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The Effects of Altruism on the Efficiency of Public Good Mechanisms

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Abstract

Previous theory has suggested that altruism in other people's utility does not affect the optimal level of public good provision if preferences can be represented by generalized quasi-linear utility. In this paper, it is shown that the conditions under which optimal provision is unaffected by altruism are narrower than is commonly recognized. Second, even when altruism does not affect optimal provision, it might still affect the ability of public good mechanisms to achieve the optimum. Altruism's effects on three public good mechanisms are considered: the voluntary contribution mechanism, the provision point mechanism, and the Groves-Ledyard tax. It is found for agents with quasi-linear utility that altruism of a "reasonable" magnitude may help, and does not hinder the efficiency of each mechanism. Implications for the experimental evaluation of public good mechanisms are also considered.

JEL classification: C92; D64; H31; H41

Keywords: Altruism, efficiency, public good mechanisms, quasi-linear utility

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1. Introduction

When Paul Samuelson (1954, p. 387) formulated the free-rider problem associated with the voluntary provision of public goods, he assumed that people were selfish. That is, each person's utility was assumed to depend only upon his own consumption of goods. When subsequent economists developed mechanisms to overcome free-riding, they also assumed that people were selfish (Groves and Ledyard 1977, p. 787, Bagnoli and Lipman 1989, p. 588). Consequently, when experimental economists have tested for free-riding or its antidotes in the laboratory, they have tried to induce selfish preferences for goods by paying subjects according to the decisions they make. They have hoped that these payments were high enough to dominate subject's other predilections, such as boredom, mischief, or altruism.

Yet in numerous tests of the voluntary contribution mechanism (VCM), substantial contributions towards public goods have been observed, often in violation of dominant (selfish) strategies to contribute nothing (Davis and Holt, 1993). For example, Rose et al. (1997) test a variant of the VCM, the provision point mechanism, where a threshold level of contributions is required for a Pareto-improving public good to be funded. The authors designed a one-shot decision with 100 subjects. 40 of the subjects had induced values for the public good that were lower than the fixed contribution fee ($.50 or $1.75 vs. $3.00).[1] Yet of these 40 subjects, 27.5% chose to contribute the $3 fee. Following the experiment, all 100

[1] The mechanism tested by Rose et al. specified that contributions in excess of the threshold would be used to provide extended payouts from the public good account. This did not change incentives for low-benefit subjects, but the complexity may have added confusion.

subjects were asked to rate the importance they attached to maximizing own earnings, and to maximizing group earnings, each on 7 point scales. Incorporating subjects' relative responses to both questions greatly increased the explanatory power of a logit regression predicting contribution towards the public good (Rose et al. 1997, 27). More generally, Andreoni (1995) has tested whether contributions in public good experiments result from kindness or confusion, and has found evidence of both.

Challenges to the assumption of pure "own-consumption" utility have also come from empirical studies of donations to charitable (public) goods. A recent large cross-sectional study of Canadians found that 80% of adults contributed to at least one charity in 1997 (Statistics Canada, 1998). The study also found that low income families gave less than high income ones in absolute levels, but three times more as a percentage of income. As Itaya et al. (1997) have noted, selfish preferences for private and (charitable) public goods would imply dollar-for-dollar crowding out of private donors by government provision. Selfish preferences would also imply that in large economies, only the richest individuals would contribute towards public goods. Empirical evidence has not supported either prediction, causing some to look for altruistic preferences as an explanation (Andreoni, 1990).

That altruism might partly explain people's contributions to public goods has led to a discussion of how it might best be modeled. Andreoni (1990) has shown that giving by low income households and a lack of complete crowding out can be predicted if individuals receive utility from the *quantity* of public goods they voluntarily contribute. Alternatively, Itaya et al. (1997) propose it could be the *value* (price x quantity) of public good contributions that brings us satisfaction. While either type of "golden glow" altruism resolves some empirical puzzles, neither can explain some additional experimental findings. Isaac et al. (1994) find that voluntary contributions per person increase with group size, and Goeree et al. (1998) find they increase in the external return to others. If contributors care only about the size of their own contributions, these contributions should not be affected by the number of beneficiaries or the degree to which others benefit.

A common alternative is to assume that an individual has "pure" altruism, or that he derives "internal" utility from the goods he consumes, and "external" utility from the internal utility others receive from the goods they consume.[2] Thus, the greater the number of other people who stand to benefit from a person's contribution to a public good, the greater the marginal benefit he will receive from that contribution.

This paper asks how this latter form of altruism will affect the performance and evaluation of common public good mechanisms. Recent theoretical work has suggested that the study of optimal public good provision has not suffered much by neglecting altruism. Eduardo Ley (1997) has shown that a Pareto efficient allocation of a public good in an economy with altruistic agents would still have been a Pareto efficient allocation with selfish agents, though with a different set of welfare weights. It is as if altruistic individuals were requesting lower social welfare weights for themselves. Ley also attempts to show that if preferences are restricted to some form of quasi-linear utility, such as the generalized version

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[2] This has been the approach adopted by, for example, Anderson et al. (1997) and Ledyard (1995).

introduced by Bergstrom and Cornes (1981), the socially optimal provision of a public good will be invariant to income distribution or welfare weights. In this case, altruism has no effect upon the socially optimal amount of a public good that should be produced (Ley, 1997, 26).[3]

So why worry further about altruism? There are three reasons. First, the conditions under which optimal public good provision is independent of altruism are narrower than Ley indicates, or than many economists recognize. Second, even when Ley's findings do hold, public good mechanisms have been developed to improve efficiency under the assumption that individuals are selfish. Such mechanisms may lose their intended efficiency properties if individuals are partially altruistic. Finally, experimentalists who try to evaluate the efficiency of public good mechanisms may be incorrectly assuming that individuals are purely selfish. Thus mechanisms may appear to lack or possess efficiency in experiments because altruism is not taken into account.

This paper will address these three issues in turn. First, the separation of altruism from optimal public good provision that is possible under quasi-linear utility will be shown to hold only for interior solutions. However, interior solutions will not occur if social welfare weights are unequal, or if altruism is not uniform across the population. Altruism can then have subtle and surprising effects on optimal provision levels, making it difficult to estimate its effects on efficiency. As a result, altruism's effect on the efficiency and evaluation of public good mechanisms are addressed only under the restrictive assumptions that welfare weights and

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[3] Bergstrom and Cornes (1981) propose the utility function $U_i(X, y_i) = v(X)y_i + u_i(X)$ as the minimum structure required to separate optimal provision of a public good from the distribution of private income. It is sometimes called generalized quasi-linear utility.

altruism are uniform. I consider altruism's effects on three public good mechanisms: the voluntary contribution mechanism (VCM), the provision point mechanism (PPM), and the Groves-Ledyard tax mechanism (GLM). Quasi-linear utility shall be employed throughout. Section 2 will discuss the problems of separating allocation and altruism. Section 3 will examine altruism's effects on the VCM, Section 4 the PPM, and Section 5 the GLM. A brief discussion and conclusion will follow in Section 6.

2. Does Quasi-linear Utility Really Separate Efficiency and Distribution?

Let us ground our discussion in a simple exchange economy with a public good. Suppose there are $i = 1,\ldots, N$ people, each with an endowment ω_i. Individuals each consume both a private good, y_i, and a public good, X. Individuals may directly consume their endowments as private goods (with p_y set to 1), or contribute some quantity x_i towards X ($= \sum_i x_i$), which is produced with constant per unit cost *mc*. Each person *i* has quasi-linear utility of the form

$$U_i = h_i(X) + y_i + \sum_{-i} \alpha_{i,-i} (h_{-i}(X) + y_{-i}) \qquad i = 1,\ldots, N \qquad . \qquad (1)$$

In other words, *i*'s utility has an *internal* component in X and y_i, with $h_i'(\cdot) > 0$ and $h_i''(\cdot) \leq 0$, and an *external* component from the utility other people derive from their consumption. In the most general case, *i* may assign a different altruism weight $\alpha_{i,-i}$ to each other person, $-i = 1,\ldots, N-1$. The choice of quasi-linear utility should be explained. As mentioned in footnote 3, Bergstrom and Cornes (1981) have proposed a more general form of (selfish) quasi-linear utility that separates the issue of optimal public good allocation from the distribution of income. General quasi-linear utility contains quasi-linear utility as a special case, but avoids the latter's unrealistic assumption that increases in wealth have no effect upon the optimal amount of the public good. However, general quasi-linear utility also loses the property of concavity, and yields fewer closed form solutions. Thus, for the illustrative analysis in this paper, I have retained the more restrictive form of (1).

Following Eduardo Ley, let us identify the socially optimal level of X, or X^{SO}, first without and then with altruism. Maximizing a (concave) social welfare function with positive weights γ_i for i = 1,..., N people, subject to an aggregate resource constraint, we have

$$\underset{\{all\ x,\ all\ y\}}{Max}\ W = \sum_i \gamma_i\,(h_i(X) + y_i) + \lambda(\sum_i \omega_i - \sum_i y_i - mc\,X) \qquad (3)$$

If we restrict ourselves to the case where positive quantities of X and y_i for all i will solve (3), the first-order conditions will be met with equality:

$$\delta W/\delta X: \qquad \Sigma\,\gamma_i\, h_i'(X) - \lambda mc = 0 \qquad (4)$$

$$\delta W/\delta y_i: \qquad \gamma_i = \lambda \qquad for\ i = 1,\ldots, N \qquad (5)$$

Substituting (5) into (4), we find that X^{SO} must satisfy the Samuelson condition,

$$\sum_i h_i'(X) = mc \qquad (6)$$

For example, if $h_i(X) = a_i \ln X$ for $i = 1,\ldots,N$, then $X^{SO} = (\sum_i a_i)/\text{mc}$.

Continuing our focus on interior solutions, we shall see that introducing altruism to selfish quasi-linear preferences has no effect on X^{SO}:

$$\underset{\{all\, x,\, all\, y\}}{Max}\ W = \sum_i \gamma_i \left(h_i(X) + y_i + \sum_{-i} \alpha_{i,-i} \left(h_{-i}(X) + y_{-i}\right)\right) + \lambda\left(\sum_i \omega_i - \sum_i y_i - mc\,X\right) \qquad (7)$$

$$\delta W/\delta X: \qquad \sum_i \gamma_i \left(h_i'(X) + \sum_{-i} \alpha_{i,-i}\, h_i'(X)\right) \ - \ \lambda mc = 0 \qquad (8)$$

$$\delta W/\delta y_i: \qquad \gamma_i = \lambda - \sum_{-i} \gamma_{-i}\alpha_{-i,i} \qquad for\ i = 1,\dots, N \qquad (9)$$

Substituting (9) into (8), and multiplying out the expression, we get

$$\lambda \sum_i h_i'(X) + \lambda \sum_i \left(\alpha_{i,-i} \sum_{-i} h_{-i}'(X)\right) - \sum_i h_i'(X) \left(\sum_{-i} \gamma_{-i}\alpha_{-i}\right) - \sum_i \alpha_i \left(\sum_{-i} \gamma_{-i}\alpha_{-i}\right)\left(\sum_{-i} h_{-i}'(X)\right) = \lambda mc \qquad (10)$$

Substituting $\lambda = \gamma_i + \sum_{-i} \gamma_{-i}\alpha_{-i}$ from (9) into the second term, and multiplying out this expression, the four terms involving $\alpha_{i,-i}$'s all cancel, leaving the same Samuelson condition as without altruism:[4]

$$\sum_i h_i'(X) = mc \qquad for\ i = 1,\dots, N. \qquad (11)$$

However, the restriction to interior solutions is misleading for quasi-linear utility.[5] For as soon as the welfare or altruism weights, γ_i , or $\alpha_{i,-i}$ differ across people, it can no longer be optimal for every individual to consume the good that enters linearly, y. In the case without altruism, the first order condition (5) will hold with equality only for the

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[4] Two terms directly cancel, and $\sum_i \alpha_i \gamma_i (\sum_{-i} h_{-i}'(X))$ is equivalent to $\sum_i h_i'(X)(\sum_{-i} \gamma_{-i}\, \alpha_{-i})$.

[5] Ley (n.4, p. 25, 1997) states that he deals only with interior solutions, but suggests that corner solutions would add no additional insights. The same oversight is made in a standard graduate microeconomics textbook (Varian, p. 419, 1992).

individual(s) assigned the largest welfare weight. Only these individuals will determine the value of λ, the social marginal utility to society of an extra unit of income:

$$\gamma_i^{max} = \lambda \tag{12}$$

Substituting (12) rather than (5) into (4), the socially optimal provision of X is given by

$$\sum_i \gamma_i h_i'(X) = mc\ \gamma_i^{max} \quad . \tag{13}$$

Thus, the optimal quantity of X depends on the distribution of income, because of the unequal assignment of welfare weights. An analogous argument holds in the case with altruism. The first order condition (9) will only hold with equality for what I shall call the "definitive" individual(s). The definitive individual is the one considered most worthy to consume y by a combination of the social planner and other altruistic agents. It is the definitive individual(s) in (9) who will determine λ, the marginal utility to society of an additional unit of income:

$$(\gamma_i + \sum_{-i} \gamma_{-i}\alpha_{-i,i})^{max} = \lambda \tag{14}$$

Substituting (14) rather than (9) into (8), the optimal provision of X is given by

$$\sum_i \gamma_i (h_i'(X) + \sum_{-i} \alpha_{i,-i} h_i'(X)) = mc\ (\gamma_i + \sum_{-i} \gamma_{-i}\alpha_{-i,i})^{max} \tag{15}$$

As long as welfare weights or altruism can vary across people, X^{SO} will be dependent on both. In particular, the optimal provision of the public good will be linked to the strength and variation of altruism in the population.

How troublesome is this link to a study of altruism upon the efficiency of public good mechanisms? Is there a simple monotonic relationship between altruism and optimal public good provision? Unfortunately, no. Heterogeneous altruism can have subtle effects on the quantity of the public good that should be provided. The following example illustrates this.

Suppose that person A is definitive, and also that he most likes the public good. If A begins to shift his own utility weights towards other agents, X^{SO} could actually increase before eventually decreasing. This is because A's growing selflessness/altruism is having two potentially offsetting effects. As A grows more altruistic, the marginal utility to society of his enjoying the public good falls. But recall that, so long as A remains definitive, he receives all residual private consumption, y. So as A grows altruistic, the marginal utility to society from his private consumption is also falling relative to using the income to fund the public good for others to enjoy. If the latter effect outweighs the former, X^{SO} can increase in A's altruism until he loses his definitive status. Then, the two influences become reinforcing, and X^{SO} starts to fall. Figure 1 provides illustrations of this and other possibilities.

If heterogeneous altruism can have ambiguous effects on the "benchmark" of optimal public good provision, it makes it difficult to gauge it's effect upon the efficiency of public good mechanisms. To keep the optimal benchmark constant, I shall only consider the effects of uniform altruism, with equal social welfare weights. These restrictions are admittedly unrealistic, and justified only on the grounds of making the subsequent analysis tractable.

3. Altruism and the Voluntary Contribution Mechanism

Recall from (6) or (11) that with uniform welfare weights and altruism, X^{SO} is

Figure 1
Heterogeneous Altruism's Effect on Optimal Public Good Provision: A Simulation

Parameters:

$N = 3$ $\qquad$ $\omega_1 = \omega_2 = \omega_3 = 10$

$U_1 = \alpha_{11}(5\ln X + y_1) + \alpha_{12}(2\ln X + y_2) + \alpha_{13}(\ln X + y_3)$ $\qquad$ where $\alpha_{11} + \alpha_{12} + \alpha_{13} = 1$

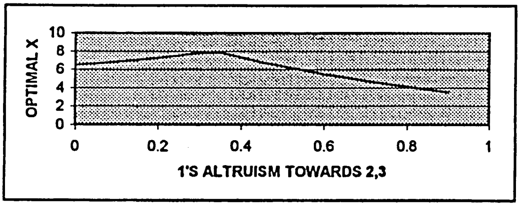
$U_2 = \alpha_{21}(5\ln X + y_1) + \alpha_{22}(2\ln X + y_2) + \alpha_{23}(\ln X + y_3)$ $\qquad$ where $\alpha_{21} + \alpha_{22} + \alpha_{23} = 1$

$U_3 = \alpha_{31}(5\ln X + y_1) + \alpha_{32}(2\ln X + y_2) + \alpha_{33}(\ln X + y_3)$ $\qquad$ where $\alpha_{31} + \alpha_{32} + \alpha_{33} = 1$

Social Welfare Function = $\gamma_1 U_1 + \gamma_2 U_2 + \gamma_3 U_3$ $\qquad$ where $\gamma_1 + \gamma_2 + \gamma_3 = 1$

Case A: $\gamma_1 = .5,\ \gamma_2 = .25,\ \gamma_3 = .25,\quad \alpha_{22} = \alpha_{33} = 1$

If $\alpha_{11} = 1$, Person 1 will be definitive. Suppose Person 1, who most likes the public good, becomes altruistic towards 2 and 3, so that α_{11} drops from 1 to .1, while α_{12} and α_{13} both rise simultaneously from 0 to .45. X^{SO} rises until Person 1 loses definitiveness when α_{11} drops below .67; X^{SO} then declines.



Case B: $\gamma_1 = .25,\ \gamma_2 = .25,\ \gamma_3 = .5,\quad \alpha_{22} = \alpha_{33} = 1$

Now if $\alpha_{11} = 1$, Person 1 will not be definitive. If Person 1 becomes altruistic towards 2 and 3 as in Case A, X^{SO} unambiguously falls.

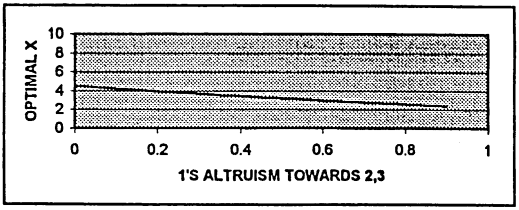
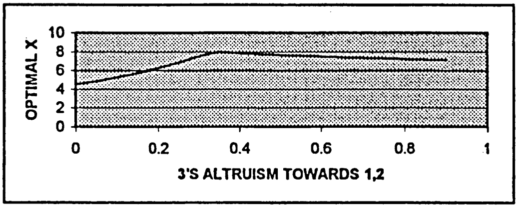


Figure 1 (Cont'd)

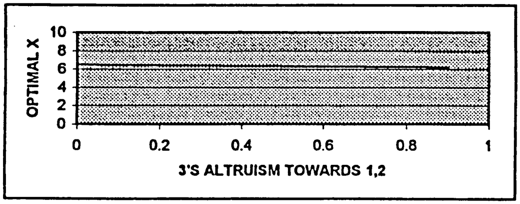
Case C: $\gamma_1 = .25,\ \gamma_2 = .25,\ \gamma_3 = .5, \quad \alpha_{11} = \alpha_{22} = 1$

If $\alpha_{33} = 1$, Person 3 will be definitive. Suppose Person 3, who least likes the public good, becomes altruistic towards 1 and 2, so that α_{33} drops from 1 to .1, while α_{31} and α_{32} both rise simultaneously from 0 to .45. X^{SO} rises until Person 3 loses definitiveness when α_{33} drops below .67; X^{SO} then declines.



Case D: $\gamma_1 = .5,\ \gamma_2 = .25,\ \gamma_3 = .25, \quad \alpha_{11} = \alpha_{22} = 1$

Now if $\alpha_{33} = 1$, Person 3 will not be definitive. If Person 3 becomes altruistic towards 2 and 3 as in Case C, X^{SO} unambiguously falls.



characterized by $\Sigma h_i'(X^{SO}) = mc$. How well does the VCM fare in achieving this? When confronted with the VCM, individuals in a population without altruism seek to

$$Max \quad U_i = h_i(X) + y_i,$$
$$subject\ to \quad mc\, x_i + y_i \le \omega_i, \qquad (16)$$
$$and\ X = \sum_i x_i \qquad for\ i = 1, \ldots, N.$$

If people differ in their tastes for the public good, there is a real possibility of corner Nash solutions. Using substitution, the first order condition with respect to x_i for each person is

$$\begin{aligned} h_1'(X) &\le mc \\ h_2'(X) &\le mc \\ &\vdots \\ h_N'(X) &\le mc \end{aligned} \qquad (17)$$

The Nash equilibrium quantity of the public good, X^*, if it exists, will be determined by a subset of individuals. The subset will consist of the individual who most likes the public good, subject to her budget constraint. If she cannot afford to unilaterally fund her desired amount, she exhausts her budget contributing, and the second most enthusiastic makes up the difference between the X he desires, and what the first person contributes. And so on. If we denote person *i* as the person who most likes the public good, this implies that the maximum possible Nash quantity X^* will be given by her first order condition, met with equality:

$$h_i'(X^*) = mc. \qquad (18)$$

From this it follows that the Nash quantity X^*, is less than the optimal level X^{SO}. To see this, rearrange the Samuelson condition from (11),

$$h_i'(X^{SO}) = mc - \sum_{-i} h_{-i}'(X^{SO}) \qquad (19)$$

Thus, $h_i'(X^*) > h_i'(X^{SO})$, which will only occur if $X^* < X^{SO}$.

On the other hand, altruistic individuals facing the VCM seek to

$$\begin{aligned} Max \quad & U_i = h_i(X) + y_i + \alpha \sum_{-i} (h_{-i}(X) + y_{-i}) \\ & subject\ to \quad mc\, x_i + y_i \leq \omega_i, \qquad\qquad (20) \\ & and\, X = \sum_i x_i \qquad\qquad for\ i = 1,\ldots, N. \end{aligned}$$

Once again, any Nash equilibrium that exists may well involve a corner solution. Using substitution, the first order condition with respect to x_i for each person is

$$\begin{aligned} h_1'(X) &+ \alpha \sum_{-i} h_{-i}'(X) \leq mc \\ h_2'(X) &+ \alpha \sum_{-i} h_{-i}'(X) \leq mc \qquad\qquad (21) \\ &\vdots \\ h_N'(X) &+ \alpha \sum_{-i} h_{-i}'(X) \leq mc \end{aligned}$$

As before, the individual *i* who desires the largest quantity of the public good will determine its maximum possible Nash quantity. Now, however, this desire may come from a strong preference for the public good itself, or from strong altruism towards others. Either way, the maximum possible Nash equilibrium X^* is identified by person *i*'s first order condition:

$$h_i'(X) + \alpha \sum_{-i} h_{-i}'(X) = mc \qquad\qquad (22)$$

Any Nash equilibrium identified by (18) will be at least as large as that identified in the case without altruism. How much larger? If individual *i* assigns a weight to each other person's internal utility equal to his own, ($\alpha = 1$), and he could afford to, he would unilaterally fund the socially optimal amount of the public good. In fact, if $\alpha = 1$ all individuals' first order