On the design of water markets in the presence of risk aversion

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Introduction

- Farmers are facing a number of risks

- Farmers are known to be risk-averse in most situations

- Various tools for risk management:
  - On-farm risk management: diversification of activities, crop choice, input choice, technology adoption etc.
  - Markets: insurance markets, forward trading, water markets etc.

- In this paper: focus on risk due to uncertainty on water supply and rainfall, and water markets
Main purpose of the paper

To better understand the role of water markets design on the decision of risk-averse farmers, through:

- the effect of initial water allocations on farmers’ choices between irrigated and non-irrigated production

- the consequences of implementing a forward market for water allocations on farmers’ decision rules (not discussed today)

Tools: theoretical model describing farmer’s behaviour + simulations calibrated on Australian data
Model features

- a risk-averse farmer receives an initial allocation of water \((e_0)\)

- there is uncertainty on the price of water \((r)\) and hence uncertainty on farmer’s wealth \((re_0)\) (background risk)

- at the beginning of the season, the farmer decides how much land to put into irrigated production \((x)\); so \((1 - x)\) is put into non-irrigated production

- net returns of the irrigated activity \((R)\) and of the non-irrigated activity \((\epsilon)\) are random because of water price uncertainty and uncertainty on rainfall patterns
Model features (cont’d)

- the farmer faces three risks that are not independent; we expect \( \text{corr}(R, r) < 0 \) and \( \text{corr}(\varepsilon, r) < 0 \)

- we want to understand how the (risk-averse) farmer chooses \( x \), i.e. the proportion of land allocated to the irrigated activity

- equivalent to a problem of portfolio selection with two risky assets and presence of a dependent background risk
Related literature

- Modelling of risk-averse farmer’s decision under production/price risk
- Role of water markets as risk management tool for farmers
- Financial literature on portfolio selection with background risk
- Dependance of market outcomes from initial allocations in the presence of uncertainty and risk aversion (markets for pollution quotas)
Timing

1. the government decides on the initial allocations \( e_0 \)

2. the farmer decides on the share \( x \) (ex ante decision with \( r \) and \( \varepsilon \) uncertain)

3. \( \varepsilon \) and \( r \) are realized

4. the farmer decides on the final quantity of irrigation water \( z \)  (ex post decision)
Farmer’s (ex-post) profit function

\[ \pi(x, z) = (pf(z) - rz) x + (1 - x)\varepsilon + re_0 \]

where

- \( x \): share of land allocated to the irrigated production,
- \( z \): quantity of irrigated water used for production,
- \( p \): the (non-random) price of the irrigated product,
- \( r \): the price of water on the market,
- \( \varepsilon \): the per acre net benefit of the non-irrigated activity,
- \( e_0 \): the initial (per acre) water allocations.

In what follows, we will consider \( R(r) = pf(z) - rz \), the per acre net benefit of the irrigated output.
Farmer’s (ex ante) optimal decision on $x^*$

Expected utility framework: the (ex ante choice) of $x$ is given by

$$\max_x V(x) \equiv E(U(w_0 + \hat{\pi}(x))$$

with $\hat{\pi}(x)$ the ex post maximized profit, $w_0$ the initial (certain) wealth, and $U(.)$ a concave utility function. The FOC is:

$$V'(x) \equiv E(U' \hat{\pi}') = 0.$$

Because of the presence of risk aversion, the initial allocation $e_0$ enters in the ex ante choice $x$ through a (random) wealth effect.
Question: what is the impact of risk aversion on the ex ante optimal choice $x^*$?

Claim

A risk neutral producer chooses $x = 1$ whenever $\bar{R} > \bar{\varepsilon}$ and $x = 0$ whenever $\bar{R} < \bar{\varepsilon}$. When $\bar{R} = \bar{\varepsilon}$ the farmer is indifferent between allocating land to irrigated and non-irrigated activities.

where $\bar{R} = ER(r)$ and $\bar{\varepsilon} = E(\varepsilon)$. 
Proposition

The farmer’s optimal decision rule under risk aversion is given by (interior solution):

\[
x^* = \frac{\bar{R} - \bar{\epsilon}}{\rho(\bar{w}(x, e_0))V(R - \epsilon)} + \frac{V(\epsilon) - \text{Cov}(R, \epsilon)}{V(R - \epsilon)} - \frac{\text{Cov}(r, R - \epsilon)}{V(R - \epsilon)}e_0
\]

where \( \rho(.) = -U''(.)/U'(.) > 0 \) is the Arrow-Pratt degree of absolute risk aversion.
The decision $x^*$ relies on the comparison of three terms:

1. **Mean effect**

   $$\frac{\bar{R} - \bar{\varepsilon}}{\rho(\bar{w}(x, e_0))V(R - \varepsilon)}.$$

The decision to allocate some land to the irrigated crop depends positively on the comparison between expected revenues $\bar{R}$ and $\bar{\varepsilon}$.

Note: a farmer who receives a low initial allocation will put a lower weight on this factor if his preferences satisfy the DARA property.
2. Variance effect

\[ \frac{V(\varepsilon) - \text{Cov}(R, \varepsilon)}{V(R - \varepsilon)}. \]

A higher variance of \( \varepsilon \) increases, ceteris paribus, \( x^* \).

Furthermore, if \( R \) and \( \varepsilon \) are negatively correlated then there is an additional reason to invest in \( x \) in order to diversify the portfolio.
3. *Covariance effect*

\[- \frac{\text{Cov}(r, R - \varepsilon)}{V(R - \varepsilon)}e_0\]

The initial allocation of water plays a role in choosing $x^*$ because its value $r$ is random and potentially correlated with the revenue difference $R - \varepsilon$.

If the background risk $re_0$ and $R - \varepsilon$ are negatively correlated, then investing in $x$ corresponds to a strategy to diversify risk.
Question: what is the role of $e_0$ on the ex ante optimal choice $x^*$?

The initial water allocation entails two effects on the optimal ex ante decision $x^*$:

1. a “pure wealth” effect which, for DARA preferences, induces the agent to put a higher weight on the expected revenue difference $\bar{R} - \bar{\varepsilon}$ relative to the variability, following an increase in $e_0$,

2. and a “background risk” effect which gives incentives to the agent to increase $x^*$ following an increase in $e_0$ if the random price of water $r$ and the revenue difference $R - \varepsilon$ are negatively correlated. This is done in order to diversify the portfolio following an increase in $e_0$. 

Question: what is the role of $e_0$ on the ex ante optimal choice $x^*$?

In the special case of CARA preferences, we have:

**Corollary**

*Under CARA preferences, an increase in $e_0$ yields the agent to increase the share of land devoted to irrigated crop when $\text{Cov}(r, R - \varepsilon) < 0$ and to decrease $x^*$ when $\text{Cov}(r, R - \varepsilon) > 0$.***

In the general case, the relationship between $x^*$ and $e_0$ is non monotonic $\rightarrow$ simulations.
Next steps

- Modelling farmer’s behaviour when forward trading of water allocations is allowed.

- Simulations calibrated on Australian data:
  - Lognormal distribution for the price of water
  - Beta distribution for the net returns of the irrigated and non-irrigated crops
  - Simulation of random variables with some degree of dependence
  - Calibration using data from ABARES irrigation farm survey and NWC data on water transactions
  - Purpose: how $x^*$ changes depending on the correlation between the three risks, the degree of risk aversion, and the level of water allocations
Main (preliminary) findings

- if the farmer is risk averse, his optimal choice of $x^*$ (land allocated to irrigated production) does depend on the level of initial allocations $e_0$

- the level of initial allocations creates a background risk due to the uncertainty on the price of water

- $e_0$ entails two effects on $x^*$: a "pure wealth" effect and a "background risk" effect

- effects cannot be signed in general (except for the CARA case)

- simulations calibrated on Australian data