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The Effect of Communicating Ambiguous Risk Information on Choice

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Decision makers are frequently confronted with ambiguous risk information about activities with potential hazards. This may be a result of conflicting risk estimates from multiple sources or ambiguous risk information from a single source. The paper considers processing ambiguous risk information and its effect on the behavior of a decision maker with α-maximin expected utility preferences. The effect of imprecise risk information on behavior is related to the content of information, the decision maker's trust in different sources of information, and his or her aversion to ambiguity.

Key words: α-Maximin Expected Utility, aggregation of expert opinions, ambiguity, Knightian uncertainty, risk communication, trust in information source

Introduction

The last two decades have witnessed increased public attention to various hazards to human health and environment. Climate change, terrorism threats, pollution, and food-borne diseases are just a few examples of hazards that are of great concern to the general public, governments, international organizations, interest groups, academics, and numerous other entities. Media outlets are filled with stories on these and other hazards and their potential consequences. Governments and interest groups around the world are now spending considerable resources on communicating information about different hazards, preventive actions, and possible responses in cases of adverse outcomes.

Communicating information about potential hazards has a substantial effect on perceptions of environmental risks (Smith et al., 1990; Loomis and duVair, 1993; Riddel, Dwyer, and Shaw, 2003; Riddel and Shaw, 2006; Leiss and Powell, 2004), consumption of potentially unsafe foods (Fischhoff and Downs, 1997; Lofstedt, 2006; Shaw, Silva, and Nayga, 2006) and many other activities with potential hazards (Slovic, 1993). Recognizing this fact, many government agencies actively use information programs as substitutes and complements for regulatory schemes. Peters, Covello, and McCallum (1997, p. 43) note that "environmental risk communication has evolved from a management concept to codified legislation." This is characteristic of many other hazards (Johnson and Slovic, 1995). Public announcements, educational programs, and labels with hazard-warning information are examples of risk communication strategies employed by government agencies. Information is transmitted not only through the policy makers' announcements but also through their actions.

Effective risk communication is complicated by the fact that many hazards are characterized by incomplete information and lack of consensus among different sources about the associated risks (Loomis and duVair, 1993). Johnson and Slovic (1995, p. 493) note that "communicating about uncertainty is necessary, because it is a reality of risk assessment and risk management." Similarly, Viscusi (1997, p. 1658) argues that "situations of diverse and conflicting risk information have become increasingly prevalent and are likely to increase in importance."

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Given that the existence of objective probabilities or the ability to determine unique subjective probabilities for many hazards is at best problematic, people often face ambiguous risk information; they are confronted with multiple risk estimates.[1,2] That is, decisions are often made under conditions where decision makers have imprecise prior information about the stochastic nature of the hazard and new information is in the form of multiple risk estimates. This is in contrast to decisions made in environments where decision makers have unique prior (objective or subjective) probabilities and new information is in the form of unique risk estimates. In the former case both *ex ante* (before receiving new information) and *ex post* (after receiving new information) decisions are made in the presence of ambiguity (or uncertainty), while in the latter case both *ex ante* and *ex post* decisions are made under conditions of risk.[3]

In this paper I consider a more general case of decisions under ambiguity and the effect of communicating ambiguous risk information. In the model a distinction is made between risk (known probabilities) and ambiguity (unknown probabilities). The economic distinction between these two environments dates to Knight (1921) and Keynes (1921). Ellsberg (1961) argued that individuals exhibit behavior sensitive to the weight of evidence about probabilities. Such behavior directly contradicts both objective and subjective expected utility theory, and, if descriptive of reality, renders expected-utility theory inappropriate for evaluating situations involving ambiguity.[4,5]

Ellsberg-type behavior has been repeatedly and routinely validated in the experimental and empirical literatures (see, e.g., Camerer, 1995). The perceived importance of these empirically validated violations has led decision theorists to develop a number of models capable of explaining Ellsberg-type behavior. In this paper the decision maker's attitude toward ambiguity is represented in terms of α-maximin expected-utility (α-MMEU) model (see, e.g., Jaffray and Philippe, 1997; Ghirardato, Maccheroni, and Marinacci, 2004; Siniscalchi, 2006). For these preferences, beliefs are represented by a set of probability distributions and prospects are evaluated by calculating a convex combination of minimum and maximum expected utilities on that set of priors. The relative weight placed on the minimum expected utility is interpreted as a measure of ambiguity aversion. A totally ambiguity averse decision maker (or a total pessimist) places all of the weight on the least desirable scenario while a totally ambiguity-tolerant decision maker (or a total optimist) places all of the weight on the most attractive scenario. In addition to explaining Ellsberg-type behavior and being tractable, α-MMEU model has the advantage that subjective expected utility, maximin expected utility (Gilboa and Schmeidler, 1989) and maximax expected utility are easily identifiable special cases.

The existing beliefs and communicated information are modeled as sets of probabilities. This representation of beliefs seems well suited for the purposes of this paper because it (1) allows for disagreement among different sources; (2) does not force a single source to pinpoint a single probability measure in order to represent her knowledge, and (3) provides additional parameters that

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[1] As a concrete example, Riddel, Dwyer, and Shaw (2003) conduct a survey of Southern Nevada households regarding the risks associated with transporting radioactive waste to the proposed Yucca Mountain repository. The authors find that a considerable proportion of respondents in their sample perceive multiple estimates of the risks of transporting radioactive waste and that belief ambiguity has a substantial effect on hypothetical choices.

[2] A decision maker's beliefs may be ambiguous not only because of the conflicting risk information coming from different sources but also as a result of individuating factors affecting the risk. Nguyen et al. (2010) conduct a study of perceptions of hazards associated with drinking water with arsenic. The respondents in their survey were told that the specific mortality risks of arsenic exposure are a function of such confounding factors as daily water consumption, smoking, exposure to second-hand smoke, the use of a filtration system, and current health status. The authors find that the amount of ambiguity perceived by the respondents is significant.

[3] The majority of early studies of the subject used the term "uncertainty" to characterize situations involving imprecise risk information (e.g., Schmeidler, 1989; Gilboa and Schmeidler, 1989). More recent literature tends to use "ambiguity" (e.g., Ghirardato and Marinacci, 2002). It should also be noted that some authors use "imprecision" or "vagueness."

[4] Even more importantly, the type of behavior that Ellsberg described also contradicts probabilistic sophistication in the sense of Machina and Schmeidler (1992).

[5] Although in most examples presented in this paper the decision makers may exhibit probability weighting similar to that in the Allais paradox (Allais, 1953; Quiggin, 1982), the paper examines only the implications of the behavioral traits exhibited in the Ellsberg experiments.

might reflect the degree of consensus or dissensus (Walley, 1982; Moral and del Sagrado, 1997). New information about the hazard is combined with existing beliefs according to the weighted linear aggregation rule Walley (1982); Cano, Moral, and Verdegay-López (1992); Moral and del Sagrado (1997). In addition to a number of desirable properties, this rule has the advantage that it allows for a simple and natural modeling of trust in an information source.

Although the results are general and applicable to a number of hazards, for concreteness' sake and due to the importance of food-borne hazards, the model is set for a consumption of a potentially hazardous food product and is interpreted using the example of a government agency announcing a range of probabilities of adverse outcome. The model generates a number of testable hypotheses. When the set of probabilities communicated by the government agency is more ambiguous than the decision maker's prior beliefs (where an increase in ambiguity of beliefs is modeled by an expansion of the set of probabilities), the agency's communication results in a decrease in consumption of the potentially hazardous food if and only if the decision maker is sufficiently ambiguity averse. Conversely, when the set of probabilities communicated by the agency is less ambiguous than the decision maker's prior beliefs, the agency's communication results in a decrease in consumption if and only if the decision maker is sufficiently ambiguity tolerant. That is, relatively optimistic decision makers respond to less ambiguous risk information by decreasing their consumption of the potentially hazardous food product. In addition to examining the effect of the decision maker's ambiguity aversion, I investigate the effects of trust in the information source and the content of new information on the decision maker's choice.

Examples of Ambiguous Environments

For many hazards, different experts, interest groups, media sources, government agencies and regulated industries have divergent viewpoints. A notable example is the debate over climate change. Morgan and Keith (1995) conducted structured interviews of sixteen leading U.S. climate scientists using "expert elicitation" methods. They found "a rich diversity of expert opinion" and concluded that "overall uncertainty about the geophysics of climate change is not likely to be reduced dramatically in the next few decades" (p. 468A). More recent studies reveal a similar picture. Another example is the controversy regarding genetically-modified foods (GMF). Some scientists, government agencies, and interest groups claim that GMFs are completely safe, while others assert that a very high likelihood exists that GMFs are harmful.

Conflicting risk information from multiple sources is not the only reason why people may face imprecise risk information. Frequently, a single source communicates multiple risk estimates. Government agencies and other sources frequently communicate hazard information in terms of ranges of probabilities (e.g., confidence intervals) and not unique numbers. We often receive messages of the type "there is at most a A% chance of contracting disease X from consuming product Y" or "mortality risk for activity X is between A% and B%." Both announcements inform us of a range of probabilities of adverse outcome. As a more concrete example, in a study by Smith and Desvousges (1990) on the effectiveness of radon risk communication programs, lifetime risk of dying from radon as a function of lifetime exposure was represented as a range of probabilities. These examples contrast with situations where precise risk information is communicated. For example, announcements like "mortality risk associated with activity X is equal to A%" and "the incidence of disease X is 15 out of 100,000 people" inform us of a unique probability of adverse outcome.

Contradictory risk estimates from multiple sources and ambiguous risk information from a single source are frequently the result of scientific uncertainty. There are numerous gaps in scientific knowledge regarding risks for many processes.[6] The National Research Council (National Research Council, 1994) observes that there are many uncertainties in risk assessment and advocates displaying "more realistic estimates of risk to show a range of probabilities." They further argue that "uncertainty analysis is the only way to combat the 'false sense of certainty,' which is caused by a refusal to acknowledge and (attempt to) quantify the uncertainty in risk predictions."[7,8]

Relationship to the Literature

This paper is related to Viscusi, Magat, and Huber (1991), Viscusi and Magat (1992), Viscusi (1997), and Cameron (2005), which examine learning process in the face of divergent risk information. Viscusi, Magat, and Huber (1991), Viscusi and Magat (1992), and Viscusi (1997) consider environmental risk information, while Cameron (2005) examines risk perceptions in the context of climate change. Viscusi and Magat (1992, p. 384) find "strong evidence of ambiguous belief aversion, even after one takes into account the full ramifications of a Bayesian learning process." Similarly, Cameron (2005, p. 64) uncovers "considerable ambiguity aversion." The present paper is at least partially motivated by these findings. In addition to "ambiguous belief aversion" (see Viscusi and Magat (1992) for a definition), these papers test whether people use Bayes law to update their beliefs. Viscusi (1997, p. 1669) discovers "patterns that were altogether inconsistent with a conventional Bayesian learning framework." Cameron (2005, p. 64) finds that "people come close to Bayesian updating for their expected values, but not for their variances."

The present work differs from these papers in a number of important respects. Prior beliefs and communicated risk information in Viscusi, Magat, and Huber (1991), Viscusi and Magat (1992), Viscusi (1997) and Cameron (2005) are represented by unique probability distributions, while here both prior beliefs and new information have more general representations in terms of sets of probabilities. That is, these papers consider aggregation of unique probability distributions while this paper examines aggregation of sets of probabilities. Furthermore, the model of preferences and updating in this paper are fundamentally different from those in the aforementioned papers. Another important distinguishing characteristic of the framework employed here is that it models both the degree of ambiguity and the decision maker's attitude to that ambiguity.

The paper is most closely related to Woodward and Bishop (1997), in which the authors apply Arrow-Hurwicz criterion (Arrow and Hurwicz, 1972) to examine the expert panel problem and derive the implications of the criterion for global warming policies. Both Arrow-Hurwicz criterion and the more general maximin expected utility model of Gilboa and Schmeidler (1989) entail total pessimism on the decision maker's part. The model in this paper allows for any degree of pessimism and constitutes a generalization of both models. Additionally, in the model of "pure uncertainty" examined by Woodward and Bishop (1997), beliefs are given by the set of all probability distributions over possible outcomes. In contrast, any convex set can represent beliefs in the model considered in this paper. Finally, Woodward and Bishop (1997) do not model trust in different information sources, one of the key elements of the present model.

[6] Consider, for example, the toxicity of industrial chemicals. In 1984, the National Academy of Sciences reported that 78% of the chemicals in the highest-volume commercial use did not have minimal toxicity testing (National Research Council, 1984). The Environmental Defense Fund and the U.S. Environmental Protection Agency (1998) much later reported that for the 3,000 highest production-volume chemicals (those with over one million pounds in commerce), 93% lack some basic chemical screening data, 43% only have basic toxicity data, 51% of chemicals on the Toxic Release Inventory lack basic toxicity information, and a large percentage of available information is based only on acute toxicity (U.S. Environmental Protection Agency, 1998).

[7] Currently, many government agencies in the United States use probabilistic risk analysis techniques to quantitatively address variability and uncertainty in risks. For example, under the 1996 Food Quality Protection Act, the U.S. EPA supported the development of tools that produce distributions of risk demonstrating the variability and uncertainty in the results (Thompson, 2002).

[8] Shaw and Woodward (2008) provide a very insightful discussion of the importance of ambiguity in environmental and resource economics. They also present a number of examples of environmental hazards characterized by ambiguity.

The Model

A decision maker allocates a fixed amount of income, I, between goods x and y, with their respective prices given by q and 1. The consumption of y involves no uncertainty about the consumer's health, and so y is referred to as 'safe'. Good y can be thought of as an aggregate of all goods except for x. Good x is a food of uncertain quality. It can be either 'bad', denoted by b, meaning that the consumer contracts a food-borne disease or contaminant, or it can be 'good', denoted by g, meaning that x does not contain any contaminant./footnoteThere is a plethora of consumer concerns associated with consumption of food products. In addition to food-borne bacteria, economic agents may also be concerned about other uncertain health effects of food consumption such as the long-term effects of certain food ingredients or the impact of consuming genetically modified foods. The model in this paper is equally applicable to these situations. Thus, the uncertainty in the model is represented by Nature choosing from the set $\theta = b, g$ which captures all possible events relevant to the decision maker's *ex post* utility. Each element θ of Θ is referred to as a state of Nature.

Food-borne hazards are recognized as a significant public health problem in the United States and abroad. Frequently, consumers have imperfect information about various food safety characteristics (Sparks and Shepherd, 1994; Frewer et al., 1996; Huffman et al., 2004; Lofstedt, 2006). Moreover, consumers' perceptions of the uncertainty surrounding these characteristics are often ambiguous (e.g., Chambers and Melkonyan, 2010).

Formally, the world is ambiguous so that the odds of different states of nature, θ, are not known with precision, and the decision maker's beliefs are characterized by a set of probability distributions over θ:

$$P = \{(p, 1-p) : p \in [\underline{p}, \overline{p}]\}, \tag{1}$$

where $0 \leq \underline{p} \leq \overline{p} \leq 1$, $\underline{p}$ and $\overline{p}$ are called lower and upper probabilities, respectively, and $[\underline{p}, \overline{p}]$ is called a probability interval.

The probabilities in P can be thought of as representing at least two factors: the decision maker's information on the possible probability distributions and his or her degree of confidence in the existing theories surrounding these probability distributions.[9] So, for example, if there are several competing hypotheses about the stochastic structure that characterize the food-borne hazard, but the decision maker is convinced that only one is truly valid, then P would be a singleton. In this case, the decision maker faces pure risk. In contrast, complete ambiguity or pure uncertainty is characterized by $[\underline{p}, \overline{p}] = [0, 1]$. This represents a situation where the decision maker has no information on the likelihood of bad state occurring other than that it falls somewhere in $[0, 1]$.

Thus, the decision maker's beliefs about the presence of food-borne pathogens are "imprecise" in the sense of Walley (1991). Following Walley (1991), I call the difference between the upper and lower probabilities a degree of imprecision and denote it by:

$$\omega \equiv \overline{p} - \underline{p}. \tag{2}$$

As the name suggests, ω is a measure of information imprecision about probability distribution over the states of Nature. When upper and lower probabilities are used to model probability assessments from different sources (or experts), the degree of imprecision measures the extent of disagreement between these sources (Walley, 1982). The variable ω can also be interpreted as a measure of ambiguity. Alternatively, some researchers have used the variance of probability assessments as a measure of ambiguity (e.g., Nguyen et al., 2010).

[9] Gajdos, Tallon, and Vergnaud (2004) provide a complementary interpretation of the set of probabilities. The decision maker in their model maximizes the minimum expected utility computed with respect to a subset of the set of initially given priors. The extent to which the set of initially given priors is reduced is a measure of aversion to information imprecision.

The decision maker has α-MMEU preferences:

$$V(x,y) = \alpha \min_{p\in[\underline{p},\overline{p}]}\{pu(x,y;b) + (1-p)u(x,y;g)\} + (1-\alpha)\max_{p\in[\underline{p},\overline{p}]}\{pu(x,y;b) + (1-p)u(x,y;g)\}, \tag{3}$$

where $\alpha \in [0,1]$ is the parameter characterizing the decision maker's attitude to ambiguity and $(u(x,y;\theta)$ is the decision maker's *ex post* utility of consuming $x \geq 0$ and $y \geq 0$ when state $\theta \in \Theta$ materializes. The α-MMEU preference structure is a generalization of Arrow-Hurwicz criterion (Hurwicz, 1952; Arrow and Hurwicz, 1972). It is also a natural generalization of the maximin and maximax decision rules (Gilboa and Schmeidler, 1989).[10]

A consumer with α-MMEU preferences has beliefs that are represented by the set of probability distributions P as above. This contrasts with subjective expected utility theory where an individual has beliefs represented by a unique probability distribution. α-MMEU reduces to an expected utility preference functional when P is a singleton.

When $\alpha = 1$, α-MMEU preferences have maximin expected utility form (Gilboa and Schmeidler, 1989) which corresponds to total ambiguity aversion (also called total pessimism) on the decision maker's part. An MMEU decision maker is totally pessimistic in the sense that, when evaluating stochastic outcomes, he or she always uses the probability distribution that yields the lowest possible expected utility over P. In the present model, an MMEU decision maker bases her actions on the smallest probability of the outcome that x does not contain any contaminant, $(1-\overline{p})$.

In contrast, $\alpha = 0$ corresponds to a decision maker with maximax expected utility preferences and reflects a situation where the decision maker is totally ambiguity tolerant. A decision maker with $\alpha = 0$ focuses all attention on the most optimistic probability distribution. Given that α-MMEU utility is a weighted linear functional of the most pessimistic and most optimistic scenarios, it is natural to call α a measure of ambiguity aversion or a coefficient of pessimism. A decision maker with a larger α is said to be more ambiguity averse (or more pessimistic).[11] A decision maker with $\alpha < 0.5$ is said to be ambiguity averse while a decision maker with $\alpha > 0.5$ is said to be ambiguity loving. Note also that a decision maker with $\alpha = 0.5$ is not ambiguity neutral. In the present model, a decision maker is ambiguity neutral if and only if the set of probabilities P is a singleton.

For the sake of simplicity, I assume that the decision maker's *ex post* utility is given by:

$$u(x,y,\theta) = v(r_\theta x + y), \tag{4}$$

where $\theta \in \{b,g\}$ and $v(\cdot)$ is a strictly increasing, concave and everywhere twice-differentiable function representing the decision maker's attitude toward risk. Thus, the decision maker is risk-averse. It is also assumed that the consumer's welfare is higher when food x is safe; parameters of the model r_b and r_g satisfy $r_g > r_b$. To avoid uninteresting corner solutions, suppose that $r_g > q > r_b$.

The analysis for a risk-neutral or risk-loving decision maker is trivial since in these cases the generic optimization problem has a corner solution. The assumption that the *ex post* utility has the functional form in (4) allows me to cast the decision maker's choice of goods x and y as a standard portfolio problem. Since the *ex post* utility in (4) is a function of a linear combination of goods x and y it may seem that assuming this functional form drives the model's results. The only reason for this assumption is to focus on the main message of the paper, how ambiguity and receiving new and

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[10] The preference functional of a decision maker with maximin expected utility preferences (Gilboa and Schmeidler, 1989) in the context of our model is given by $V(x,y) = min_{p\in[\underline{p},\overline{p}]}\{pu(x,y;b) + (1-p)u(x,y;g)\} = \overline{p}u(x,y;b) + (1 - \overline{p}u(x,y;g)$, where the last equality follows from $u(x,y;g) > u(x,y;b)$ for all (x,y). In contrast, the preference functional of a decision maker with maximax expected utility preferences is given by: $V(x,y) = max_{p\in[\underline{p},\overline{p}]}\{pu(x,y;b) + (1-p)u(x,y;g)\} = \underline{p}u(x,y;b) + (1 - \underline{p}u(x,y;g)$, where the last equality follows from $u(x,y;g) > u(x,y;b)$ for all (x,y).

[11] Note that"more ambiguity averse" is a comparative rather than absolute notion. Thus, a more ambiguity averse decision maker may very well be ambiguity loving.

possibly ambiguous information affect choice, rather than to divert the reader's attention by delving into the mechanics of the cross-partial effects (between x and y) that would need to be invoked to present the main findings.

Substituting the budget constraint $I = qx + y$ into (4), the decision maker's *ex post* utility can be written as:

$$v((r_\theta - q)x + I). \tag{5}$$

The decision maker's preference functional can then be written as:

$$U(x) \equiv v((r_g - q)x + I) - [\alpha\overline{p} + (1-\alpha)\underline{p}][v((r_g - q)x + I) - v((r_b - q)x + I)]. \tag{6}$$

The expression in (6) reveals that an individual with α-MMEU preferences acts, in the present and a number of more general frameworks, exactly the same as an expected utility maximizer facing a level of risk characterized by an objective probability density function $\dot{p} \equiv [\alpha\overline{p} + (1-\alpha)\underline{p}]$. The difference between the two expressions is that under risk decision-maker welfare is a function of the unique probability density function $\dot{p}$, while under ambiguity decision maker welfare depends upon the coefficient of pessimism and the most pessimistic and most optimistic probabilistic scenarios as embedded in $\dot{p}$. This close nexus between the two models also implies by reverse reasoning that any behavior attributable to risk aversion in an expected-utility framework with a degree of riskiness given by some probability density function is also explainable as a mixture of risk aversion and ambiguity aversion for a different belief structure. This fundamental identity between the two models arises from the fact that the expected-utility model posits, arbitrarily, a single belief structure and thus by definition attributes all reactions to uncertainty as arising from risk considerations.

There is also a close mathematical relationship between the present model and the second-order probability approach to represent ambiguity (Marschak et al., 1975; Camerer and Weber, 1992). If one interprets the degrees of pessimism and optimism as the second-order probabilities over the first-order probabilities $\underline{p}$ and $\overline{p}$, then the model in this paper becomes a second-order probability model with expected utility preferences. However, the parameters of this alternative model will have a very different behavioral content. Note also that an appropriate interpretation of the trust parameters introduced in the next section, the degree of ambiguity aversion, α, and the lower and upper probabilities yields a similar mathematical equivalence between the two models.

It is straightforward to verify that $U(x)$ is strictly increasing and strictly concave in x. In what follows assume an interior solution. The optimal consumption of x in this case is implicitly given by:

$$\frac{1 - \alpha\overline{p} - (1-\alpha)\underline{p}}{\alpha\overline{p} + (1-\alpha)\underline{p}} = \frac{(q - r_b)v'((r_b - q)x + 1)}{(r_g - q)v'((r_g - q)x + 1)}, \tag{7}$$

which is Borch's (1962) condition adjusted for the more general α-MMEU preferences. Thus, the α-MMEU decision maker chooses her consumption bundle exactly like an expected utility maximizer with a belief that the probability of bad state is equal to $[\alpha\overline{p} + (1-\alpha)\underline{p}]$. This observation allows for a considerable simplification of the analysis via adoption of standard expected utility theory results. It is also informative to write condition (7) in the form:

$$\frac{1}{\underline{p} + \alpha\omega} - 1 = \frac{(q - r_b)v'((r_b - q)x + 1)}{(r_g - q)v'((r_g - q)x + 1)}. \tag{8}$$

It follows immediately from (7) that consumption of food x is a strictly decreasing function of $[\alpha\overline{p} - (1-\alpha)\underline{p}]$. This, in turn, implies that:

Proposition 1:

(a) Consumption of food x is a decreasing function of both the lower probability $\underline{p}$ and upper probability $\overline{p}$.

(b) A more ambiguity averse decision maker–i.e. a decision maker with a larger index of ambiguity aversion α–consumes a smaller amount of food x.

When the decision maker is totally ambiguity averse ($\alpha = 1$), she completely ignores the lower probability of the bad outcome and her consumption depends only on the largest probability of bad state, $\overline{p}$. In contrast, a total optimist bases her decisions on the lowest probability of bad state. When the decision maker is neither totally ambiguity averse nor totally ambiguity tolerant, her consumption is a strictly decreasing function of both the lower and upper probabilities.

The second part of Proposition 1 is also intuitive. A more ambiguity averse (or, equivalently, less ambiguity-loving) decision maker places a relatively large weight on the pessimistic scenario and as a result consumes less of the food of uncertain quality. The comparative statics for the other parameters of the model are standard and are omitted due to space considerations.

Belief Revision

Consider the decision maker's processing of information communicated by a second party. A government agency, an interest group, a media source, an academic, or a relative can all be sources of information. Results are interpreted using the example of a government agency (called agency) although it should be clear that nothing in the analysis hinges upon that interpretation.

The agency informs the decision maker about the possible probabilities of bad state. That is, the agency announces the set of probabilities $[\underline{p}^c, \overline{p}^c]$ to the decision maker. Alternatively, one could interpret the model of belief revision as reflecting a situation where there are two sources of information with one of the sources communicating lower probability $\underline{p}^c$ while the other communicating upper probability $\overline{p}^c$. I focus on the former interpretation for the sake of compactness.

Drawing from the literature on the aggregation of expert opinions, it is assumed that the decision maker aggregates this new information and her prior beliefs $[\underline{p}, \overline{p}]$ according to the weighted linear aggregation rule (Walley, 1982; Cano, Moral, and Verdegay-López, 1992; Moral and del Sagrado, 1997):

$$[\underline{p}^A, \overline{p}^A] = [\underline{t}\underline{p}^c + (1 - \underline{t})\underline{p}, \overline{t}\overline{p}^c + (1 - \overline{t})\overline{p}], \tag{9}$$

where $\underline{p}^A$ and $\overline{p}^A$ denote the combined (or aggregate) lower and upper probabilities of bad state, while parameters $\underline{t} \in [0,1]$ and $\overline{t} \in [0,1]$ are naturally interpreted as the decision maker's trust in the lower and upper probabilities communicated by the agency. It is assumed that the values of the trust parameters are such that $\overline{t}\overline{p}^c + (1 - \overline{t})\overline{p} \geq \underline{t}\underline{p}^c + (1 - \underline{t})\underline{p}$. This inequality holds when, for example, $\overline{t} = \underline{t}$. When the probabilities $\underline{p}^c$ and $\overline{p}^c$ come from different sources, $\underline{t}$ is the decision maker's trust in the source that announces $\underline{p}^c$ while $\overline{t}$ reflects trust in the source that announces $\overline{p}^c$. Appendix A contains further discussion of the properties of the weighted linear aggregation rule and alternative methods for combining new information with existing beliefs.

The trust parameters $\underline{t}$ and $\overline{t}$ are likely to depend on a number of factors, including the decision maker's propensity to trust others, trustworthiness of the source of information, and the content of new information and its relationship to the decision maker's prior beliefs. A decision maker who tends to trust others will have larger $\underline{t}$ and $\overline{t}$. Similarly, when the source of information is trustworthy a decision maker will place a larger relative weight on the new information. Finally, one may expect that when there is a large disparity between the decision maker's existing beliefs and new information, the latter will be trusted less. To accommodate this latter potential interaction, one would have to explicitly model the dependence of trust on the decision maker's existing beliefs and new information. We leave this extension of the model to future empirical and theoretical research.

When the decision maker uses the same weight for both the lower and upper probabilities (i.e. $\underline{t} = \bar{t} \equiv t$), aggregation rule (9) assumes the form:

$$(10) \qquad [\underline{p}^A, \overline{p}^A] = t[\underline{p}^c, \overline{p}^c] + (1-t)[\underline{p}, \overline{p}] = [t\underline{p}^c + (1-t)\underline{p}, t\overline{p}^c + (1-t)\overline{p}],$$

where t is interpreted as the decision maker's trust in the information provided by the government agency. Thus, for the aggregation rule (10) the decision maker has the same amount of trust in the lower and upper probabilities. For this reason, (10) is called a constant weighting rule. In contrast to (10), the aggregation rule (9) allows for the weights assigned to the lower and upper probabilities to vary so that the government agency may be relatively credible for one of these probabilities and completely untrustworthy for another.

Trust has a central role in the formulation because of its importance in risk communication for various hazards, including those associated with consumption of foods (Frewer et al., 1996; Huffman et al., 2004; Lofstedt, 2006), environmental hazards (Peters, Covello, and McCallum, 1997) and many other hazards. Moreover, risk communication failures are frequently attributed to lack of trust (Slovic, 1993, 1999; Leiss and Powell, 2004).[12]

Before moving to the analysis of the effect of new information on choice, a comment is in order. One may question whether the results and insights of the simple two-state model can be extended to more general frameworks. Appendix B contains a generalization of the model to the case with an arbitrary finite number of states of Nature and provides a sketch of how the results for the two-state case can be generalized.

The Effect of New Information on Choice

The decision maker combines the new information with her existing beliefs according to (6 and subsequently makes her consumption decision. As was demonstrated, consumption of food x is a strictly decreasing function of $[\alpha\overline{p} + (1-\alpha)\underline{p}]$. Hence, communication of $[\underline{p}^c, \overline{p}^c]$ results in a decrease in consumption of food x if and only if:

$$(11) \qquad \alpha\overline{p} + (1-\alpha)\underline{p} \leq alpha\overline{p}^A + (1-\alpha)\underline{p}^A,$$

where $\underline{p}^A$ and $\overline{p}^A$ are given by (9). This immediately implies that:

Proposition 2:

Communication of $[\underline{p}^c, \overline{p}^c]$ results in a decrease in consumption of the potentially hazardous food if and only if:

$$(12) \qquad \alpha\bar{t}[\overline{p} - \overline{p}^c] \leq (1-\alpha)\underline{t}[\underline{p}^c - \underline{p}].$$

[12] For an analysis of the literature on trust in various information sources, see for example Chryssochoidis, Strada, and Krystallis (2009).

Table 1. The Effect of New Information on Choice

Relationship between prior beliefs and communicated information	Change in consumption
$\underline{p}^c \leq \underline{p} \leq \overline{p} \leq \overline{p}^c$	$\downarrow$ iff large α and/or large $\bar{t}$ and/or small $\underline{t}$
$\underline{p} \leq \underline{p}^c \leq \overline{p}^c \leq \overline{p}$	$\downarrow$ iff small α and/or small $\bar{t}$ and/or large $\underline{t}$
$\underline{p} \leq \underline{p}^c \leq \overline{p} \leq \overline{p}^c$	$\downarrow$
$\underline{p} \leq \overline{p} \leq \underline{p}^c \leq \overline{p}^c$	$\downarrow$
$\underline{p}^c \leq \underline{p} \leq \overline{p}^c \leq \overline{p}$	$\uparrow$
$\underline{p}^c \leq \overline{p}^c \leq \underline{p} \leq \overline{p}$	$\uparrow$

Notes: "$\downarrow$" and "$\uparrow$" denote a decrease and increase in consumption, respectively.

Proposition 2 implies that under constant weighting with $\underline{t} = \bar{t} > 0$ (i.e., when the decision maker's trust in the upper and lower probabilities is the same), communicating $[\underline{p}^c, \overline{p}^c]$ results in a decrease in consumption of the potentially hazardous food if and only if:

$$\alpha[\overline{p} - \overline{p}^c] \leq (1 - \alpha)[\underline{p}^c - \underline{p}]. \tag{13}$$

Thus, if the decision maker uses constant weighting, then consumption decreases or increases following communication of $[\underline{p}^c, \overline{p}^c]$ are independent of the decision maker's trust in the agency's information. In other words, under constant weighting the qualitative effect of communication of ambiguous risk information depends on the content of information and not on whether that information is trusted. Hence, when probabilities $\underline{p}^c$ and $\overline{p}^c$ arrive from different sources and the decision maker trusts them equally, whether the decision maker will decrease or increase her consumption following these announcements depends on the relationship between $\underline{p}^c$ and $\overline{p}^c$ and the decision maker's existing beliefs but not on the trust in the two sources. Under non-constant weighting, the qualitative effect of uncertainty communication depends on trust in both the lower and upper probabilities. In what follows the general case of non-constant weighting function is considered unless explicitly stated.

Proposition 2 also implies that if the decision maker is totally ambiguity averse, communicating $[\underline{p}^c, \overline{p}^c]$ results in a decrease in consumption of the potentially hazardous food if and only if $\overline{p} \leq \overline{p}^c$. Since a totally ambiguity averse decision maker places all of the weight on the upper probability of adverse outcome, communication of $[\underline{p}^c, \overline{p}^c]$ results in a decrease in consumption of the potentially hazardous food if and only if the upper probability of adverse outcome communicated by the government agency is greater than the decision maker's prior upper probability. In other words, the only thing that matters for the total pessimist is the relationship between the maximum prior probability of adverse outcome (the decision maker's worst possible prior scenario) and the maximum probability communicated by the agency (the worst possible scenario announced by the agency). The reverse is true for the absolute optimist, who places all of the weight on the upper probability of good state. Specifically, if the decision maker is totally ambiguity tolerant, communication of $[\underline{p}^c, \overline{p}^c]$ results in a decrease in consumption of the potentially hazardous food if and only if $\underline{p} \leq \underline{p}^c$.

It also follows from Proposition 2 that the effect of new information on the decision maker's consumption depends on the relationship between the new information $[\underline{p}^c, \overline{p}^c]$ and the decision maker's existing beliefs $[\underline{p}, \overline{p}]$. There are six permutations of possible rankings of these lower and upper probabilities (see Figure 1). The following corollary describes the effect of communication of

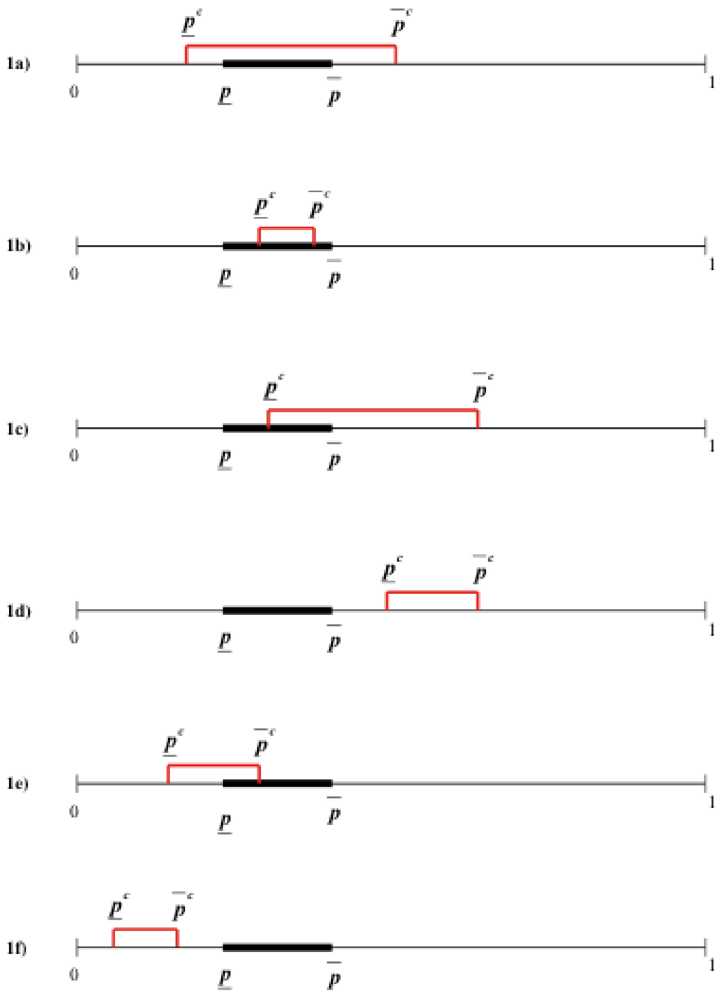

Figure 1. Prior Beliefs and Communicated Information

$[\underline{p}^c, \overline{p}^c]$ on consumption for each of these scenarios. Table 1 summarizes the results reported in the corollary.

Corollary 1:

(a) When $[\underline{p}^c, \overline{p}^c] \supseteq [\underline{p}, \overline{p}]$ (Figure 1a), communication of $[\underline{p}^c, \overline{p}^c]$ results in a decrease in consumption of the potentially hazardous food if and only if:

(14) $$\frac{\alpha}{1-\alpha}\frac{\overline{t}}{\underline{t}}(\overline{p}^c - \overline{p}) \geq (\underline{p} - \underline{p}^c);$$

that is, if and only if the decision maker is sufficiently ambiguity averse (large α) and/or the measure of trust in the agency's upper probability is sufficiently large (large $\overline{t}$) and/or the measure of trust in the agency's lower probability is sufficiently small (small $\underline{t}$).

(b) When $[\underline{p}^c, \overline{p}^c] \supseteq [\underline{p}, \overline{p}]$ (Figure 1b), communication of $[\underline{p}^c, \overline{p}^c]$ results in a decrease in consumption of the potentially hazardous food if and only if:

$$\frac{\alpha}{1-\alpha}\frac{\bar{t}}{\underline{t}}(\overline{p} - \overline{p}^c) \leq (\underline{p}^c - \underline{p}); \tag{15}$$

that is, if and only if the decision maker is sufficiently ambiguity tolerant (small α) and/or the measure of trust in the agency's upper probability is sufficiently small (small $\bar{t}$) and/or the measure of trust in the agency's lower probability is sufficiently large (large $\underline{t}$).

(c) When $\underline{p} \leq \underline{p}^c \leq \overline{p} \leq \overline{p}^c$ (Figure 1c) or $\underline{p} \leq \overline{p} \leq \underline{p}^c \leq \overline{p}^c$ (Figure 1d), communication of $[\underline{p}^c, \overline{p}^c]$ results in a decrease in consumption of the potentially hazardous food.

(d) When $\underline{p}^c \leq \underline{p} \leq \overline{p}^c \leq \overline{p}$ (Figure 1e) or $\underline{p}^c \leq \overline{p}^c \leq \underline{p} \leq \overline{p}$ (Figure 1f), communication of $[\underline{p}^c, \overline{p}^c]$ results in an increase in consumption of the potentially hazardous food.

Parts (a) and (b) of Corollary 1 cover the scenarios where the probability intervals $[\underline{p}, \overline{p}]$ and $[\underline{p}^c, \overline{p}^c]$ are nested in each other. The nested probability interval is called less ambiguous.[13] Part (a) of Corollary 1 represents situations where the set of probabilities $[\underline{p}^c, \overline{p}^c]$ communicated by the agency is more ambiguous than the decision maker's prior beliefs $[\underline{p}, \overline{p}]$. In contrast, in part (b) the probability interval communicated by the agency is less ambiguous than the decision maker's prior beliefs.

When the information announced by the agency is less imprecise than the decision maker's prior beliefs (part (a) of Corollary 1), the new information contains both more and less attractive possibilities than the probabilistic scenarios contemplated by the decision maker prior to the agency's communication. Since a sufficiently pessimistic decision maker places more weight on the less attractive scenario announced by the agency she decreases her consumption (part (a) of Corollary 1). Similarly, if the decision maker's trust $\bar{t}$ in the "bad news", represented by $\overline{p}^c$, is relatively high and/or her trust in the "good news", represented by $\overline{p}^c$, is relatively small, the decision maker will reduce her consumption of the hazardous food product. Note that part (a) also encompasses the cases where the decision maker has unambiguous prior beliefs and the agency's announcement results in *ex post* ambiguous beliefs (when the agency announces an imprecise range of probabilities and the decision maker has at least some trust in the agency's communication).

At first glance the results of part (b) may seem counterintuitive, but a close examination reveals the factors that drive the result. A relatively ambiguity tolerant decision maker places a larger weight on the lower than on the upper probability. Since $\underline{p}^c \geq \underline{p}, \overline{p}^c \leq \overline{p}$, and a relatively ambiguity tolerant decision maker places a greater weight on the part of the agency's communication that points to a greater hazard ($\underline{p}^c$), a sufficiently ambiguity tolerant decision maker decreases consumption of the potentially unsafe food. The impact of the trust parameters on the qualitative effect of new information on consumption of x may also seem counterintuitive. Note, however, that the "bad news", represented by $\overline{p}^c$, is more positive than the decision maker's prior belief $\overline{p}$, while the "good news", represented by $\underline{p}^c$, is more negative than the decision maker's prior belief $\underline{p}$. Hence, the decision maker will decrease her consumption of x if she places relatively little amount of trust in the more positive news $\underline{p}^c$ and/or has relatively high amount of trust in the more negative news $\underline{p}^c$.

[13] Note that if one of the sets $[\underline{p}, \overline{p}]$ and $[\underline{p}^c, \overline{p}^c]$ is nested in the other then the nested set has a smaller degree of imprecision. The reverse is obviously not true; inclusion of one of the sets in the other is not implied by a smaller degree of imprecision of the former set. The terminology "more ambiguous" and "less ambiguous" is reserved for characterizing nestedness of the sets and not the relationship between their degrees of imprecision.

In the two cases considered in part (c) of Corollary 1 the lower and upper probabilities announced by the agency are larger than the corresponding prior probabilities of the decision maker. Thus, the agency's announcement unambiguously informs of a greater hazard and results in a reduction in consumption of the potentially unsafe food. Part (d) of Corollary 1 covers the reverse case, where the lower and upper probabilities are smaller than the corresponding prior probabilities of the decision maker.

To gain further insight into Corollary 1, suppose the decision maker completely trusts the agency's announcements ($\underline{t} = \bar{t} \equiv 1$)). The decision maker's prior and posterior beliefs then coincide, respectively, with the agency's prior and posterior assessments of possible probabilities of bad state. The agency may over time acquire knowledge that will allow the decision maker to narrow down the set of possible theories about the stochastic process governing the occurrence of bad state. Part (b) of Corollary 1 covers these scenarios. In contrast, part (a) reflects situations where the agency's and hence the decision maker's beliefs about the stochastic process become more ambiguous.

To demonstrate the difference in the mechanisms affecting behavioral responses to ambiguous and unambiguous risk information, I analyze two polar cases; the degree of imprecision in the first case is equal to its largest feasible value of 1, while in the second it is equal to the smallest feasible value of 0. First, consider the case where the agency announces that it lacks any information on the odds of bad state. That is, the agency's announcement is completely ambiguous, $[\underline{p}^c, \overline{p}^c] = [0,1]$, and the degree of imprecision is equal to 1. It may seem unrealistic that the agency will literally make such an announcement. Still, it is plausible that some types of announcements may be interpreted as $[\underline{p}^c, \overline{p}^c] = [0,1]$.[14] The probability interval $[\underline{p}^c, \overline{p}^c] = [0,1]$ could also reflect a situation where one source claims that the food is perfectly safe while the other claims that it is contaminated.

Corollary 2:

If the agency announces $[\underline{p}^c, \overline{p}^c] = [0,1]]$, then:

(a) the totally ambiguity averse decision maker reduces consumption of good x;
(b) the totally ambiguity loving decision maker increases consumption of good x;
(c) the decision maker reduces consumption of good x if and only if:

$$\frac{\alpha}{1-\alpha}\frac{\bar{t}}{\underline{t}}(1-\overline{p}) \geq \underline{p}; \tag{16}$$

that is, if and only if the decision maker is sufficiently ambiguity averse (large α) and/or the measure of trust in the agency's upper probability is sufficiently large (large $\bar{t}$) and/or the measure of trust in the agency's lower probability is sufficiently small (small $\underline{t}$) and/or her prior lower and upper probabilities are sufficiently small (small $\underline{p}$ and $\overline{p}$).

Note also that under constant weighting the decision maker reduces consumption of the potentially hazardous food in response to $[\underline{p}^c, \overline{p}^c] = [0,1]$ if and only if $\alpha - \underline{p} \geq \alpha\omega$; that is, if and only if the prior degree of imprecision is sufficiently small.

Finally, consider the effect of risk communication on consumption (i.e. a change in consumption resulting from communication of unambiguous risk information) in the form of a single probability of bad state, $\underline{p}^c = \overline{p}^c \equiv p^c$.

Corollary 3:

[14] One could even conjecture that announcements of "no comment" and "will report after a thorough investigation" types can sometimes be perceived as $[\underline{p}^c, \overline{p}^c] = [0,1]$.

Suppose $\underline{p}^c = \overline{p}^c \equiv p^c$ and $\underline{p} \leq p^c \leq \overline{p}$. Then the decision maker decreases consumption of the potentially hazardous food if and only if:

$$\lambda \overline{p} + (1 - \lambda)\underline{p} \leq p^c \tag{17}$$

where $\lambda \equiv \frac{\alpha\bar{t}}{\alpha\bar{t}+(1-\alpha)\underline{t}}$. Thus, the decision maker reduces consumption of the potentially hazardous food if and only if the decision maker is sufficiently ambiguity loving (small α) and/or the measure of trust in the agency's upper probability is sufficiently small (small $\bar{t}$) and/or the measure of trust in the agency's lower probability is sufficiently large (large $\underline{t}$).

It follows from Corollaries 2 and 3 that pessimists and optimists react differently in these two polar cases. Relatively pessimistic decision makers reduce consumption in response to completely ambiguous announcements but increase consumption in response to risk communication. Optimists react differently. They increase consumption in response to completely ambiguous announcements but reduce consumption in response to communication of precise probabilities. Relatively optimistic decision makers place most of the weight on the lower probability of adverse outcome. Since communication of p^c results in an increase in the lower probability $(\underline{p}^A \geq \underline{p})$, optimists reduce their consumption in response to risk communication. Finally, decision makers that put relatively little trust in "bad news" and relatively high trust in "good news" increase consumption in response to completely ambiguous announcements but decrease consumption in response to risk communication.

Discussion and Conclusion

Decision makers are frequently confronted with vague risk information about activities with potential hazards. This may be a result of conflicting risk estimates coming from multiple sources or ambiguous risk information coming from a single source. Understanding how people view a hazard and react to new information when existing beliefs and new information are given by non-unique probabilities is important for both positive and normative reasons. In this paper I have analyzed the effect of ambiguous risk information on the decision maker's behavior. The effect of imprecise risk information on behavior is related to the content of information, the decision maker's trust in different sources of information, and her aversion to ambiguity (imprecision of information).

For many hazards, decision makers often overestimate likelihoods of adverse outcomes. In a study of perceptions of risks associated with transporting radioactive waste to the proposed Yucca Mountain repository, Riddel, Dwyer, and Shaw (2003) and Riddel and Shaw (2006) find that the risk estimates for both "certain" respondents (those who report a unique probability of adverse outcome) and "uncertain" respondents (those who report a range of probabilities of adverse outcome) are thousands of times higher than the risks found in the Department of Energy studies. Under these circumstances, federal and state agencies might find it advantageous to communicate the hazards of nuclear waste transport to the public with the objective of mitigating the adverse economic effects of nuclear waste transport. A public education program in this case will correspond to the scenario characterized in part (d) of Corollary 1, where, compared to the decision maker's prior beliefs, the agency's announcement unambiguously informed the decision maker of a lower hazard of adverse outcome.

Translating the results in the preceding sections into the framework of Riddel, Dwyer, and Shaw (2003) and Riddel and Shaw (2006), we obtain that the net benefit to their decision maker from staying in her/his present location will increase as a result of this communication and the increase will be larger for those decision makers that have relatively high trust in the agency's communication. Moreover, if the outside source communicates risk the increase will tend to be larger for decision makers that are relatively more ambiguity averse (or pessimistic). The intuition behind the latter result is as follows. A relatively pessimistic decision maker places most of the weight on the least desirable probabilistic scenario. Hence, following an announcement that the likelihood of an adverse outcome is smaller than any probabilistic scenario she was entertaining prior to the communication, this decision maker will revise her probability of adverse outcome by a larger amount than a more optimistic decision maker.[15]

There are also a variety of circumstances where decision makers receive new information that contains both more and less attractive possibilities than their prior beliefs. Cameron (2005) examines the perceptions regarding climate change where respondents were informed of average temperature forecasts made by the government scientists and environmental groups for the 2011-2020 period. In all of the treatments, temperature forecasts made by the environmental groups exceed those of the government scientists. Moreover, the average prior temperature forecast by respondents falls between these two values. Thus, the relationship between the respondent's prior beliefs and the information provided by the government scientists and environmental groups roughly corresponds to the scenario examined in part (a) of Corollary 1. Cameron (2005) finds that following the receipt of new information the respondents' temperature forecast on average increases. The results in the present paper suggest that this type of response is expected when respondents are sufficiently ambiguity averse (or pessimistic) and/or the measure of trust in the environmental group's forecast is relatively large and/or the measure of trust in the government scientists' forecast is relatively small.

The paper's results also provide a number of other insights into the potential effects of different risk communication strategies. Consider, for example, an agency that decides between informing the public about the "best guess" of the likelihood of some food-borne hazard versus the whole range of probabilistic scenarios. Suppose, for concreteness, that both the agency's "best guess" and its range of probabilistic scenarios fall within the set of probabilities characterizing the decision makers' prior beliefs. Consider two behavioral extremes. First, suppose that all of the decision makers in the population targeted by the risk communication strategy are complete pessimists. Then, both communication strategies will lead to an increase in the consumption of the food product with the "best guess" strategy resulting in a larger increase. On the other hand, if all of the decision makers are complete optimists then both strategies will lead to a decrease in the consumption of the food product and the strategy where the agency announces the whole range of probabilistic scenarios will result in a smaller decrease in consumption.

The developed methodology has a wide range of other applications. In addition to those discussed above, the methodology seems to be applicable to decision making regarding occupational choice, prevention of terrorist attacks, financial investments, and education choices. Contingent valuation methods, which are explicitly or implicitly based on the expected utility model, constitute another important application. Frequently, subjects in these studies have little prior information regarding the characteristics of an environmental amenity (ambiguous prior beliefs), and the information provided by the researcher may send an ambiguous signal to the subjects. It is important to understand how this ambiguity will affect reported values of environmental amenities. Thus, the next natural step in this research agenda is to test predictions of the model using naturally occurring or experimental data.

Finally, one may wonder whether the model lends itself to empirical applications. In fact, the main reason for developing a simple two-state model was to have a framework readily amenable to empirical analysis. A number of recent studies develop methodologies for using laboratory and

[15] Formally, suppose the outside source communicates risk, so that $\underline{p}^c = \overline{p}^c \equiv p^c$, that unambiguously informs the decision maker of a lesser hazard than her prior beliefs ($p^c < \underline{p}$). Assume also that $\underline{t} = \bar{t} \equiv t$. The change in the weighted probability as a result of receiving new information is given by $-t(\alpha\overline{p} + (1-\alpha)\underline{p} - p^c) < 0$. The effect of the degree of ambiguity aversion on the change in the weighted probability as a result of communication of new information is equal to $-t(\overline{p} - \underline{p}d\alpha < 0$. Hence, a relatively pessimistic decision maker will decrease her probability of adverse outcome by a larger (in absolute terms) amount.

naturally-occurring data to elicit decision makers' beliefs and their attitudes toward ambiguity (see, e.g., Halevy, 2007; Ahn and Kariv, 2009; Andersen et al., 2010). These methods can be applied to the present model to elicit these behavioral traits. The next step in an empirical application of the model would be to recover the trust parameters. This could be potentially accomplished by providing the subjects with ambiguous risk information and subsequently eliciting the subjects' new willingness to pay for a good or amenity of uncertain quality.[16] The relationship between the original and new willingness to pay will provide estimates of the trust parameters.

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[16] A similar procedure is utilized by Melkonyan and Rollins (2010), who conduct wine-tasting experiments to estimate the effect of trust in wine ratings and other information on willingness to pay for a bottle of wine.

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Appendix A: Properties of the Weighted Linear Aggregation Rule

The method of weighted linear aggregation of convex sets of probabilities (Walley, 1982; Cano, Moral, and Verdegay-López, 1992; Moral and del Sagrado, 1997) is a generalization of the linear opinion pool (Laplace, 1951; Stone, 1961) to sets of probability distributions. In addition to aggregation of single probability functions and convex sets of probabilities, this method has been used to aggregate belief functions (Moral and del Sagrado, 1997). The weighted linear aggregation method is based on the Dempster-Shafer theory of evidence (Dempster, 1968; Shafer, 1976) and is fundamentally non-Bayesian (Nau, 2002).

The weighted linear aggregation method satisfies a number of attractive properties. It is independent of the order of aggregation. It also satisfies the unanimity property: the combined set of probabilities coincides with the decision maker's prior beliefs if the latter is equal to the set of probabilities communicated by the government agency. That is, if $[\underline{p}^c, \overline{p}^c] = [\underline{p}, \overline{p}]$, then $[\underline{p}^A, \overline{p}^A] = [\underline{p}, \overline{p}]$. The restriction of aggregation rule (9) to singleton sets of probability distributions (i.e. the linear opinion pool) satisfies the marginalization property. The latter is a requirement that the marginal probability distribution obtained from the following two procedures be the same; (i) individual probability distributions are first combined to form a single probability distribution and then some marginalization is performed, and (ii) the marginalization on individual probability distributions is performed first and then the resulting marginal distributions are combined into a single probability distribution.

The weighted linear aggregation of convex sets is one of the many methods to aggregate sets of probabilities. Alternatives to the weighted linear aggregation include the union and intersection of sets of probability distributions. Both methods have a number of undesirable properties. For example, the union of sets of probabilities fails to produce more informative aggregates, while the intersection of sets of probabilities may be empty. Recently, Nau's (2002) introduced two quasi-Bayesian methods for aggregating imprecise probabilities elicited from a group of experts in terms of their betting rates. Both methods consider reconciling differences in the experts' opinions through information exchange. Due to their game-theoretic nature, Nau (2002)'s methods are not readily applicable to the analysis in this paper. Troffaes (2003) proposed a second-order hierarchical model for aggregating expert opinions represented as imprecise probabilities. His approach is based on a number of assumptions that cannot be justified within the framework of the present paper.

An alternative natural candidate for combining new information with existing beliefs for the problem considered here is the prior-by-prior Bayesian updating rule (Gilboa and Schmeidler, 1993; Epstein and Schneider, 2003; Chambers and Melkonyan, 2009). This rule involves updating all priors via Bayes law. Under this approach, specification of the agency's possible announcements and the decision maker's prior beliefs are critical determinants of the effect of new information. Within the general model considered here, there is little ground to place any types of restrictions on the set of possible announcements by the government agency and the decision maker's prior beliefs. Another challenge of this approach is the modeling of trust. It seems that the most natural way to model trust in a framework with prior-by-prior Bayesian updating will be by enriching the state space with a random variable reflecting the agency's trustworthiness. This would lead to a richer space with a more realistic dynamic trust parameter. This addition, however, would considerably complicate the analysis. It would be interesting to examine the effect of uncertainty communication in a model where beliefs are updated by using these "Bayes-type" rules and compare these models' predictions with the present analysis. It would be also at least as important to empirically test the predictive power of these models and the weighted linear aggregation rule examined in the present paper. I leave exploration of these ideas to future research.

Appendix B: The General Model

Let $\Theta = \{1, ..., S\}$ denote a finite state space. Random variables are defined as maps from the state space to $\Re$, the set of reals. Δ denotes the set of all additive probability measures over Θ. A generic element of Δ is denoted by $\pi = (\pi_1, ..., \pi_S)$, where π_S is the probability of elementary event $\{S\}$. The decision maker allocates a fixed amount of income, I between a riskless good, y, and a risky good, x. The prices of the riskless and risky goods are 1 and q, respectively. The decision maker's *ex post* utility in state s is given by $v(r_S x + y)$. The states of nature correspond to different quality levels of good x. Suppose also that $r_1 < r_2 < \cdots < r_S$. Substituting the budget constraint into $v(r_S x + y)$, the consumer's *ex post* utility can be written as $v(z_S)$ where $z_S \equiv (r_S - q)x + 1$. The decision maker's α-maximin expected utility preference functional can be written as:

$$\text{(A1)} \qquad W(v(z)) = \alpha \min\left\{\sum_{S=1}^{S} \pi_S v(z_S) : \pi \in P\right\} + (1-\alpha)\max\left\{\sum_{S-1}^{S} \pi_S v(z_S) : \pi \in P\right\},$$

where P is a nonempty, closed, and convex subset of Δ, $\alpha \in [0,1]$ and $v(z) \equiv [(z_1, ..., v(z_S)]$. P represents the decision maker's prior beliefs. Let:

$$\text{(A2)} \qquad \begin{array}{l} \overline{\pi}(x) \in argmin\left\{\sum_{S=1}^{S} \pi_S v((r_S - q)x + 1) : \pi \in P\right\}, \\ \underline{\pi}(x) \in argmax\left\{\sum_{S=1}^{S} \pi_S v((r_S - q)x + 1) : \pi \in P\right\}, \end{array}$$

Note also that to economize on notation I have subsumed the dependence of the functions $\overline{\pi}$ and $\underline{\pi}$ on $(r_1, r_2, ..., r_S)$, q, I, and P. The decision maker's preference functional can now be written as:

$$\text{(A3)} \qquad \begin{array}{c} \alpha \sum_{S=1}^{S} \overline{\pi}_S(x) v((r_S - q)x + I) + (1-\alpha)\sum_{S=1}^{S} \underline{\pi}_S(x) v((r_S - q)x + I) = \\ \sum_{S=1}^{S} [\alpha\overline{\pi}_S(x) + (1-\alpha)\underline{\pi}_S(x)] v((r_S - q)x + I). \end{array}$$

Thus, similarly to the two-state case, the α-MMEU decision maker for this more general setup chooses her consumption bundle exactly like an expected utility maximizer with beliefs given by the unique probability distribution $[\alpha\overline{\pi}(x) + (1-\alpha)\underline{\pi}(x)]$. Note that in the general case both probability distributions $\overline{\pi}$ and $\underline{\pi}$ may depend on the consumption of good x, which is not the case for the two-state model. However, for a range of widely used belief structures, P, for example, when P is equal to the core of a supermodular capacity (see Schmeidler (1989) for a definition), both $\overline{\pi}$ and $\underline{\pi}$ will be independent of x as well as of all of the parameters of the model. When this independence holds, the optimal choice of consumption of good x is given by:

$$\text{(A4)} \qquad \sum_{S=1}^{S} [\alpha\overline{\pi}_S + (1-\alpha)\underline{\pi}_S]((r_S - q)x + 1),$$

which is a "natural" generalization of condition (7).

The communicated information is represented by the set $P^c \subseteq \Delta$. The decision maker's prior beliefs and the communicated information are aggregated according to the constant weighting rule:

$$\text{(A5)} \qquad P^A = tP^c + (1-t)P = \{\pi^A : \pi^A = t\pi^c + (1-t)\pi, \text{ for some } \pi^c \in P^c \text{ and } \pi \in P\}$$

which is a natural extension of the constant weighting rule (10) for $S = 2$. Moral and del Sagrado (1997) impose additional structure on the sets of prior probabilities and the communicated information to introduce more general aggregation rules. One of these more general rules corresponds to the non-constant weighting rule (9).

The decision maker's preference functional after receiving the new information and updating her beliefs is given by:

$$\text{(A6)} \qquad \begin{aligned} &\alpha \ \min\left\{\textstyle\sum_{S=1}^{S} \pi_S v(z_S) : \pi \in P^A\right\} + (1-\alpha) \ \max\left\{\textstyle\sum_{S=1}^{S} \pi_S v(z_S) : \pi \in P^A\right\} = \\ &\textstyle\sum_{S=1}^{S} [\alpha \overline{\pi}_S^A(x) + (1-\alpha)\underline{\pi}_S^A(x)] v((r_S - q)x + I), \end{aligned}$$

where:

$$\text{(A7)} \qquad \begin{aligned} &\overline{\pi}^x(x) \in argmin\left\{\textstyle\sum_{S=1}^{S} \pi_S v((r_S - q)x + 1) : \pi \in P^A\right\}, \\ &\underline{\pi}^A(x) \in argmax\left\{\textstyle\sum_{S=1}^{S} \pi_S v((r_S - q)x + 1) : \pi \in P^A\right\}, \end{aligned}$$

Similarly to the case of an *ex ante* preference functional, if both the decision maker's prior beliefs and the agency's announcement are equal to the cores of some supermodular capacities, so that P^A is also a core of a supermodular capacity, then $\overline{\pi}^A$ and $\underline{\pi}^A$ will be independent of consumption of good x and the parameters of the model. In this case, it is straightforward to generalize the results of the two-state case to the general framework in this Appendix. This discussion should not be interpreted that the general model is intractable unless one imposes an additional structure on beliefs. The only complication associated with analyzing general belief structures stems from the fact that both prior and posterior beliefs will vary with consumption of x and need to be accommodated appropriately in the determination of the first-order conditions. This seems to be a rather minor hurdle.