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INTEREST RATE AND OUTPUT PRICE UNCERTAINTY AND

INDUSTRY EQUILIBRIUM FOR NONRENEWABLE RESOURCE

EXTRACTING FIRMS

by

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Abstract

We establish convexity of a nonrenewable resource extracting agent's value function in the future interest rate, a random variable. A preference by the agent for future interest rate uncertainty follows. A rational expectations, m identical firm industry equilibrium is characterized and the links between interest rate uncertainty and output price uncertainty are investigated.

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INTEREST RATE AND OUTPUT PRICE UNCERTAINTY AND INDUSTRY EQUILIBRIUM FOR NONRENEWABLE RESOURCE EXTRACTING FIRMS

We established that a non-renewable resource extracting firm with orthodox convex extraction costs preferred output price uncertainty in Hartwick and Yeung [1985]. Here we establish the same result for future interest rate uncertainty and move on to characterize a rational expectations industry equilibrium for m identical firms. Interest rate uncertainty induces output price uncertainty in an m-firm industry so the interest rate shock is a convenient avenue to introduce output price uncertainty at the level of the industry. Of note is the fact that a realized low rate of interest induces an upward jump in industry price and vice versa for a realized high rate of interest. In an industry model, the uncertainty is realized at the level of the firm as a pair of observations, a specific interest rate and a specific output price. In this case, appealing to rational expectations, the output price is a deterministic function of the realized interest rate.

Interest Rate Uncertainty and Non-renewable Resource Extraction

We begin with an investigation on the optimal strategy of a resource owner facing interest rate uncertainty. Let s denote the owner's initial endowment of a homogeneous non-renewable resource stock and r the current rate of interest. At time T_a, a new interest rate will be announced. The probability distribution of interest rate is as follows: the probability of the interest rate being equal to r_i is π_i and the expected mean rate is equal to $\bar{r}$. Hence

$$\sum_{i=1}^{n} \pi_i r_i = \bar{r} \quad \text{and}$$

$$\sum_{i=1}^{n} \pi_i = 1 \tag{1}$$

Let the instantaneous objective function of the resource owner be denoted by

$$f(q(t)) \tag{2}$$

where $q(t)$ is the quantity of the resource extracted.

The function $f(q(t))$ becomes $[P(t)q(t) - c(q(t))]$ for a risk-neutral competitive resource firm facing a given price path $P(t)$ and a cost function $c(q(t))$. For a monopolistic resource firm, the function f becomes $[P(q(t)) \cdot q(t) - c(q(t))]$ where $P = P(q(t))$ is the market demand of the resource. Finally, for a social planner, the instantaneous objective may be represented by the net social welfare brought about by resource consumption in $u(q(t))$.

The objective of the resource owner is to maximize the expected present value of discounted instantaneous objective,

$$\int_0^{T_a} f(q(t))e^{-rt}dt + \sum_{i=1}^{n} \pi_i \int_{T_a}^{\tau_i} f(q_i(t))e^{-r_i t}dt \tag{3}$$

subject to the resource constraint

$$\int_0^{T_a} q(t)dt + \int_{T_a}^{\tau_i} q_i(t)dt = s \qquad \text{for} \quad i=1,2,\ldots n \tag{4}$$

where τ_i is the time when the resource is completely exhausted.[1]

For a given interest rate r and a certain level s, define the function

$$V(s;r) = \max_{q(t)} \int_0^{\tau} f(q(t))e^{-rt}dt \tag{5}$$

subject to

$$\int_0^{\tau} q(t)dt = s \tag{6}$$

<u>Lemma 1</u>: The function $V(s;r)$ is convex in r.

<u>Proof</u>: Define three interest rates r_1, r_2 and $\hat{r}$ such that

$$\hat{r} = \alpha r_1 + (1-\alpha)r_2 \qquad (0 < \alpha < 1) \tag{7}$$

Denote optimal extraction path which maximizes (5) given s and $\hat{r}$ subject to (6) by $\hat{q}(t)$. We have then

$$\int_0^{\hat{\tau}} f(\hat{q}(t))e^{-\hat{r}t}dt - \alpha \int_0^{\hat{\tau}} f(\hat{q}(t))e^{-r_1 t}dt - (1-\alpha) \int_0^{\hat{\tau}} f(\hat{q}(t))e^{-r_2 t}dt$$

$$= \int_0^{\hat{\tau}} f(\hat{q}(t))[e^{-\hat{r}t} - \alpha e^{-r_1 t} - (1-\alpha)e^{-r_2 t}]dt \tag{8}$$

For $t>0$ and r_1 and r_2 distinct and positive the term $[e^{-\hat{r}t} - \alpha e^{-r_1 t} - (1-\alpha)e^{-r_2 t}]$ in (8) is negative since e^{rt} is convex in r. For $t=0$, the term in square brackets is zero. Hence (8) is negative.

Let $q_1(t)$ and $q_2(t)$ denote the optimal extraction path when the interest rate is equal to r_1 and r_2 respectively. One can infer that

$$\int_0^{\tau_1} f(q_1(t))e^{-r_1 t}dt \geq \int_0^{\hat{\tau}} f(\hat{q}(t))e^{-r_1 t}dt$$

and

$$\int_0^{\tau_2} f(q_2(t))e^{-r_2 t}dt \geq \int_0^{\hat{\tau}} f(\hat{q}(t))e^{-r_2 t}dt$$

Using the result stated in equation (8), one obtains

$$V(s;\hat{r}) - V(s;r_1) - V(s;r_2) < 0.$$

Hence the function $V(s;r)$ is strictly convex in r. Q.E.D.

In the Appendix, we establish that for the case of our resource extracting firm facing simultaneous output price and interest rate uncertainty, the latter two being independently distributed, a preference for uncertainty obtains.

Theorem 1:

A resource extracting agent prefers facing a stochastic interest rate with mean $\bar{r}$ beyond T_a to facing a certain interest rate $\bar{r}$ beyond T_a.

Proof: In order to prove Theorem 1, one has to show that the maximized value function stated in (3) is greater than the maximized value function for a program with interest rate equal to r up to T_a and equal to $\bar{r}$ for time $t \epsilon (T_a, \infty)$.

Let $\bar{s}_1$ be the optimally chosen cumulative level of extraction up to time T_a for the program with interest rate equal to r and $\bar{r}$ for the time intervals $[0, T_a]$ and (T_a, ∞) respectively. The maximized value function can be expressed as

$$V(s;r,\bar{r},T_a) = V(\bar{s}_1;r,T_a) + e^{-rT_a} V(s-\bar{s}_1;\bar{r}) \tag{9}$$

Since s_1 is optimally chosen, $\dfrac{\partial V(\bar{s}_1;r,T_a)}{\partial \bar{s}_1} = e^{-rT_a} \dfrac{\partial V(s-\bar{s}_1;\bar{r})}{\partial \bar{s}_1}$ (10)

Let s_1 be the optimally chosen cumulative level of extraction up to time T_a which maximizes the value function (3) and hence the maximized value can be expressed as

$$V(s;r,T_a,\underline{\pi}_i,\underline{r}_i) = V(s_1;r,T_a) + e^{-rT_a} \sum_{i=1}^{n} \pi_i V(s-s_1;r_i) \tag{11}$$

where $\underline{\pi}_i = (\pi_1,\pi_2,\ldots\pi_n)$ and $\underline{r}_i = (r_1,r_2,\ldots r_n)$

Once again, since s_1 is optimally chosen

$$\frac{\partial V(s_1;r,T_a)}{\partial s_1} = e^{-rT_a} \sum_{i=1}^{n} \pi_i \frac{\partial V(s-s_1;r_i)}{\partial s_1} \tag{12}$$

Invoking the fact that s_1 is optimally chosen (for the uncertain program), one infers that

$$\begin{aligned} V(s;r,T_a,\underline{\pi}_i,\underline{r}_i) &= V(s_1;r,T_a) + e^{-rT_a} \sum_{i=1}^{n} \pi_i \; V(s-s_1;r_i) \\ &\geq V(\bar{s}_1;r,T_a) + e^{-rT_a} \sum_{i=1}^{n} \pi_i \; V(s-\bar{s}_1;r_i) \end{aligned}$$

Recalling that V is strictly convex in r, one can infer that

$$\begin{aligned} V(\bar{s}_1;r,T_a) + e^{-rT_a} \sum_{i=1}^{n} \pi_i \; V(s-\bar{s}_1;r_i) &> V(\bar{s}_1;r,T_a) + e^{-rT_a} \; V(s-\bar{s}_1,r) \\ &= V(s;r,\bar{r},T_a) \end{aligned}$$

Q.E.D.

An optimally chosen s_1 has to satisfy condition (12). This leads to the basic intertemporal zero arbitrage condition at T_a, the date of the onset of the uncertain output price.

Lemma 2: If $\{q(t)\}$ defines an optimal interior solution then

$$f'(q(T_a^-)) = \sum_{i=1}^{n} \pi_i f'(q_i(T_a^+) \tag{13}$$

where $f'(q(T_a^-))$ is the left hand limit of $f'(\cdot)$ as t approaches T_a and $f'(q_i(T_a^+))$ is the right hand limit given that the interest rate is equal to r_i

Proof: We substitute in (12) to obtain this result. First we observe that for a candidate path $\{q(t)\}$

$$f'(q(t)) = f'(q(T_a^-))e^{-r(T_a-t)} \qquad t \in (0,T_a) \tag{14A}$$

If there exists a $f'(q_i(T_i))$ which satisfies transversality condition of $\lim_{t \to T_i} f'(q_i(T_i)) \cdot q_i(T_i) = 0$, then

$$f'(q_i(t)) = f'(q_i(T_i))e^{-r_i(T_i-t)} \qquad t \in (T_a,T_i) \tag{14B}$$

These are zero intertemporal arbitrage conditions for path $\{q(t)\}$ on either side of T_a.

The left hand side of (12) can be written as

$$\int_0^{T_a} e^{-rt} f'(q(t)) \frac{dq(t)}{ds_1} dt$$

which using (14A) becomes

$$f'(q(T_{\bar{a}}))e^{-rT_a}\int_0^{T_a}\frac{dq(t)}{ds_1}\,dt \qquad (15)$$

and $\dfrac{dq(t)}{ds_1} = \dfrac{dq(t)}{df'(q(t))}\cdot\dfrac{df'(q(t))}{dq_{T_a^-}}\cdot\dfrac{dq_{T_a^-}}{ds_1}$ where $\int_0^{T_a}\dfrac{dq(t)}{ds_1}\,dt = 1.$

The term on the right hand side of (12) becomes using the same procedure

$$e^{-rT_i}f'(q_i(T_i))\int_0^{T_i-T_a}\frac{dq(t)}{d(s-s_1)}\,dt \qquad (16A)$$

Now $\dfrac{dq(t)}{d(s-s_1)} = \dfrac{dq(t)}{df'(q(t))}\cdot\dfrac{df'(q(t))}{d(T_i-T_a)}\cdot\dfrac{d(T_i-T_a)}{d(s-s_1)}$ where $\int_0^{T_i-T_a}\dfrac{dq(t)}{d(s-s_1)}\,dt = 1.$

Using (14B) one can express (16A) as

$$e^{rT_a}f'(q_i(T_{\bar{a}}^{+}))\int_0^{T_a-T_i}\frac{dq(t)}{d(s-s_1)}\,dt \qquad (16B)$$

Substitution of (15) and (16B) into (12) yields (13).

Q.E.D.

Consider an example. For the case of a competitive firm facing a constant price p and a linear marginal extraction cost schedule $bq(t)$, we have in (14) and (15) respectively

$$p-bq(t) = [p-bq(T_{\bar{a}})]e^{-r(T_a-t)} \qquad t\ \epsilon\ (0,T_a) \qquad (17)$$

$$p-bq_i(t) = pe^{-r_i(T_i-t)} \qquad t\ \epsilon\ (0,T_i) \qquad (18)$$

We obtain $\dfrac{dmc(q(t))}{dq(T_{\bar{a}})} = be^{-r(T_a-t)}$ and $\dfrac{dmc(q(t))}{dT_i} = r\ p\ e^{-r_i(T_i-t)}$. We now define $s-s_1 \equiv \int_0^{T_a}q(t)dt$. Now using (17), we obtain $q(T_{\bar{a}}) = -\dfrac{p}{b}\left[\dfrac{rT_a + e^{-rT_a} - 1}{1-e^{-rT_a}}\right] +$