Estimating the Impact of Safety First Risk Constraints on Risk-Income Frontiers[1/]

by

Glenn A. Helmers, Larry J. Held and Myles J. Watts[2/]

Incorporating risk into behavioral models of the agricultural firm continues to generate widespread interest. The objective of risk modeling is to provide a better framework to understand the adjustment process of the firm compared to simple profit maximization models. Vital to the construction of firm risk models is the conceptual nature of risk in the context of the choices available to the firm. Traditionally enterprise choice modeling has defined risk as a variability concept. Within such a framework, a number of minor modifications have been suggested to improve risk models. Less modeling effort has been placed on the safety first risk concept--the concern over disaster occurrences in a given year. The objective of this paper is to investigate the impact of including safety first restrictions on traditional models of income-variability trade-offs.

The mean-variance analysis of risk behavior is generally traced to Markowitz while the associated Quadratic Programming model was developed by Freund. Hazell has suggested a linearized risk model (MOTAD) which minimizes absolute deviations as an alternative to Quadratic Programming and its use has become commonplace. For both Quadratic Programming (E,V analysis) and MOTAD (E,A analysis) risk is defined as deviations from expected income with the basic foundation for income-variability trade-offs found in expected utility theory.

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[2/] The authors are respectively Professor, University of Nebraska; Assistant Professor, University of Wyoming; and Assistant Professor, Montana State University.

A number of risk model modifications have been suggested in the literature including Separable Programming (Thomas, et al), Focus-Loss (Boussard and Petit), Marginal Risk Constraints (Chen and Baker) and Diagonalized Separable Programming (McCarl and Tice) and others.

A widely different concept of risk is safety first where profit is maximized subject to the probability that profits exceed a given level (Baumol, Day, and Pyle and Turnovsky). Thus in this case profit differences between alternative plans become important only under certain conditions--those conditions being that the top priority is survival and survival must first be assured. It may be argued that such a concept of risk is preferable to variability under conditions where the firm faces financial constraints. Less applied work has been completed using safety first risk concepts. An example of its use is provided by Musser, et al.

In this paper we specify safety first constraints as limits to annual deviations in a MOTAD model. The addition of these constraints limits the solution to those organizations where annual deviations cannot exceed specified levels. By so constraining the model, it is seen that a trade-off surface between annual and total deviations result or in other words, annual disaster deviations versus total risk.

The investigation of the sensitivity of E,A and E,V frontiers has received recent attention including near optimals (Schurle and Ervin), different measures of dispersion (Persaud and Mapp) and the use of total versus random variances in E,V analysis (Adams, et al and Bravo-Ureta and Helmers). This paper examines an alternative dimension to the income-variability frontier resulting in a surface over which annual and total deviations substitute.

Framework and Method

A conventional MOTAD E,A frontier forms the basic departure point. Such a hypothetical frontier defines the efficient set of choices between income and deviations and is shown in Figure 1. As a contrast, the alternative frontier resulting from annual deviation constraints is hypothesized as Figure 2. Only the negative deviations are so constrained in this model. The differences between the two frontiers may then be viewed as a surface in three dimensions where an additional dimension of annual deviations is varied. It is expected that the effect of annual constraints is effective only over certain ranges. That is, examining annual constraint levels above levels existing in conventional MOTAD solutions is irrelevant. Further a limit is reached for any income level beyond which annual deviations cannot be reduced without infeasible solutions occurring.

The exact process of outlining the surface follows:

1) Develop the conventional E,A frontier, minimizing total deviations for selected levels of expected income.
2) Maximize expected income for selected arbitrary levels of annual deviations (50,000; 40,000; 30,000; 20,000; 10,000; and 5,000). An alternative frontier is thus generated as that suggested in Figure 2.
3) Investigate trade-offs for the resultant surface for alternative expected incomes where as annual deviations are decreased, increased total deviations are observed.

Farm Setting

The representative farm is a 400-acre furrow irrigated crop farm in Eastern Wyoming and Western Nebraska. Sugar beets, dry beans, corn for grain, corn for silage and alfalfa are the alternative crops considered. Cost of production data were secured from Agee and Stephenson for 1978 and extrapolated to 1980 levels using GNP deflators (t_1 - t_6) from prices paid indices. County average yield data along with annual average crops prices and the above

Figure 1. Conventional MOTAD

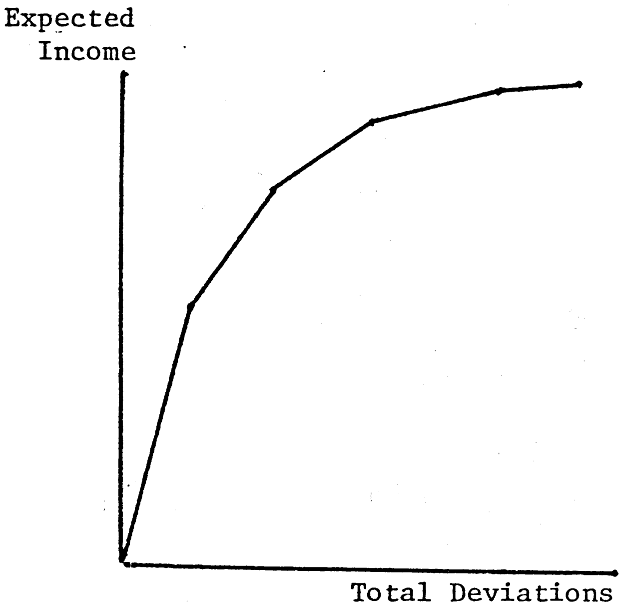
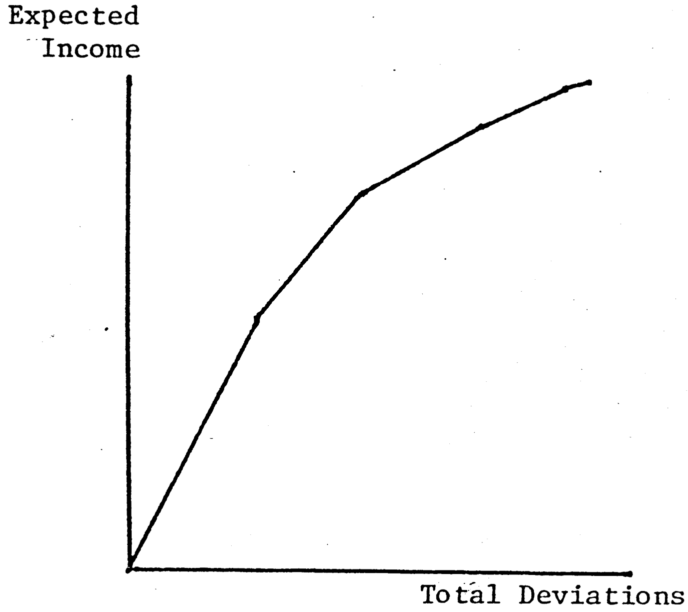


Figure 2. Hypothesized Constrained MOTAD



cost series determined annual gross margins for 1975-80 (t_1 - t_6). All margins were converted to real margins with 1980 as the base.

The study farm is considered as a two-person business. Labor constraints for five periods were considered: 1) early spring (Espr) March 16 - April 30, 2) late spring (Lspr) May 1 - June 15, 3) summer (Sum) June 16 - Aug. 15, 4) late summer (Lsum) Aug. 16 - Sept. 30, and 5) fall (Fall) Oct. 1 - Nov. 15. Constraints exist for both hours of labor and hours of field operation (man and fld referring to labor hours and field operations hours respectively). Input requirements for each crop and resource availabilities are given in Table 1. Capital was not considered to be a constraint. More detail regarding the farm setting can be found in Zink and Held.

Correlation coefficients of expected income between enterprises as well as the mean and standard deviations of expected income for alternative enterprises are presented in Table 2. It can be noted that sugar beets has the

Table 1. Resource Requirements and Gross Margins for 1975-80 (t_1 - t_6) for Representative Farm.

Row	Resource Available	Beets	Beans	Silage	Corn	Alfalfa
Land (ac)	400	1.00	1.00	1.00	1.00	1.00
Esprman (hr)	1084	2.97	1.77	1.82	1.85	1.90
Esprfld (hr)	805	2.72	1.56	1.61	1.63	.67
Lsprman (hr)	1127	1.08	1.09	1.25	1.28	.25
Lsprfld (hr)	768	1.04	.99	1.20	1.22	.08
Summan (hr)	1611	2.84	3.71	3.08	3.14	.96
Sumfld (hr)	1230	.57	.88	.80	.83	.36
Lsumman (hr)	1232	2.39	3.91	4.43	.64	1.23
Lsumfld (hr)	904	.19	3.15	3.64	.08	.15
Fallman (hr)	1084	5.68			1.74	
Fallfld (hr)	897	5.30			1.58	
t_1 $		118.39	- 64.95	6.93	95.35	17.88
t_2 $		-120.18	-150.31	73.27	1.96	46.54
t_3 $		- 59.93	43.54	29.98	-38.50	- 3.90
t_4 $		-112.07	-109.41	-82.37	-49.95	-22.14
t_5 $		31.55	101.54	-37.24	-46.82	- 8.47
t_6 $		142.23	179.56	9.40	37.98	-29.92

highest return followed by dry beans, silage, corn (grain) and alfalfa respectively. Beans has the highest variabilility of all crops, with corn having a higher variability than silage. Thus, some exceptions exist relative to the generally observed positive relationship of income and variability of agricultural enterprises.

Results

Figures 3 and 4 depict the conventional and constrained MOTAD solutions for the example farm with the frontier points and enterprise organizations presented in Tables 3 and 4. In addition to the maximum income point, only those frontier points corresponding to the six annual deviation limits

Table 2. Income Correlation Coefficients, Average Expected Income, and Standard Deviation of Enterprise Incomes.

Enterprise	Beets	Beans	Silage	Corn	Alfalfa
Beets	1.00	.659	-.013	.69	-.327
Beans		1.00	-.093	-.017	-.669
Silage			1.00	.376	.694
Corn				1.00	.284
Alfalfa					1.00
Expected Income $	489.18	314.07	242.91	172.01	122.41
Income Standard Deviation $	114.75	129.02	53.99	57.81	28.14

discussed earlier (50,000, 40,000,...,5000) are shown rather than the usual procedure of parametrically varying income by arbitrary and constant changes. It can be seen that the general results correspond to the hypothesized effects. The top three frontier locations (150,022, 149,416, and 143,380 dollar incomes) are seen to be identical solutions for both MOTADs. These correspond to unlimited, 50,000, and 40,000 annual deviation limits. Land becomes slack at incomes below $62,445 for the conventional MOTAD frontier, hence scale changes in organization occur below that level (the six solution points, zero, and the maximum income point are connected in a linearized fashion).

The general pattern of organization for both frontiers is comparable. As income is reduced below the maximum point, beets first slightly increase in acreage, then decline. Beans consistently decline in acreage although remaining in solution longer for the conventional MOTAD than the constrained MOTAD (except for $123,986 income). Silage disappears more rapidly from the constrained solutions. Corn only appears once in each while alfalfa acreage shows a similar pattern for each frontier.

Figure 3. Conventional MOTAD E-A Frontier

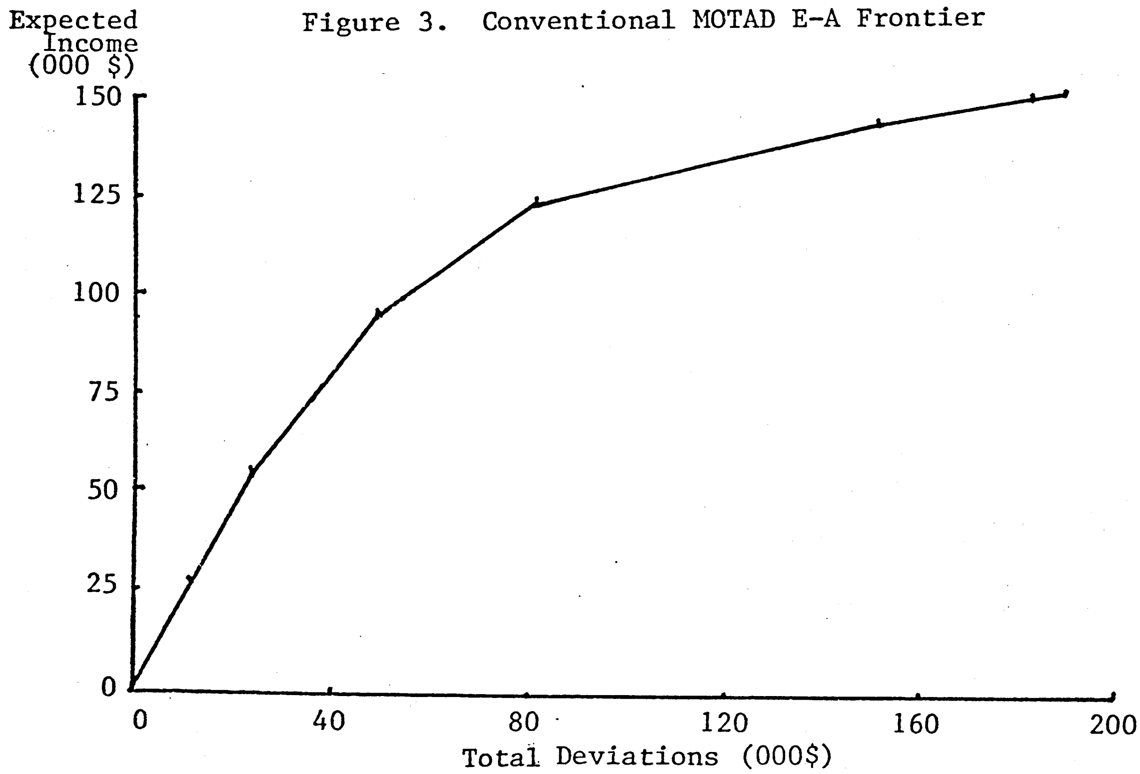


Figure 4. Constrained MOTAD E-A Frontier

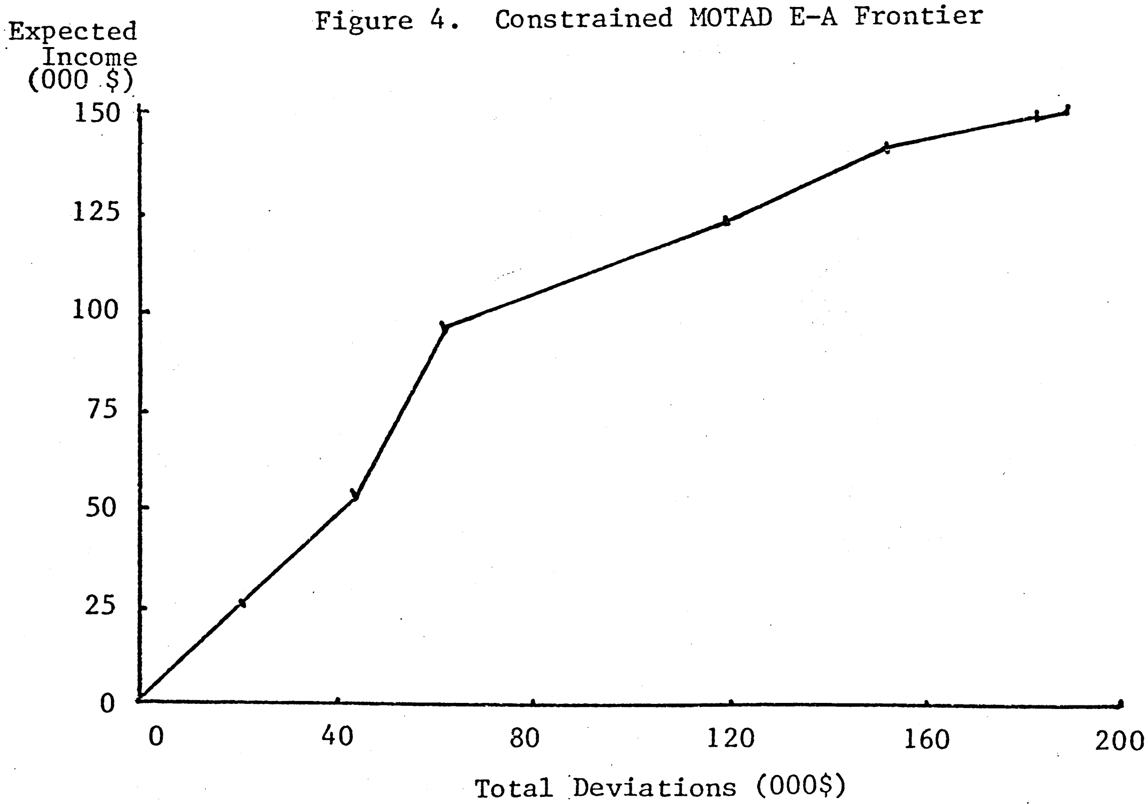


Table 3. Conventional MOTAD Enterprise Solutions for Representative Farm.

Expected Income ($)	Total Deviations ($)	Maximum Annual Deviation ($)	Enterprise Levels (acres)	
150,022.47	188,235.94	51,072.25	Beets	164.95
		(t_2)	Beans	207.89
			Corn	14.40
			Alfalfa	12.76
149,416.50	182,834.00	50,000.00	Beets	169.25
		(t_2)	Beans	197.33
			Silage	4.64
			Alfalfa	28.79
143,379.56	151,299.62	40,000.00	Beets	169.25
		(t_2)	Beans	130.80
			Silage	60.36
			Alfalfa	39.60
123,986.00	80,086.76	32,723.22	Beets	145.95
		(t_4)	Silage	178.34
			Alfalfa	75.70
94,413.63	49,509.02	24,757.91	Beets	68.51
		(t_4)	Beans	48.89
			Silage	90.89
			Alfalfa	191.71
52,262.47	23,533.59	11,768.86	Beets	10.13
		(t_4)	Beans	39.49
			Alfalfa	285.14
26,131.24	11,766.80	5,884.43	Beets	5.07
		(t_4)	Beans	19.74
			Alfalfa	142.57

The difference between the two frontiers is basically centered on alfalfa and silage. The constrained MOTAD has greater alfalfa acreage and less silage acreage compared to comparable conventional MOTAD solutions. This is due to alfalfa having the lowest negative deviation of all crops for year t_4. Year t_4 becomes the year with the effective annual constraint when solutions have high levels of alfalfa and silage. For solutions with high levels of beets and beans year t_2 becomes the year of the effective annual deviation constraint. It should be noted that the difference in total deviations for frontier solutions varies from nearly 40,000 at the $123,986 income level to approximately

Table 4. Constrained MOTAD Enterprise Solutions for Representative Farm.

Expected Income ($)	Total Deviations ($)	Maximum Annual Deviation ($)	Enterprise Levels (acres)	
150,022.47	188,235.94	51,072.25 (t_2)	Beets	164.95
			Beans	207.89
			Corn	14.40
			Alfalfa	12.76
149,416.50	182,834.00	50,000.00 (t_2)	Beets	169.25
			Beans	197.33
			Silage	4.64
			Alfalfa	28.79
143,379.56	151,299.62	40,000.00 (t_2)	Beets	169.25
			Beans	130.80
			Silage	60.36
			Alfalfa	39.60
123,986.11	119,851.70	30,000.00 (t_2,t_4)	Beets	169.25
			Beans	64.27
			Silage	5.23
			Alfalfa	161.25
94,413.63	61,089.74	20,000.00 (t_4)	Beets	123.92
			Alfalfa	276.08
52,262.47	42,982.01	10,000.00 (t_4,t_6)	Beets	23.20
			Alfalfa	334.22
26,131.24	21,491.00	5,000.00 (t_4,t_6)	Beets	11.60
			Alfalfa	167.11

11,500 at the $94,414 income level to approximately 19,500 at the $52,262 income level.

In Figure 5 the results of the two MOTAD frontiers are presented using the dimensions of annual deviations and total deviations. A particular substitution relationship ($123,986 income) is presented in Table 5. As annual deviation limits are decreased by given amounts, increased levels of total deviations are required. This process of decreasing annual deviation limits is seen to involve consistently increased alfalfa acreage, consistently reduced silage acreages, small but consistently increased beet acreage and the entrance of bean acreage when annual deviations are decreased to $30,500 and $30,000.

Figure 5. Tradeoff Between Annual and Total Deviations From Conventional and Constrained MOTAD Solutions.

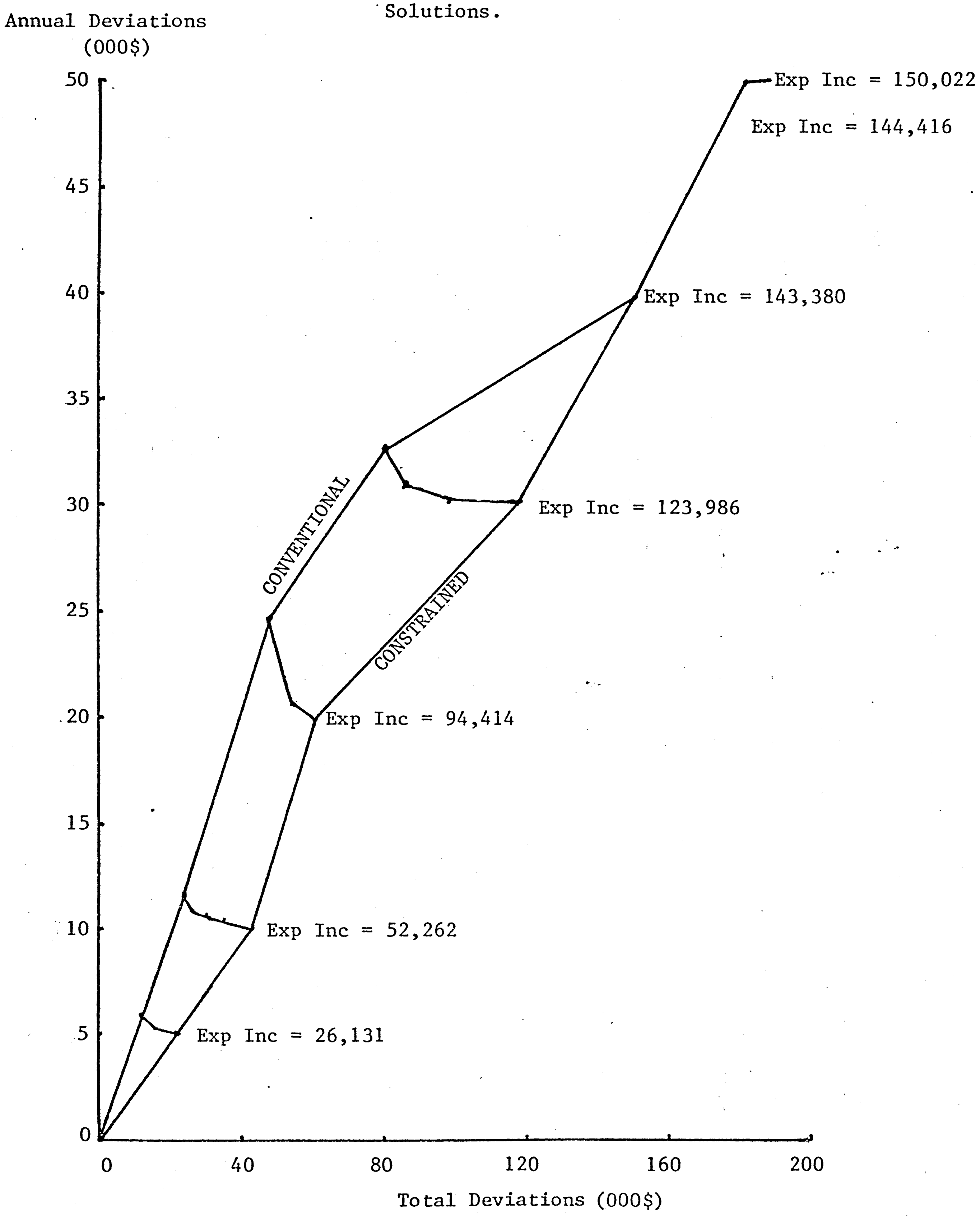


Table 5. Tradeoff Between Annual and Total Deviations for 123,986 Dollar Income Level.

Total Deviations ($)	Annual Deviations ($)	Enterprises (acres) Beets	Beans	Silage	Alfalfa
80,086.76	32,723.22	145.95		178.34	75.70
80,977.21	32,500.00	148.34		171.07	80.59
82,971.76	32,000.00	153.70		154.77	91.53
84,966.30	31,500.00	159.05		138.48	102.47
86,960.85	31,000.00	164.41		122.18	113.41
92,024.38	30,500.00	169.25	5.63	98.49	126.63
119,848.61	30,000.00	169.25	64.26	5.24	161.25

The high levels of alfalfa, enabling limits on t_4 to be reached, allow the higher income bean enterprise to increase because of the counteracting deviation influence of beans and alfalfa in year t_2.

Conclusions

It has been demonstrated that trade-offs between annual deviations and overall variability can exist in E,A analysis. This indicates that should annual limits on risk exist in terms of concern over large losses in a particular year, risk-income efficient frontiers are modified by such constraints. The determination regarding the disaster level (here described by negative deviations) may be a subjective or objective choice. The trade-off surface generated by the procedure developed in this paper would be expected to vary according to the co-variability of gross margins and loss dominance of activities in crucial MOTAD time periods. Where survival is crucial perhaps because of fixed loan repayments or a delicate financial position, such constraints may be useful in behavioral models of the firm.

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