PRICE ENDOGENOUS MATHEMATICAL PROGRAMMING MODELS
AND INTEGRABILITY: AN ALTERNATIVE APPROACH

by

Mark A. Peters*
and

Thomas H. Spreen

*Mark A. Peters is a Graduate Research Assistant and Thomas H. Spreen is a Professor, Food and Resource Economics Department, University of Florida, Gainesville, FL.
PRICE ENDOGENOUS MATHEMATICAL PROGRAMMING MODELS

AND INTEGRABILITY: AN ALTERNATIVE APPROACH

by

Mark A. Peters

and

Thomas H. Sreen

*Mark A. Peters is a Graduate Research Assistant and Thomas H. Sreen is a Professor, Food and Resource Economics Department, University of Florida, Gainesville, FL.
ABSTRACT

The methodology utilized in much of the demand systems literature sets out to empirically test consumer theory in a logical positivist manner, but, inevitably the theory fails. If Karl Popper's agenda were consistently applied the theory would cease to exist. Yet, economists are uneasy about destroying old dictums. So, empirically unsupported theoretical constraints are imposed on the data. If the theory is assumed to always hold (an apriorist philosophy) then it should not be tested, and should be imposed from the beginning.
DEMAND SYSTEMS: A NOTE ON METHOD

INTRODUCTION

It has been pointed out by Mark Blaug (1980) and Donald McCloskey (1983) that economists implicitly or explicitly rely on logical positivism as their philosophical doctrine for research. Initially, logical positivists generated synthetic statements that could be proven or disproven empirically. Karl Popper though, felt that verification of a theory was too easy and that falsification was the true test of a theory's merit. Popper used falsification as the foundation for his Demarcation Principle between science and non-science. Science was the business of generating potentially falsifiable synthetic statements. Mark Blaug offers support for Popper's contention: "you can never demonstrate that anything is materially true but you can demonstrate that some things are materially false, a statement which we may take to be the first commandment of scientific methodology" (1980, p.12). Popper reasoned that theories had to be subjected to the most severe empirical tests that could be devised. Theories that were disproven were to be discarded or at least altered. Sprio Latiss has said, "The criterion of goodness of a theory is a function of its falsifiability together with its success in withstanding attempts to falsify it... economists should try and devise from their theories empirical propositions of a high degree of specificity describing conditions the non occurrence of which would falsify the entire theory" (1980, pp. 7-8).

As a logical positivist one can distinguish between three types of theories, (i) theories that have withstood serious tests, (ii) testable theories that await testing and (iii) untestable theories (Hutchison, 1988, p. 177). The first category is analogous to testing the law of demand and the negative slope of a demand function for normal goods. Obviously, the most credence is given to the most empirically unfalsified theories, and this conforms well to the law of demand. A.W. Coats gives a more detailed explanation of 'good' theory by listing eight attributes, the last three
of which are, "congruence with reality" (the ability to explain some empirical knowledge), "testability" (the ability to create synthetic statements which can be falsified) and "relevance to the expectations of the scientific community" (1980, p. 56). It is my contention (and that of Coats) that these three attributes of theory are the most important to economists. This is the point of the story since if falsification is an essential part of our research then we must be prepared to face the consequences. Popper was vehemently opposed to the practices of scientists who qualified results (which were at variance with theory) with ad hoc explanations and counter arguments.

Ludwig von Mises argued that economic theory is true by definition, thereby making tests redundant. The foundation of Mises' philosophy, called apriorism, rests with the assumption of the rationality of human beings and that through introspection economists can arrive at analytic statements about human behaviour. Analytic statements are true by definition since they are the product of pure deduction. Thus, the fundamental propositions of economics are true or hold independently of any experiential knowledge. According to Mises:

Experience tells us something we did not know before and could not learn but for having had the experience. But the characteristic feature of a priori knowledge is that we cannot think of the truth of its negation or of something that would be at variance with it... If we qualify a concept or a proposition as a priori, we want to say: first, that the negation of what it asserts is unthinkable for the human mind and appears as nonsense; secondly, that this a priori concept or proposition is necessarily implied in our mental approach to all the problems concerned... Their truth or validity cannot be proved or refuted... (1962, p. 18)

Furthermore, Mises firmly believed that statistics were merely historical facts describing past happenings for a given area and a given number of people. To him econometrics was a waste of time: "As a method of economic analysis econometrics is a childish play with figures that does not contribute anything to the elucidation of the problems of economic reality" (1962, p.63). So, if one
were a true apriorist, econometrics would be meaningless, or at best merely a statistical characterization of past economic circumstances.

It should be obvious that apriorism and logical positivism are not complementary to one another in doing empirical research. It would seem strange to see both philosophies resident in any study, yet, all too often economists fail to recognize their use of diametrically opposed methodologies. The fundamental purpose of this paper is to show that this is in fact the case in recent work on consumer theory.

DEMAND SYSTEMS AND CONSUMER THEORY

Through the pioneering efforts of economists such as H. Shultz and J.R.N. Stone, the theoretical works of Marshall, Slutsky and Hicks have become falsifiable; falsifiable in the logical positivist sense that the propositions of theory can be tested empirically and refuted. Recent advances in demand systems research lend support to this observation, since economists seem to be forever developing new ways to test consumer theory. In this vein one can trace the birth of demand systems analysis from Stone in 1954 to the AIDS model of Deaton and Muellbauer (1980).

What flexible functional forms (such as the AIDS) seek to do is allow the data to speak for itself, thus enabling comprehensive tests of the theory. Table 1 summarizes the six axioms of consumer theory along with four propositions of demand theory. Proposition one of demand theory is always assumed to hold, while propositions two through four are of somewhat lessor importance. Proposition two, homogeneity, implies that consumers do not suffer from money illusion, while the third proposition, symmetry, results from the Slutsky equation. The negativity restriction stated in proposition four ensures that the own substitution effect of a good is negative (that is, if price rises then less is demanded). Satisfaction of the homogeneity condition does not
Table 1
CONSUMER AND DEMAND THEORY

AXIOMS OF CONSUMER THEORY

1. REFLEXIVITY - EACH BUNDLE IS AS GOOD AS ITSELF
2. COMPLETENESS - ANY TWO BUNDLES CAN BE COMPARED AND RANKED
3. CONSISTENCY - IF A IS PREFERRED OVER B, AND B IS PREFERRED OVER C, THEN A IS PREFERRED OVER C
4. CONTINUITY - "SMOOTH" INDIFFERENCE CURVES
5. NONSATIATION - UTILITY IS NONDECREASING IN ALL ARGUMENTS AND INCREASING IN AT LEAST ONE
6. CONVEXITY - THE UTILITY FUNCTION IS QUASI-CONCAVE

PROPOSITIONS OF DEMAND THEORY

1. ADDING UP - THE BUDGET CONSTRAINT IS SATISFIED
2. HOMOGENEITY - NO MONEY ILLUSION
3. SYMMETRY - CROSS PRICE EFFECTS ON HICKSIAN DEMANDS ARE EQUAL
4. NEGATIVITY - MATRIX OF HICKSIAN PRICE EFFECTS IS NEGATIVE SEMIDEFINITE
imply that symmetry will be satisfied, however, symmetry is a stronger restriction than homogeneity. Symmetry enforces consistency of the consumer's choices, while homogeneity reflects the budget constraint. Negativity is perhaps the most important restriction since it ensures that Hicksian demands are negatively sloped. If symmetry and negativity are falsified then the plausibility of Axioms one to five of consumer theory are diminished greatly since the consumer is no longer behaving "rationally" (See Appendix).

Weak separability is another interesting consequence of demand analysis, since it is necessary for the construction of a two stage budgeting process. Weak separability implies that consumers order their choice set in a manner akin to a utility tree. Thus, the consumer allocates fixed sums of money across broad groups of goods such as food, clothing, housing, and transportation. Subsequently, the consumer reallocates income to individual items such as beef, chicken, milk, eggs, and so on in the food group, and also separately chooses his allocation of gym socks, shoes, and underwear from the clothing group. The price of underwear has no direct effect on the quantity of sirloin steak bought, except that if the price of underwear were to drop appreciably, more real income would be available to spend: possibly on sirloin steak. This is the principle of weak separability. It is common practice to assume that goods being analyzed are weakly separable from those omitted from the study, but some researchers are recognizing the importance of ad hoc groupings and are formally testing for weak separability. While weak separability does not in itself constitute proof or falsification of demand theory, it influences the structure of preferences and the underlying utility function. Weak separability is a powerful assumption since one can seemingly ignore all extraneous cross commodity effects (but not if it is testable!).
were a true apriorist, econometrics would be meaningless, or at best merely a statistical characterization of past economic circumstances.

It should be obvious that apriorism and logical positivism are not complementary to one another in doing empirical research. It would seem strange to see both philosophies resident in any study, yet, all too often economists fail to recognize their use of diametrically opposed methodologies. The fundamental purpose of this paper is to show that this is in fact the case in recent work on consumer theory.

DEMAND SYSTEMS AND CONSUMER THEORY

Through the pioneering efforts of economists such as H. Shultz and J.R.N. Stone, the theoretical works of Marshall, Slutsky and Hicks have become falsifiable; falsifiable in the logical positivist sense that the propositions of theory can be tested empirically and refuted. Recent advances in demand systems research lend support to this observation, since economists seem to be forever developing new ways to test consumer theory. In this vein one can trace the birth of demand systems analysis from Stone in 1954 to the AIDS model of Deaton and Muellbauer (1980).

What flexible functional forms (such as the AIDS) seek to do is allow the data to speak for itself, thus enabling comprehensive tests of the theory. Table 1 summarizes the six axioms of consumer theory along with four propositions of demand theory. Proposition one of demand theory is always assumed to hold, while propositions two through four are of somewhat lessor importance. Proposition two, homogeneity, implies that consumers do not suffer from money illusion, while the third proposition, symmetry, results from the Slutsky equation. The negativity restriction stated in proposition four ensures that the own substitution effect of a good is negative (that is, if price rises then less is demanded). Satisfaction of the homogeneity condition does not
A CHARACTERIZATION OF THE LITERATURE

The past two decades have seen an immense growth in the literature as applied to demand estimation. High performance computing power at a reasonable cost has put this sort of analysis within reach of virtually anyone. The LES (Linear Expenditure System) has been superseded in most cases by the flexible functional forms, (such as the AIDS, translog, and Rotterdam models) because of the restrictiveness of the Stone-Geary utility function.\textsuperscript{2} With the new tools economists have simultaneously ventured forth into the realm of quantification and falsification.

Table 2 summarizes ten studies conducted between 1975 and 1986. Various demand systems were utilized to examine diverse commodities in varying degrees of disaggregation, and of course, to test consumer theory. Six countries are represented in this example, yet only one study unequivocally accepted consumer theory with a single model (Clements and Johnson, 1983). Christensen, Jorgensen and Lau (1975) implemented the translog functional form and tested it with U.S. data over a lengthy period. Symmetry was tested and the theory failed. The authors concluded: "Our results rule out this alternative interpretation and make possible an unambiguous rejection of the theory of demand" (Christensen et al, 1975, p. 381). Interestingly, no one else inferred this when demand theory was falsified by empirical testing.

Table 3 illustrates that in the thirteen instances where symmetry was tested it was rejected ten times, while homogeneity was rejected five out of ten times. The literature contains a battery of arguments as to why the theory has failed: "the failure of homogeneity is not a new result...and can be ascribed to a number of possible causes..." (Deaton and Muellbauer, 1980, p. 320), indeed, the list of explanations abound. An interesting caveat was advanced by Baldwin et al:
<table>
<thead>
<tr>
<th>Author</th>
<th>Demand System</th>
<th>Data and Estimation Period</th>
<th>Model Use</th>
<th>Goods Estimated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Christensen, Jorgensen and Lav (1975)</td>
<td>Translog</td>
<td>U.S. data annual 1929-1972</td>
<td>Introduce Translog and apply various tests to functional form</td>
<td>3 Goods (1) durables (2) nondurables (3) Other services</td>
</tr>
<tr>
<td>Deaton and Muellbauer (1980)</td>
<td>AIDS</td>
<td>U.K. data annual 1954-1974</td>
<td>Introduce AIDS and apply various tests to functional form</td>
<td>8 Goods (1) food (2) clothing (3) housing (4) fuel (5) drink &amp; tobacco</td>
</tr>
<tr>
<td>Clements and Johnson</td>
<td>Rotterdam</td>
<td>Australian data annual 1955-1977</td>
<td>Compute welfare costs of alcohol taxes, and determine why wine consumption has drastically increased</td>
<td>3 Goods (1) beer (2) wine (3) spirits</td>
</tr>
<tr>
<td>Goddard (1983)</td>
<td>LES, AIDS</td>
<td>Canadian data annual/quarterly 1949-1981</td>
<td>Analyze shifts in aggregate food consumption. To predict future food demand levels to 1987.</td>
<td>3 Goods First Stage- (1) current consumption (2) savings Second Stage - (1) Food at home (2) Food away from home (3) Non food goods &amp; services</td>
</tr>
<tr>
<td>Swamy and Blinwanger (1983)</td>
<td>Translog</td>
<td>Indian data annual 1956-1975</td>
<td>The three flexible functional forms were tested for their &quot;suitability&quot;, then the T.L. was chosen on basis of tests for the analysis</td>
<td>5 Goods (1) rice (2) wheat (3) inferior cereals (4) pulses (5) other commodities</td>
</tr>
<tr>
<td>Blanciforti and Green (1983)</td>
<td>AIDS</td>
<td>U.S. data annual 1948-1978</td>
<td>Estimate a dynamic AIDS model to incorporate habit effects.</td>
<td>11 Goods Including Food Food is disaggregated further: (1) meats (2) cereals (3) fruits &amp; vegs (4) misc.</td>
</tr>
<tr>
<td>Borooah (1985)</td>
<td>Rotterdam</td>
<td>U.K. data annual 1954-1981</td>
<td>Hypothesis tested is that demand theory is &quot;not meaningful&quot; at broad category level</td>
<td>48 Goods With 11 Broad Categories</td>
</tr>
<tr>
<td>Kim (1986)</td>
<td>Translog</td>
<td>Korean data annual</td>
<td>Apply a demand system to an LDC</td>
<td>5 Goods (1) food (2) housing (3) energy (4) clothing (5) misc.</td>
</tr>
<tr>
<td>HOMOGENEITY</td>
<td>TESTS PERFORMED</td>
<td>NEGATIVITY</td>
<td>SEPARABILITY</td>
<td></td>
</tr>
<tr>
<td>-------------------</td>
<td>-----------------</td>
<td>------------</td>
<td>--------------</td>
<td></td>
</tr>
<tr>
<td>imposed</td>
<td>reject</td>
<td>not tested</td>
<td>not tested</td>
<td></td>
</tr>
<tr>
<td>reject</td>
<td>reject</td>
<td>reject</td>
<td>not tested</td>
<td></td>
</tr>
<tr>
<td>imposed</td>
<td>reject</td>
<td>not tested</td>
<td>not tested</td>
<td></td>
</tr>
<tr>
<td>accept</td>
<td>accept</td>
<td>accept</td>
<td>not tested</td>
<td></td>
</tr>
<tr>
<td>AIDS reject</td>
<td>AIDS reject</td>
<td>not tested</td>
<td>not tested</td>
<td></td>
</tr>
<tr>
<td>accept on T.L.</td>
<td>accept</td>
<td>not tested</td>
<td>not tested</td>
<td></td>
</tr>
<tr>
<td>imposed G.L., N.Q.</td>
<td>reject G.L., N.Q.</td>
<td>not tested</td>
<td>not tested</td>
<td></td>
</tr>
<tr>
<td>reject &amp; out of 11</td>
<td>not tested</td>
<td>not tested</td>
<td>not tested</td>
<td></td>
</tr>
<tr>
<td>(impose for results)</td>
<td>not tested in Food AIDS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>reject static</td>
<td>reject static</td>
<td>not tested</td>
<td>not tested</td>
<td></td>
</tr>
<tr>
<td>accept dynamic</td>
<td>accept dynamic</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-reject across categories</td>
<td>-reject across categories</td>
<td>not tested</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-accept within categories</td>
<td>-accept within categories</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>imposed</td>
<td>reject</td>
<td>accept</td>
<td>test of housing &amp; energy separable from others rejected</td>
<td></td>
</tr>
</tbody>
</table>
Table 3
SUMMARY OF THEORETICAL TESTS ON EMPIRICAL DEMAND MODELS

<table>
<thead>
<tr>
<th>TEST</th>
<th>NUMBER OF TIMES TESTED</th>
<th>NUMBER OF TIMES REJECTED</th>
</tr>
</thead>
<tbody>
<tr>
<td>HOMOGENEITY</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>SYMMETRY</td>
<td>13</td>
<td>10</td>
</tr>
</tbody>
</table>
The likelihood ratio test is more likely to lead to a rejection of the symmetry hypothesis when based upon the estimator used by Barten and Byron than when based on the maximum likelihood estimator. This was cogently argued by Deaton... Moreover this study has calculated the test statistics proposed by Deaton and still found a strong rejection of the symmetry restrictions by all of them." (1983, p. 86-87)

So it would seem that based on the two aforementioned tests demand theory has been adequately falsified in the logical positivist sense, and yet it still persists.

Weak separability while intriguing and of noteworthy significance for a two stage budgeting process is rarely tested and is implemented by assumption (in the few studies which acknowledge its existence) in the majority of studies. Kim (1986) tested for weak separability between housing-energy and food-clothing-miscellaneous groupings and rejected the hypothesis.

An Inconsistent Methodology

Glenn Johnson observes that "Econometrics is logically positivistic in the sense of combining theory or logic with observational data to produce descriptive knowledge" (1986, p. 82). Furthermore, the presuppositions of theory are employed to create synthetic testable hypotheses that are capable of refuting the theory. In the demand system literature this is what has been done, this is Popper's logical positivism. Yet, once serious doubts are cast upon theory an about face is made by the majority of researchers. Instead of believing their own results they assert that theory should hold in all instances and thus the theory is "aprioristic" knowledge.

Figure 1 is a schematic of the procedure which has just been described. Empirical demand models hinge upon the axioms and propositions stated in Table 1; this is the "core" of the theory. These axioms and propositions are then utilized to construct refutable hypotheses in the form of statistical tests. These statistical tests invariably refute theory, yet, a "do loop" (of test, re-specify, and re-test) most often subjugates the research to technical cookbookery. The theory neatly
switches from being a posteriori (able to be refuted) to being a priori, so that it is true by definition.\(^3\)

Why is this wrong? It is wrong because the research is somehow rooted in subjecting theory to empirical tests and then once the feat has been accomplished its consequences are not given serious attention. Even worse, after testing the theory and having the restrictions fail some economists see no problem with imposing the restrictions anyway: "...the fact that many economists would argue that homogeneity and symmetry of aggregate demand should be taken as the maintained hypothesis (as evidenced by the widespread use of the LES), the Linear Approximate AIDS estimates are reported with symmetry and homogeneity imposed" (Goddard, 1983, p. 300).

Economists must be held accountable for their research results, if symmetry, homogeneity, negativity and weak separability are all "maintained hypothesis" and are hence a prioristic knowledge, why then are they treated as synthetic statements and subsequently tested?

\[A \text{ Degenerating Research Programme}\]

According to Lakatos, a scientific research programme: is "theoretically progressive" if it predicts some new and hitherto unexpected fact, and, is "empirically progressive" if some of these novel facts are actually confirmed by experience.\(^4\) Theoretical progressiveness must be present in each step of the research programme, while empirical progressiveness is necessary only intermittently. In contrast, "if the programme is characterized by the endless addition of \textit{ad hoc} adjustments that merely accommodate whatever new facts become available, it is labelled 'degenerating'." (Blaug, 1980, p. 156). Two propositions of demand theory (as given in Table 1), namely symmetry and negativity have been repeatedly refuted in the empirical literature. These
propositions are part of the central core of the research programme, but economists treat these results as if only the so-called protective belt of the theory was affected.\textsuperscript{5}

The scientific research programme of empirical demand system analysis (if it has been correctly portrayed in this investigation) has suffered from the inability to satisfy its underlying purposes. Indeed, \textit{ad hoc} explanations abound as to why the theory is falsified. The research programme may be providing some new unexpected facts, however, they are too often undesirable unexpected facts, thereby leading to this programme's degeneration.

CONCLUSIONS

The methodology of the demand system research programme is an odd mixture of logical positivism and apriorism. The results of the demand system approach have not corresponded well with theory, yet, economists are extremely wary of believing their own results. Demand analysis has also become something of a growth industry as economists have rushed out to apply each new model in succession and invariably arrive at the same results: the theory is in trouble.

The flawed methodology of these economists has put them on an endless roller coaster ride, and no one knows how to get off. The models are complex, and undoubtedly ensure employment for the econometricians who must solve a multitude of statistical predicaments. From a philosophical standpoint the research programme is degenerating. Obviously a wearisome "do loop" of testing-refutation, re-specification, re-testing-refutation is not the solution to this dilemma. Barring unconditional abandonment of received theory, could we not examine the philosophical-methodological nightmare that we have created, and propose that apriorism is indeed a step in the right direction? If this is so, then the restrictions could be imposed outright. If this is so, then perhaps we could abandon the "do loop" of this ceaseless programme and get back to providing plausible answers without testing refutable hypotheses.
APPENDIX

Think of a "utility tree". Weak separability allows the grouping of related commodities together at the exclusion of others.

Let $U = U(X_{\text{Food}},X_{\text{Housing}},X_{\text{Transport}},X_{\text{Clothing}})$

Max $U$ S.T. $P_F X_F + P_H X_H + P_T X_T + P_C X_C = M^T$

where $X_{\text{Food}} \equiv$ quantity of food consumed

$X_{\text{Housing}} \equiv$ quantity of housing consumed

etc.

$P_F \equiv$ price food

with weak separability between groups F,H and T,C we have:

$$\max U = F(U_1(X_F,X_H), U_2(X_T,X_C))$$

S.T. $X_F P_F + X_H P_H = M^1, X_T P_T + X_C P_C = M^2$

where $M^1 + M^2 = M^T$

$U_1$ and $U_2$ are "sub-utility functions" of $U()$ we now have a 2 stage budgeting process, allocations are made to 2 groups of goods $U_1$ and $U_2$ (1st stage) then that amount ($M^1,M^2$) is divided up amongst the respective goods in the group.

Weak separability implies that a change in the quantity consumed of any good outside of $U_1$ say has no effect on the MRS between goods in $U_1$. 
NOTES

1. It must be remembered that two aggregation problems may be involved in rejection of symmetry and negativity: the first is improper aggregation across commodities, and the second is improper aggregation across individuals.

2. The Stone-Geary utility function causes all goods to be normal and net substitutes for one another.

3. This is the avenue taken by the aforementioned studies except for Christensen et al (1975).

4. These terms are from Blaug (1980, p. 156), and are originally attributed to Imre Lakatos.

5. The protective belt of a theory embodies the non-essential replaceable propositions of the theory. These can be discarded without affecting the central core of the theory.
REFERENCES


