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A Note on Cost Functions and the Regression Fallacy

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Random variation in output may lead to serious bias in cost function estimates. Despite the availability of simple, consistent instrumental variable estimators to deal with this problem, most empirical studies appear to have ignored this errors-in-variables problem, or dealt with it in an *ad hoc* manner. In this note, an instrumental variable approach is suggested, and applied to the estimation of cost-output relationships for a sample of irrigated citrus farms. Ordinary least squares was found to result in a statistically significant bias. Although this bias did not alter the economic interpretation of the results for the sample used, the bias is likely to be more serious for samples with greater chance variation in output. Thus, it is argued that all cost function studies should explicitly deal with this problem.

Studies of farm size economies utilizing cost-output relationships have almost invariably obtained L-shaped average cost curves, without any evidence of increasing-cost ranges of output (Anderson and Powell 1973). Under the deterministic theory of the firm, optimum farm output occurs at the minimum of a U-shaped average cost curve. Thus, this result implies that farm size is indeterminate, leaving the theory seriously incomplete as a framework or modelling the behaviour of the firm.

The apparent lack of completeness in the theory may be due to deficiencies in the empirical evidence or may point to the need for a more complete theoretical framework. Since the deterministic theory has proved to be an extremely powerful tool for modelling firm behaviour, the empirical evidence should be examined very closely before moving towards a more complete, and presumably more complex, theoretical framework, such as that suggested by Quiggin (1982).

A great deal of the available empirical evidence on size economies, and on other aspects of production technology, is based upon estimated cost functions (Anderson and Powell 1973; McKay, Lawrence and Vlastuin 1980). A problem of errors-in-variables arises in the estimation of cost functions, and can lead to biases, such as the 'regression fallacy' discussed by Walters (1960) and Stigler (1966). The purpose of this note is to point out the potentially serious effects of this bias, and to suggest an approach for dealing with it.

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A simple application is included to investigate the importance of the problem for estimates of size economies. The sample used should be less seriously affected than most other such samples, and so the results provide a lower-bound estimate of the effects of the problem.

In the following section, the nature and effects of the regression fallacy are reviewed. Then, in the third section, the instrumental variable estimator is explained and justified. This estimator is applied in the fourth section, and the conclusions are given in the final section.

The Regression Fallacy

The regression fallacy arises where an explanatory variable is measured with an error. While the output variable considered in economic theory is the expected level of output, the output variable actually observed differs from this by an error resulting from unanticipated output fluctuations. Using actual output as a proxy for expected output thus introduces measurement errors. These errors-in-variables are likely to be particularly serious in agricultural industries where most resources are allocated at the beginning of the growing season before the *ex post* level of output is known and where output tends to be more variable than in other industries.

Whenever the explanatory variables in a regression are measured with an error, it can be shown that the parameter estimates obtained by the application of ordinary least squares will be biased (Johnston 1972, pp. 281–2). It is possible to obtain entirely specious estimates of size economies when cost function estimates are sought. If, for instance, all firms had identical costs c and expected output x, expected average cost in the absence of output fluctuations would be given by the constant $ac = (c/x)$. If output were, instead, subject to random fluctuations, a fallacious estimate of size economies could be obtained by fitting a rectangular hyperbola to observed average costs, *i.e.:*

(1) $AC = a + c/x.$

Where input prices vary within the sample, they should be included in the estimated cost function; this allows estimates of the production technology to be obtained by duality (Binswanger 1974). Since the asymptotic bias of OLS in the presence of measurement errors depends upon the entire data matrix (Judge *et al.* 1980, p. 515), the coefficients on the price variables may also be biased because of the measurement errors on the output variable.

One approach to the problem introduced by the regression fallacy is to reject the use of the cost function approach in favour of alternative approaches such as the production function (e.g. Vlastuin, Lawrence and Quiggin 1982).

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[1] Some inputs can be varied as conditions change within the season, but these inputs are believed to be the exception.

While this approach is appealing, it is vulnerable to specification bias because of the paucity of data on some input variables, such as managerial skill. Griliches (1957) has shown that specification bias may result in biased estimates of the individual regression coefficients and of the scale elasticity. In addition, the scale elasticities which it provides are not measures of size economies unless the production function is homothetic (Hanoch 1975), or unless the scale elasticities are evaluated only along an expansion path. Where the estimation problems introduced by the regression fallacy can be overcome, the use of cost functions appears to offer considerable advantages, both for the traditional purpose of estimating size economies and for investigating the structure of production (Varian 1978). An approach to overcoming this problem is discussed in the next section.

The Instrumental Variable Estimator

Although the regression fallacy is generally acknowledged in empirical cost-function studies, it does not appear to have been related to the econometric literature on estimation in the presence of measurement errors. Two general approaches to estimation in the presence of measurement errors have been suggested in this literature (Johnston 1972, pp. 283–91; Judge *et al.* 1980, pp. 520–50):

(i) the maximum likelihood approach, and

(ii) the instrumental variable approach.

While the maximum likelihood approaches are of interest, they rely upon the introduction of extraneous information about the error variances (Judge *et al.* 1980, p. 528), and such information is unlikely to be available in cost function studies.

Despite the availability of consistent instrumental variable estimators, most cost function studies have used relatively *ad hoc* approaches to the problem. One approach has been to use a single input as the measure of output (e.g., Gibbon 1974; Longworth and McLeland 1972). Another approach has been to average the results for a number of years (Stoeckel 1974) or for a number of farms in cross-section (McKay 1974). In estimating a cost function with a sample in which prices varied, Binswanger (1974, p. 381) excluded the level of output on the assumption that scale effects are Hicks neutral. McKay, Lawrence and Vlastuin (1980, p. 60) omitted the output variable by assuming separability of the cost function, which is equivalent to imposing homotheticity on the production function (Silberberg 1978, p. 308). In a recent cost function study, Ball and Chambers (1982) did not impose these restrictions but included the output variable without taking account of its stochastic nature.

The potential biases involved in the above procedures can be avoided by using an instrumental variable procedure to deal with the regression fallacy directly. The basic instrumental variable approach to dealing with measurement error requires the use of a set of instruments which are uncorrelated with both the measurement error in the explanatory variables and with the error term in the equation, but which are correlated with the true values of the explanatory variables. If a matrix, Z, of such instrumental variables can be found, then the estimator:

$$(2)\ b_1 = (Z'X)^{-1} Z'y$$

will be a consistent estimator of β for the model $y = X\beta + v$.

The selection of the instruments depends upon the type of data available. In a cross-sectional study, the input variables are logical candidates for use as instruments since their levels are largely determined by decisions made at the start of the growing season. The set of predetermined input levels should be strongly correlated with expected output and may be used to generate predicted values of output which are independent of chance output fluctuations. In a time-series analysis, lagged values of output could also be used to form the instruments. Judge *et al.* (1980, pp. 535–43) note some other variables, such as discrete grouping variables, which might be used as instruments.

Since there is more than one input variable, a generalized form of the instrumental variable estimator must be used. This estimator is defined as:

$$(3)\ b_2 = [X'Z\,(Z'Z)^{-1}\,Z'X]^{-1}\ X'Z\,(Z'Z)^{-1}\,Z'y$$

and is equivalent to two-stage least squares (Judge *et al.* 1980, p. 533). In the example which follows, a 2SLS estimator has been used to investigate the seriousness of the bias which results from the use of OLS.

Estimation

A cross-sectional sample for 1979–80 of irrigated region citrus farms drawn from the Australian Horticultural Industry Survey (BAE 1980) was used in the analysis. Because relative output prices were somewhat unrepresentative in 1979–80, an aggregate output value index was calculated using the average real prices (in 1979–80 dollars) prevailing during the previous four years. Only farms with at least 75 per cent of output value derived from citrus were included, in order to confine the sample to farms with similar technology. After exclusion of one obvious outlier, the sample contained 30 observations. Inputs were measured at 1979–80 prices in categories of Labour, Materials and Services, Capital, and Land. The opportunity cost of land and capital was measured using an assumed real interest rate of 3 per cent. The data set used is available on request from the author. This sample is fairly typical of the samples which have been used to investigate size economies (*e.g.*, Longworth and McLeland 1972; Stoeckel 1974; Martin 1977) except that it contains only irrigated farms, for which unanticipated variations in output are likely to be smaller than for dry-land farms.

Initial estimates of the average cost-output relationship were obtained by direct application of OLS to this relationship. The resulting estimates were made for a range of functional forms. Based on the overall fit and the significance of individual coefficients, the inverse function and the log-linear function were the most preferred estimates and only these two are presented in Table 1.

Examination of the residuals from both of these equations suggested that the assumption of homoscedasticity was appropriate. Both of these functions are consistent with the L-shaped curve which has been observed in most studies, although the inverse function would suggest a diminishing cost-output elasticity, while the log-linear form implies a constant elasticity. The nonparametric test suggested by Rao and Miller (1971, p. 109) was used in an attempt to discriminate between the explanatory power of these two functional forms, but the null hypothesis of equality could not be rejected, even at the 10 per cent level.

Table 1: *Cross-sectional Cost-Output Relationships for Citrus Farms* 1979–80 [a]

Ordinary-least squares—	
1. $AC = 0.66 + 12693.05\ Q^{-1}$ (14.16) (10.39)	$R^2 = 0.79$
2. $lnAC = 3.50 - 0.32\ ln\ Q$ (6.93)(−7.27)	$R^2 = 0.65$
Two-stage least squares—	
1. $AC = 0.68 + 11937.68\ Q^{-1}$ (14.26) (9.39)	$R^2 = 0.76$, $F(1,27)$ for bias of *OLS* = 14.12
2. $lnAC = 3.09 - 0.29\ ln\ Q$ (5.91)(−6.23)	$R^2 = 0.58$, $F(1,27)$ for bias of *OLS* = 20.97

[a] Figures in parentheses below the coefficient estimates are t-statistics for the hypothesis that Coeff. = 0.

The two functional forms presented were re-estimated using the 2SLS estimator with combinations of the input variables used as first-stage regressors. In order to avoid unnecessary restrictions on the form of the estimated functions, the first-stage regressions were specified as reasonably flexible functional forms. Thus, the logarithmic function was estimated using a translog production function in the first-stage. The first-stage regressions for the inverse function utilized a functional form which was selected to provide a suitable approximation to the inverse of farm output, *i.e.:*

$$(4)\quad Q = a_o + \Sigma_i\, a_i X_i + \tfrac{1}{2}\Sigma_i \Sigma_j b_{ij}\, X_i^{-\frac{1}{2}}\, X_j^{-\frac{1}{2}}$$

where X_i, X_j are the inputs defined above ($i, j = 1, \ldots, 4$.).

The results obtained using the 2SLS estimator are also presented in Table 1. In both of the equations the 2SLS estimates have a smaller (negative) slope than the OLS estimates. The elasticity of cost with respect to output falls from −0.32 to −0.29 in the log-linear case, and from −0.096 to −0.090 in the inverse function case (at the sample means). While this difference is in the expected direction, it has very little effect on the interpretation of the coefficients in this case.

It is important to be able to determine whether the difference between the OLS and 2SLS estimates is statistically significant. One approach to testing this is to augment the ordinary least squares regressions with the residuals from the first-stage regressions. If the OLS and 2SLS estimates have the same limiting values, then the included residuals will have no additional explanatory power. Thus, the test is based on the null hypothesis that the coefficients on these variables are equal to zero.

For the inverse function and the log-linear function, this test can be undertaken using the *t*-statistic for the single error term included. This *t*-statistic had a value of −3.76 for the inverse function, and −4.58 for the log-linear function. Both of these are significantly different from zero at the 1 per cent level. For other functional forms, such as the quadratic, more than one additional term would be added and an F-test would be required. The estimated F-statistics equivalent to the *t*-statistics given above are presented in Table 1. Both are substantially above the critical value of 7.72 at the 1 per cent level of significance.

Figure 1: AVERAGE COST AS A FUNCTION OF FARM OUTPUT: INVERSE FUNCTION

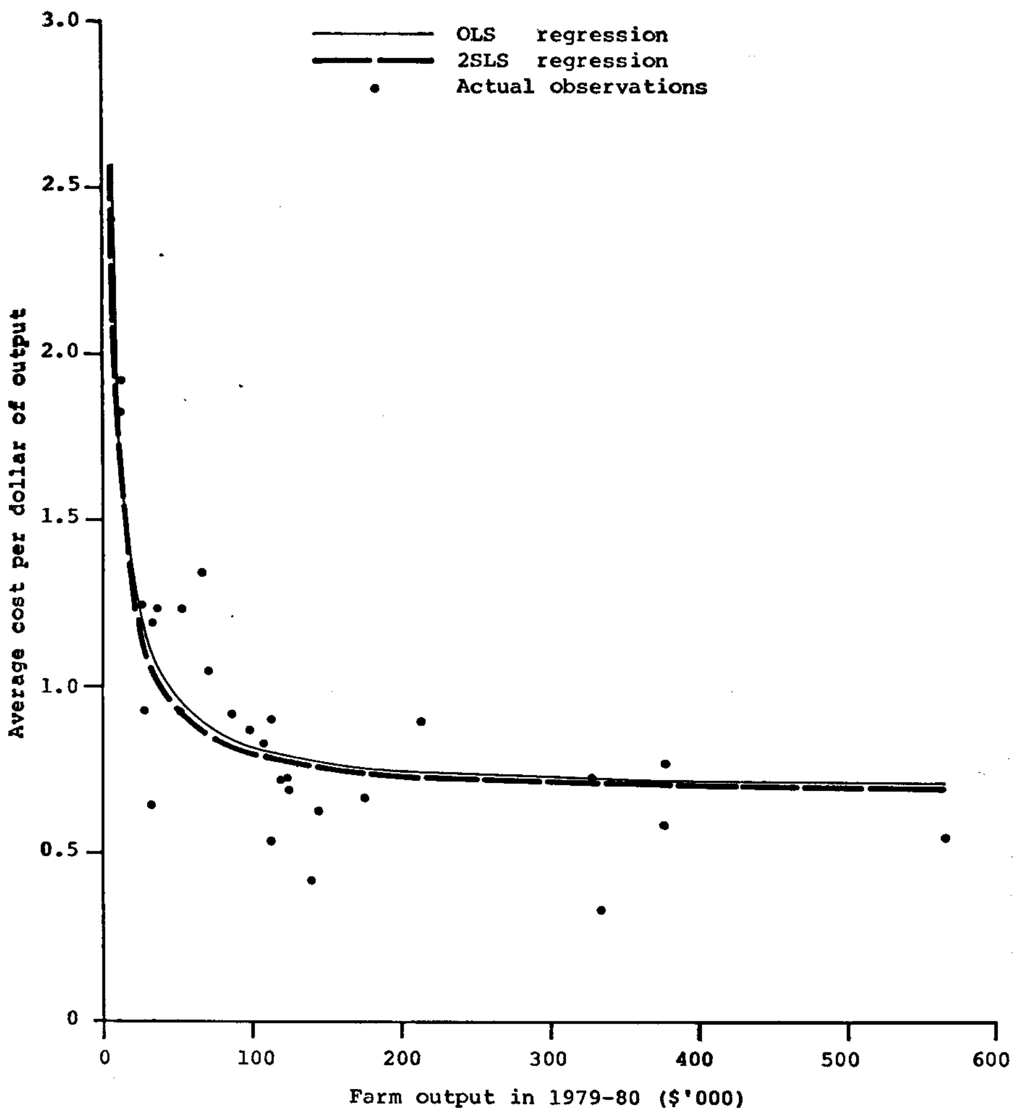
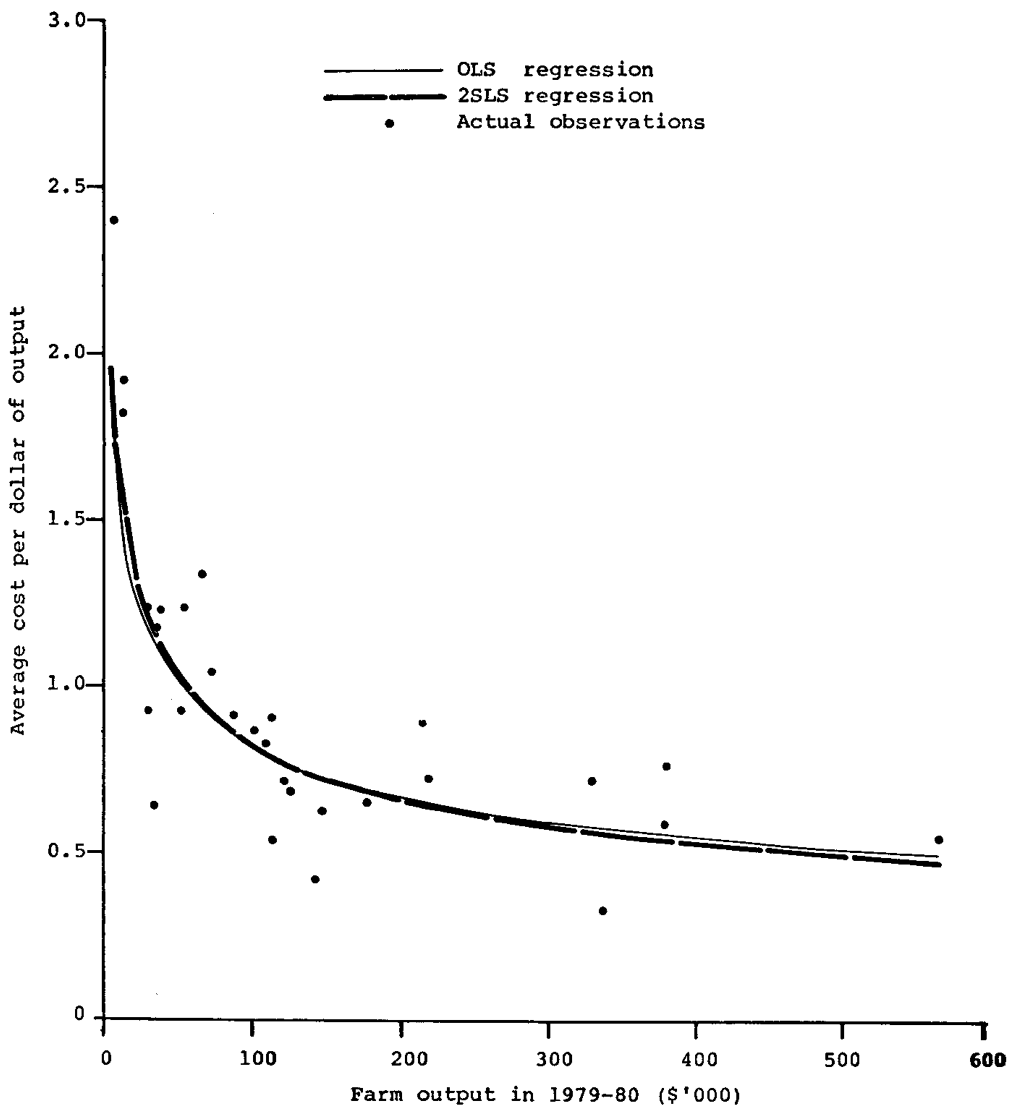


Figure 2: AVERAGE COST AS A FUNCTION OF FARM OUTPUT: LOG-LINEAR



The OLS and the 2SLS estimates have been plotted for the inverse and log-linear functional forms in Figures 1 and 2. Examination of these superimposed plots makes it clear that the difference between the resulting estimates is unlikely to be significant in an economic sense, even though it is statistically significant. Since the data set used in this analysis did not include any variation in input prices, it was not possible to investigate whether the coefficients on price variables (and, hence, the estimated elasticities of factor substitution) would be noticeably biased. This issue would be worth investigating in future studies which utilize time-series data.