A conceptual model is formulated that shows that a downward sloping supply function may exist for a profit maximizing firm facing a cash-flow constraint. The necessary requirement is that at least one factor must be a non-cash input. The model is tested using analysis of variance on two groups of producers from farm record data, one group facing a binding budget constraint the other group not. The results indicate that farms facing a cash flow constraint increase output more than farms not restricted by a cash flow constraint in response to a price decrease.

Assertions have been made recently that if the price of milk is reduced farmers will react by producing more milk. These statements can be found in farm magazines (e.g., Hoards Dairyman), survey research (e.g., Bancroft and Young), farm organization meetings (e.g., Strickler), as well as heard in many rural coffee shops. Proponents often justify this contention by pointing to the recent trends of decreasing milk prices simultaneously with increasing milk production. Economists are quick to correctly point out the flaw in this logic, which confuses increases in milk supply with supply response to price changes. The standard reply by economists is that increases in production are more likely due to increases in technology, decreases in input prices, or other supply shifters rather than a decrease in milk price (see, for example, Paarlberg).

Is it possible that a profit maximizing dairy farmer would increase production in response to a decrease in the milk price, holding other determinants of supply constant? The objective of this paper is to demonstrate conceptually that the answer to this question may be yes under certain conditions. The key requirements are that the firm faces a binding cash-flow constraint and that at least one factor be a non-cash input. Empirical evidence is then presented using farm record data. The farms from these records are split into two categories, cash-flow binding and non-binding groups, and analysis of variance is used to test whether the two groups have statistically different changes in milk production in response to a price decrease consistent with the conceptual model.

Previous Research

Economists usually attribute a downward sloping supply response to irrational farmers or to farmers who are politically rational, i.e., they argue this line of reasoning to prevent decreases in support prices aimed at reducing excess production. Various explanations have been offered as to why milk supply response may appear to be inelastic, or even negative, especially at the market level. The first is the failure to properly isolate milk supply shifters. For example, continuous technological change will occur even when output prices are falling and will cause an outward shift in the supply function. The visible effect may be a lower price and greater output. Proper estimation procedures, however, should correct for technological change (Cochrane).

Another phenomenon especially relevant to agriculture is the very inelastic asymmetric nature of factor demands. In periods of declining output prices, little or no adjustments in the employment of factors will accompany the decline in output price, even if factor prices remain constant. D. Gale Johnson postulated that farm land, labor, and machinery utilization would change little in response to a decline in output price, although recent work by Savada and Chambers reject this hypothesis in its strict form. Similarly, the fixed asset theory implies that once purchased, assets are fixed in production over a wide range of output prices because of low salvage values (Glenn L. Johnson). Milk production, which is characterized by a very large proportion of fixed inputs, is a situation where the fixed asset theory may apply.
Just and Zilberman also demonstrate that even if the expected output price increases, the variability in the price may be great enough to result in a negative supply response if risk averse operators shift to alternative commodities. In addition, recent research by Lee and Chambers rejects the notion that farmers’ objective is to maximize unconstrained profits. Rather, their empirical results imply that expenditure-constrained profit maximization is more indicative of farmers’ actual behavior. Unlike their model, however, we distinguish between both cash and non-cash (non-purchased) variable inputs.

The Model

For simplicity assume that milk is being produced with two variable inputs by an increasing strictly concave production function

\[ y = f(x_1, x_2 | Z), \]

where \( x_1 \) is a purchased variable input, \( x_2 \) is a non-purchased variable input, and \( Z \) are fixed inputs.

Although the objective of the farm family is to maximize profits, they are faced with a net cash-flow constraint

\[ py - r_1 x_1 \geq B, \]

where \( B \) is the net cash flow required for debt payment, family living needs, and other cash requirements, \( p \) is the price of output \( y \), and \( r_1 \) is the price of input \( x_1 \) (and \( r_2 \) is the implicit price or opportunity cost of input \( x_2 \)).

The optimization problem is:

\[
\begin{align*}
\max & \quad \pi = pf(x_1, x_2) - r_1 x_1 - r_2 x_2, \\
\text{s.t.} & \quad pf(x_1, x_2) - r_1 x_1 \geq B.
\end{align*}
\]

The Lagrangian becomes

\[ L = pf(x_1, x_2) - r_1 x_1 - r_2 x_2 + \lambda \left( -B - r_1 x_1 + pf(x_1, x_2) \right). \]

The Kuhn-Tucker conditions are

\[
\begin{align*}
\frac{\partial L}{\partial x_1} &= pf_1 - r_1 - \lambda r_1 + \lambda pf_1 \leq 0, \\
x_1 &\geq 0, \quad \frac{\partial L}{\partial x_1} x_1 = 0, \\
\frac{\partial L}{\partial x_2} &= pf_2 - r_2 + \lambda pf_2 \leq 0, \\
x_2 &\geq 0, \quad \frac{\partial L}{\partial x_2} x_2 = 0,
\end{align*}
\]

First assume that positive inputs are used, \( x_1, x_2 > 0 \), but the cash-flow constraint is not binding so that \( \lambda = 0 \). The two first order necessary conditions then become the typical profit maximization conditions.

\[
\begin{align*}
pf_1 &= r_1, \\
pf_2 &= r_2.
\end{align*}
\]

Now assume the cash-flow constraint is binding so that \( \lambda > 0 \). The first order necessary conditions then become

\[
\begin{align*}
pf_1 (1 + \lambda) &= r_1 (1 + \lambda), \\
pf_2 (1 + \lambda) &= r_2, \quad \text{and} \\
pf - r_1 x_1 &= B.
\end{align*}
\]

The first condition collapses to the typical profit maximization result that \( f_1 = r_1/p \). The marginal product of input \( x_1 \), however, may depend upon the use of \( x_2 \), which will be shown to be used in a greater quantity than with no binding cash-flow constraint. If \( x_2 \) is a technical complement for \( x_1 \), then the marginal product of \( x_1 \) will be greater. This implies a greater use of \( x_1 \) than with no binding constraint. The second condition can be rewritten as \( f_2 (1 + \lambda) = r_2/p \) so the marginal product for \( x_2 \) would be lower than the non-constrained profit maximization use of \( x_2 \). Assuming a concave production function then more of \( x_2 \) would be used. Thus, the existence of a binding cash-flow constraint may cause the farmer to use more \( x_2 \) and more of \( x_1 \) if \( x_2 \) is a technical complement for \( x_1 \).

Comparative Statics

In order to determine the change in output (milk) as the price of milk is reduced it is necessary to perform comparative statics. Treating \( x_1, x_2 \) and \( \lambda \) as variables and \( r_1, r_2 \) and \( p \) as parameters and assuming a binding cash-flow constraint (equality), the total differential of the first order conditions become

\[
\begin{align*}
pf_{x_1} dx_1 + pf_{x_2} dx_2 + f dp = dr_1 \\
pf_{x_1} dx_1 + pf_{x_2} dx_2 + f dp + \lambda pf_{x_1} dx_1 \\
+ \lambda pf_{x_2} dx_2 + \lambda f dp + \lambda pf_{x_1} dx_1 \\
pf_{x_1} dx_1 + pf_{x_2} dx_2 + f dp - x_1 dr_1 - r_1 dx_1 = 0
\end{align*}
\]

Assuming no change in input prices (\( dr_1 = 0 \))
and \(dr_2 = 0\), solving for \(dx_1\) and \(dx_2\) results in (see appendix):

\[
dx_1 = \frac{dp(f_1f_2 - f_12f)}{-pf_1f_2},
\]

\[
dx_2 = \frac{-fdp}{pf_2}.
\]

Assuming a decrease in a positive output price and a strictly increasing concave production function \((f_1 > 0, f_2 > 0, f_{11} < 0, f_{22} < 0)\), then for \(f_{12} < 0\) (technically competitive inputs), \(dx_1 < 0\). A decrease in output price will reduce the use of \(x_1\). However, if \(f_{12} > 0\) (technically complements), then \(dx_1 > 0\) for \(f_{12} > f_1f_2\), or a decrease in output price will increase the use of \(x_1\). In contrast, a decrease in output price will always cause an increase in the use of \(x_2\), \(dx_2 > 0\), regardless of the sign of \(f_{12}\).

It is interesting that the change in \(x_1\) or \(x_2\) does not appear to depend upon the price of either \(x_1\) or \(x_2\). The price of \(x_2\) is not included because it is immaterial in fulfilling the binding budget constraint since it is not purchased. As shown in the appendix however, the price of \(x_1\) was eliminated by substituting \(p_1f_1\) for \(r_1\). The importance of \(r_1\) is still inherent in the reduced form.

The expressions for \(dx_1\) and \(dx_2\) can be substituted into the differential of the production function, \(dy = f_1 dx_1 + f_2 dx_2\), to determine the change in output that occurs with a decrease in output price. The result is (see appendix)

\[
dy = \frac{dp(ff_{11}f_2 - ff_1f_{12} + f^2f_2)}{-pf_1f_2}.
\]

The change in \(y\) will be positive for a decrease in output price if and only if the term in parentheses is negative

\[ff_{11}f_2 - ff_1f_{12} < -f^2f_2.\]

Thus an inverse milk supply response is possible especially if \(f_{12} > 0\) (technically complements). If milk price falls it has been shown that more of the non-purchased variable input \((x_2)\) will be used. If the inputs are complements then more \(x_1\) will also be used leading to an output increase. In contrast, if the inputs are technical substitutes output will only increase if the increased use of the non-purchased input \((x_2)\) increases output more than the decreased output from the reduced use of the purchased input \((x_1)\). This might happen if the price of \(x_1\) is relatively low so that its marginal product is low.

An example of these concepts might be a shift from two to three times a day milking. This might be accomplished with no changes in fixed investment, but family labor (unpaid) might be used more extensively and additional feed purchased or produced. The labor and feed may be technical complements in this case, thereby causing an increase in output and feed usage.

### Estimating Supply and Demand Functions

The functional form of the supply and demand functions depend upon the functional form of the underlying technology (production function). However, certain characteristics of those functions can be gleaned from the three first order conditions. First, any supply or demand function should contain the variables \(r_1, r_2, p\) and \(B\) since these are all in the F.O.C.s. Second, the functions would be homogeneous of degree zero in these four variables since multiplying the four variables by a scalar \(t\) does not alter the F.O.C.s. However, the functions would not be homogeneous of degree zero in the input and output prices only, unless the constraint is not binding (unconstrained profit maximization). In empirical estimation, therefore, it would be imperative to determine whether the cash-flow constraint was binding.

### Empirical Evidence

Ideally, to test the validity of this model one could estimate separate supply functions for the binding and non-binding groups, i.e.,

\[S^B = f(P^m, P_i, T, PR, B)\]

\[S^{NB} = f(P^m, P_i, T, PR)\]

where, \(S^B\) is the supply function for the binding group, \(S^{NB}\) is the supply function for the non-binding group, \(P^m\) is the milk price, \(P_i\) is the factor price for input \(i\), \(T\) is technology, \(PR\) is profitability of substitute commodities or off-farm employment to milk production, and \(B\) is a budget constraint measure. After estimating the two supply functions, one could statistically test whether the price elasticity for the binding group was more inelastic (or negative) than the nonbinding group. This approach requires appropriate data on input prices and profitability measures of the firm’s competing enterprises. However, if the model is a valid representation then the comparative statics demonstrated that farmers facing a binding cash-flow constraint would increase output more or decrease output less in response to an output price decrease compared to a group of farmers not facing a cash-
flow constraint if they face the same prices and opportunities. A cash-flow constrained group would also increase their utilization of non-cash inputs to a greater extent than a non-constrained group of farms in order to meet their cash flow constraint in response to a price decrease. The constrained group may also increase their use of cash inputs more than the unconstrained group if they complement non-cash inputs, but this is an empirical question.

These hypotheses were tested using a sample of New York dairy farmers who participated in the Cornell Dairy Farm Business Summary during both 1984 and 1986 (Smith, Knoblauch, and Putnam). This is a convenient time period to use because annual milk prices consistently fell between 1984 and 1986. All producers in the summary who participated in the 1984–85 Milk Diversion Program were deleted from the sample. This was done so that changes in production between 1984 and 1986 were not biased by this voluntary supply control program. The two year lag was judged sufficient time for dairy farmers to make production adjustments yet short enough that technology increases were minor. Also, the input prices did not change a great deal between 1984 and 1986. The index of prices paid by New York dairy farmers fell only slightly from 155 (1977 = 100) in 1984 to 149 in 1986. Hence, milk supply shifters in New York remained relatively stable for the period (New York Economic Handbook 1987).

The farmers were separated into two groups depending upon whether or not they faced a binding cash-flow constraint. The binding group was defined as all firms in the business summary that had planned debt payments greater than or equal to cash available. Planned debt payments for the upcoming year are provided by each summary participant. Cash available is calculated by subtracting cash farm expenses from cash farm receipts, adding back interest paid, and subtracting net personal withdrawals from the farm (which is personal withdrawals and family expenditures less nonfarm income and nonfarm money borrowed). If a farm’s budget constraint was binding in 1984 or 1986, the farm was classified in the budget binding category. All other farms were placed into the budget nonbinding category. Of the 248 farms, 145 had binding cash-flow constraints (Table 1). That may appear to be a large proportion of participants facing a cash-flow constraint, especially when these farmers appear to be good managers (Kauffman and Tauer). Yet, a number of farmers may overestimate planned debt payments hoping to pay off all operating debt (no refinancing). Others may not actually require the amount estimated for living expenses. The percentage changes in: (1) milk marketing, (2) milk price, and (3) total operating cash expenses between 1984 and 1986 were calculated for all farms in the sample. Total operating cash expenses include: hired labor, feed, machinery, livestock, crops, real estate, interest, and expansion livestock cash expenses. Although total non-cash accounting expenses were available they would not have been a good measure of non-cash economic expenses. Economic depreciation could not be calculated since useful lives of assets, or utilization rates were not known. Asset values were also estimated by the participants and financially stressed operators may have incentives to overestimate these values. Since both binding and non-binding groups decreased their

---

1 The 1986 business summary did not indicate whether participants were accepted into the Dairy Termination Program (DTP), another supply control program initiated in 1986. However, it is highly unlikely that farmers accepted into this program would have participated in the 1986 summary program. A 1986 analysis of their dairy business would not have been particularly useful since the DTP required exit from dairying for five years.

---

### Table 1. Average Values of Selected Farm Characteristics for 1986 and Percentage Changes from 1984 through 1986 for Binding and Non-Binding Cash-Flow Constraint Groups

<table>
<thead>
<tr>
<th>Item</th>
<th>Entire Sample</th>
<th>Binding Cash-Flow Constraint</th>
<th>Non-Binding Cash-Flow Constraint</th>
<th>F-Ratio*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Farms</td>
<td>248</td>
<td>145</td>
<td>103</td>
<td></td>
</tr>
<tr>
<td>Number of Milk Cows</td>
<td>98.0</td>
<td>104</td>
<td>88.8</td>
<td>1.9</td>
</tr>
<tr>
<td>Production Per Cow</td>
<td>15,698</td>
<td>15,626</td>
<td>15,801</td>
<td>0.3</td>
</tr>
<tr>
<td>Age (Years)</td>
<td>45.5</td>
<td>44.3</td>
<td>47.1</td>
<td>3.7</td>
</tr>
<tr>
<td>Education (Years)</td>
<td>13.3</td>
<td>13.4</td>
<td>13.1</td>
<td>1.3</td>
</tr>
<tr>
<td>Percent Change Milk Price</td>
<td>-6.4</td>
<td>-6.3</td>
<td>-6.6</td>
<td>0.3</td>
</tr>
<tr>
<td>Percent Change Milk</td>
<td>9.4</td>
<td>10.9</td>
<td>7.3</td>
<td>3.4</td>
</tr>
<tr>
<td>Production</td>
<td>0.0</td>
<td>1.1</td>
<td>-1.4</td>
<td>1.7</td>
</tr>
</tbody>
</table>

*The critical value for the F-ratio (1,494) at the 10% significance level is 2.75.
machinery purchases during this period, tax depreciation was significantly reduced, especially for the binding group. All operators indicate their own labor usage in months, and almost all enter 12 months, so changes in unpaid labor utilization could not be measured.

Analysis of variance was used to test the hypotheses of equal means between groups for the percentage change in output, price, and total cash expenses. The following F-ratio was used to test the null hypotheses:

\[ F = \frac{\sum_{k=1}^{k} n_i (\bar{X}_i - \bar{X}_j)^2 / (k - 1)}{\sum_{i=1}^{k} \sum_{j=1}^{n} (X_{ij} - \bar{X}_i)^2 / k(n - 1)} \]

where:

- \( n_i = \) number of observations in group \( k \);
- \( \bar{X}_i = \) mean of group \( k \);
- \( \bar{X} = \) mean of all groups;
- \( k = \) number of groups \( (k = 2) \);
- \( n = \) number of observations in all groups;
- \( X_{ij} = \) value of \( j \)-th farm, \( i \)-th group.

Average values for selected farm characteristics for the two farm groups are displayed in Table 1. The number of milk cows for each group was not statistically different from each other. In addition, production per cow for both groups was not statistically different. Hence, the budget binding and non-binding groups may be operating under the same technology, i.e., production function.

Not surprisingly, the binding cash-flow constraint farms were younger than the non-binding groups. The average age of the binding groups was 44.3 years, while the average age of the non-binding groups was 47.1 years. It should be expected that younger farmers are generally more financially constrained than older farmers, who may have little or no debt. The younger farmers may also be expanding their business, which could be manifested as milk production increases from 1984 through 1986. Other farm characteristics, e.g. education level, are not statistically different.

The percentage change in the milk price from 1984 to 1986 was quite similar for both groups, 6.3 and 6.6% decreases, respectively, which is reflected in the extremely low F-ratio of 0.3 (Table 1). This makes comparisons of changes in production response convenient, since no adjustment is necessary to reflect comparable changes in price between groups.

The average increase in output for the total 248 farmers was 9.4 percent between 1984 and 1986. As hypothesized, the budget binding sample group, on average, increased output by more than the non-binding budget sample group. The mean of the former group (10.9%) compared with the mean of the latter group (7.3%) was statistically different at the 10% significance level (Table 1). Therefore, the empirical evidence supports the hypothesis that farmers facing cash-flow constraints have different production responses to a price decrease than producers whose cash-flow constraint is nonbinding. Of course, some of this may be because the binding group is younger and hence more likely to expand than the non-binding group. The budget binding sample group increased its cash expenses 1.1% while the non-binding budget sample group decreased its cash expenses by 1.4%. However, the F-ratio for this difference was not statistically significant at the 10% significance level so little credence can be placed on this difference.

Summary

The conceptual model in this paper has shown that under certain conditions a firm may exhibit both a downward sloping supply curve and profit maximizing behavior. The necessary conditions for this are that the firm faces a cash flow constraint and at least one variable factor of production be a non-cash input. Intuitively, the model results do not appear to be a great detraction from observed practices of some dairy farmers. It is not uncommon for producers to increase milk production by increasing their non-cash inputs in response to a price decrease, especially farmers in tight cash flow situations. A common practice is to increase milkings from twice to three times a day to generate additional revenue.

The empirical evidence presented in this study did not refute the conceptual model. Using dairy farm record data, it was shown that farmers facing cash-flow constraints, on average, increased milk production more than farmers whose cash-flow constraint was nonbinding in response to a price decrease. While the former group achieved this higher increase in milk production by a higher average increase in the utilization of non-cash inputs than the latter group, the change in non-cash input use between groups was not found to be statistically different. While the empirical evidence presented in this study is not enough to validate a negative milk supply response, we cannot say that at least some farmers do not behave this way.

Future empirical research of this model would be useful. In particular, the use of a more complete data set that has detailed information on cash and non-cash input prices and utilization over time would
be a valuable venture. Such a data set would allow for estimation of a supply curve for each group of farmers. Comparisons of price elasticities could then be made between farmers facing a binding cash-flow constraint and farmers whose cash-flow constraint is nonbinding. This was not attempted in this article due to a lack of reliable data on input prices. A temporal data set would also allow empirically estimating the dynamics of the adjustment process to determine the duration of any downward sloping supply. As Rosen has discussed, a number of system shocks can occur that results in a backward-bending initial supply and elastic long-run supply. His work and others were on the beef sector where increasing production assets (cows and bulls) necessitates reducing current slaughtered production. A recent paper by Chang and Stefanou shows that a model of intertemporal profit maximization can also lead to negative supply response. Allowing for asset fixity and asymmetric adjustment of quasi-fixed factors of production, their simulations of various milk price decreasing scenarios suggest an oscillating supply response that eventually flattens out. Thus, some periods show a negative milk supply response. While modeling and estimating the temporal supply functions for the two groups might yield a more definitive test of the validity of our model, the empirical procedures followed here are quite useful since they suggest that in the short-run the model cannot be refuted.

Appendix

The total differential of the first order condition can be more succinctly written in matrix form as:

\[
\begin{bmatrix}
\frac{df}{dt} \\
\frac{d^2f}{dt^2}
\end{bmatrix}
= \begin{bmatrix}
(pf_{11}, pf_{12}) \\
(pf_{21} + \lambda pf_{22}, pf_{22}, pf_{22}) \\
(pf_{11} - r_1, pf_{22})
\end{bmatrix}
\begin{bmatrix}
\frac{dx_1}{dt} \\
\frac{dx_2}{dt}
\end{bmatrix}
\]

Using muMath, a symbolic mathematics software, the solutions for \(dx_1\) and \(dx_2\) with \(dr_1 = 0\) and \(dr_2 = 0\) are

\[
\begin{align*}
\frac{dx_1}{dt} &= \frac{f_1 f_2 dp}{(pf_{12} f_1 - pf_{11} f_2 - f_{12} r_1)} - \frac{f_1 dp}{(pf_{12} f_1 - pf_{11} f_2 - f_{12} r_1)} \\
\frac{dx_2}{dt} &= \frac{f_1 f_2 dp}{(pf_{12} f_1 - pf_{11} f_2 - f_{12} r_1)} - \frac{f_1 f_2 dp}{(pf_{12} f_1 - pf_{11} f_2 - f_{12} r_1)}
\end{align*}
\]

Since we have more than artificial intelligence it became obvious that these could be simplified further.

\[
\begin{align*}
\frac{dx_1}{dt} &= \frac{f_1 f_2 dp}{(pf_{12} f_1 - pf_{11} f_2 - f_{12} r_1)} - \frac{f_1 dp}{(pf_{12} f_1 - pf_{11} f_2 - f_{12} r_1)} \\
\frac{dx_2}{dt} &= \frac{f_1 f_2 dp}{(pf_{12} f_1 - pf_{11} f_2 - f_{12} r_1)} - \frac{f_1 f_2 dp}{(pf_{12} f_1 - pf_{11} f_2 - f_{12} r_1)}
\end{align*}
\]

References


