance of time effects yields an F ratio of 1.14.³ The corresponding tabled value for the $F_{.05}(5, 48) = 2.40$ suggests that time effects are not statistically significant. Thus, the procedure is to focus solely on firm effects.

The most general model is expressed by equation (5). It is first tested for homogeneity of the output

³See Johnston (pp. 192-296) for a discussion of the tests used.

TABLE 1. PRODUCTION COEFFICIENTS

Independent Variables ^a	FIXED EFFECTS EQUATIONS						RANDOM EFFECTS EQUATION		RANDOM COEFFICIENTS EQUATION	
	Column 1 No Firm Or Time Effects		Column 2 Firm Effects		Column 3 Firm and Time Effects		Column 4		Column 5	
	Coefficient	Standard Error	Coefficient	Standard Error	Coefficient	Standard Error	Coefficient	Standard Error	Coefficient	Standard Error
Intercept	2.455	(.487)	1.860	(.687)	5.105	(1.74)	2.854	(.419)	1.631	(1.623)
Capital (X ₁)	.338	(.094)	.274	(.097)	.234	(.100)	.326	(.106)	.488	(.367)
Labor (X ₂)	.540	(.092)	.688	(.119)	.415	(.187)	.515	(.088)	.494	(.424)
F ₁			---		---					
F ₂			-.513	(.125)	-.528	(.125)				
F ₃			-.503	(.134)	-.237	(.185)				
F ₄			-.385	(.137)	-.086	(.198)				
F ₅			-.442	(.134)	-.600	(.156)				
F ₆			-.354	(.127)	-.402	(.128)				
F ₇			-.313	(.134)	-.376	(.138)				
F ₈			-.656	(.142)	-.367	(.201)				
F ₉			-.762	(.149)	-.388	(.237)				
F ₁₀			-.646	(.135)	-.815	(.160)				
F ₁₁			-.228	(.135)	-.425	(.167)				
t ₁					.079	(.102)				
t ₂					.213	(.112)				
t ₃					.300	(.150)				
t ₄					.361	(.188)				
t ₅					.461	(.224)				
t ₆					---					
Return to Scale	.873	(.044)	.962	(.058)	.649	(.163)	.842		.983	(.787)
R ²	.86		.93		.94		---		---	
Sum of Squared Errors	4.878		2.485		2.222		---		---	

^aThe dependent variable is:

Z_{it} = dollar value added = revenue from plant sales plus changes in plant inventory value minus current inputs costs other than labor.

The independent variables are:

X₁ = dollar value of capital service = annual depreciation and an interest charge of 8% on the capital investment,

X₂ = the annual wages = wages paid by the firm,

F_i corresponds to the firm effects specified in equations (4) and (5),

t_t corresponds to the time effects specified in equations (4) and (5).

elasticities, i.e., are they the same for each firm. The resultant F ratio for this test is 2.18. In choosing a significance level for this test, it is important to note that significance of the intercept shifts from one firm to the next must also be tested. Thus, for an overall significance level of 5 percent one part of that needs to be apportioned to the output elasticity test. A choice of 1 percent is convenient and indicative of the relatively high cost incurred in terms of generality by specifying different elasticities and rejecting the null hypothesis of homogeneous (equal) output elasticities.

Given the homogeneous elasticities, proceeding requires the conditionally imposed restrictions that elasticities are the same for each firm and the intercept shifts must be tested. This corresponds to a (conditional) test on the equality of the intercepts (α_i). The resultant F ratio in this case is 5.10. The tabled F value for 10 and 50 degrees of freedom is 2.70 at the 1 percent level, thus rejecting the null

hypothesis of all firms having the same intercept, conditional on their having the same elasticities.

The set of estimates on which most weight is placed are presented in column 2 of Table 1. They include an intercept shift for each firm; but all elasticities are the same across firms; and time effects are not included. Column 1 is included for comparative purposes, illustrating the results when all observations are pooled with no firm effects taken into account.⁴

IMPLICATIONS

Since *a priori* reasoning and statistical tests suggest that the "fixed effects" model (analysis of covariance) with only intercept changes across firms is the appropriate model, inferences are drawn from this set (column 2 of Table 1). It is clear from estimates for labor and capital in Table 1 that the coefficient magnitudes are dependent on this choice.

⁴Columns 4 and 5 are included to illustrate the results under the random effects model and the random coefficients model. We place less credence on these, however.

On the other hand, it is reassuring that in all cases the elasticities were positive and the returns to scale were consistently less than 1.

Returns to Scale

As noted in column 2 (Table 1), the returns to scale estimate is just under unity, .962. The t ratio for this estimate as compared to unity is $-.65$, thus failing to reject the null hypothesis of constant returns to scale.⁵ We find a point estimate in the area of decreasing returns, although it is not statistically distinguishable from constant returns.

Marginal Returns

The marginal returns for each input are presented in Table 2. They are derived from equations (3). As noted there, R_1 and R_2 would be unity in equilibrium indicating that an additional dollar of expenditure on the input returns an additional dollar. Based on the estimates of our preferred set (column 2), the estimated marginal return per dollar of capital is (Table 2):

$$.274 \frac{179,872}{30,182} = 1.63$$

Similarly, the marginal return per dollar of labor is:

$$.688 \frac{179,872}{90,219} = 1.37$$

Since the above marginal returns estimates are based on estimated parameters, it is important to

TABLE 2. MARGINAL RETURN ESTIMATES FOR FIXED EFFECTS EQUATIONS^a

	Column 1 No firm or time effects	Column 2 Firm effects	Column 3 Firm and time effects
Capital (X_1)	2.01	1.63	1.39
Labor (X_2)	1.08	1.37	.827

^aCalculated as $R_1 = \hat{\beta}_1 \frac{PY}{PK}$ and $R_2 = \hat{\beta}_2 \frac{PY}{wL}$ where output and inputs are evaluated at the geometric means.

know whether or not they are significantly different from unity. Individual t-tests are not particularly relevant since it is relevant only to ask whether the firm is in or out of equilibrium, not whether it is in equilibrium with respect to each input separately. This suggests a joint F test on the restriction that the marginal returns be equal to unity in each case, i.e., $R_2 \equiv 1 \equiv \hat{\beta}_2 \frac{PY}{wL}$ or $\hat{\beta}_2 = \frac{wL}{PY}$. The implied F ratio is 13.90 with 2 and 53 degrees of freedom. The closest tabled values are $F_{.05}(2, 50) = 3.18$ and $F_{.01}(2, 50) = 5.06$, indicating rejection of the null hypothesis. Since both labor and capital have marginal returns greater than unity, increases in labor and capital are warranted for the average firm.

SUMMARY AND CONCLUSIONS

The purpose of this study was to evaluate the returns to scale and possible deviations from optimal resource allocation through a production function for a cross-section of foliage nurseries over time. Statistical support was found for the hypothesis that the firms operate under similar technologies, i.e., they have the same production function parameters.

Although returns to scale are not found to be statistically different than unity, point estimates for the fixed effects model are all less than unity. Applying parameter estimates to resource allocation problems, labor and capital increases would improve the profit position of the average firm. This is consistent with growth of the firms over the time period. Since estimates refer to the average firm and time period, one must be cautious about extrapolating this justification of firm growth very far into the future. The conclusion is, however, supportive of the expansion activity observed over the time period under consideration. From the standpoint of providing information to the industry, an annual updating of such estimates might be warranted to check for indications that further growth may not be advisable.

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⁵The estimated standard error for the returns to scale estimate is .058 where $cov(\hat{\beta}_1, \hat{\beta}_2) = -.010$.

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