Pricing Commodity Options under Markov Regime Switching GARCH Processes

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Introduction

The key to pricing agricultural commodity futures options is to find the underlying stochastic process of futures prices. There are two prominent features in corn futures prices in the era of biofuels production. First, prices more often exhibit excess kurtosis and negative skewness. Recent corn prices have risen persistently and significantly, and have seen to break the historical price record due to the rising demand from ethanol production. Second, the corn futures volatility has exhibited a structural change. Rapid expansion of corn-based ethanol production has made corn and energy markets more connected, thus causing volatility spillovers from energy markets to corn markets. However, these two features are not captured in the B-S model. Moreover, none of the previous studies incorporates all features simultaneously into option pricing models.

Objectives

The objective of this study is to fill the gap by developing an option pricing model that incorporates these features in commodity futures price dynamics. We adopt the student's t distribution to model the excess kurtosis. As Baillie and Myers (1991) have found, a student t density for the conditional distribution of price changes can do a good job in capturing such fat-tailed properties. Since negative skewness could be generated by the leverage effect—a negative correlation between current returns and futures returns volatility, we introduce the Exponential GARCH (EGARCH) or GJR GARCH (Threshold GARCH) model to cope with the skewness. Finally, we propose a Markov Regime-Switching GARCH (MRS-GARCH) process that allows for volatility process to switch from one GARCH process to another. In addition to the t distribution and the variations of GARCH models, we also incorporate a classic Gaussian distribution into a basic GARCH framework to assess the relative performance of the models by its ability to forecast the observed options price in corn futures contracts.

Models

Single Regime GARCH Models

$$y_t = 100[\log(F_t) - \log(F_{t-1})]$$
$$y_t = \mu + \varepsilon_t$$

Error Types	GARCH Types
t-ditribution $\varepsilon_t \vert I_{t-1} \sim td(0, h_t, v)$	*GARCH* $h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}$
Normal Distribution $\varepsilon_t \vert I_{t-1} \sim N(0, h_t)$	*EGARCH* $\log(h_t) = \alpha_0 + \alpha_1 \left\vert \frac{\varepsilon_{t-1}}{h_{t-1}} \right\vert + \xi \frac{\varepsilon_{t-1}}{h_{t-1}} + \beta_1 \log(h_{t-1})$
	GJR-GARCH $h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 [1 - \mathcal{L}_{\{\varepsilon_{t-1}>0\}}] + \xi \varepsilon_{t-1}^2 \mathcal{L}_{\{\varepsilon_{t-1}>0\}} + \beta_1 h_{t-1}$

NOTE: F_t is corn futures price; v is the degree of freedom of t-distribution; $\mathcal{L}_{\{.\}}$ is the indicator function which is equal to one when the argument is true and zero otherwise.

Reference

Klaassen, F. 2002. " Improving GARCH Volatility Forecasts." *Empirical Economics* 27(2):363-394.

Myers, R.J., and S.D. Hanson, 1993. "Pricing Commodity Options when Underlying Futures Price Exhibits Time-Varying Volatility." *American Journal of Agricultural Economics* 75, 121-130.

Markov Regime-Switching GARCH Models

$$y_t = 100[\log(F_t) - \log(F_{t-1})]$$
$$y_t = \mu^{(i)} + \varepsilon_t \qquad i = 1,2$$

Error Types	MRS-GARCH Types
t1-ditribution $\varepsilon_t \vert I_{t-1} \sim td(0, h_t^{(i)}, v)$	$h_t^{(i)} = \alpha_0^{(i)} + \alpha_1^{(i)} \varepsilon_{t-1}^2 + \beta_1^{(i)} E_{t-1}[h_{t-1}^{(i)} \vert s_t]$
t2-ditribution $\varepsilon_t \vert I_{t-1} \sim td(0, h_t^{(i)}, v^{(i)})$	$E_{t-1}[h_{t-1}^{(i)} \vert s_t] = \sum_{j=1}^{2} \tilde{p}_{ji,t-1} \left[(\mu^{(j)})^2 + h_{t-1}^{(j)} \right] - \left[\sum_{j=1}^{2} \tilde{p}_{ji,t-1} \mu^{(j)} \right]^2$
Normal Distribution $\varepsilon_t \vert I_{t-1} \sim N(0, h_t^{(i)})$	$\tilde{p}_{ji,t} = \Pr(s_t = j \vert s_{t+1} = i, I_{t-1}) = \frac{p_{ji} \Pr(s_t = j \vert I_{t-1})}{\Pr(s_{t+1} = i \vert I_{t-1})} = \frac{p_{ji} p_{j,t}}{p_{i,t+1}}$ where i, j = 1,2. The Transition Matrix $P = \begin{bmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{bmatrix} = \begin{bmatrix} p & (1-q) \\ (1-p) & q \end{bmatrix}$

NOTE: $t1$-distribution means the model with constant degrees of freedom; t2-distribution means the model can allow the degrees-of-freedom parameters to switch between the two regimes; s_t is a state variable. We follow Klaassen (2001) to specify the MRS-GARCH model.

Estimation

The tables present results of estimating one using only corn May-contract futures data from 5/14/2003 to 3/14/2008. The estimation results show that the largest log-likelihood is given by the MRS-GARCH with Student's t distribution, where the degrees of freedom switch across the two volatility regimes. Based on the estimation, the multi-step-ahead volatility forecasts can be calculated.

Maximum Likelihood Estimates of Single Regime GARCH Models

	t-ditribution			*Normal Distribution*		
	GARCH	*EGARCH*	*GJR*	*GARCH*	*EGARCH*	*GJR*
μ	-0.0036 (-0.0920)	-0.0196 (-0.5090)	-0.0060 (-0.3210)	0.0629 (1.2980)	0.0993 (2.0420)	0.0644 (1.2950)
α_0	0.4852 (3.5930)	-0.0653 (-3.7190)	0.5180 (11.0100)	1.5452 (6.9850)	0.0868 (1.5660)	2.0000 (8.9500)
α_1	0.1666 (4.3500)	0.1140 (4.1760)	0.1995 (2.7000)	0.2371 (12.6350)	0.3917 (8.6380)	0.1674 (3.1390)
ξ	-	0.0162 (0.8700)	0.1353 (3.3320)	-	-0.0113 (-0.4270)	0.3229 (12.0190)
β_1	0.6598 (9.4510)	0.9786 (102.0350)	0.6494 (31.4820)	0.2854 (3.1810)	0.6532 (10.0520)	0.1086 (1.1920)
ν	5.4495 (9.7120)	4.9328 (10.2580)	5.3859 (249.6000)	-	-	-
$Log(L)$	-2247.4109	-2233.3071	-2247.0465	-2455.1730	-2370.7253	-2365.0221

Simulation

We use Monte Carlo simulation method to pricing options under different GARCH specifications. We make 20000 independent random draws to get the realization of the futures price in any period before maturity.

Evaluation

We use a simple root mean-square error (RMSE) criterion to determine how well each alternative option pricing model estimates actual market prices. The model with the smallest RMSE estimates market price the best. The out-of-sample evaluation is implemented between 3/15/2008 to 4/25/2008.

Maximum Likelihood Estimation of MRS-GARCH Models

	t-ditribution		*Normal Distribution*
	MRS-GARCH-t1	*MRS-GARCH-t2*	*MRS-GARCH*
μ^1	-0.2075 (-0.4550)	0.2243 (0.1470)	-0.0034 (-0.0850)
μ^2	0.0748 (0.3850)	-0.1139 (-0.0850)	0.7895 (1.6980)
$\alpha_0^{(1)}$	0.2309 (1.0690)	1.6181 (1.8710)	0.0650 (0.7160)
$\alpha_0^{(2)}$	0.0356 (0.0770)	0.0000 (0.0000)	10.0000 (7.1360)
$\alpha_1^{(1)}$	0.3688 (0.3430)	0.0000 (0.0000)	0.0000 (0.0000)
$\alpha_1^{(2)}$	0.0000 (0.0000)	0.1187 (0.2500)	0.9284 (2.8610)
$\beta_1^{(1)}$	0.5060 (3.7700)	0.5100 (0.6820)	0.7372 (14.6700)
$\beta_1^{(2)}$	0.9891 (0.9110)	0.8742 (0.9900)	0.0000 (0.0000)
p	0.9865 (10.3110)	0.9193 (0.7020)	0.9317 (62.1830)
q	0.9950 (19.3130)	0.9650 (4.9380)	0.0000 (0.0000)
v^1	5.7129 (9.3140)	3.1173 (1.1780)	
v^2		16.6564 (16.6610)	-
$Log(L)$	-2223.2273	-2219.7808	-2251.9334

NOTE: each GARCH Model has been estimated with a normal and a student's t distribution. The superscripts indicate the regime. T-statistics are in parentheses.

RMSE for Alternative Models Used to Predict Market Option Premiums

	Normal Distribution				*t-ditribution*				
Strike	*GARCH*	*EGARCH*	*GJR*	*MRS*	*GARCH*	*EGARCH*	*GJR*	*MRS-t1*	*MRS-t2*
370	0.86	0.88	0.82	0.85	0.86	0.87	0.88	0.86	0.80
400	0.84	0.79	0.70	0.71	0.81	0.79	0.81	0.79	0.97
420	0.97	0.85	0.72	0.73	0.93	0.88	0.89	0.91	1.18
450	1.60	1.29	0.88	0.96	1.57	1.46	1.42	1.43	1.69
470	2.35	1.84	1.03	1.17	2.41	2.25	2.22	2.09	2.09
500	3.80	2.81	1.55	1.39	4.17	3.91	3.84	3.42	2.56
520	4.83	3.43	2.14	1.46	5.60	5.26	5.24	4.53	2.78
550	6.44	4.30	3.20	1.70	8.20	7.73	7.78	6.68	3.04
570	7.60	4.92	3.93	2.01	9.99	9.61	9.69	8.76	3.48
600	7.76	4.96	4.01	2.05	11.39	11.17	11.19	10.72	3.83
620	6.19	4.22	3.31	1.66	7.30	7.21	7.25	7.13	3.19
650	3.64	2.83	2.10	1.20	3.63	3.54	3.55	3.51	2.16
Mean	3.77	2.68	1.97	1.28	4.51	4.34	4.35	4.05	2.25

Conclusion

The EGARCH or GJR-GARCH model with either a normal or student's t distribution estimates market option prices better than does the basic GARCH model. Thus the GARCH model accounting for the skewness properties in the true distribution of futures price changes appears to result in superior predictive performance.

GARCH-type models with a normality assumption always perform better than models with a student's t distribution. These results are consistent with Myers and Hansen (1993), which found that the normality assumption did not represent the deficiency in Black's model of pricing commodity options.

MRS-GARCH with a normal distributional error has the smallest root mean squared errors among all the GARCH-Type with different error distributions, which suggests that it does really outperform all other models in forecasting option price in multi-step horizons.

When an option is at-the-money, it is difficult to precisely predict option prices