

On the Relevance of Open Market Operations

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On the Relevance of Open Market Operations

ABSTRACT

This paper reexamines the role of open market operations for short-run effects of monetary policy. Money demand is induced by a cash constraint, while the central bank supplies money exclusively in exchange for securities, discounted with a short-run nominal interest rate. We consider a legal restriction for open market operations by which only government bonds are eligible, whereas private debt is not accepted as collateral for money. Supply of eligible securities is bounded by assuming fiscal policy to ensure government solvency. The model provides an endogenous liquidity premium on non-eligible assets and liquidity effects of money supply shocks regardless whether prices are flexible or set in a staggered way. Nominal interest rate policy is always associated with a uniquely determined price level and rational expectations equilibrium. It is further shown that an intuitive equivalence principle between money supply and interest rates arises in this case.

JEL classification: E52, E32.

Keywords: Inside Money, Liquidity Puzzle, Risk-free Rate Puzzle, Ricardian Fiscal Policy, Price Level and Equilibrium Determinacy.

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1 Introduction

Central banks in most industrial countries exert control over money via open market operations, where money is supplied in exchange for risk free securities discounted with a short-run nominal interest rate. Hence, the costs of cash acquisition depend on the current discount rate and the availability of collateral. Monetary theory, however, has not reached a consensus on the effects of open market operations and even claimed open market operations to be irrelevant, as for example shown in Wallace (1981) and Sargent and Smith (1987), or more recently in Eggerston and Woodford (2003).³ In accordance with the latter view, most contributions to the monetary policy literature disregard an explicit specification of open market operations and assume that money is injected via lump-sum transfers. In this paper open market operations are (re)introduced in a standard monetary business cycle framework and it is shown that the relevance of open market operations depends on whether the set of eligible securities is restricted or not. In particular, when only government bonds are accepted in open market operations, the liquidity puzzle is solved and the equilibrium features an endogenous liquidity premium on non-eligible securities. Further, price level indeterminacy and equilibrium multiplicity are then ruled out for nominal interest rate policy.

This paper presents a model where money is demanded by households due to a Clower (1967)-constraint, while money supply is conducted in form of outright sales/purchases and repurchase agreements, where money and interest bearing securities are exchanged. The amount of money supplied in open market operations equals the discounted value the securities. Households can decide on whether to carry over money from one period to the other or to repurchase the securities. The former corresponds to the conventional specification of money, where money is treated as a store of value. Due to the assumption that interest bearing securities are nominally state contingent, households are indifferent between the two types of money holdings. The analysis focuses on the case where money is exclusively held under repurchase agreements, which relates to the specification of money in Drèze and Polemarchakis (2000) and Dubey and Geanakoplos (2003) and can be interpreted as inside money that serves as a pure medium of exchange.⁴

Households' financial wealth comprises contingent claims on other households and government bonds carried over from the previous period. In each period they can acquire money from the central bank via repurchase agreements. We explicitly take into account that real world central banks' behavior is typically characterized by an asset acquisition policy, by which eligible securities are restricted to a set of assets with high credit quality. The US Federal Reserve, for example, exclusively accepts securities issued by the treasury, federal agencies, as well as acceptances or bank bills, which meet high credit

³Dupor (2001), on the contrary, shows that open market operations matter if they are associated with a non-Ricardian fiscal policy regime.

⁴At the same time, it avoids Hahn's (1965) paradox even though money is held over a finite horizon.

quality standards (see Meulendyke, 1998).⁵ In the model, a legal restriction is imposed which constrains money supply in that only government bonds can be used in open market operations. The crucial assumption is that households internalize not only the goods market (cash) constraint, but also this money market constraint when they decide on their optimal plan. Then there exist a rational expectations equilibrium where private debt yields a higher interest than public debt and the money market restriction is binding, such that the outstanding stock of government bonds relates to the amount of money supplied in open market operations.⁶

In order to facilitate comparisons with the literature on New Keynesian macroeconomics, the model further allows for prices to be set by monopolistically competitive (retail) firms in a staggered way. When there is no legal restriction on open market operations, money supply can be interpreted as being conducted according to the real bills doctrine. In this case the set of linearized equilibrium conditions is isomorphic to the consensus monetary business cycle model, i.e., the standard New Keynesian model applied in Clarida et al. (1999) or Woodford (2003). When open market operations matter, a monetary injection raises prices and reduces the nominal interest rate regardless whether prices are flexible or sticky. Hence, it generates a liquidity effect, that is repeatedly found in the data (see Hamilton, 1997, or Christiano et al., 1999) and is hardly generated by conventional sticky price models (see Christiano et al., 1997, or Andrés et al., 2002), where the nominal interest rate tends to increase with money supply due to higher expected inflation. While the liquidity effect can – at least temporarily – be generated by allowing for segmentations and information asymmetries in asset markets (see Christiano et al., 1997, and, Alvarez et al., 2002), the solution for the ‘liquidity puzzle’ in this paper crucially relies on the availability of eligible securities. As the evolution of public debt is constrained to ensure government solvency, a rise in the money growth rate must necessarily be accompanied by a decline in its price, i.e., the nominal discount (repo) rate. When prices are set in a staggered way, the model further predicts real activity to increase and the spread between the interest rates on private and public debt to decrease with a monetary expansion, if households are risk averse. Hence, the spread can be interpreted as a liquidity premium on non-eligible securities, providing an explanation for the ‘risk-free rate puzzle’ (Weil, 1989) in the spirit of the strategy presented by Bansal and Coleman (1996).

When the central bank controls the repo rate, which equals the nominal interest rate on government bonds, the analysis discloses that some unpleasant features of conventional models relate to the irrelevance of open market operations therein. If, however, the money market constraint is binding, nominal interest rate policy, which is allowed to react to changes in inflation, is

⁵Recent asset acquisition policy of the US Federal Reserve can even be summarised to ‘Treasury-only’ (see e.g. Broadus and Goodfriend, 2001).

⁶Public debt then exhibits a liquidity value, which is for example directly assumed in Bansal and Coleman (1996), Canzoneri et al. (2000), and Lahiri and Vegh (2003).

always associated with a uniquely determined price level and rational expectations equilibrium regardless whether prices are flexible or sticky.⁷ Hence, equilibrium uniqueness does not require the fulfillment of the so-called Taylor-principle, as in the case where open market operations are irrelevant (see Woodford, 2001), implying that the central bank can already stabilize the economy by setting the nominal interest rate rather than being constrained to control the (expected) real interest rate. However, the central bank should refrain from setting the interest rate in a highly reactive way when debt interest payments are not completely tax financed. Otherwise, it would heavily burden public debt obligations, which can interfere with interest rate policy and can even give rise to instability of the equilibrium path due to debt interest spirals.⁸ Macroeconomic stability then requires monetary policy to consider the evolution of public debt.

The stability analysis leads to the last result derived in this paper, which regards the relations between interest rates, money supply, and inflation. On the one hand, it is always possible to construct an interest rate rule that implements a sequence of non-accommodating money growth rates, which one would expect to characterize a monetary policy regime aiming at stabilizing inflation (see McCallum, 1999). On the other hand, these interest rate rules are associated with a unique rational expectations equilibrium only if the money market constraint is binding. In contrast, for the standard New Keynesian model, where open market operations are irrelevant, an interest rate policy that is associated with equilibrium (multiplicity) determinacy implements a sequence of money growth rates, which are (decreasing) increasing with inflation.

The remainder is organized as follows. Section 2 develops the model. In section 3 we present results for the flexible and sticky price version for money growth policy. Section 4 concludes.

2 The model

The outline of the model Household-firm units are endowed with government bonds, money, and claims on other households carried over from the previous period. They produce a wholesale good employing labor from all households. Aggregate uncertainty is due to monetary policy shocks, which are realized at the beginning of the period. Then goods are produced and asset markets open, where households can trade in all assets without restrictions. Money demand is induced by assuming that purchases of consumption goods are restricted by a liquidity constraint. The central bank supplies money exclusively via open market operations. Here, the supplied amount of money

⁷Given that tax policy is assumed to ensure government solvency these results do not relate to similar findings in the literature where fiscal policy is specified in a non-Ricardian way, as, e.g., in Woodford (1994) or Benhabib et al. (2001).

⁸The destabilizing effect of aggressive interest rate policy due to 'debt-interest spirals' is also found by Leith and Wren-Lewis (2000) in a framework where public debt is non-neutral due to overlapping generations.

equals the discounted value of interest bearing assets, which are deposited at the central bank.⁹ Then the goods market opens. After goods have been traded, households can repurchase the securities from the central bank. The remaining amount of money is carried over to the next period. In order to allow for a nominal rigidity, we introduce monopolistically competitive retail firms, who differentiate the wholesale goods and set their prices in a staggered way. Given these assumption, the log-linear approximation of the model nests the standard New Keynesian model presented in Clarida et al. (1999).

Households Lower (upper) case letters denote real (nominal) variables. There is an infinite number of time periods t ($t = 0, 1, 2, \dots$). Let $s^t = (s_0, \dots, s_t)$ denote the history of events up to date t and $g(s^t|s^{t-1})$ denote probability of state s_t and, thus, of the history s^t conditional on the history s^{t-1} at date $t - 1$. The initial state, s^0 , is given so that $g(s^0) = 1$. There is a continuum of perfectly competitive household-firm units distributed uniformly over $[0, 1]$. In each period t a household $j \in [0, 1]$ consumes a composite good $c(j, s^t)$ and supplies working time $l(j, s^t) = \int_0^1 l^k(j, s^t) dk$ to household-firm units, where $l^k(j, s^t)$ denotes the working time of household j in firm k . It produces a wholesale good $x(j, s^t)$ using the technology

$$x(j, s^t) = \int_0^1 l^j(k, s^t) dk, \quad (1)$$

and sells the wholesale good to retail firms charging a price $P^w(s^t)$ per unit. Household j is assumed to maximize the expected value of the discounted stream of utility stemming from consumption and leisure, which is given by

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t g(s^t) u(c(j, s^t), l(j, s^t)), \quad \beta \in (0, 1), \quad (2)$$

where β denotes the subjective discount factor. The instantaneous utility function u is assumed to be strictly increasing in consumption c , strictly decreasing in working time l , strictly concave, twice continuously differentiable with respect to both arguments, satisfies the usual Inada conditions, and is additively separable.

We separate the household problem into a temporal and an intertemporal part. In the *temporal* part they make their optimal decisions on production and on the composition of consumption. Profit maximizing leads to the following demand for labor $l^j(k, s^t)$

$$P(s^t)w(s^t) = P^w(j, s^t), \quad (3)$$

where $P(s^t)$ denotes the aggregate price level and $w(s^t)$ the real wage rate. Let $c(j, s^t)$ be consumption of a composite good which is defined as a CES aggregate of differentiated goods $y^j(i, s^t)$, which are bought from retailers

⁹Equivalently, it can be assumed that financial intermediaries engage in open market operations on the behalf of the households.

indexed with $i \in [0, 1]$: $c(j, s^t)^{\frac{\epsilon-1}{\epsilon}} = \int_0^1 y^j(i, s^t)^{\frac{\epsilon-1}{\epsilon}} di$, where $\epsilon > 1$ is the constant elasticity of substitution between any two retail goods. Let $P(i, s^t)$ denote the price of the retail good $y^j(i, s^t)$ and the price of the composite good $P(s^t)$ be given by $P(s^t)^{1-\epsilon} = \int_0^1 P(i, s^t)^{1-\epsilon} di$. Minimizing costs for purchasing a unit of the composite good leads to the following optimal demand for the retail good $y^j(i, s^t)$:

$$y^j(i, s^t) = \left(\frac{P(i, s^t)}{P(s^t)} \right)^{-\epsilon} c(j, s^t). \quad (4)$$

The *intertemporal* part unfolds as follows. In what follows the index j is disregarded, except for the supply side variables, as households are identical. At the beginning of period t households are endowed with financial wealth $A(s^{t-1})$ which comprises government bonds holdings $B(s^{t-1})$, claims on other households $D(s^{t-1})$, and money holdings $M^H(s^{t-1})$: $A(s^{t-1}) = B(s^{t-1}) + D(s^{t-1}) + M^H(s^{t-1})$. Both interest bearing assets are assumed to be nominally state contingent leading to a payoff in period t equal to $R^d(s^t)D(s^{t-1})$ and $R(s^t)B(s^{t-1})$.

Before agents trade in assets or goods, the aggregate shocks arrive, goods are produced, and wages are credited on checkable accounts at financial intermediaries. Then households enter the assets market, where they can trade with other households and the treasury in an unrestricted way. After the asset market is closed, households can participate in open market operations, where they can exchange interest bearing assets $B^c(s^t)$ for new money $I(s^t)$. The amount $I(s^t)$ injected by the central bank equals the discounted value $B^c(s^t)/R(s^t)$:

$$I(s^t) = \frac{B^c(s^t)}{R(s^t)}. \quad (5)$$

Hence, the exchange (repo) rate in open market operations equals the gross nominal interest rate on government bonds. The exchange restriction (5) is assumed to hold for all types of open market operations, namely outright sales and purchases as well as repurchase agreements, which both are for example also applied by the US Federal Reserve. The fraction of money held under repurchase agreements, which is denoted by $M^R(s^t)$, is only held until the end of the period, when the repurchase agreements are settled. Hence, $M^R(s^t)$ is a flow variable and can be interpreted as inside money, as it is the counterpart of securities temporarily deposited at the central bank. Money injections thus satisfy $I(s^t) = M^R(s^t) + M^H(s^t) - M^H(s^{t-1})$.

After having acquired money from the central bank, households enter the goods market. Here, they rely on the total amount of money $M(s^t) \equiv M^H(s^t) + M^R(s^t)$, i.e., money held under outright sales/purchases $M^H(s^t)$ and held under repurchase agreements $M^R(s^t)$ and on checkable accounts as means of payment. These accounts consist of the individual labor income $P(s^t)w(s^t)l(j, s^t)$ less the wage outlays for the own firm $P(s^t)w(s^t) \int_0^1 l^j(k, s^t) dk$.

Hence, the purchase of goods is subject to the following liquidity constraint:

$$P(s^t)c(s^t) \leq M(s^t) + \left[P(s^t)w(s^t)l(j, s^t) - P(s^t)w(s^t) \int_0^1 l^j(k, s^t)dk \right]. \quad (6)$$

The modification of the Clower (1967) constraint, i.e., the term in the square brackets, is introduced to avoid the cash-credit good distortion between consumption and leisure.¹⁰ Applying a standard cash-in-advance constraint would, by the nominal interest rates distorting the consumption-leisure decision, unnecessarily complicate the analysis. The avoidance of this distortion is responsible for the model to nest the standard New Keynesian model, which is for example applied in Clarida et al. (1999).¹¹

Households receive cash by selling its product $x(s^t)$ in the goods market and in form of profits of retail firms $P(s^t) \int_0^1 \omega(i, s^t)di$, and have to pay a lump sum tax $P(s^t)\tau(s^t)$. After the goods market is closed, inside money $M^R(s^t)$ is used by the households to repurchase securities from the central bank. Household j 's budget constraint is given by

$$\begin{aligned} & D(s^t) + B(s^t) + M^H(s^t) + (R(s^t) - 1) (M^R(s^t) + M^H(s^t) - M^H(s^{t-1})) \quad (7) \\ & \leq R(s^t)B(s^{t-1}) + R^d(s^t)D(s^{t-1}) + M^H(s^{t-1}) - P(s^t)c(s^t) - P(s^t)\tau(s^t) \\ & \quad + P(s^t)w(s^t)l(j, s^t) - P(s^t)w(s^t) \int_0^1 l^j(k, s^t)dk + P^w(s^t)x(s^t) + P(s^t) \int_0^1 \omega(i, s^t)di, \end{aligned}$$

The main novel feature of the model is that the market for money is assumed to be constrained, similar to the goods market. Considering that asset acquisition policy of most real world central banks, including the US Federal Reserve or the Bank of England, is restricted to a set of high credit quality securities, a legal restriction on open market operations is imposed by which only government bonds are accepted by the central bank as eligible securities:

$$B^c(s^t) \leq B(s^t). \quad (8)$$

It is further assumed that households are aware of the fact that their access to cash is restricted by their holdings of government bonds. This restriction is would be irrelevant when they can issue private debt earning an interest rate not higher than the interest rate on government bonds. However, as the monetary authority (directly or indirectly) controls the latter, a positive spread $R^d(s^t) > R(s^t)$ cannot generally be ruled out, so that they internalize the constraint (8), which can rewritten as

$$M^R(s^t) + M^H(s^t) - M^H(s^{t-1}) \leq B(s^t)/R(s^t), \quad (9)$$

when they derive their optimal decisions. Maximizing (2) subject to the constraints for goods market (6), the asset market (7), the money market (9),

¹⁰This specification closely follows Jeanne (1998).

¹¹The cash-credit good distortion would cause the nominal interest rate to enter the aggregate supply constraint, i.e., the New Keynesian Phillips curve.

and a no-Ponzi-game condition

$$\lim_{i \rightarrow \infty} \sum_{s^{t+i}} g(s^{t+i}) A(s^{t+i}) \prod_{v=1}^i R^d(s^{t+v})^{-1} \geq 0, \quad (10)$$

for a given initial value of total nominal wealth $A(s^0) > 0$ leads to the following first order conditions for consumption, leisure, holdings of private and public debt, and for money holdings:

$$u_c(s^t) = \lambda(s^t) + \psi(s^t), \quad (11)$$

$$u_l(s^t) = -u_c(s^t)w(s^t), \quad (12)$$

$$\lambda(s^t) = \beta E(s^t) \left[R^d(s^{t+1}) \lambda(s^{t+1}) / \pi(s^{t+1}) \right], \quad (13)$$

$$\eta(s^t) = \beta E(s^t) \left[\lambda(s^{t+1}) \left(R^d(s^{t+1}) - R(s^{t+1}) \right) / \pi(s^{t+1}) \right], \quad (14)$$

$$\psi(s^t)|_{MR} = (R(s^t) - 1) \lambda(s^t) + R(s^t) \eta(s^t) - \xi(s^t), \quad (15)$$

$$\begin{aligned} \psi(s^t)|_{MH} = & R(s^t) \lambda(s^t) - \beta E(s^t) \left[R(s^{t+1}) \lambda(s^{t+1}) / \pi(s^{t+1}) \right] \\ & + R(s^{t+1}) \eta(s^{t+1}) - \beta E(s^t) \left[R(s^{t+1}) \eta(s^{t+1}) / \pi(s^{t+1}) \right] \end{aligned} \quad (16)$$

where $E(s^t)$ denotes the expectation operator conditional on the information in period t , $\pi(s^t) = P(s^t)/P(s^{t-1})$ the rate of inflation, λ the shadow price of wealth, ψ the Lagrange multiplier on the goods market constraint (6), and η the Lagrange multiplier on the money market constraint (9). The multiplier ξ further measures if money is held under repurchase agreements $M^R(s^t) \geq 0$. The optimum is further characterized by the constraints (7), (6), and (9),

$$\xi(s^t) \geq 0, \quad \xi(s^t) M^R(s^t) = 0, \quad (17)$$

$$\eta(s^t) \geq 0, \quad \eta(s^t) [b(s^t) - R(s^t)m(s^t)] = 0, \quad (18)$$

$$\psi(s^t) \geq 0, \quad \psi(s^t) \left[m(s^t) + w(s^t)l(j, s^t) - w(s^t) \int_0^1 l^j(k, s^t) dk - c(s^t) \right] = \mathbf{0} \quad (19)$$

where $b(s^t) \equiv B(s^t)/P(s^t)$ and $m(s^t) \equiv M(s^t)/P(s^t)$, and (10) holding with equality, which provides the transversality condition.

Retailer There is a monopolistically competitive retail sector with a continuum of retail firms indexed on $i \in [0, 1]$. Each retail firm, owned by the households, buys a quantity $x^i(j, s^t)$ of the wholesale good produced by household j at price $P^w(s^t)$. To minimize distortion induced by liquidity constraints, it is assumed that households buy coupons for the differentiated consumption goods providing retail firms with cash, which they use to purchase the wholesale good. We assume that a retailer is able to differentiate the wholesale good without further costs. The differentiated retail good $y(i, s^t) = \int_0^1 x^i(j, s^t) dj$ is then sold at a price $P(i, s^t)$. We assume that retailers set their prices according to Calvo's (1983) staggered price setting model. The retailer changes its price when it receives a signal, which arrives in a given period with probability $(1 - \phi)$, where $\phi \in [0, 1]$. A retailer who does

not receive a signal adjusts its price by the steady state aggregate inflation rate $\bar{\pi}$, such that $P(i, s^t) = \bar{\pi}P(i, s^{t-1})$. A retailer who receives a price change signal in period t chooses a price $\tilde{P}(i, s^t)$ to maximize the expected sum of future discounted profit streams given by

$$\sum_{v=0}^{\infty} \sum_{s^{t+v}} (\beta\phi)^v q(s^{t+v}, s^t) \tilde{\omega}(i, s^{t+v}, s^t), \quad (20)$$

where $q(s^{t+1}, s^t) \equiv \frac{\lambda(s^{t+1})/P(s^{t+1})}{\lambda(s^t)/P(s^t)} g(s^{t+1}|s^t)$ denotes the stochastic discount factor and $\tilde{\omega}(i, s^{t+v}, s^t)$ real profits in period $t+v$ for own prices not being adjusted after period t : $P(s^t) \tilde{\omega}(i, s^{t+v}, s^t) = \tilde{P}(i, s^t) g(i, s^{t+v}) - P^w(s^{t+v}) \int_0^1 x^i(j, s^{t+v}) dj$. Maximizing (20) subject to the demand function (4), taking the price $P^w(s^t)$ of the wholesale good, the aggregate final goods price index $P(s^t)$ and the initial price level $P(s^0)$ as given, yields the following first-order condition for $\tilde{P}(i, s^t)$

$$\tilde{P}(i, s^t) = \frac{\epsilon}{\epsilon - 1} \frac{\sum_{v=0}^{\infty} \sum_{s^{t+v}} (\beta\phi)^v q(s^{t+v}, s^t) x(s^{t+v}) P(s^{t+v})^\epsilon \pi^{-\epsilon v} P^w(s^{t+v})}{\sum_{v=0}^{\infty} \sum_{s^{t+v}} (\beta\phi)^v q(s^{t+v}, s^t) x(s^{t+v}) P(s^{t+v})^\epsilon \pi^{(1-\epsilon)v}}. \quad (21)$$

where $x(s^{t+v}) \equiv \int_0^1 x^i(j, s^{t+v})$. Using the simple pricing rule for the remaining fraction ϕ of the firms ($P(i, s^t) = \bar{\pi}P(i, s^{t-1})$), the price index for the final good P_t evolves recursively over time. In a symmetric equilibrium the price level satisfies $P(s^t)^{1-\epsilon} = \phi (\bar{\pi}P(s^{t-1}))^{1-\epsilon} + (1-\phi) \tilde{P}(s^t)^{1-\epsilon}$, which can be rewritten as:

$$1 = \phi (\bar{\pi}\pi(s^t)^{-1})^{1-\epsilon} + (1-\phi) [\tilde{P}(s^t)/P(s^t)]^{1-\epsilon}. \quad (22)$$

Public sector The public sector consists of a fiscal and a monetary authority. The monetary authority supplies money in open market operations in exchange for government bonds and transfers the seigniorage to the fiscal authority. The budget constraint of the central bank is given by

$$M^H(s^t) + (R(s^t) - 1)I(s^t) = M^H(s^{t-1}) + P(s^t)\tau^c(s^t),$$

where τ^c denotes transfers to the fiscal authority. The latter issues risk free one period bonds earning a gross nominal interest rate $R(s^t)$, collects lump-sum taxes τ from the households and receives the transfer from the monetary authority τ^c :

$$R(s^t)B(s^{t-1}) = B(s^t) + P(s^t)\tau^c(s^t) + P(s^t)\tau(s^t). \quad (23)$$

We consider two monetary policy regimes, which differ with regard to the choice of the operating target being controlled according to simple rules. The first regime is characterized by the central bank controlling the supply of money $\mu(s^t) \equiv M(s^t)/M(s^{t-1})$. In the second regime, which is analyzed in the last part of the paper, the central bank applies the nominal discount (repo) rate $R(s^t)$ as the operating target.

The fiscal policy regime is characterized by the following simple rule which relates interest rate payments on outstanding debt to tax receipts and transfers from the central bank:

$$\vartheta(R(s^t) - 1)B(s^{t-1}) = P(s^t)\tau(s^t) + P(s^t)\tau^c(s^t), \quad \vartheta \in (0, 1]. \quad (24)$$

The fiscal policy parameter ϑ governs the portion of government expenditures covered by tax receipts. It can thus serve as a measure for fiscal responsiveness, as, for example, a high value of ϑ indicates fiscal austerity. Using the fiscal policy rule (24) to eliminate the transfers in the budget constraint (23) leads to the following rule for the supply of public debt

$$B(s^t) = [(1 - \vartheta)(R(s^t) - 1) + 1] B(s^{t-1}). \quad (25)$$

Hence, a higher value for the fiscal policy parameter ϑ reduces the growth rate of government bonds. As $\vartheta > 0$ is assumed, it follows immediately from (25) that solvency of the public sector is guaranteed as

$$\lim_{i \rightarrow \infty} \sum_{s^{t+i}} g(s^{t+i}) [B(s^{t+i}) + M(s^{t+i})] \prod_{v=1}^i R(s^{t+v})^{-1} = 0 \quad (26)$$

is always satisfied. In other words, public policy is Ricardian (see Benhabib et al., 2001). It should be noted that this specification of fiscal policy contrasts the one applied in Dupor (2001), where open market operations are defined as 'holding fiscal policy constant in the face of a government asset exchange' (see Sargent and Smith, 1987, page 91), implying that public policy is non-Ricardian.

Equilibrium Given that households are identical, in equilibrium $D(s^t) = 0$, $l(j, s^t) = l(s^t)$, $x(j, s^t) = x(s^t)$, $P^w(j, s^t) = P^w(s^t)$, and $l^j(k, s^t) = l(s^t)$, and as retail firms behave symmetrically: $\tilde{P}(i, s^t) = \tilde{P}(s^t)$, $\omega(i, s^t) = \omega(s^t)$, and $y(i, s^t) = y(s^t)$. Market clearing further implies $y(s^t) = x(s^t)$, $y(s^t) = c(s^t)$, and $a(s^t) = b(s^t)$, where $a(s^t) \equiv A(s^t)/P(s^t)$. In what follows we restrict our attention on the cases where the goods market constraint is binding, $c(s^t) = m(s^t)$. For this, it is sufficient that the nominal interest rate on government bonds $R(s^t)$ larger than one (see 15) such that $\psi(s^t) > 0$.

Definition 1 *A rational expectations equilibrium of the model is a set of sequences $\{\lambda(s^t), \psi(s^t), \eta(s^t), \xi(s^t), c(s^t), l(s^t), y(s^t), P(s^t), P^w(s^t), \tilde{P}(s^t), \pi(s^t), w(s^t), x(s^t), m^H(s^t), m^R(s^t), b(s^t), R(s^t), \mu(s^t)\}_{t=0}^{\infty}$ satisfying the households' first order conditions (11)–(19) combined with (6) and (9), the aggregate version of the production function (1), the labor demand condition (3), the conditions (21), (22), and $\pi(s^t) = P(s^t)/P(s^{t-1})$ for the evolution of aggregate prices, the retail goods production, $x(s^t) = y(s^t)$, the aggregate resource constraint, $y(s^t) = c(s^t)$, the fiscal policy rule (25), and the transversality condition (10 holding with equality), for a given monetary policy rule for $\mu(s^t)$ or $R(s^t)$ and initial values $A(s_0) > 0$ and $P(s_0) > 0$.*

3 Results

In this section the role of open market operations for short-run macroeconomic effects of monetary policy is examined. It starts by establishing households' indifference between accumulating money or holding money (intratemporally) under repurchase agreements. Using this property, the remainder of the paper focuses on the case where money is exclusively held under repurchase agreements. The first sample of results are then derived for flexible prices. The last part of this section examines the effects of open market operations under rigid prices. To lighten the notion, the reference to the state is suppressed in what follows.

3.1 The role of open market operations

In order to acquire money, which serves as a means of payment, households have to engage in open market operations. Here, the central bank supplies an amount of money equal to the discounted value of interest bearing securities, which are deposited at the central bank. At the end of the period, after the goods market is closed, households can either repurchase these securities from the central bank or they can carry over money to the next period, such that the securities are held by the central bank. The foregone interest by holding money instead of debt exactly equals the additional cost of money acquisition under repurchase agreements. This property is responsible for households to be indifferent between both types of money, M^H and M^R . This result is summarized in the following proposition.

Proposition 1 (Indifference) *Households are indifferent between carrying over money from one period to the other and holding money intratemporally under repurchase agreements.*

Proof. In order to establish the claim in the proposition it has to be shown that the multiplier ξ_t on the non-negativity constraint $M_t^R \geq 0$ is equal to zero. Eliminating the multiplier ψ_t on the cash constraint (6) in the first order conditions for money (15) and (16), gives $\xi_t = -\lambda_t + \beta E_t [R_{t+1}\lambda_{t+1}/\pi_{t+1}] + \beta E_t [R_{t+1}\eta_{t+1}/\pi_{t+1}]$. Further applying the first order condition for private debt (13) and government bonds (13) proofs that $\xi_t = 0$. ■

The indifference between the two types of money holdings, measured by the multiplier ξ_t on the non-negativity constraint $M_t^R \geq 0$, critically hinges on the assumption that government bonds are nominally state contingent. If, for example, it is assumed that their payoff in period $t + 1$ equals $R_t B_t$, implying that they are not nominal (though, still real) state contingent, the multiplier on money holdings under repurchase agreements is in general not equal to zero. Given that the assumed payoff structure induces households to be indifferent between accumulating money or holding money temporarily, $\xi_t = 0$, the following assumption is introduced without any loss of generality.

Assumption 1 *Money is exclusively held under repurchase agreements, $M_t^H = 0 \forall t > 0$, and the initial value of money held by the households is equal to zero, $M_0^H = 0$, such that $M_t = M_t^R \forall t$.*

Assumption 1, which will be applied throughout the remainder of the paper, substantially simplifies the subsequent analysis, as money can be treated as a flow variable. It will be shown that the model features two fundamentally different versions depending on the relevance of open market operations, i.e., on whether the money market constraint (9) enters the set of equilibrium conditions as an equality or an inequality. When open market operations are not legally restricted by (8), which demands that only government bonds are eligible, open market operations are obviously irrelevant as money can be acquired in exchange for securities, which can be issued by the households themselves.

Even if open market operations are legally restricted by (8), they are irrelevant as long as households' government bonds holdings are sufficiently large such that $B_t \geq B_t^c$ always holds. Given the timing of events in the model, households can afford the latter when government bonds earn the same interest as private bonds ($R_t = R_t^d$). In this case, households can freely issue private debt to invest costlessly in government bonds to any amount. In contrast, when the interest rate on government bonds is smaller than the interest rate on private debt, this strategy becomes costly and households are willing to minimize holdings of government bonds. Due to the existence of the money market constraint, which reads $M_t^R (= M_t) \geq B_t/R_t$ under Assumption 1, a positive spread $R_t^d > R_t$ arises in equilibrium, which associated with a positive liquidity value of government bonds, indicated by a positive multiplier $\eta_t > 0$. In this case, the open market constraint (9) is binding, $B_t = B_t^c$, indicating that households are only willing to hold government bonds equal to the desired amount of money times the actual discount rate, $B_t = R_t M_t$. This result is summarized in the following proposition.

Proposition 2 (Money market constraint) *The money market constraint is binding, $M_t = B_t/R_t$, if the interest rate spread between private and public debt is expected to be positive $E_t[R_{t+1}^d - R_{t+1}] > 0$.*

Proof. Given that $\lambda_t > 0$ is ensured by (11) and (19), the first order condition (14) implies that the multiplier η_t is strictly positive if $E_t[R_{t+1}^d - R_{t+1}] > 0$. Then the complementary slackness condition (18) demands the open market constraint to hold with equality. ■

Whether the open market constraint is binding or not has substantial consequences for the determination and the evolution of government bonds, interest rates, money, and for consumption. Suppose that the cash constraint (6) is binding and that the expected spread between the interest rate on private debt and the interest rate on government bonds is positive. According to the result in proposition 2 the open market constraint then demands that money

and, thus, consumption is linked to real government bonds $c_t R_t = B_t/P_t$. If, however, the interest rate spread equals zero, $R_t = R_t^d$, the money market constraint becomes irrelevant and the amount of securities traded in open market operations B_t^c is not directly linked to public debt. This case corresponds to the conventional specification of monetary business cycle models.

3.2 Money supply and interest rates under flexible prices

In this subsection the role of open market operations for the relation of money supply and interest rates is examined. In particular, we are interested in the ability of the model to generate a liquidity effect. While the liquidity effect is (for the short-run) commonly found in empirical contributions (see Eichenbaum, 1992, or Hamilton, 1997), it can hardly be reproduced in monetary business cycle models, without referring to segmentations or information asymmetries in asset markets (see Lucas, 1990, Fuerst, 1992, Christiano and Eichenbaum, 1992, or Alvarez et al., 2002). Nevertheless, the success of these strategies to resolve the so-called liquidity puzzle in general depends on parameter restrictions which decide on the ability of the effects, brought about by the particular frictions, to dominate the expected inflation effect of a monetary injection, that tends to raise the nominal interest rate. On the contrary, it is shown that an unanticipated increase in money supply is always associated with a liquidity effect, when the money market constraint is binding.

Consider the case where prices are flexible, i.e., the probability of a retailer receiving a price signal is equal to one ($\phi = 0$), and that the central bank exogenously controls the supply of money via open market operations. The growth rate, $\mu_t = m_t \pi_t / m_{t-1}$, is assumed to be stochastic and satisfies $\mu_t = \mu^{1-\rho} \mu_{t-1}^\rho \exp(\varepsilon_t^\mu)$, where $\rho \in [0, 1)$ and $\mu \geq 1$. The innovations ε_t^μ , i.e., money supply shocks, are assumed to have an expected value equal to zero and to be serially uncorrelated. As prices are flexible, the real wage rate is constant and equals the inverse of the retailers' markup, $w_t = P_t^w / P_t = \frac{\epsilon-1}{\epsilon}$, which immediately implies together with $c_t = l_t$ that consumption is uniquely pinned down by (12) and, thus, constant.¹² Further, suppose that the nominal interest rate on government bonds exceeds one, $R_t > 1$, implying that the cash constraint is binding, $m_t = c_t$, and that the rate of inflation equals the growth rate of money, $\pi_t = \mu_t$. Then the response of the nominal interest rate(s) on a money supply shock, $\varepsilon_t^\mu > 0$, critically hinges on whether the open market constraint is binding or not.

When, the interest rate spread is expected to be equal to zero, $R_t = R_t^d$, the money market constraint is irrelevant, $\eta_t = 0$. Combining the first order conditions for money, which then reads $\psi_t = (R_t - 1) \lambda_t$, for consumption (11) and for bonds (13), gives the consumption Euler equation

$$u_{ct} = \beta R_t^d E_t [u_{ct+1} / \pi_{t+1}]. \quad (27)$$

As consumption is constant the nominal interest rate satisfies $R_t = E_t \pi_{t+1} / \beta$.

¹²This property is actually a virtue of avoiding the cash-credit good distortion between consumption and leisure by applying the modified cash constraint (6).

Hence, for serially correlated money growth rates, $\rho > 0$, an expansionary money supply shock leads to a rise in the nominal interest rate, due to the so-called expected inflation effect (see Christiano et al., 1997). If, on the other hand, the money market constraint is binding, the inverse relation between money supply and the repo rate becomes crucial. For a liquidity effect to occur, it is, however, essential that fiscal policy is assumed to be Ricardian, i.e., to ensure government solvency by satisfying $\vartheta > 0$.

Suppose that the money market constraint holds with equality, $M_t = B_t/R_t$. Then the stock of government bonds outstanding relates to the supply of money and, for a binding goods market constraint, $c_t = m_t$, to consumption expenditures, $b_t = c_t R_t$. Applying the supply rule for government bonds (25), which reads in real terms $\pi_t b_t = [(1 - \vartheta)R_t + \vartheta] b_{t-1}$, and using that consumption is constant under flexible prices, leads to the following relation between money supply and the nominal discount rate R :

$$\frac{R_t}{(1 - \vartheta)R_t + \vartheta} = \frac{R_{t-1}}{\mu_t}. \quad (28)$$

Equation (28) indicates that a rise in the money growth rate μ_t is associated with a decline in the nominal interest rate R_t , provided that we assumed the fiscal authority to satisfy $\vartheta > 0$. If, however, $\vartheta = 0$ would have been assumed, which implies that fiscal policy is non-Ricardian, then a money injection would leave the current interest rate unchanged. Solvency of government policy, which is here ensured by $\vartheta > 0$, thus serves as a bound for the supply of eligible securities and is therefore responsible for the price of money, i.e., the nominal discount rate, to decline when its supply rises.

In order to provide an exact solution of the model, we apply a local analysis of log-linear approximation to the model at the steady state. The steady state is characterized by constant values for c , π , a , m , R^d , and R given by: $u_c(\bar{c})/[-u_l(\bar{c})] = \epsilon/(\epsilon - 1)$, $\bar{c} = \bar{m}$, $\bar{\pi} = \mu$, and $R^d = \mu/\beta$, regardless whether the money market constraint is binding or not. Hence, the steady state of the model is always consistent with the 'monetary facts' of McCandless and Weber (1995).¹³ If the money market constraint is binding, $\bar{\eta} > 0$, the steady state repo rate satisfies

$$\bar{R} = (\mu - \vartheta)/(1 - \vartheta) \text{ and } \bar{a} = \bar{c}\bar{R}. \quad (29)$$

Otherwise, $\bar{\eta} = 0$, the steady state discount rate satisfies $\bar{R} = \bar{R}^d$. The existence of a steady state with a binding money market constraint, requires the central bank to choose a small average money growth rate μ and the fiscal authority to finance a minimum amount of debt obligations with taxes. The conditions for binding constraints in the money and the goods market, which immediately follow from the steady state condition (29) and $\bar{R}^d = \mu/\beta$, are presented in the following proposition

¹³In particular, money is always neutral in the long-run.

Proposition 3 (Steady state) *Suppose that the fiscal policy is sufficiently responsive such that $\vartheta \geq 1 - \beta$. Then there exists a steady state with a binding constraints for the money and the goods market if the central bank chooses an average money growth rate $\mu \in (\beta, \tilde{\mu})$, with $\tilde{\mu} \equiv \vartheta/[1 - (1 - \vartheta)/\beta] > 1$.*

Suppose that public policy satisfies the conditions in Proposition 3 and that the support of ε is sufficiently small such that the money market constraint always binds, $\eta_t > 0$. Then, by log-linearizing (28) at the steady state the fundamental solution for the nominal interest rate can be shown to be the unique solution according to the criterion of Blanchard and Kahn (1980). As the nominal interest rate is not a predetermined variable, this requires the difference equation (28) to exhibit an unstable eigenvalue. Once the solution for the log-linearized model with a binding money market and cash constraint is derived it can immediately be seen that an unambiguous liquidity effect arises. The fundamental solution for the repo rate is presented in the following proposition, where \hat{x}_t denotes the percentage deviation of a generic variable x from its steady state value \bar{x} : $\hat{x}_t \equiv (x_t - \bar{x})/\bar{x}$.

Proposition 4 (Liquidity effect) *The fundamental solution of the log-linear approximation to the model at the steady state with $\mu \in (\beta, \tilde{\mu})$ is the unique solution and generates a liquidity effect by:*

$$\hat{R}_t = -(\mu/\vartheta)\hat{\mu}_t \quad (30)$$

Proof. Log-linearizing equation (28) at the steady state with $\eta > 0$ and $\psi > 0$, leads to $\hat{R}_t = (\mu/\vartheta)\hat{R}_{t-1} - (\mu/\vartheta)\hat{\mu}_t$. Given that $\mu \geq \vartheta$ by assumption, the eigenvalue is unstable and, thus, the unique solution reads $\hat{R}_t = -(\mu/\vartheta)\hat{\mu}_t$ and implies an unambiguous liquidity effect. ■

Hence, the model is able to generate a liquidity effect if the money market constraint is binding, while the so-called 'liquidity puzzle' arises when open market operations are irrelevant. In both cases, the consumption Euler equation predicts that the nominal interest rate on private debt rises with the inflation rate. However, in the latter case both interest rates are identical, whereas the repo rate behaves inversely in the former case. It will be shown in the subsequent section that the nominal interest rate R^d will also decrease with money supply when prices are not completely flexible.

The fundamental solution for the nominal interest rate given in (30) further implies that there exists a simple equivalence between the applied money supply rule and an exogenous interest rate policy: An interest rate peg is equivalent to a constant money growth policy. This feature leads to the last analysis in this subsection, which concerns the determinacy of the price level. As it is well known, interest rate policy can easily lead to price level indeterminacy if it is not set highly contingent to the state of the economy. In particular, an interest rate peg is commonly associated with price level indeterminacy if fiscal policy is assumed to be Ricardian (see, e.g., Benhabib et al.,

2001). For an interest rate peg in this model, the existence of a nominal anchor, i.e., the stock of nominal government liabilities, depends on whether the money market constraint is binding or not. Hence, price level indeterminacy can be resolved if open market operations matter. This result is summarized in the following proposition.

Proposition 5 (Price level determinacy) *Suppose that the cash constraint is binding and that the central bank pegs the nominal discount rate $R_t = R$. Then the price level is (in)determined if the money market constraint is (not) binding.*

Proof. Financial wealth is predetermined and satisfies $A_0 > 0$ and $A_t = B_t$ in equilibrium. Hence, it evolves, by (25), according to $A_t = \alpha^t A_0$, where $\alpha \equiv (1 - \vartheta)R + \vartheta > 0$. When $\eta_t = 0$, an interest rate peg fixes the inflation rate by $\pi = R\beta$ and the growth rate of real financial wealth is given by $a_t/a_{t-1} = \alpha/(R\beta)$, while its level cannot be determined. For $\eta_t > 0 \Rightarrow m_t = a_t/R$ and $m_t = c_t$, real financial wealth equals $a_t = a = Rc$, such that the price level is uniquely determined by: $P_t = A_t/a_t = \alpha^t A_0/(cR)$. ■

The reason why the price level can be determined when the money market constraint binds relies on the property that government bonds provide liquidity services. This finding relates to the result in Canzoneri et al. (2000), where price level indeterminacy is resolved by assuming that government bonds directly enter a cash-in-advance constraint.

3.3 Staggered price setting

In this section, the analysis of monetary policy effects is extended to the case where prices are not completely flexible, $\phi > 0$. The model then additionally features an aggregate supply constraint, i.e., the so-called New Keynesian Phillips curve, stemming from the partial price adjustment of retailers. Log-linearizing (21) and (22), the evolution of the inflation rate can then be summarized by the following aggregate supply constraint (see Yun, 1996): $\hat{\pi}_t = \chi \widehat{mc}_t + \beta E_t \hat{\pi}_{t+1}$, where $\chi \equiv (1 - \phi)(1 - \beta\phi)\phi^{-1} > 0$ and $mc_t = P_t^w/P_t (= w_t)$ denotes the retailers' real marginal costs. The equilibrium of the log-linear approximation to the model at a steady state with $\bar{R} > 1$, $\sigma \equiv -\bar{u}_c/(\bar{u}_{cc}\bar{c}) > 0$, and $v \equiv \bar{u}_l/(\bar{u}_{ll}\bar{l}) > 0$, and the exogenous monetary policy rule (30) is defined as follows.

Definition 2 *A rational expectations equilibrium of the log-linear approximation to the sticky price model at the steady state with $\sigma, v > 0$ and $\bar{R} > 1$ is a set of sequences $\{\hat{c}_t, \hat{m}_t = \hat{c}_t, \hat{\pi}_t, \hat{R}_t, \hat{a}_t\}_{t=0}^\infty$ satisfying and (30),*

$$\hat{c}_t = \begin{cases} \hat{a}_t - \hat{R}_t & \text{if } \eta_t > 0 \\ E_t \hat{c}_{t+1} - (\hat{R}_t - E_t \hat{\pi}_{t+1})/\sigma & \text{if } \eta_t = 0 \end{cases}, \quad (31)$$

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \gamma_1 \hat{c}_t, \quad (32)$$

$$\hat{a}_t = \hat{a}_{t-1} + \gamma_2 \hat{R}_t - \hat{\pi}_t, \quad (33)$$

where $\gamma_1 \equiv \chi(\sigma + \nu) > 0$, $\gamma_2 \equiv (\mu - \vartheta)/\mu \in [0, 1)$, and the transversality condition for a given initial value $a_0 = A_0/P_0 > 0$.

The equilibrium conditions listed in Definition 2 reveal that real financial wealth and, thus, the real value of government bonds outstanding only affects consumption and inflation in the case where open market operations matter ($\eta_t > 0$). Otherwise ($\eta_t = 0$), the equilibrium sequences of consumption, inflation, real balances, and the nominal interest rate are completely unaffected by real wealth, given that they can already be determined by (30)-(32). This version of the model, (31) and (32), is in fact isomorphic to the canonical New Keynesian model as for example applied in Clarida et al. (2000) or in Woodford's (2003) textbook. In this version, real wealth and, thus, real public debt can recursively be determined by (33). The public financing decision, which is represented by the feedback parameter ϑ governing the ratio of tax to debt financing, is therefore irrelevant, implying that Ricardian equivalence applies.

In what follows the effects to a monetary policy shock $\widehat{\mu}_t$, which can either be interpreted as money injection or as a decline in the nominal discount rate (see 30), are examined in the version with a binding money market constraint, $\eta_t > 0$, given in Definition 2. This version of the model exhibits exactly one predetermined variable $a_{t-1} = A_{t-1}/P_{t-1}$, such that the state space is given by $s_t = (a_{t-1}, \mu_t)$. Hence, the fundamental solution reads

$$\widehat{a}_t = \delta_a \widehat{a}_{t-1} + \delta_{a\mu} \widehat{\mu}_t, \quad \widehat{\pi}_t = \delta_{\pi a} \widehat{a}_{t-1} + \delta_{\pi\mu} \widehat{\mu}_t, \quad \widehat{m}_t = \widehat{c}_t = \delta_{ca} \widehat{a}_{t-1} + \delta_{c\mu} \widehat{\mu}_t, \quad (34)$$

and (30). The characteristic polynomial of the model reveals that there exists exactly one stable eigenvalue, indicating saddle path stability. Hence, the fundamental solution (34) is the unique stable solution of the model. The following proposition summarizes this result and presents sign restrictions for the coefficients in (34).

Proposition 6 (Fundamental solution) *The fundamental solution of the model given in Definition 2 with $\eta_t > 0$ is the unique solution and is characterized by (i) $\delta_{ca} = \delta_a$ and $\delta_{c\mu} > 0$; (ii) $\delta_{\pi a} \in (0, 1)$ and $\delta_{\pi\mu} > 0$ if $\vartheta > \underline{\vartheta}$, where $\underline{\vartheta} \equiv 1 - \beta\gamma_1/(1 - \delta_a + \gamma_1) < 1$; (iii) $\delta_a \in (0, 1)$ and $\delta_{a\mu} < 0$; and (iv) $\partial \widehat{R}_t / \partial \widehat{\mu}_t < 0$ and $\partial(\widehat{R}_t^d - \widehat{R}_t) / \partial \widehat{\mu}_t < 0$ if $\sigma > 1$.*

Proof. See Appendix 5.1.

According to the properties of the fundamental solution presented in Proposition 6, the model predictions about monetary policy effects on real activity and prices qualitatively accord to evidence from vector autoregressions (see Christiano et al., 1999). To be more precise, part (i) of Proposition 6 predicts that consumption (output) and real balances decline in response to a monetary contraction, $\widehat{\mu}_t < 0$, whereas part (ii) reveals that the price reaction is

not unambiguous. For inflation to decline in response to a monetary contraction, the degree of fiscal responsiveness should be sufficiently large, $\vartheta > \underline{\vartheta}$. For example, the parameter values, $\beta = 0.99$, $\phi = 0.75$, and $\sigma = \nu = 3$, lead to $\underline{\vartheta} \simeq 0.5$. Otherwise, the associated rise in the nominal interest rate on bonds (see part (iv)), would cause the treasury to increase their liabilities. A stationary sequence of public debt would then require the inflation rate to rise in the future to deflate public debt. As retailers set their prices in a forward looking way, inflation would then also rise in the impact period. Hence, a small degree of the feedback of debt on taxes, $\vartheta < \underline{\vartheta}$, can serve as an explanation for an inverse price response to a monetary policy shock, which is commonly found in vector autoregressions, known as the price-puzzle (see Sims, 1991).

The model further predicts that real wealth declines in response to a monetary injection (see part (iii) of Proposition 6), which is mainly caused by the surge in inflation. Regarding the return on interest bearing assets, part (iv) of Proposition 6 shows that the model generates a liquidity effect and a spread, $\widehat{R}_t^d - \widehat{R}_t$, which rises with a monetary contraction if agents are risk-averse, $\sigma > 1$. Given that only government bonds can be exchanged for money in open market operations, this spread can be interpreted as a liquidity premium and behaves in an intuitive way: A decline in money supply raises the willingness of households to liquidate their securities, such that the liquidity value of government bonds and, thus, the premium on private debt rises.

Corollary 1 *A binding money market constraint is associated with an endogenous liquidity premium if households are risk-averse.*

In the last part of this section, the central bank is assumed to set the interest rate contingent on endogenous variables. In particular, interest rate setting is considered to depend on the realizations of the current inflation rate $\widehat{R}_t = \rho_\pi \widehat{\pi}_t$, which is applied in recent studies on the determinacy properties of interest rate rules (see Benhabib et al., 2001), and can be viewed as a simple version of the rule proposed by Taylor (1993). As prices are rigid, a stabilization of inflation rates is in fact a welfare enhancing policy strategy (see Woodford, 2003), implying that the inflation elasticity should be positive, if fiscal policy is sufficiently responsive, $\vartheta > \underline{\vartheta}$. Otherwise, a rise in the nominal interest rate intended to stabilize inflation would cause the opposite, as shown in part (ii) of Proposition 6. Hence, a binding money market constraint gives rise to an interaction of fiscal and monetary policy.

As the paper aims to provide a positive rather than a normative analysis, we disregard the implications of policy interaction for households' welfare and focus on the local dynamic properties. While an interest rate peg was shown to lead to saddle path stability, the same property is not guaranteed for the case where the nominal interest rate is set highly reactive to changes in inflation. In particular, the upper bound for an inflation elasticity, which ensures saddle path stability, depends on the fiscal responsiveness, measured

by the feedback parameter ϑ of the tax rule (24). This result is summarized in the following proposition.

Proposition 7 (Real determinacy) *Suppose that the central bank sets the discount rate according to $\widehat{R}_t = \rho_\pi \widehat{\pi}_t$, where $\rho_\pi \geq 0$. Then the rational expectations equilibrium path of the model in Definition 2 with a binding money market constraint is (i) uniquely determined, and (ii) converges to the steady state if and only if $\rho_\pi < \overline{\rho}_\pi$, where $\overline{\rho}_\pi \equiv [(1 - \vartheta)R + \vartheta]/[(1 - \vartheta)R] > 1$.*

Proof. See Appendix 5.2.

To get an intuition for the result part (ii) in Proposition 7, consider that the central bank chooses a high inflation elasticity ρ_π and inflation rises due to a fundamental shock. If tax policy is highly reactive to the evolution of public debt (high ϑ), then the real value of public debt will be reduced by higher prices. If, on the other hand, the fiscal policy regime finances only a small fraction of its debt obligations by taxes, then the associated rise in the nominal interest rate R_t can lead to a rise in real public debt. This, however, corresponds to a rise in the real value of eligible securities held by the households. Thus, households raise their consumption expenditures as the increase in public debt eases the money market constraint. Hence, a highly aggressive interest rate policy might lead to debt interest spirals when the fiscal feedback is too small. The upper bound $\overline{\rho}_\pi$ given in Proposition 7 further reveals that saddle path stability is guaranteed if the fiscal authority runs a balanced budget policy ($\vartheta = 1$). On the contrary, a non-Ricardian regime ($\vartheta = 0$) would require a passive interest rate policy ($\rho_\pi < 1$) to escape explosiveness.¹⁴ In any case, the model is always associated with a unique rational expectation equilibrium (see part (i) of Proposition 7).

Regarding the relation of interest rates and money supply, the two versions of the model further reveal a remarkable principle. As already shown in Proposition 4, there is an intuitive equivalence between interest rates and money supply when the money market constraint is binding. For example, it predicts that an Taylor-type interest rate rule satisfying $1 < \rho_\pi (< \overline{\rho}_\pi)$, is associated with a money supply which responds negatively to current inflation. Hence, money supply is negatively related to inflation and thus mirrors an anti-inflationary stance of the central bank, $\partial \widehat{\mu}_t / \partial \widehat{\pi}_t = -\rho_\pi \vartheta / \mu$.

The conventional version of the model ($\eta_t = 0$), however, behaves quite differently in this regard. The relation between interest rates and money supply is then based on the consumption Euler equation (27). Its linearized version (31) together with the cash constraint, leads to the following relation between money growth rates and inflation:

$$\eta_t = 0 \Rightarrow \partial \widehat{\mu}_t / \partial \widehat{\pi}_t = (\rho_\pi - 1) \sigma^{-1} + 1. \quad (35)$$

¹⁴A similar outcome can occur in a sticky price model with overlapping generations (see Leith and Wren-Lewis, 2000).

According to (35), an active interest rate setting, $\rho_\pi > 1$, is associated with accommodating money growth rates $\partial\widehat{\mu}_t/\partial\widehat{\pi}_t > 0$. On the contrary, an interest rate rule which implements a non-accommodating money supply, violates the so-called Taylor principle and – as shown by Woodford (2001) in an isomorphic model – allows for multiple rational expectations equilibria. This result is summarized in the following proposition.

Proposition 8 (Equivalence) *A central bank setting the nominal interest rate according to $\widehat{R}_t = \rho_\pi\widehat{\pi}_t$ can only implement a sequence of non-accommodating money growth rates $\partial\widehat{\mu}_t/\partial\widehat{\pi}_t \leq 0$ on an unique rational expectations equilibrium path if the money market constraint is binding.*

The result presented in Proposition 8 implies that a central bank can only implement a sequence of constant money growth rates on a saddle stable equilibrium path if the money market constraint binds. If a central bank aims at stabilizing the economy, 'one would expect sensible policy behavior to involve a negative value' for $\partial\widehat{\mu}_t/\partial\widehat{\pi}_t$ (see McCallum, 1999, page 623). This, however, is for the conventional specification of monetary policy, $\eta_t = 0$, associated with the central bank setting the nominal interest rate in a passive way, $\rho_\pi < 1$, which allows for arbitrary expectations to be self-fulfilling.

4 Conclusion

Are open market operations really irrelevant for macroeconomic dynamics, as commonly assumed in recent business cycle theory? In this paper it is shown that, when money is the counterpart of discounted securities deposited at the central bank, the relevance of open market operations depends on whether the set of eligible securities is constrained or not. A legal restriction on open market operations is introduced, by which only government bonds are accepted as collateral, while households are assumed to internalize this money market constraint, similar to a goods market (cash-in-advance) constraint. An otherwise standard New Keynesian model then exhibits an equilibrium where non-eligible securities are associated with an endogenous liquidity premium and money supply is inversely related to the nominal discount interest rate. As a consequence, the liquidity puzzle is solved, given that fiscal policy is assumed to be Ricardian such that the supply of eligible securities, i.e., public debt, is not unbounded. Households' financial wealth provides a nominal anchor and the price level as well as the rational expectations equilibrium is always uniquely determined when the central bank sets the nominal interest rate, regardless whether prices are flexible or sticky.

5 Appendix

5.1 Proof of proposition 6

In order to examine the eigenvalues of the model with $\eta_t > 0$ given in Definition 2, it is reduced to a 2×2 system in real wealth, which is a predetermined variable, and inflation:

$$\mathbf{M}_0 \begin{pmatrix} \hat{a}_t \\ E_t \hat{\pi}_{t+1} \end{pmatrix} = \mathbf{M}_1 \begin{pmatrix} \hat{a}_{t-1} \\ \hat{\pi}_t \end{pmatrix} + \mathbf{M}_e \hat{\mu}_t \quad (36)$$

$$\text{where } \mathbf{M}_0 = \begin{pmatrix} \gamma_1 & \beta \\ 1 & 0 \end{pmatrix}, \quad \mathbf{M}_1 = \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}, \quad \mathbf{M}_e = \begin{pmatrix} -\gamma_1 \pi / \vartheta \\ 1 - \pi / \vartheta \end{pmatrix}.$$

The characteristic polynomial of $\mathbf{M}_0^{-1} \mathbf{M}_1$ is $f(X) = X^2 - \frac{\beta + \gamma_1 + 1}{\beta} X + \frac{1}{\beta}$. Given that $f(0)$ is equal to $1/\beta$ and, therefore, strictly positive and $f(1)$ is negative $f(1) = -\gamma_1/\beta < 0$, the model exhibits one eigenvalue lying between zero and one, $X_1 \in (0, 1)$ and one unstable eigenvalue, $X_2 > 1$.

As there is only a single stable eigenvalue, the fundamental solution (34) is the unique stable solution of the model. Using the general form in (34) to replace the endogenous variables in the equilibrium equations (31)-(33), leads to the following conditions for the undetermined coefficients δ_a , $\delta_{\pi a}$, $\delta_{a\mu}$, and $\delta_{a\mu}$

$$\begin{aligned} \gamma_1 \delta_a + \beta \delta_{\pi a} \delta_a - \delta_{\pi a} &= 0, & \delta_a - 1 + \delta_{\pi a} &= 0, \\ \gamma_1 \delta_{a\mu} + \beta \delta_{\pi a} \delta_{a\mu} + \gamma_1 (\mu / \vartheta) - \delta_{\pi \varepsilon} &= 0, & \delta_{a\mu} + \delta_{\pi \mu} + (\mu - \vartheta) / \vartheta &= 0. \end{aligned} \quad (37)$$

where δ_a is the single stable eigenvalue of the model, $\delta_a = X_1$. Manipulating the conditions in (37), gives the following impact multiplier on inflation and real wealth

$$\delta_{\pi \mu} = [\vartheta \gamma_1 + (\vartheta - \mu)(1 - \delta_a) \beta] / \Gamma, \quad \delta_{a\mu} = -[\gamma_1 \mu + (1 - \beta \rho)(\mu - \vartheta)] / \Gamma < 0,$$

where $\Gamma \equiv \vartheta[\gamma_1 + \beta(1 - \delta_a) + (1 - \beta \rho)] > 0$ and $\delta_{\pi a} = 1 - \delta_a$ with $\delta_{\pi a} \in (0, 1)$. The impact multiplier on inflation, $\delta_{\pi \mu}$, is strictly positive if $\vartheta \gamma_1 + (\vartheta - \mu)(1 - \delta_a) \beta > 0$. Using that μ is assumed to be strictly smaller than $\bar{\mu} \equiv \vartheta / [1 - (1 - \vartheta) / \beta]$, it follows that $\delta_{\pi \mu}$ is strictly positive if $\vartheta > 1 - \beta \gamma_1 / (1 - \delta_a + \gamma_1)$. The coefficient $\delta_{a\varepsilon}$ is further used together with the solution for \hat{c}_t , $\hat{c}_t = \delta_a \hat{a}_{t-1} + (\delta_{a\mu} + \mu / \vartheta) \hat{\mu}_t$, to derive the impact multiplier on consumption and real balances $\delta_{c\mu}$, which reads

$$\delta_{c\mu} = [\vartheta(1 - \beta \rho) + \mu \beta(1 - \delta_a)] / \Gamma > 0.$$

With these solutions and $\hat{R}_t = -(\mu / \vartheta) \hat{\mu}_t$, one can determine the response of the interest rate spread, $\hat{R}_t^d - \hat{R}_t$, by using with the consumption Euler equation, $\hat{c}_t = E_t \hat{c}_{t+1} - (\hat{R}_t^d - E_t \hat{\pi}_{t+1}) / \sigma$. Replacing consumption with the structural relation $\hat{c}_t = \hat{a}_t - \hat{R}_t$ and applying the fundamental solution gives $\hat{R}_t^d = -[(\sigma - 1)(1 - \delta_a) \delta_{a\mu} + \sigma \mu / \vartheta] \hat{\mu}_t + \delta_a [\sigma(\delta_a - 1) + \delta_{\pi a}] \hat{a}_{t-1}$, such that $\partial(\hat{R}_t^d - \hat{R}_t) / \partial \hat{\mu}_t = -(\sigma - 1)(1 - \delta_a) \delta_{a\mu}$. Hence, the spread declines with $\hat{\mu}_t$ if and only if $\sigma > 1$, which completes the proof of proposition 6. ■

5.2 Proof of proposition 7

When the discount rate is set according to $\widehat{R}_t = \rho_\pi \widehat{\pi}_t$, the matrices of the 2×2 model in (36) are unchanged except for the second column of \mathbf{M}_1 . Its elements are now given by $M_1^{(1,2)} = 1 + \gamma_1 \rho_\pi$ and $M_1^{(2,2)} = \gamma_2 \rho_\pi - 1$. The characteristic polynomial therefore changes to

$$f(X) = X^2 - [(\gamma_1 \rho_\pi + 1) - \gamma_1 (\gamma_2 \rho_\pi - 1) + \beta] \beta^{-1} X + (\gamma_1 \rho_\pi + 1) \beta^{-1}.$$

Apparently, $f(X)$ is strictly positive at $X = 0$, $f(0) = (1 + \gamma_1 \rho_\pi) / \beta > 0$. At $X = 1$, its sign depends on ρ_π : $f(1) = \gamma_1 (\gamma_2 \rho_\pi - 1) / \beta$. If $\rho_\pi < 1 / \gamma_2 = \bar{\pi} / (\bar{\pi} - \vartheta)$, the model exhibits one stable and one unstable eigenvalue, indicating a saddle path configuration. If $\rho_\pi \geq 1 / \gamma_2$, there are either two stable or two unstable eigenvalues. To discriminate between the two cases, the slope $X = 1$ is considered: $f'(1) = \beta^{-1} \{ \gamma_1 [(\gamma_2 - 1) \rho_\pi - 1] - (1 - \beta) \}$, revealing that $f'(1) < 0$, given that $\gamma_2 \in [0, 1)$. Thus, both eigenvalues exhibit a real part larger than one. Therefore, equilibrium indeterminacy cannot occur, while, using $\bar{\pi} = (1 - \vartheta)R + \vartheta$, saddle path stability prevails if and only if $\rho_\pi < [((1 - \vartheta)R + \vartheta) / ((1 - \vartheta)R)]$. ■

6 References

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