FORWARD CONTRACTING WITH UNCERTAIN INPUT SUPPLIES:
A RISK PROGRAMMING APPROACH

by

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Abstract

Forward contracting with uncertain input supplies and prices was investigated and a decision technique developed. Uncertain input supplies increased the risk in forward contracting. A case study of a large peanut marketing firm demonstrated the effect of uncertain input supplies on the optimal proportion of expected production to forward contract.
First handlers of agricultural commodities encounter inseparable input-supply and product disposal risks when buying and selling the commodity. Uncertain supplies from farms affect the proportion of expected production a handler can safely forward contract to manufacturers. A shortage of input supplies could leave the handler unable to fulfill forward contracts while a surplus of farm supplies might cause product disposal difficulties in a period of depressed prices.

Several authors have studied issues related to first handler's risk. Buccola investigated forward contracting of a fixed quantity of a single commodity under differing pricing provisions. Barry and Willman applied multiperiod mean-variance analysis to determine forward contracting strategies with credit constraints. Bond and Thompson demonstrated that nonlinear input costs make the decision maker's risk-aversion a determinant of the recommended hedging ratio. Other researchers (Hiefner, Johnson, and Ward and Fletcher) have concentrated on optimal hedging ratios with futures contracts. A restrictive premise underlying the above papers is production certainty (Rolfo). In particular, first handlers are concerned with production uncertainty attributed to uncertain input supplies.

Risk programming with uncertain input supplies has concentrated on variations of chance-constrained programming (CCP). Charnes and Cooper, and Tintner presented CCP as a method of protecting against insufficient acquisition of essential inputs that have free disposal. The magnitude of the CCP safety margin, however, has not been linked to the firm's utility function (Falatoonzedeh, Conner,
Considerable parameterizing must occur with CCP if uncertain prices are also present.

Lambert and McCarl recently presented a risk-programming formulation based on sampling. The formulation relaxed constraining assumptions on the utility function and the distributions of uncertain parameters. This study presents a multiperiod stochastic programming method, also based on sampling, for determining the proportion of expected production a firm should forward contract. The firm's uncertain input supplies, uncertain prices received, and risk preferences are considered. An hypothesis is advanced that limiting uncertain input supplies increase the risk associated with forward contracting and may cause a risk-averse manager to forward contract only at a price above his expected market price. Validity of the hypothesis is demonstrated with a case study of peanut buying and shelling.

Background

For an input supply to be considered uncertain, two requirements must be met. First, the input must have been purchased or endowed at an earlier period but yield an uncertain flow of services in the future production period. Second, the firm must not be able to purchase more of the input during the production period. Rainfall, solar radiation, and available field hours are traditional examples of farm inputs with uncertain supplies.

During the future production period, the amount acquired of such an aleatory input is not a decision variable; the firm must take what occurs. In the earlier period, however, the firm might influence either the endowment or the firm's future requirements for that input. For a commodity marketing firm, an example of a decision variable that affects future input requirements might be forward
contracting of the output. Forward contracting occurs prior to harvest and is associated with the long run risk and profit position of the firm rather than with immediate production decisions.

Five important factors influence the proportion of expected future production to forward contract: input supply uncertainty, price uncertainty, the correlation between prices and input supply, the firm’s risk preferences, and the difference of the expected cash price minus the forward contract price (the expected basis). A balance needs to be determined. Research should be able to quantify that in general, a smaller (or more negative) basis implies more forward contracting and greater input supply uncertainty implies less forward contracting for a risk-averse firm. The technique presented next maximizes expected utility while considering all five factors.

The Technique

The distribution of profits in the presence of limiting uncertain inputs is not transparent. Let Y be an aleatory input with a given probability distribution. Profits (π) are a function of Y, prices, the level of forward contracting, and other non-stochastic inputs. The distribution of profits is then dependent on the distributions of Y and prices. In typical mathematical programming models price risk is reflected in the objective function and input supply risk is contained in the right-hand-side (rhs). Assuming prices have a normal distribution, the variance of profits in the presence of only uncertain prices is merely a linear combination of squared prices. However, the path of inputs from the rhs through the technical matrix to the objective function presents a more complicated formulation. The input supply must limit production to affect profits. Furthermore, infeasible solutions might occur at extreme input levels.
The goal is to determine the level of forward contracting that maximizes the expected utility of profits. \( E(U) = \int U^* F(\pi) d(\pi). \)

Freund, using facilitating premises, demonstrated maximization of this formula with respect to the objective function variables using quadratic programming. However, maximizing with respect to the level of forward contracting requires a different approach. The approach in this study is based on the fact that a continuous profit distribution can be approximated by a discrete sampling procedure.

The optimization procedure requires several components. A profit maximizing linear programming (LP) model is needed which incorporates the time span and production activities influenced by input supply uncertainty. The parameters of the probability distributions of input supply and uncontracted prices are used to generate random samples that could represent actual prices and input supplies. If prices and input supply are correlated, the generated sample values must reflect this correlation (Schuer and Stoller, Clements et al). Finally, the firm's utility function with respect to profits is required. Firms may be considered to know or to describe their utility functions while others must be estimated with elicitation.

The steps in the procedure were as follows. First, forward contracting is set to a specific level. A profit was determined using one sample of the generated random prices and input supply in the linear programming model. Repeated solutions were obtained with the remaining random values until sufficient sample profits were given to adequately describe the profit distribution. One hundred trials appeared adequate for the case study; one thousand trials did not change the distribution significantly. An expectation formula was then applied to the utility function and estimated profit sample points to determine an expected utility for that specific level of
forward contracting. All the previous steps were then repeated in a systematic search of all levels of forward contracting. An expected utility for each forward contracting level was furnished and a maximum determined.

An advantage of using a discrete approximation is that no restrictions are placed on the distributions of uncertain parameters (Lambert and McCarl). Uncertain input supplies are accounted for in the profit distribution. Also, random number generators exist for most classes of probability distributions. A desire to account for any correlation between prices and input supply, however, may limit the analysis to the multivariate normal.

The linear programming model and technique are useful for forward contracting of sales. After the stochastic inputs are acquired, input supply will no longer be uncertain. More conventional methods such as mean-variance analysis could be applied to the LP model to analyze the effects of uncertain prices.

An obstacle in this technique is the volume of linear programming tableaus that need to be maximized. To overcome the computing hurdles, the revised simplex method using the product form of the inverse (Gass) was written in vectorized Fortran for use with a CDC Cyber 205 computer. This algorithm and computer proved to have the speed necessary to do iterations in a reasonable amount of time. In the next section, the above approach is applied to determine the amount of shelled peanuts a large peanut sheller should forward contract to further processors of peanut products.

**Application to a Peanut Sheller**

Peanuts are the major limiting aleatory input in a peanut sheller's production function. Knowledge of farm yields is necessary to formulate strategies and plans for marketing. However, annual
yields have fluctuated as much as fifty percent from average and current techniques for forecasting peanut yields are not of sufficient accuracy for early-season (prior to August) marketing analysis. Marketing decisions must be made with imperfect information.

Current peanut program provisions require a sheller to contract with farmers prior to July 31 in order to export peanuts. However, the typical contract between a sheller and a farmer states that the farmer is not required under penalty to deliver the contracted poundage of peanuts. In effect, farmers are not penalized for being an uncertain input supply. In fact, evidence indicates that farmers may underplant on acreage needed to fulfill the contract if export prices are low. The sheller, on the other hand, must purchase all the contracted peanuts and, if profitable, will purchase all the peanuts produced by the contracted farms.

In-shell peanuts qualify as an uncertain input supply. The only established source of in-shell peanuts is from farms. These peanuts are purchased or endowed prior to the mandated farm contracting deadline of July 31 but are not delivered until after harvest in September. Farm plantings, yields, and subsequent deliveries are unpredictable. The market among peanut handlers for in-shell peanuts is insignificant; peanuts purchased from other handlers are already shelled. Overcapacity in the shelling industry creates a desire for each firm to shell all the peanuts it handles.

Peanut marketing is described by McArthur et al., Miller, and Fletcher et al. The Food Security Act of 1985 continued a two-tiered pricing system with domestic production quotas and import restrictions. According to price support policy, all contracted
additional peanuts must either be exported, or crushed for oil and meal. Only quota peanuts may be sold domestically.

The marketing period for one year's crop is approximately fifteen months long and lasts from September of the current year to November of the following year. The harvest season (Sept-Dec) is the period when farm deliveries are ordinarily made. During the first months of the following year (Jan-Aug) in-shell peanuts come only from storage.

An appropriate model of the flows of peanuts through the firm and the time of their occurrence was achieved through linear programming techniques (LP). The model is a multiperiod multiproduct planning model with a one crop year horizon. The flow of peanuts through the firm is depicted in Figure 1. The combination of farm production, forward contracts, and open market sales and purchases of shelled peanuts that maximize the firm's marketing margin with regard to the current peanut program is determined. Old and future crops are connected in this model through the beginning and ending inventories. The supply of farm peanuts was the major restriction to firm expansion.

Figure 1. Flowchart of Inventory, Farm Purchases, Buy-ins, and Sales.
The objective function was arranged into four major sections. The additions market, the quotas market, buybacks, and purchases of farm peanuts have a total of 42 activities. The additional and quota markets for shelled peanuts deal with the sales and purchases among handlers and sales to manufacturers. In the model, additional and quota peanuts can each be sold four ways, either for cash or by forward contract for delivery in either Sep-Dec or Jan-Aug.

Buy-ins are purchased only from other domestic peanut handlers and are analogous to sales. Intraperiod arbitrage is disallowed in the model by adjusting the buy-in prices to equal to the sale prices plus a standard one percent brokerage fee. However, interperiod arbitrage is allowed.

Farmers produce peanuts classified three ways by the peanut program and deliver them to shellers in the most profitable order. First, domestic quota peanuts with a high support price (above domestic free market equilibrium) are delivered. These are followed by contracted additional peanuts with a lower price (near international free market levels). Last to be delivered are uncontracted additionals which must go into the CCC loan program.

Farm deliveries of the three types of peanuts create three stochastic input supplies in the rhs. The sheller was required to purchase all contracted additional and quota peanuts. Uncontracted additionals could be used as buybacks if profitable or left with the CCC if unprofitable (Table 2).

<table>
<thead>
<tr>
<th>A1</th>
<th>A2</th>
<th>A3</th>
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<td>0</td>
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<td>1</td>
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where: A1 = Export uncontracted additional buybacks
A2 = Domestic uncontracted additional buybacks
A3 = Additional farm purchases
A4 = Quota farm purchases
Forward contracting in each period was restricted to be a proportion of expected production of shelled peanuts. The structure of the constraints was:

Forward Contracts to Sep-Dec = $S_1 \cdot$ expected additional deliveries.
Forward Contracts to Jan-Aug = $S_2 \cdot$ expected additional deliveries.

Analogous constraints were imposed on quota forward contracting. The variables $S_1$ and $S_2$ are the proportion forward contracted and were parameterized from 0 to 100. An empirical example is presented next.

**Data**

The above model and risk management technique were applied to a large sheller with 1986 peanut program regulations and price and yield expectations. Some parts of the data were provided by a case study firm, while other parts were estimated from industry publications. Estimates were both historical and subjective.

The covariance matrix included the stochastic cash prices and farm yields. For a large sheller with contracted farmers located over a wide area a negative correlation exists between farm yields and prices. No correlation was assumed to exist between export and domestic prices. Export prices varied more than quota prices suggesting that additional price risk is greater than domestic price risk.

The utility function was defined to be the satisfaction of the management gained from the profits of peanut shelling. The negative exponential function ($U = 1 - \exp(-0.0000008 \cdot \text{profits})$) was a close representation of and was substituted for a utility function suggested by the research department of the case study firm.

**Results**

Sheller profits are not a monotonic function of farm production. Positive profits were made on expected deliveries of quotas and...
contracted additions. However, the firm incurred losses on the deliveries above the amount expected. No free disposal of the excess peanuts was available. Thus, the sheller had a two-sided input supply risk and would maximize profits at a finite input supply level. In fact, the case study sheller experienced lower profits after the record high farm production of 1985 than after the major 1980 drought.

An initial stochastic analysis omitted input supply risk. Farm deliveries were fixed at expected levels but sales prices were allowed to vary. The generated profit distribution appeared symmetric and was not rejected as a normal distribution using the Kolmogorov-Smirnov modified D statistic (D= 0.097 with n=100). Inclusion of uncertain farm deliveries, however, visibly skewed the distribution to left; it was rejected as a normal distribution (D= 0.184). The sheller had possibilities of large losses without proportionate possibilities of large gains. Additionally, the dispersion of profits was greater with the inclusion of uncertain farm deliveries.

The next step was to analyze the expected basis to determine price levels at which the sheller should forward contract. The expected cash price was set equal to the forward contract price minus the basis. A small but positive basis was found only for domestic peanuts forward contracted to Sept-Dec. Domestic peanuts have minimal input supply risk.

Export peanuts have significant input supply risk and forward contracting will not maximize utility when the forward contract price is less than the expected cash price. In Figure 3 the basis for Sept-Dec export peanuts was parameterized so that the forward contract price was 5 to 25 percent above the expected cash price (the basis was increasingly negative). For example, the middle curve in Figure 3 represents a forward contract price 15 percent above the expected cash price. At this basis, the sheller maximizes utility by
Figure 3. Utility of expected export production forward contracted to Sept-Dec
forward contracting 20 percent of expected export production to Sept-Dec. The utility curves initially rise because of the potentially greater revenues and decreased price risk from forward contracting. The utility curves then fall due to uncertain farm deliveries and the subsequent uncertainty of shelled export peanut production. The sheller has to be compensated for assuming the risk that farm production may not be sufficient to cover the forward contracts. The sheller should not forward contract export peanuts to Sept-Dec if the basis is zero or positive. Similar results held for forward contracting export peanuts to Jan-Aug.

The amount of expected farm production to forward contract was strongly influenced by farm production uncertainty. Although price variability was greater for export peanuts than domestic quota peanuts, the risk management technique suggested that only a small amount of additionals be forward contracted. Since quotas are delivered to the sheller first, the input supply risk of domestic peanuts was found to be minimal. The risk-averse sheller might forward contract all of its expected production of domestic peanuts at a positive or small negative basis (expected cash minus forward contract price). Input supply risk is concentrated in export peanuts. The results suggested that, even with an extremely favorable basis, at most about 30 percent of expected additional production should be forward contracted for export delivery in Sept-Dec and 21 percent for delivery in Jan-Aug.

Conclusions

This study has demonstrated that in the presence of uncertain inputs (and thus an unknown quantity of production), a risk-averse buyer-seller may offer to forward contract sales only at a level above the expected market price. Input supply uncertainty adds risk to forward contracting and the seller will
demand compensation for assuming the risk of insufficient product supply to fulfill the contract.

An analytical tool was presented that can evaluate management decisions in the presence of both uncertain prices and input supplies. Sampling was combined with linear programming to describe a profit distribution. This approach accounted for the effects of uncertain input supplies on the distribution of profits. Although normality was assumed for all initial distributions, the profit distribution was skewed to the left.

Uncertain supplies of limiting inputs are sources of risk that need consideration in forward contracting decisions. As government farm policies trend toward reducing excess production, uncertain input supplies will appreciate in significance. This study has given some insights in modeling input supply uncertainty and a technique that can be extended to other situations. Studying risk incidence may increase efficiency in the entire food marketing chain.
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