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Chairman: K. O. Campbell *Australia* *Rapperteur*: I. G. Simpson *U.K.*

Development of a Decision Rule for Farm Planning under Uncertain Conditions

E. HANF
Arbeit aus der Abteilung Wirtschaftslehre des Landbaues der Universität Hohenheim, W. Germany

1. *Introduction*

In the last years Linear Programming has been used more and more in planning farms. With growing experience, by continued application, and by improvement of the approaches in individual cases, effective help is given for decision making. With the increasing range of application, the demands made on the quality of results have also increased.

1.1 *Planning approach and uncertainty*

Special problems arise by the necessary estimations of values of certain planning data at future points of time.

In order to conceive a planning model the farmer or the farm manager, who has to make decisions, requires information on the production factors available, for instance land, labour, capital and certain intermediate products, which are produced and used on the farm itself. Furthermore, information is required about the limitations of production resulting from the production factors available.

In a Linear Programming approach the restrictions of production may be subdivided in the following groups (9. p. 47–49)

(a) land use and rotation of crops
(b) labour capacity in time periods and for special tasks
(c) stables and storage capacity, machine capacities, money restrictions (e.g. liquidity or minimal income)
(d) intermediate products (e.g. fodder and cattle, diet restrictions)

The production activities may be sub-divided in a similar manner as follows:

(a) marketable crop production
(b) intermediate and secondary products and their internal transfer
(c) marketable animal production
(d) activities of purchase and sale

Some of the capacities, like labour in a time period, the length of which depends on weather conditions, some of the technical coefficients like labour demands of activities and natural returns of fodder crops, further on the natural returns of the activities and the prices of products and factors are subject to uncertain predictions. Restrictions, that means rows of the matrix of coefficients, in which coefficients can't be predicted with certainty and therefore are assumed to be stochastically variable, are called stochastic restrictions (9, p. 51).

In practical application the internal transfer losses are taken into consideration by risk allowances which correspond to the expected losses. This insurance effect may be justified, if the individual occurrences which lead to losses repeat themselves often enough and expected losses hence can be measured and estimated. This practice becomes questionable, however, when a short-term plan is conceived with a unique decision.

Summarizing, it can be said, although not all data required for planning are objects of stochastical fluctuation, the following restrictions are especially affected:

(a) the restrictions of financial character, like liquidity etc.,
(b) the restrictions of labour force,
(c) the restrictions of fodder cropping and diet conditions,

and the activities of sale and purchase, especially the objective function.

The variability of coefficients, hence affects the objective function the matrix of technical coefficients and the capacities.

Linear Programming is essentially based on the existence of determined vectors and a determined matrix. The method thus generates in normal cases a single optimal solution. This solution is optimal, provided that the event, described by the vectors of objective function, p, and the capacities, b, and by the matrix A, would occur in time of realization.

In reality, however, there is a large number of possible events which result in different coefficients and elements of p, A and b. Thus, there are a lot of Linear Programs, the individual solutions of which are not necessarily optimal for another combination of data.

If all stochastic restrictions of a program are collected (7, p. 680) to a partial matrix A_2 of A, the problem can be written as

$$\max\ Z = p'x$$

under $$A_1 x \leq b_1 \text{ and } A_2 x \leq b_2,$$

A_1, b_1 being deterministic, A_2, b_2 being stochastically variable.

This subdivision is not necessary from the point of view of the following discussion, therefore it is assumed, that all restrictions inclusive, the capacity coefficients and the vector of the objective function may be subjects of stochastical fluctuations.

1.2 *The intention of the following discussion*

The following discussion is based on the subdivision of the planning process (9, p. 9) in the steps information, estimation, planning and decision making.

The intention is to derive a decision rule or a decision function which is appropriate with respect to the object, that is the agricultural firm and with respect to the most commonly used method of planning a farm, that is the method of Linear Programming, and with respect to the special problems of uncertainty characterizing the agricultural production.

It has made use of the fact, that with modern computer programs a great number of optimal solutions of similar Linear Programming problems may be generated in a relatively short time. The step 'planning' is understood as generating alternative approaches and solutions, the decision making as the process of the selection of one solution or organisation for realization.

The results are more general. The application of the decision rule is not restricted to the special case of the farm as the object and not to the method of Linear Programming, but these aspects are not specially investigated and described here.

2. *The decision process*

The decision process (15) may now be defined as the selection of one organisation under a set of actions under consideration, the selected action should be optimal in relation to a decision criterion or to a preference relation about a defined action space.

2.1 *The decision situation*

The events at the time of realization of the selected action are characterized by a set of combinations of p, A and b. Thus a space of events is defined,

$$E = \left\{ E_i = (p_i, A_i, b_i),\ i = 1, I_o \right\}$$

which includes all possible combinations of data.

Especially it is possible to compute to each problem arising from a special event E_{i_o}, an optimal solution of the problem

$$\max\ Z = p_{i_o}\ x \ \text{and}\ A_{i_o}\ x \leqslant b_{i_o},\ x \geqslant O.$$

This solutions should be called x_{i_o}.

These optimal solutions, computed for the different events of the space of events, form a meaningful action space.

2.2 *Conclusions for an appropriate decision rule*

The following consequences arise from the fact, that an organisation could be realized or an action selected, which is wrong with respect to the actual event in the time of realization, because it is not optimal with respect to this event.

(1) Because of lower prices and lower natural returns losses may arise; on the other hand by underestimating prices or returns, additional profits may arise.

(2) If the actual technical coefficients or capacities differ from those of the planning approach, a lack of production factors like fodder or labour may arise. Of course, unexpected occurrences can lead to unused capacities.

Under these circumstances and if one is able to realize only one of the actions, a preference relation is required, which should take into consideration the following aspects,

(1) profits not made cause a regret, but in serious cases they can endanger the existence of the farm. The lost profits under different conditions from those on which the plan is based should be regarded as a disadvantage of the organisation,
(2) additional profits under the same conditions may diminish risk, they should be regarded as an advantage of the organisation,
(3) the lack of factors causes additional costs due to their purchase and due to higher prices,
(4) excess factors might cause extra costs, e.g. storage costs, they might cause additional income, e.g. the sale of fodder or the use of labour out of the farm, or they can be completely neutral like labour, which cannot be used elsewhere,
(5) the additional profits or expenses, which arise due to wrong actions, cannot be compared according to their absolute value, but only in relation to a standard value characterizing each organisation.

2.3 *Special aspects*

2.31 *Infeasibility costs*

As a result of the fact that a lack of production factors can be caused by wrong estimation of technical co-efficients or capacities, organisations may have been realized, which violate a restriction, which has actually occurred. The costs to overcome the lack are called 'infeasibility costs' or 'penalty costs' (7. p. 680; 14, p. 463).

Often these costs may be higher than the planned costs of procurement. Under certain circumstances it is impossible to procure lacking factors, this corresponds to infinitely high infeasibility costs. In practice it will often be impossible to determine the actual prices of lacking factors, so that the infeasibility costs are to be inserted as parameters of an individual attitude to risk into the preference relation. In cases in which they can be regarded as factor prices, they also depend on the economic environment and it is reasonable to complement the definition of events by a vector k of the infeasibility costs, so that

$$E_i = (p_i, A_i, b_i, k_i)$$

2.32 *The probability of events*

In order to widen the range of application it is necessary to make assumptions or statements on the probability structure on the set of events. In the sense of this discussion it is not important whether these probabilities are measured as objective distributions or guessed as degrees of belief (9, p. 35). In order to use all information available and with respect to the fact that a unique decision is to be made it is assumed–in the sense of a 'degree of belief'–hypothesis–that for each event E_i of the set E exists a probability π_i with $\Sigma\pi_i = 1$ and that the events are statistically independent. The latter is at first a problem of defining the space of events.

3. An appropriate decision rule

3.1 General remarks

In the preceding chapters the characteristics of the decision problem were given. Some general remarks have to be made of the action space and of the principle used to get an appropriate decision rule.

3.11 Various action spaces

An action x_j is understood as a plan of organisation, the components of which are the levels of realization of various production alternatives. The action x_j should be a feasible solution of the problem $Ax \leqslant b$ under certain assumptions about the constellation of data in p, A and b. In general there should not be made special assumptions on the source of the knowledge of these actions. Thus various action spaces A are conceivable. Preferred is the one formed by those actions which are optimal with respect to the events under consideration. The task of the preference functions, called H(x), to be determined on the action space A, is the selection of optimal organisations according to A and H.

3.12 The preference function or decision rule

The decision rule should be constituted in a manner such that the optimal organisation maximizes the function. It consists of three additive components F(x), K(x), N(x), which correspond to varying aspects of the problem of decisions under uncertain conditions, outlined above.

(1) The part F(x) takes into account the costs of regret, which are the profits not made or additionally made by the possibly wrong organisations, expected on all events under consideration for the time of realization. It is a function of the actions x_j as variables and with the vectors of objective function (p_i) and the probabilities of events (π_i) as constants.

(2) The part K(x) describes the expected infeasibility costs of action x_j on all events, with A_i, b_i, k_i and the probabilities π_i as constants.

(3) The part N(x) evaluates the standard value characterizing each organisation. It is important to make the amounts of F(x) and K(x) comparable for different actions. The functional dependence is similar to that of F(x).

The preference function H(x) consisting of these elementary parts would be in general terms

$$H(x_j) = N(x_j) + F(x_j) + c\ K(x_j)$$

with c as a constant which has the values 0 and 1 according to whether the infeasibility costs are neglected or not.

With respect to an action space A an organisation x_j is preferable to x_k if $H(x_j) \geqslant H(x_k)$. The rule becomes operational if the symbols N(x), F(x) and K(x) are thought of as special functions.

3.2 *The specification of the components of H(x)*

The further discussion will examine special assumptions about N, F and K, which lead to varying decision rules.

In most cases different functions correspond to different attitudes to risk. In the following discussion it is assumed, that the action space is defined by actions which are optimal with respect to the different elements of the finite space of events.

3.21 *The part of the expected costs of regret*

Under the assumption event E_{i_o} would occur, an optimal solution of the deterministic Linear Program

max $Z = p_{i_o}$ x under the conditions $A_{i_o}x \leq b_{i_o}$ and
$x \geq$ exists

Cases of degeneration, unbounded or infeasible solutions should be excluded. If a different event occurs then more or less profits may result. There are two possibilities to evaluate the expected deviations.

(a) The expected costs of regret might be computed by comparing the results of an organisation x_{i_o} if all events would occur corresponding to their probability. This means

$$F(x_{i_o}) = \sum_i \pi_i \, \emptyset \, (p_i x_{i_o} - p_{i_o} x_{i_o})$$

is to be computed, with Ø as an operator symbol discussed later. This value should be called the risk of the decision.

(b) The expected costs of regret might be computed by comparing the results of organisation x_{i_o} if all events would occur and the corresponding optimal action would have been selected. This means

$$F(x_{i_o}) = \sum_i \pi_i \, \emptyset \, (p_i x_i - p_{i_o} x_{i_o})$$

is to be computed. This value should be called the error of decision.

The operator symbol Ø (y) may be thought of as one of the functions (3,14)

I) $\emptyset(y) = y$, II) $\emptyset(y) = y^3$, III) $\emptyset(y) = \begin{cases} y & \text{if } y < O \\ O & \text{else} \end{cases}$

IV) $\emptyset(y) = \begin{cases} y^2 & \text{if } y < O \\ O & \text{else} \end{cases}$ V) $\emptyset(y) = -/y/$, VI) $\emptyset(y) = y^2$

(among other possibilities).

In the cases V and VI it is assumed, that positive and negative deviations from the planning data are risks of the same manner.

If y is a profit variable it is reasonable to think of the risk-diminishing aspects of positive deviations. This aspect becomes meaningful if unsymmetric distributions are used to describe the probabilistic structure on the space of events. Therefore in the following discussion $\emptyset(y) = y$ is to be the preferred type of measure. Under this statement the risk of decision becomes

$$F(x_{io}) = (E(p) - p_{io})\, x_{io}$$

where $E(p) = \sum_i \pi_i p_i$ applies, the error of decision becomes

$$F(x_{io}) = E(Z) - p_{io} x_{io}$$

where $E(Z) = \sum_i \pi_i\, p_i x_i$ applies.

3.22 *The part of the expected infeasibility costs*

If action x_{io} were realized and event E_k occurred then the vector

$$r_k\ (x_{io}) = A_k\ x_{io} - b_k$$

is a measure of the possibility of realization of organisation x_{io}. The organisation is realizable if $r_k(x_{io}) \leqslant 0$. If there are positive components of r_k, then it means, that restrictions have been violated and additional amounts of factors must be purchased.
Defining (3, p. 83)

$$a \div b\ \frac{1}{2}\ (a\text{-}b+/a\text{-}b/) = \begin{cases} a\text{-}b & \text{if } a > b \text{ applies.} \\ 0 & \text{if } a \leqslant b \text{ applies.} \end{cases}$$

and with k_k as the vector of infeasibility costs corresponding to event E_k, then

$$K(x_{io}) = \sum_k \pi_k k'_k\ (A_k x_{io} \div b_k)$$

are the expected infeasibility costs provided that action x_{io} was realized.

3.23 *The standard value N(x)*

Rational reasons for the choice of certain functions to compute a standard value do not exist. The following however, are reasonable characteristics for the organisation:

(a) the average incomes or profits which may be achieved over a number of years with a constant organisation, that means $N(x_{io}) = E(p)x_{io}$

(b) the incomes or profits which may be achieved under most favourable conditions, that means $N(x_{io}) = p_{io}\ x_{io}$.

(c) the incomes or profits, which may be achieved under most favourable conditions, weighted with the probability of the corresponding events, that means $N(x_{io}) = \pi^s_{io} p_{io} x_{io}$, where π^s_{io} is the sum of the probabilities of those events whose objective vector is p_{io}.

3.3 *Synthesis of the decision rule H(x)*

Combining the alternatives for the additive parts of the preference function a number of decision rules results, which differ from each other in form and content and generate different optimal decisions. The following rules are conceivable. They have the general form

$$H(x_j) = a \,.\, E(Z) + v'x_j - K(x_j)$$

where the vector v′ can be defined in a different manner.

If a = 0 applies, alternatively

$v = E(p)$ or $v = (2E(p)-p_j)$ or $v = (E(p)-(1-\pi_j{}^s)p_j)$ applies,

and if a = 1 is true, alternatively

$v = 0$ or $v = (E(p)-p_j)$ or $v = (\pi_j{}^s-1)p_j$ applies

The discussion of the alternative results which are achieved with different forms of the decision rules shows that the differences correspond to different points of view with respect to risk. It is remarkable, that the stated forms of preference function are derived under the assumption, that the operator symbol O is used as Ø (y) = y and that there are more possibilities to define other decision rules using another definition of Ø (y).

3.4 *Action space and infeasibility costs*

Remember the fact, that the problem

$$\max Z = p'x \text{ under } A\,x = b,\ x = 0$$

with stochastic restrictions could be turned to the problems

$$\max Z = p_i'x \text{ under } A_1 x \leq b_1 \text{ and } A_{2i}x \leq b_{2i},\ x \geq 0$$

where A_1, b_1 are deterministic, while p_i, A_{2i}, b_{2i}, $i = 1, \ldots I_o$ correspond to the elements of the space of events, the index i stands for the alternative events E_i. If one uses as action space the organisations which are optimal with respect to the events, $A_1x = b_1$ applies for all these actions. If the action space is defined over a set of actions, each of them is feasible for at least one event, then $A_1x = b_1$ applies, too. But this is not necessarily right if the action space is constructed at random or so-to-speak untested with respect to the deterministic part of the restrictions. There are two ways, among others, of treating infeasibility in the last case.

(a) If one does not want those elements of the action space to be considered which violate the deterministic restrictions, the infeasibility costs of these restrictions must be set up with so high values, that violating one of these restrictions can never lead to more favourable risk measures than using any feasible solution.

(b) Should it, on the other hand, be reasonable, that some or all of the deterministic restrictions can be exceeded, then the deterministic part may be formally treated as the stochastic part of the inequalities.

Rather important is another aspect of the decision function H(x). Examining (9, p. 80) the mathematical properties of the function H(x), it can be shown, that under special circumstances H(x) has an absolute maximum which is not a vector x of the given action space, but is a linear combination of some optimal solutions corresponding to the elements of the space of events. Proceeding from these facts it seems to be reasonable to maximize the function H(x) independent from a given action space. Those forms of the decision rule, which don't refer for a special co-ordination of a vector p_j to an organisation x_j under consideration allow this free maximization.

It is obvious (9, p. 94) that a linear combination of these elements

$$x_o = \sum_k \lambda_k x_k,\ 0 \leqslant \lambda_k \leqslant 1,\ \sum_k \lambda_k = 1$$

does not violate the deterministic restrictions of the problems, if they are not violated by the x_k. If this applies, the problem of free maximization

$$\max_x H(x)$$

can be written in the form

$$\max_\lambda H(\sum_k \lambda_k x_k) = \max_\lambda H(X\lambda)$$

with the vector $\lambda = (\lambda_k)$ and the matrix $X = (x_1, x_2, ..., x_k)$, with $0 \leqslant \lambda_k \leqslant 1, \sum_k \lambda_k = 1$.

The function H(x) is then maximized over a convex set of points P, defined by

$$P = \left\{ x/x = \sum_k \lambda_k x_k \quad 0 \leqslant \lambda_k \leqslant 1, \sum_k \lambda_k = 1 \right\}$$

where the vectors x_k are the optimal solutions corresponding to the events under consideration.

4. *The individual attitude to risk*

The decision maker's individual attitude to risk influences the decision process in a relevant way. The interdependence between individual and the uncertain situation and its influence on decision making cannot be measured.

By determining certain coefficients and formal aspects of the decision rule these trans-rational components of the decision process may be simulated. Through certain prior decisions a subjective attitude to risk is prejudiced and, therefore, in this way certain alternatives of action are indirectly preferred. The parameters of the decision rule which are influenced by the subjective standpoint with respect to uncertainty are not to be regarded as constants generally, but they are the object of a control process between decision maker and the results of using the decision function. These parameters must be thought of as variable or rather corrigible.

The necessary prior decisions have to be done on different levels. As they generate principally different actions as optimal with respect to the resulting decision rule, they can be considered as reflecting individual attitudes to risk. The following aspects are striking.

(a) The first prior decision concerns the set of events, which are taken into consideration. The events which are not considered, that means, which are regarded as impossible or improbable by the decision maker, or the deviations from those under consideration, which are neglected, are an expression of the subjective standpoint.

(b) Should the distribution on the space of events not be capable for measurement or objective observations but for subjective estimations, a further possibility of representation of individual aspects is given in the subjectively estimated values of the probability distribution.

(c) In connection with the set of events the selected action space also reflects subjective points of view. Excluded actions, based on *a priori* judgements, are not examined for their relative superiority in relation to the decision function.

(d) In the general form

$$H(x) = aN(x) + bF(x) - c\,K(x)$$

there are degrees of freedom in setting the coefficients a, b, c at zero or one. Meaningful alternatives for the triple (a, b, c) are, for example,

1. (1,0,0) 2. (0,1,0) 3. (1,0,1) 4. (1,1,1).

The resulting rules depend of course from the choice with respect to the functions N, F, K. Basically, however, varying selections of a,b,c prefer varying actions. In table 1 some rules which can be compared with well known decision criteria are shown. Under different assumptions rules may be deduced which are comparable with the Hurwicz–or the Bayes Criterion, the Niehans-Rule or the methods of Madansky (14) and Evers (7).

(e) As pointed out the choice of functions N(x), F(x) and K(x) influences the resulting decision rule and therefore the preferred actions, especially the choice of the operator symbol Ø in F(x) has consequences.

(f) When considering the infeasibility costs, prior decisions about the function K(x) can be influenced by fundamental attitudes to the problem. Principally different judgement of positive and negative deviations from planned data corresponds to the decision for one of the basic forms

$$K(x) = k_k\ (A_k x - b_k) \text{ respectively } K(x) = k_k\ (A_k x \div b_k).$$

(g) The infeasibility costs, as far as they are regarded as calculatory costs, are an important instrument with respect to individual attitude to risk. Providing for a parametric variability of the costs they can be a measure of stability of a decision. If the optimal decision changes in response to small change of infeasibility costs, the decision would be called unstable.

The seven possibilities outlined before open many degrees of freedom, the consequences of which can't be described here in detail. But it is remarkable, that some of the assumptions lead to well known decision criteria.

As it has been hown above, for instance in table 1, neglecting infeasibility costs leads to some so-called classical decision criteria. Using $N(x) = E(p) \cdot x$ and setting the infeasibility costs constant, that means

TABLE 1

Some special forms of $H(x) = a\,N(x) + b\,F(x) + c\,K(x)$
with $K(x) = \sum_k \pi_k\, k_k\, (A_k\, x \div b_k)$

No.	Selected values for a	b	c	Selected form for $N(x_j)$	$F(x_j)$	Rule, maximize on x_j	Remarks
1.	1	0	0	$p_j x_j$		$p_j x_j$	Hurwicz–Criterion with $\lambda = 1$
2.	1	0	0	$(\sum_k \pi_k p_k) x_j$		$E(p) x_j$	Commonly used in Linear Programming, corresponds to the Bayes–Criterion (expected value of a special result)
3.	1	0	0	$(\pi_j p_j x_j)$		$(\pi_j p_j x_j)$	
4.	0	1	0		$(E(p)-p_j) x_j$	$(E(p)-p_j) x_j$	$(E(p)-p_j)$ corresponds to the 'opportunity costs' of the decision for x_j, likely to the Niehans–Criterion
5.	1	0	1	$p_j x_j$		$p_j x_j - K(x_j)$	If for all indices k, $p_k = p$ and $k_k = k$, the criterion becomes the slack-method of Madansky or, neglecting the set-up-costs to the approach of W. Evers
6.	1	0	1	$E(p) x_j$		$E(p) x_j - K(x_j)$	
7.	1	1	0	$p_j x_j$	$(E(p)-p_j) x_j$	$E(p) x_j$	see under 2.
8.	1	1	1	$p_j x_j$	$(E(p)-p_j) x_j$	$E(p) x_j - K(x_j)$	see under 6.

independent from the different events, an approach similar to that of Madansky or neglecting the so-called set-up-costs similar to Evers' approach. Neglecting infeasibility costs, using $N(x) = E(p)\ x$ and using a quadratic function instead of a linear one for the operator symbol Ø in F(x) and going from a discrete action space to a continuous one leads to a decision function which is similar to the approaches describing the problem of decision under uncertainty with a quadratic criterion function with variance-covariance-matrix in the quadratic part of the objective function (8).

5. *Some final remarks*

There are two critical points in decision making with help of decision rules. The first is the treatment of seldom events, the second is the consideration of liquidity in investment problems.

5.1 *Seldom events*

Seldom events in this connection are events with low probability and extremely high returns. A disadvantage of the most classical decision rules is that they induce optimal decisions in the manner of a player. To avoid this irregular and often uneconomical behaviour a very simple way is to add a minimum restriction

$$a = p'x$$

to the normal secondary conditions of the Linear Programming Problem. In this condition p is the vector of the objective function, x is the action and a is a variable similar to minimum income. By using an appropriate value of infeasibility costs corresponding to this restriction, the optimal decision is directed to reasonable actions.

5.2 *The liquidity problem*

Liquidity is an important aspect of investment problems, which are dynamic problems. It is clear, that there are formally no additional difficulties if the secondary conditions used to compute the infeasibility costs are taken from a Dynamic Linear Programming approach instead of a static approach. There are two ways to overcome the problem of illiquidity.

The first possibility is to formulate common liquidity restrictions and to define high infeasibility costs for violating one of them Thus all organisations or actions are excluded which don't ensure liquidity under all circumstances.

Under special conditions it may not be very meaningful to exclude organisations which violate the liquidity restriction under very improbable combinations of data. In these cases a second way is preferable.

If the event E_j is characterized by the vectors p_j, b_j, k_j, these are objective function, capacities and infeasibility costs respectively, and the matrix A_j of technical co-efficients and if π_j is the probability of this event, and if F is the value of minimum of income or profit including the farmer's personal expenses and fixed costs, then the value

$$Z_j(x_i) = p_j x_i - K_j(x_i) - F$$

is computable for all events j and all actions x_i. The term $K_j(x_i)$ does not mean the expected value, but the actual value of infeasibility costs if event E_j occurs and action x_i has been realized. Let J be the set of indices j for which

$$Z_j(x_i) = 0 \text{ holds, then } \pi(i) = \sum_{j \in J} \pi_j$$

could be called the probability of survival of action i. By determining a certain, subjectively set limit π_o, all organisations, the probability of survival of which is smaller than the critical value π_o, can be excluded.

Very high infeasibility costs for the liquidity restrictions correspond to the critical value $\pi_o = 1$.

Conclusion

The intention of the discussion was to derive an appropriate decision rule, appropriate with respect to the object of planning, the farm with respect to the planning approach, Linear Programming, and the special properties of uncertainty of agricultural production. The result was a generally formulated decision function, from which reasonable decision rules and under special assumptions some well known criteria functions, which are similar to them, were derived. The prior decisions for special forms of functions can be thought of as an expression of the subjective attitude to risk. The feasibility of the special problems of liquidity and seldom events by application of the decision function was shown.

The operationality of the method depends on computability and the possibilities to define a space of events and the probability distribution on it.

With most programs for Linear Programming series of solutions may be generated by changing coefficients with subroutines. Using the preceding solution as a basis for the next step a great number of organisations, which are optimal with respect to different events, may be computed and stored with little additional computing time. With special programs it is possible to select the optimal action with respect to the decision function in a second step.

The possibility of defining a space of events and a possibility distribution on it is limited by lack of information. The more events are concerned, the more difficulties arise to define the distribution. In certain cases, using all objective and subjective information available, it will be possible to find an appropriate approach.

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Obtaining Acceptable Farm Plans under Uncertainty

P. B. R. HAZELL and R. B. HOW
Cornell University, U.S.A.

Linear programming is widely recognized as a method of determining a profit maximizing combination of farm enterprises that is feasible with respect to linear constraints on land, labor, equipment and financial resources, and to such fixed restrictions as rotational requirements and available market outlets. The technique has greatest potential for complex farm organizations, and yields considerable information of value to management in addition to the optimal farm plan. But linear programming solutions to enterprise combination problems have often been rejected because the approach has failed to consider uncertainty and leads to a higher degree of enterprise specialization than farm managers are willing to accept. We define uncertainty to consist of situations where our knowledge of future events is limited to estimates both of possible outcomes and their relative frequencies.

Such uncertainty may arise in forecasted yields, costs or prices for individual activities, in activity requirements for fixed resources, or in the fixed constraint levels for the farm This paper will be concerned with uncertainty in gross margins (gross returns net of variable costs) which result only from uncertainty in activity yields, costs or prices. Methods suggested to handle such uncertainty include modifications of linear programming, quadratic programming, game theory, simulation, dynamic programming and statistical decision theory. These approaches differ primarily in the assumed decision criteria, but the relationships between them have not been fully explored.

Utilizing various decision criteria we propose to develop a general class of short run farm planning models that have in common an expected (forecasted) income, and a measure of income uncertainty. These models are similar in that they generate sets of farm plans for the range of all possible expected incomes, and are efficient in that the measure of income uncertainty is minimized for each expected income level. Efficiency in this sense also implies that for each level of income uncertainty, the expected total income is maximised. Classifying the models in this way aids in their comparison, and facilitates the selection of an appropriate model for a given situation.

Decision Criteria

When uncertainty exists in future enterprise returns the kinds of things we can say about any given farm enterprise return depend on our knowledge of the nature of possible outcomes for each enterprise. We may or may not know the relative expected frequency of different possible outcomes. When we know the relative frequency of different outcomes we can use measures such as the variance, semi-variance or mean absolute deviation as criteria to describe income uncertainty. When we do not, as is often the case in games against Nature, we resort to such criteria as the Wald maximin or Savage regret.

Markowitz [4, 5] and others have explored at length the concept of expected income–variance (or E-V) efficient plans in the choice of investment portfolios using quadratic programming. From our attempts to apply this procedure to farm planning and the problems we encountered grew the interest in the general class of models generating two parameter sets of farm plans.

The objective criteria of these models can all be evaluated in monetary terms. As such they are all extreme simplifications of a farm operator's utility function which might be regarded as a composite of socio-economic factors, including expected income and some measure of income uncertainty. Rather than attempt to measure such functions and prescribe the plan that provides the greatest utility, we intend, given the present state of knowledge regarding the measurement of utility functions, to simply describe methods of developing alternative farm plans from which farmers may choose.

We have taken this approach to develop efficient sets of farm plans for a 400 acre farm in New York growing 20 to 25 different vegetable crops for market, and for a 4,000 acre farm in Florida on which 10 different vegetable crops were grown. The managers of these two large businesses found considerable value in the opportunity to consider alternative farm plans each with varying levels of expected income and associated income uncertainty.

Specification of Income Distributions

In short run planning models farm overhead costs are usually assumed constant for the length of the planning horizon. The income distribution of a farm plan is then totally specified by the total gross margin distribution. Necessary data for the two parameter approach therefore include information about the gross margin distribution of each farm activity, and the interrelationships between these distributions. The interrelationships might be summarized as correlation or covariance coefficients or, if the distributions are finite, by specifying the mutually exclusive and discrete sets (or vectors) of possible gross margin outcomes with or without their probabilities of occurrence.

Necessary data must generally be obtained from time series or cross-sectional samples unless subjective expectations are already available from the farm operator. Subjective information, unless inconsistent with objective data, is to be preferred in that farm operators are more likely to accept plans that conform to their expectations. However, subjective information can be very difficult to obtain for a complex farm operation, particularly with respect to the dispersion and interrelationships between activity gross margins. A practical compromise to these problems may be to use sample data at the outset, but to make adjustments where possible according to subjective information available from the farm operator.

Regardless of the estimation procedure, it is generally necessary to assume that derived parameters take on their population values. Although statistical inference concerning the population characteristics may be possible given sufficient sample observations, it is not particularly useful in the ultimate selection of a farm plan. This places a heavy demand on the available data and encourages use of unbiased and efficient sample estimates of expected income and income uncertainty in defining two parameter models. When relying on sample data for estimates of income uncertainty, the sample can be regarded as a finite set of mutually exclusive vectors of possible gross margin outcomes, and this allows some latitude in methods of describing interrelationships between activity gross margins [1].

General Definition of Efficient Two Parameter Models

Assuming sample gross margin data at the outset, we can define the general class of models generating two parameter sets of farm plans as follows.

$$\text{Minimize } R = \phi \left\{ \left\{ c_{hj} \,\middle|\, h=1 \text{ to } s \right\},\ x_j \,\middle|\, j=1 \text{ to } n \right\} \tag{1}$$

in words: to minimize R, a measure of total gross margin uncertainty, defined as a set function o of the farm activity levels (x_j) and associated sample gross margin distributions $\left\{ c_{hj} \,\middle|\, h=1 \text{ to } s \right\}$; where c_{hj} is the h^{th} sample observation of the j^{th} activity gross margin, and there are s such observations.

Such that $\sum_{j=1}^{n} f_j x_j = \lambda$ $(\lambda = 0 \text{ to } 00)$ (2)

such that the expected total gross margin, the sum of the activity levels times their forecasted gross margins (f_j), equals λ, a scalar to be parameterized from zero to unbounded.

And $\sum_{j=1}^{n} a_{ij} x_j \leqslant b_i$ (for all i, i=1 to m) (3)

and total fixed activity requirements for the i^{th} constraint, the sum of the unit activity requirements a_{ij} for the constraint i times the activity levels x_j, do not exceed the level of the i^{th} constraint b_i, for all i.

And $x_j \geqslant 0$ (for all j, j=1 to n) (4)

and all activity levels x_j are non-negative.

In summary we wish to minimize a measure of total gross margin uncertainty (1) over all possible levels of expected total gross margin (2) while maintaining feasibility with respect to the linear farm constraints (3) and non-negative values of the activity levels (4).

We can further group this class of models depending on whether or not the total gross margin outcomes are symmetrically distributed about the expected value for a given farm plan. This is a significant classification since we expect farmers to be primarily concerned with negative deviations around expected income and not willing to sacrifice the opportunity for large positive deviations unnecessarily. An empirical problem arises in ascertaining whether the total gross margin distributions are symmetric.* Models suitable for the asymmetric case are obviously acceptable for the symmetric case, but the converse does not hold.

Choice of a particular model for a given farm situation should depend largely on the measure of uncertainty most relevant to the farm operator [1]. For many farm situations no one measure will be most appropriate, and we suggest the following criteria for evaluating alternatives.

(a) The computational ease of the solution procedure. Linear programming codes are available to most researchers with access to a computer, while quadratic programming codes that permit parametric programming are not as widely available and are

*Whereas necessary conditions for symmetry cannot be stated in any generality we do know that if either a) the individual activity gross margin distributions are approximately normal or b) if the sample observations are independent and sufficient numbers of activities enter farm plans so that the Central Limit Theorem is applicable, then approximate normality will hold. A reasonable course of policy might be to take normality as synonomous with symmetry, knowing that if symmetry occurs without normality then little would have been lost by using a model suitable for the asymmetric case.

frequently of limited dimensions and uncertain performance. Special purpose simulation models require much time and effort to develop.

(b) The gross margin data requirements of the uncertainty criterion. The ultimate goal of complete information about activity outcomes and their probability of occurrence and interrelationships may be difficult to realize, but some degree of estimation may be possible. We may, however, have to be content with information about possible gross margin outcomes without specifying relative frequency of occurrence. This is likely to occur, for example, when using a small sample of data and in the absence of subjective probability estimates.

(c) The ease of incorporating subjective information into estimates of expected gross margins, their frequency distributions and interrelationships. Expected income may be easily modified in all models of this class, but the ease of modifying the uncertainty criterion varies.

Models for Symmetrically Distributed Farm Plan Incomes

1. *Expected Income – Variance Criterion*

Let V denote the variance of the total gross margins of a farm plan, then

$$R = V = \frac{1}{s-1} \sum_{h=1}^{s} \left[\sum_{j=1}^{n} (\tilde{c}_{hj} - f_j)\, x_j \right]^2$$

$$= \sum_{j=1}^{n} \sum_{k=1}^{n} x_j\, x_k\, \hat{\sigma}_{jk}$$

is an unbiased and efficient estimator of the population variance. Where:

c_{hj} denotes the h^{th} sample gross margin observation for the j^{th} activity (c_{hj}) adjusted by scalar addition or subtraction in accordance with any changes made to the forecasted gross margins (f_j) from the sample means in incorporating subjective information about their values.[2]

$\hat{\sigma}_{jk}$ denotes the estimated covariance coefficient of gross margins between the j^{th} and k^{th} activities when $j \neq k$, the estimated variance coefficient of gross margins for the j^{th} activity when $j=k$.

*It is assumed that subjective estimates of the most likely gross margins pertain to measures of central tendancy, and not to specific predictions for one year [1]. Such estimates are equivalent to scalar addition or subtraction on the initial sample observations.

If V is now minimized subject to the constraints (2), (3) and (4), this is the familiar quadratic programming model developed by Markowitz [4, 5]. This model is appealing since probability statements are readily derived for the likelihood of occurrence of different income levels for a farm plan in the final set providing the total gross margins are approximately normally distributed. Further, the variance V is totally specified by the variance-covariance coefficients, and when subjective information is available for these values, the criterion does not depend on specification of the possible sets of gross margin outcomes and associated probabilities. We have found computational problems however, in solving large dimension models with available computer codes. The data requirements are also considerable.

2. *Expected Income–Mean Absolute Deviation Criterion*

Let A denote the mean absolute total gross margin deviation for a farm plan. Then

$$R = A = \frac{1}{s} \sum_{h=1}^{s} \left| \sum_{j=1}^{n} (\tilde{c}_{hj} - f_j) x_j \right|$$

is an ubiased estimator of the population mean absolute deviation.

With some manipulation A can be minimized subject to constraints (2), (3) and (4) by parametric linear programming [1]. Rigid specification of sets (or vectors) of possible activity gross margin outcomes together with their relative frequencies are required. It total gross margin distribution can be expected to be approximately normal then the mean absolute deviation may be regarded as a linear alternative to the variance in deriving efficient income–variance farm plans [1]. Further, given normality then probability statements for the income levels of efficient farm plans may be derived through use of Herrey's H statistic [1, 3], or by calculating the variance ex post [1].

3. *Expected Income–Wald Maximin Criterion*

Let U be the worst possible total gross margin that could occur with a farm plan from the sample data. Then replace R by the following system of linear equations.

$$\text{Maximize } U \quad \text{Such that} \quad \sum_{j=1}^{n} c_{hj} x_j \geqslant U \qquad \text{(for all } h, h=1 \text{ to } s)$$

When solved subject to the constraints (2), (3) and (4) this is the maximin parametric game model [2]. Note that the activity gross margins c_{hj} need not be adjusted for any changes in the forecasted gross margins f_j, but if the forecasted values are changed it may be necessary to add a new constraint requiring that the forecasted total gross margin exceed the worst possible outcome [2].

This model can be solved by parametric linear programming. The maximin gross margin U initially increases with expected income, reaches a maximum and then decreases until the final linear program solution is obtained. The solution for the maximum value of U corresponds to that of the resource constrained maximin game model developed by McInerney [6]. All solutions are efficient in that the maximum worst possible outcome is ensured for each expected level of income.

The model avoids specific assumptions about gross margin probabilities when subjective values are available for the forecasted gross margins, and uses data about possible outcomes only. This can lead to less satisfactory consideration of the interrelationships between activity gross margin distributions. The model requires specification of the finite sets (or vectors) of possible gross margin outcomes, and these must capture gross margin interrelationships.

4. *Expected Income–Savage Regret Criterion*

Define regret as the difference in income between a farm plan evaluated for a set (or vector) of possible gross margin outcomes and the value of an optimal linear program solution solved and evaluated for the same gross margins. In particular, let g_h denote the total gross margin of the optimal linear program solution solved and evaluated for the h^{th} sample observation of activity gross margins, then we can define this measure of regret more formally as $g_h - \sum_{j=1}^{n} c_{hj}x_j$.

Let W be the largest such regret value that could occur with a farm plan from the sample data. Then replace R by the following system of linear equations.

Minimize W

Such that $$g_h - \sum_{j=1}^{n} c_{hj}x_j \leqslant W \qquad \text{(for all h, h=1 to s)}$$

When solved to the constraints (2), (3) and (4) this is the regret parametric game model [2]. Again note that the activity gross margins c_{hj} need not be adjusted for any changes in the forecasted values f_j, but if the forecasted values are changed it may be necessary to add a new constraint requiring that regret for the forecasted total gross margin does not exceed W.

This model can be solved by parametric linear programming. The minimax regret W initially decreases with expected income, reaches a minimum and then increases until the final linear program solution is obtained. The solution for the minimum value of W corresponds to that of the non-parametric resource constrained regret game model [2]. The model avoids specific assumptions about gross margin probabilities, when subjective values are available for the forecasted gross margins, but requires specification of finite sets (or vectors) of possible gross margin outcomes which must capture gross margin interrelationships.

Models for Asymmetrically Distributed Farm Plan Incomes

1. *Expected Income–Semi-Variance Criterion*

Let $\widehat{V}$ denote the semi-variance, mean squared negative deviation, of the total gross margins of a farm plan, then

$$R = \hat{V} = \frac{1}{s-1} \sum_{h=1}^{s} \left[\text{Minimum} \left(\sum_{j=1}^{n} (\tilde{c}_{hj} - f_j) x_j, 0 \right) \right]^2$$

is an unbiased estimator of the population semi-variance. When minimized subject to the constraints (2), (3) and (4) this is the semi-variance model proposed by Markowitz [4].

This model can only be handled by Monte Carlo simulation techniques [4] which are not likely to be a practical procedure for farm planning researchers. Unlike the variance, the semi-variance is not specified by variance-covariance coefficients, but like other criteria depends on specification of the possible sets of gross margin outcomes and associated probabilities.

2. *Expected Income–Mean Absolute Negative Deviation Criterion*

Let $\hat{A}$ denote the mean absolute value of the negative total gross margin deviations about the expected value for a farm plan, then

$$R = \hat{A} = \frac{1}{s} \sum_{h=1}^{s} \left| \text{Minimum} \left(\sum_{j=1}^{n} (\tilde{c}_{hj} - f_j) x_j, 0 \right) \right|$$

is an unbiased estimator of the population mean absolute negative deviation.

With some manipulation $\hat{A}$ can be minimized subject to constraints (2), (3) and (4) by parametric linear programming to yield a set of farm plans providing minimum mean absolute negative deviation for any given level of expected income [1]. However, because the sum of the negative deviations around the mean is always equal to the sum of the positive deviations, this criterion is equivalent to the expected income–mean absolute deviation criterion and leads to identical results. In other words, the expected income–mean absolute deviation criterion is appropriate for symmetric or asymmetric income distributions, and the only justification for considering the expected income–mean absolute negative deviation criterion is that it suggests a smaller and more efficient parametric linear programming model for obtaining solutions [1].

3. *Expected Income–Wald Maximin and Savage Regret Criteria*

These models are directly applicable to the asymmetric situation.

Comparison of Models

These models all have the following general characteristics.

(a) A set of farm plans are obtained that are efficient in the sense that for all levels of possible expected income an associated measure of income uncertainty is minimized.

(b) The forecasted gross margins need not be the original sample means buy any values deemed the most likely outcomes.

(c) The models are defined as linear in the farm constraints, or as extensions of the deterministic linear programming model. If the measure of uncertainty is also linear they can generally be solved by parametric linear programming.

(d) The solution for maximum λ is always the linear programming solution for the forecasted gross margins.

The models differ in the following important respects.

(a) The variance model requires a special quadratic programming algorithm and the semi-variance model requires simulation, while the others can all be handled with linear programming codes containing the parametric option.

(b) The variance, semi-variance, mean absolute deviation and mean absolute deviation models require data about the relative frequency of possible activity gross margin outcomes. This places heavy demands on available data but permits more satisfactory consideration of the interrelationships between activity gross margins. The Wald Maximin and Savage Regret models avoid explicit assumptions about the probabilities, though these can be incorporated.

(c) The variance is totally specified by the variance-covariance coefficients, and subjective information can easily be incorporated. All the other models require incorporating subjective information for the uncertainty criterion by defining important gross margin outcomes and associated probabilities.

(d) When total gross margin distributions are expected to be approximately normal the mean absolute deviation may be regarded as a linear alternative to the variance in deriving farm plans efficient with respect to expected income and variance. This permits use of available linear programming computer codes.

Conclusions

Only a few of many possible efficient two parameter models for obtaining sets of farm plans under uncertainty have been presented. Our purpose has been to indicate a general framework of analysis that a research worker or farm advisor might use to select a model most appropriate to his needs and available data. We conclude at this stage that the most useful models are those combining expected income levels with the mean absolute deviation and the Wald maximin criteria. Both models are suitable regardless of the shape of income distributions and both can be solved with available linear programming computer codes that contain the parametric feature. The maximin parametric model seems most appropriate when data are limited to a few sets of possible gross margin outcomes, such as a small sample, whereas the mean absolute deviation model yields more information and is to be preferred when knowledge of gross margin probabilities are available.

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An Approach to Farm Planning Under 'Ambiguity'

Y. MARUYAMA and T. KAWAGUCHI
Kyoto University Japan and University of Essex, U.K. and Kyushu University, Japan.

I. *Introduction*

The state of a farmer's knowledge regarding his own farm planning: Very seldom is he in the state of complete ignorance that is assumed in many game-theoretic decision models under uncertainty. Some estimates of probability distributions, e.g. in the form of his own observations or of his neighbour's, including those of government experimental stations, of the relevant variables are available. However, these estimates are not always very reliable, so that he is not completely confident of the associated objective or subjective risks. Thus, his state of knowledge appears to simulate what Ellsberg calls the 'ambiguity' [4].

The role of a farm planning model under uncertainty is understood to process whatever information the farmer has regarding his alternative economic opportunities, say, his procurement, production and marketing opportunities, and derive on his behalf the implications of this information relative to his ultimate objective, e.g., the maximization of expected utility, which are not often obvious at first sight, and thereby help him make a more sensible decision whenever possible. The derived implications, of course, cannot be more reliable than the basic informational inputs.

The purpose of this paper is to formulate a farm planning model under 'ambiguity', relate it to other decision making models under uncertainty, and apply it to a representative Eastern North Carolina farm due to Freund [5] so as to illustrate some of its further aspects. Throughout this paper the parametric or comparative statics analysis is emphasized, which is necessary (a) because of the apparent difficulty of revealing the farmer's preference with respect to uncertainty and (b) the insufficient reliability of the basic informational inputs.

For the sake of expository convenience, the subsequent sections will be developed in a close relation to Markowitz's portfolio analysis [8], which was popularized by Heady and Candler as 'risk programming' [6] to the agricultural economics profession. The risk programming model is a simplest form of program planning under uncertainty, which deals in uncertainties only in the form of objective or subjective risks, and incorporates them only in the coefficients of objective function.

II. *Formulation of the model*

The information available to a farmer usually consists of (a) a number of time and/or cross-sectional series of net returns from a unit of individual alternatives,

$$(1)\ r_{tj}, t = 1, \ldots, T; j = 1, \ldots, n,$$

where the first suffix denotes the t^{th} series, and the second the j^{th} alternative; (b) similar series of resource requirements or productions per unit level of individual alternatives,

$$(2)\ a_{tij}, t = 1, \ldots, T; i = 1, \ldots, m; j = 1, \ldots, n,$$

where an additional suffix i denotes the i^{th} resource; (c) a similar series of resource availabilities,

$$(3)\ b_{ti}, t = 1, \ldots, T; i = 1, \ldots, m,$$

and (d) a similar series of objective or subjective probabilities associated with these series, whichever apply to the case,

$$(4)\ y_t \geqq 0, \Sigma_{t=1}^{t=T} y_t = 1.$$

The downward variability in the net return: The mathematical expectations of these series of information,

$$(5)\ r_j = \Sigma_{t=1}^{t=T} r_{tj} y_t, j = 1, \ldots, n,$$

$$(6)\ a_{ij} = \Sigma_{t=1}^{t=T} a_{tij} y_t, i = 1, \ldots, m; j = 1, \ldots, n,$$

$$(7)\ b_i = \Sigma_{t=1}^{t=T} b_{ti} y_t, i = 1, \ldots, m,$$

are utilized as basic informational inputs in the ordinary linear programming models. Besides these, the variances and covariances between r_{tj}'s are employed as measures of variability in r_j's in the risk programming model. Here, let us instead define an alternative measure of variability in r_j's and see what this much extraordinariness will imply. The deviation of individual r_{tj}'s from their expectation r_j's,

$$(8)\ v_{tj} = r_j - r_{tj}, t = 1, \ldots, T; j = 1, \ldots, n,$$

may serve as measures of variability as well. Our rationale for emphasizing the downward as compared with the directionless absolute deviations or variabilities coincides with Markowitz's for introducing the semi-variances [8].

Problem (I): The downward variability in the net return from a program $x_1, \ldots, x_n$ in the t^{th} series, where $x_j \geqq 0$ ($j = 1, \ldots, n$) denotes the level of commitment to the j^{th} alternative,

$$(9)\quad d_t = v_{t1}x_1 + \ldots + v_{tn}x_n, t = 1, \ldots, T.$$

The relations (4) and (8) imply that d_t cannot be negative. The downward variability in the net return from this program is defined as max $(d_1, \ldots, d_T)$. The constraint to confine it within $d \geqq 0$ is equivalent to the following set of inequalities.

$$(10)\quad d_t = v_{t1}x_1 + \ldots + v_{tn}x_n \leqq d, t = 1, \ldots, T.$$

Additional constraints specifying the technical, institutional or behavioural feasibility of this program are introduced, in an ordinary way, in terms of a_{ij}'s and b_i's.

$$(11)\quad a_{i1}x_1 + \ldots + a_{in}x_n \leqq b_i, i = 1, \ldots, m.$$

The expected net return from this program,

$$(12)\quad E = r_1x_1 + \ldots + r_nx_n.$$

The problem of maximizing E subject to the feasibility constraints (11) will be referred to as Problem (I). Let us assume that a solution to this problem exists and that the corresponding value of E is equal to E.

Problem (II) and E-V_d efficiency locus: The problem of maximizing E subject to (10) and (11) is referred to as Problem (II). The minimum value of d that guarantees E to E is denoted as d. The minimum feasible value of d under the constraints (10) and (11) is denoted as d, and the maximum value of E corresponding to d, where d is regarded as a parameter, is denoted as E. The optimum programs for all values of $d \geqq d$ can be obtained by means of the parametric linear programming (PLP for short). See Table 1 for the results of application to the representative Eastern North Carolina farm. From the construct of PLP it follows that E is a continuous, monotonically non-decreasing and concave function of d. Furthermore, it is strictly increasing for $d < d < d$. The locus of E(d) thus obtained consists of a number of straight line segments, and looks as a whole much like the familiar E–V or E–σ efficiency loci in risk programming. For contrast let us refer to the E–d locus thus obtained as E–V_d efficiency locus. See Figures 1 and 3, which show the E–V_d and E–σ efficiency loci for the above-mentioned farm, respectively.

Problem (III) and the minimum expected net return: The net return from a program $x_1, \ldots, x_n$ in the t^{th} series,

$$(13)\quad k_t = r_{t1}x_1 + \ldots + r_{tn}x_n, t = 1, \ldots, T.$$

TABLE 1

d	0.00($\underline{d}$)	2.91	3.31	3.42	5.00	12.53	28.95	43.23(d_0)	52.39($\bar{d}$)	Hundred Dollars
E	0.00($\underline{E}$)	25.31	26.77	27.07	31.42	47.67	70.58	87.69(E_0)	91.31($\bar{E}$)	"
k	–	–	–	–	–	–	–	44.46($\bar{k}$)	38.92($\underline{k}$)	"
Potatoes X1	0.00	2.25	3.20	3.32	4.98	13.42	14.91	16.45	26.55	Acres
Corn X2	0.00	7.25	10.02	10.44	16.49	34.04	30.16	26.50	0.00	"
Beef X3	0.00	50.50	46.78	46.24	38.53	12.54	14.93	8.57	32.26	"
Cabbage X4	0.00	0.00	0.00	0.04	0.70	3.53	14.91	24.92	27.74	"
Z_1	–	–	–	–	–	–	–	38.46	48.42	Hundred Dollars
Z_2	–	–	–	–	–	–	–	39.69	47.54	"
⋮	–	–	–	–	–	–	–	⋮	⋮	"
Z_{17}	–	–	–	–	–	–	–	21.60	40.47	"

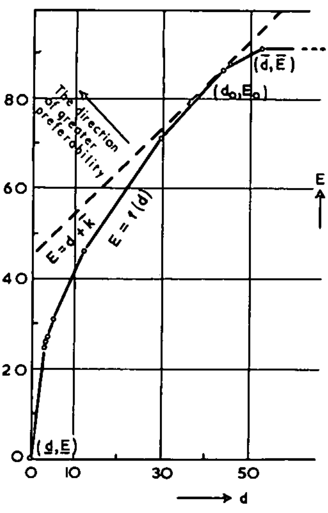

Figure 1.

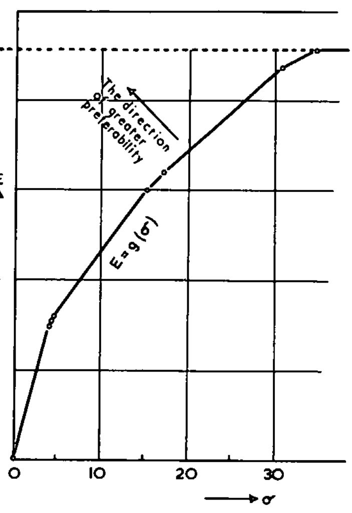

Figure 3.

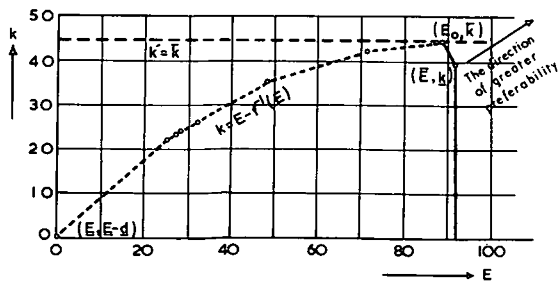

Figure 2.

The constraint of requiring the return from this program not to fall short of a specified minimum level, say k, can be expressed in terms of k_t.

$$(14)\quad k_t = r_{t1}x_1 + \ \ldots\ + r_{tn}x_n \geqq k,\ t = 1,\ \ldots,\ T.$$

The minimum level of net return k is meant to reflect the importance of the subsistence or similar levels of income in many farm planning problems. See, e.g., Wharton [10]. Let us refer to the problem of maximising E subject to (11) and (14) as Problem (III). The optimum programs corresponding to various values of k can be obtained by means of PLP. However, this is not really necessary as is shown in the sequel. The set of solutions to the Problem (II) contains as a subset those to the Problem (III). See Table 1.

Relationship between Problems (II) and (III): The recourse to the relation (8) suggests that the constraints (10) and (14) can be rewritten as

$$(10a)\quad k_t \geqq E - d,\ t = 1,\ \ldots,\ T,$$

$$(14a)\quad d_t \geqq E - k,\ t = 1,\ \ldots,\ T.$$

These two relations suggest a close connection between Problems (II) and (III). Let us express by the function E = f(d) the E–V_d efficiency locus in Figure 1. Then the monotonicity of f for $\underline{d} < d < \overline{d}$ assures that its inverse d = $f^{-1}(E)$ exists and that f^{-1} is continuous, monotonically increasing and convex for $\underline{E} < E < \overline{E}$. Therefore, the function k = E - $f^{-1}(E)$ is continuous and concave for $E < E < E$. It attains its maximum in the interval $E_1 \leqq E \leqq E_2$ where the curve E = f(d) has a slope of one. In Figure 1 the interval $E_1 \leqq E \leqq E_2$ reduces to a single point E_0 (= E_1 = E_2).

The E–k efficiency locus: In case f is differentiable, the above result can be proved easily as follows. In case f consists of a number of straight line segments, a similar proof requires a set-theoretic discussion.

$$dk/dE = 1 - df^{-1}(E)/dE = 0, \text{ hence}$$

$$df^{-1}(E)/dE = 1.$$

k is strictly increasing for $\underline{E} < E < E_1$ and strictly decreasing for $E_2 < E < \overline{E}$. It looks much like Baumol's L = E - Kσ curves [1], except that k = E - $f^{-1}(E)$ consists of a number of straight line segments as is shown in Figure 2. The relationship between the E–k efficiency locus and the curve k = E - $f^{-1}(E)$ should be obvious. The former corresponds to a part of the latter for $E_2 \leqq E \leqq E$. Thus, the relationship between the E–V_d efficient set and the E–k efficient set of programs is very similar to that between Markowitz's E–σ efficient set and Baumol's E–L efficient set of portfolios. Just like Baumol's E–L efficiency criterion, the E–k efficiency criterion may help reduce the farmer's decision-making task by providing him with a smaller efficient set of programs from among which he must still choose.

III. *Discussion of the model*

The constrained game: Multiplying both sides of the inequalities (14) by y_t and summing up overall t, we have

$$(15)\ \Sigma_{t=1}^{t=T}\ \Sigma_{j=1}^{j=n}\ y_t r_{tj} x_j \geqq k\,\Sigma_{t=1}^{t=T}\ y_t = k.$$

By (5) this relation can be rewritten as

$$\Sigma_{j=1}^{j=n}\ r_j x_j \geqq k.$$

The existence of a solution to Problem (I) implies the existence of a maximum of k. Let it be denoted as $\overline{k}$. If we regard the left-hand side of (15) as the pay-off in a game, $\overline{k}$ proves to be equal to the value of a constrained game, (a) where the pure strategy $x_1, \ldots, x_n$ of the maximizer, the farmer, is subject to the feasibility constraints (11), and (b) where the mixed strategy $y_1, \ldots, y_T$ of the minimizer, Nature, is subject to the regular conditions (4). Thus, optimum programs corresponding to $\overline{k}$ guarantee the maximum security while those corresponding to $\underline{k}$ (= $\overline{E}$ - $\overline{d}$) guarantee the maximum expected net return $\overline{E}$. The theory of constrained game, which was first developed by Charnes [2], has recently been applied to the farm planning problem by Imamura [7] and McInerney [9].

Ellsberg's decision rule with a confidence parameter p: In the situation of 'ambiguity' where some estimates of y_t's are possible but they are not completely reliable, Ellsberg [4] suggests a decision rule to choose such strategies that maximize the following index (as rewritten in our notation):

$$(16)\ U = p\overline{E} + (1-p)\overline{k},\ 0 \leqq p \leqq 1,$$

where p denotes the decision maker's degree of confidence in the estimate of y_t's. If p = 0, optimum programs corresponding to $\overline{k}$, if p = 1, those corresponding to E, respectively, will be chosen. If $0 < p < 1$, those represented by the intermediate points on the E–k efficiency locus will be chosen. In other words, the ratio p/(1–p) reflects the rate of substitution between $\overline{E}$ and $\overline{k}$ in the decision maker's preference system with respect to uncertainty. Thus a close correspondence between the E–k efficiency locus and Ellsberg's decision rule is ascertained.

Probability that the minimum expected net return falls short of a specified level: By use of slack variables $z_t \geqq 0$, t = 1, ..., T, the constraints (14) can be rewritten as equalities,

$$(14b)\ k_t = r_{t1}x_1 + \ \ldots\ + r_{tn}x_n = z_t + k,\ t = 1, \ \ldots,\ T.$$

The PLP procedure determines all the values of z_t for $\underline{k} < k < \overline{k}$. See Table 1. Then the sum $z_t + k$ can be compared with the arbitrary level of net return h. The sum of y_t's over all t such that $z_t + k < h$ is equal to the probability that the minimum expected return from the program $x_1, \ldots, x_n$ falls short of h. Such a probability will provide useful information in many farm planning problems where the consideration of the subsistence or similar levels of income is important. The usefulness of such a probability is, of course, dependent upon the farmer's confidence in the estimate of y_t's.

Variabilities in the resource requirements and availabilities can be similarly defined as v_{tj} in (8). The upward variability in case of the resource requirements and the downward variability in case of the resource availabilities respectively will be of more interest than their counterparts for similar reasons as in the case of unit level net returns.

$$(17)\quad s_{tij} = a_{tij} - a_{ij},\ t = 1,\ \ldots,\ T;\ i = 1,\ \ldots,\ m;\ j = 1,\ \ldots,\ n,$$

$$(18)\quad q_{ti} = b_i - b_{ti},\ t = 1,\ \ldots,\ T;\ i = 1,\ \ldots,\ m.$$

The variation in the resource requirements of a program $x_1, \ldots, x_n$ can be similarly constrained as in (10).

$$(19)\quad u_{ti} = s_{tij}x_1 + \ \ldots\ + s_{tin}x_n \leqq u_i,\ t = 1,\ \ldots,\ T;\ i = 1,\ \ldots, m.$$

The variabilities in b_i's can be used to specify the relevant ranges of the values of u_i's, but they would be more conveniently suppressed in the formulation of (19). Their implications with respect to the optimum programmes will be derived as the latter corresponding to various values of u_i's are obtained by means of PLP procedure.

Problem (IVA) [(IVB)]: The problem of maximizing E subject to (10), [(14)], (11) and (19) is referred to as Problem (IVA) [(IVB)]. There are a tremendous number of combinations of d [k] and u_i's. Accordingly, there are as many possible PLP runs to try. It would be economically infeasible to try them all. However, some of them, especially those associated with the resources of severely limited supply, say, hay or labour time during the peak seasons in certain cases, may be worth trying.

IV. *Application to the Representative Eastern North Carolina Farm Due to Freund*

Data for the unit level net returns series are adapted from his unpublished Ph.D. thesis by his courtesy. Data for the resource requirements and availabilities, together with the variances and covariances of the unit level net returns are adapted from his *Econometrica* article [5]. They are not reproduced here for lack of space.

The results of application are shown in Table 1, and are summarized in Figures 1 and 2, along with Figure 3, where the results of risk programming analysis on the same farm are summarily shown for contrast. The risk programming analysis is performed by means of the Wolfe-Dantzig modified simplex algorithm for quadratic programming [3].

V. *Conclusion*

In the foregoing sections *a farm planning model under ambiguity* has been formulated, developed, discussed and then applied to a representative farm.

The apparent prevalence of ambiguity in many farm planning problems may suffice to endorse the relevance of the proposed model. The difficulty of revealing a farmer's preference relative to uncertainty together with the insufficient reliability of the basic informational inputs, both inherent in the farm planning under ambiguity, may lead us to find particular uses in *the parametric features* of the proposed model.

Furthermore, the formal simplicity of the present approach offers *an economy of information* through a more direct utilization of the basic informational inputs, in particular, compared with the risk programming analysis where, for instance, the information of the unit level net returns are transformed into variances and covariances before analysis. This economy of information may be significant especially under ambiguity, where the amount of information is very severely limited. To be definite, let us assume a plausible set of values, say, five, forty and sixty for T, m and n. Then the proposed model makes a full use of 300 r_{tj}'s, 1200 a_{tij}'s and 200 b_{ti}'s, while the risk programming model requires 3660 variances and covariances among 300 r_{tj}'s, i.e., a tremendous stretching of the basic informational inputs. Whereas it finds only a very sparing use, at the rate of one-fifth, of 1200 a_{tij}'s and 200 b_{ti}'s.

An economy of computation, closely connected with the above economy, can also be enjoyed with the proposed model. Should we work with Problem (II) in the above, and solve the corresponding risk programming problem by means of the Wolfe-Dantzig modified simplex method, the dimension of simplex tableau for the proposed model will be 45 x 60 while that for the risk programming model will be 100 x 100, assuming for T, m and n the same set of values in the above. Furthermore, the proposed model can be employed if only a PLP code is available which is definitely more readily available than a quadratic programming code.

The formal simplicity of the proposed model also makes possible a *symmetrical analysis of the variabilities in the resource requirements and availabilities,* that is virtually impossible in many other decision making models under uncertainty, including the risk programming model.

Finally, the proposed model offers a very important set of information as a by-product of the main analysis. *The probabilities that the minimum expected returns from alternative programs fall short of a certain specified level* will prove to be useful in many farm planning problems where the consideration of the subsistence or similar levels of income is crucial. However, let it be noted that the usefulness of such information depends on the farmer's confidence in the basic informational inputs. In general, such probabilities may also be somewhat more easily understood as measures of risk by an average farmer than the standard deviations or variances.

All these features of the proposed model may add to its recommendability.

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SPECIAL GROUP B REPORT

Following the paper by Hazell and How, the first of the three papers presented at this meeting, the question was raised as to whether the models discussed represented an approach which could be regarded as consistent with the utility functions of farmers. The authors, however, doubted whether the measurement of utility functions was far enough advanced for them to be directly incorporated but argued that the two parameter models such as that using the mean absolute deviation represented an approximation to these. In the general discussion on all three papers, it was pointed out that the utility function of a farmer was not constant but changed, for instance, with each change in the capital assets held. The temporary nature of the planning coefficients and resource levels was instanced by one speaker as an argument against the more elaborate programming methods. He claimed that the subdivision of activities into the largest possible number of 'tasks' while involving very large matrices enabled solutions to be reached by standard linear programming procedures which took account of the uncertainty aspects.

There was some discussion of the problems involved in arriving at a full set of probabilities for use in the programming models. It was pointed out that probabilities based on time series would not correspond to realised probabilities. More attention needed to be given to this particularly in view of the rapidity of technical change.

Difficulties in the use of probability models were seen by one speaker in situations where the use of many transfer activities was appropriate. This applied particularly to quadratic programming. Simulation models might be an alternative. It was claimed that the mean absolute deviation model was compatible with transfer activities.

The need to distinguish between risk and uncertainty was made. The term 'ambiguity' used by Maruyama and Kawaguchi was considered to be appropriate as a term going rather beyond the strict definition of uncertainty. Reference was made to the difficulties posed by uncertainty attributable to political and social factors.

Speakers from centrally planned countries questioned the aims and value of the individual farm models and the extent to which these could have a general applicability. The situations facing individual farmers were perhaps so complex as to be beyond solution. Central planning using price changes, bonuses and insurance removed to a large extent the uncertainties and risks facing the individual farmer. However, other speakers in reply said that the methods were helpful. Even if the solutions are not always implemented the models provided a basis for a logical approach to the problems. Other reasons apart, it was necessary to check on the validity of the policies advocated by farm extension workers.

Among the participants in the discussion were J. L. Dillon *Australia*, S. S. Johl *India*, U. Renborg *Sweden*, C. D. Throsby *Australia*, J. Anderson *Australia*, D. Vlador *Bulgaria*, P. K. Ray *F.A.O.* V. A. Tichonov *U.S.S.R.* Z. Griliches *U.S.A.* V. Stankov *Bulgaria*.